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PENETRATION OF REALISTIC POTENTIAL BARRIER OF H–I FUSION*

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The calculations of heavy-ion fusion cross sections in the above and sub-barrier regions are presented. A barrier penetration model with matrix method for the calculations of transmission coefficients through real one-dimensional barrier of nucleus-nucleus potential is used. The renormalization parameter N_S of Satchler–Love M3Y nuclear potential and critical radius R_{cr} are the two parameters of the model. A very good description of the experimental fusion cross sections for lp-shell colliding ions was obtained for the full measured energy range. It was found that both parameters N_S and reduced critical radius r_{cr} are very similar for all systems considered and are energy independent.

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1. Introduction

The barrier penetration model (BPM) of the fusion of two nuclei is based on the assumption that fusion take place when the potential barrier of colliding ions is passed [1]. There are some practical applications of the BPM in which the different approximations of the real effective nucleus-nucleus potential have been done. The crucial point in the BPM are the calculations of the transmission coefficients $T_l^{\text{fus}}(E)$. The usually used approach is the method proposed by Hill and Wheeler [2]. They obtained the

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analytic expression for the $T_l^{\rm fus}(E)$ by approximating the effective potential in the neighbourhood of the barrier maximum by parabola. Haider and Malik added a Coulomb tail to the parabolic shape of the nucleus-nucleus potential [3]. The double barrier method was proposed by Descouvemont *et al.* [4]. These calculations can not explain the experimental fusion excitation function in the full measured energy range. The only exception is the work of Descouvemont *et al.* [4] in which the experimental data was fitted by using the shallow Saxon–Woods shape of the nuclear potential. However it is known from recent paper of Wada and Horiuchi [5] that for ^{16}O+^{16}O system the equivalent local interaction should take the form of a deep potential which is consistent with the Pauli principle.

It can be generally concluded that $\sigma_{\rm fus}(E)$ is not satisfactory well reproduced by the BPM model, in both energy regions, *i.e.* sub- and above-the Coulomb barrier, simultaneously.

The purpose of the present paper is to show that one-dimensional BPM gives satisfactory results in $\sigma_{\rm fus}(E)$ description if: *(i)* the potential barrier is well approximated and *(ii)* the nuclear potential is realistic. We applied the matrix method [6] to calculate the $T_l^{\rm fus}(E)$ through the realistic potential barrier. It is assumed that the incoming particle penetrates the potential barrier and the fusion occurs when the critical distance $R_{\rm cr}$ is reached. So the parameters of the model are $R_{\rm cr}$ and the parameters of nuclear interaction. The success of the microscopic double folded Satchler–Love M3Y potential [7] leads us to use it (with the suitable renormalization factor) as a nuclear part of effective interaction of colliding nuclei.

The results of the BPM calculations of the fusion cross section for the reactions: ^{12}C + ^{12}C, ^{14}N + ^{12}C, ^{12}C + ^{16}O, and ^{16}O + ^{16}O are presented. The calculations of the $\sigma_{\rm fus}(E)$ were performed for the full measured energy range starting from the sub-barrier region up to several times the Coulomb barrier energy.

The model is shortly outlined in Sect. 2. In Sect. 3 the results of calculations and discussion are presented. The paper is concluded in Sect. 4.

2. The model

In this section we shall outline the framework of the model. The details have been described elsewhere [6].

The one-dimensional, effective interaction potential $V_{\rm eff}(r,l)$ of two colliding nuclei is a sum of nuclear $V_{\rm N}(r)$, Coulomb $V_{\rm C}(r)$ and centrifugal $V_l(r,l)$ terms, *i.e.*

$$V_{\rm eff}(r,l) = V_{\rm N}(r) + V_{\rm C}(r) + V_l(r,l)\,. \tag{1}$$

It is assumed that the fusion occurs when the some critical distance $R_{\rm cr}$ is reached by the incoming particle. Before that the projectile penetrates the potential barrier, for a given angular momentum l, what is described by the transmission coefficients $T_l^{\rm fus}(E)$. The fusion cross section $\sigma_{\rm fus}(E)$ can be calculated, when $T_l^{\rm fus}(E)$'s are known, according to the formula from Ref. [4]

$$\begin{aligned}\sigma_{\rm fus}(E) &= \sum_{l=0}^{\infty} \sigma_{\rm fus}(l,E) \\ &= \frac{\pi}{k^2}\sum_{l=0}^{\infty}\left(1+\frac{(-1)^{1+2I}\delta}{(2I+1)}\right)(2l+1)T_l^{\rm fus}(E)\,,\end{aligned} \tag{2}$$

where k is the wave number of the incoming particle, I is its spin, and $\delta = 0$ ($\delta = 1$) for nonidentical (identical) nuclei. For the calculations of $T_l^{\rm fus}(E)$'s the matrix method is proposed. It can be presented in the following steps (for the details see Ref. [6]).

(i) The potential curve $V_{\rm eff}(r,l)$ for a given angular momentum l is divided in the region $R_{\rm cr} \le r \le R_{\rm cut}$ into N equal parts (see Fig. 1). $R_{\rm cut}$ is the distance beyond which the nuclear potential $V_{\rm N}(r)$ becomes negligible ($R_{\rm cut} = 15$ fm in the calculations).

(ii) $T_l^{\rm fus}(E)$ for a given l can be expressed by the following formula:

$$T_l^{\rm fus}(E) = \frac{|A^-|^2}{|Z^-|^2}\,, \tag{3}$$

where: Z^- (Z^+) is the amplitude of the incoming (outgoing) wave function in $r = R_{\rm cut}$. It is expressed by the regular F_l and G_l Coulomb wave functions. A^- (A^+) is the amplitude of the incoming (outgoing) wave function in $r = R_{\rm cr}$.

The amplitudes A^- and A^+ are obtained from the following matrix equation

$$\begin{pmatrix} A^+ \\ A^- \end{pmatrix} = \prod_{i=1}^{N} \widehat{T}_i \begin{pmatrix} Z^+ \\ Z^- \end{pmatrix}. \tag{4}$$

The elements of the matrix $\widehat{T}_i$ ($i = 1, \cdots, N$) depend on the height and width of the i-th part of the potential barrier, and also on the energy of the projectile.

(iii) In the limit $N \to \infty$ one obtains the staircase function which is a good approximation of the real potential barrier (see Fig. 1). The transmission coefficients $T_l^{\rm fus}(E)$ given by Eq. (3) tend towards the transmission coefficients through the real potential barrier.

It should be stressed here that this method can be applied for the calculations of transmission coefficients for any shape of the static nuclear potential barrier.

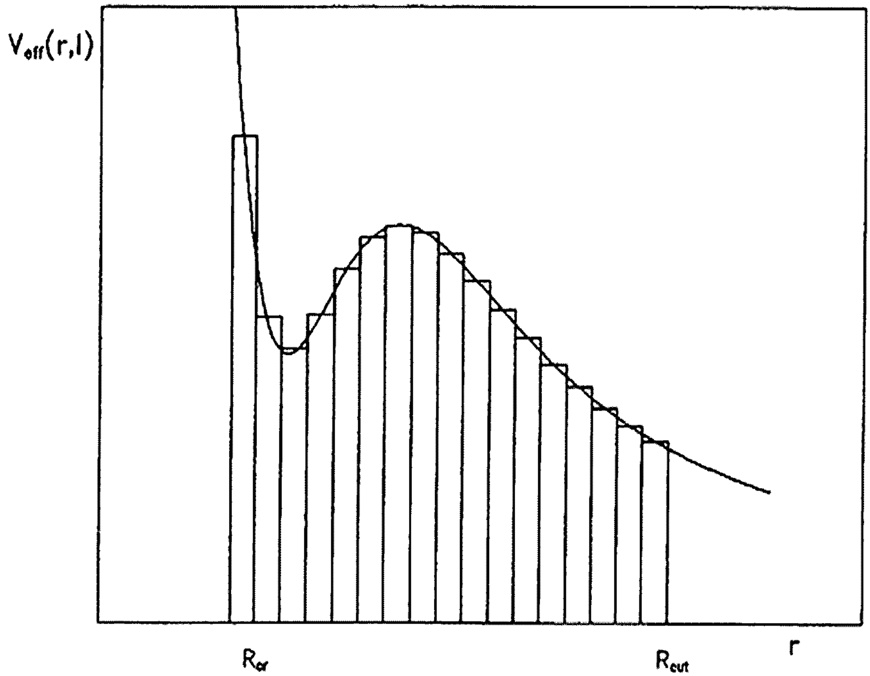

Fig. 1. The shape of the effective potential $V_{eff}(r,l)$ barrier for a given angular momentum l.

3. Results and discussion

The fusion cross section calculations were done for the following 1p-shell systems: $^{12}C + ^{12}C$, $^{14}N + ^{12}C$, $^{12}C + ^{16}O$, $^{16}O + ^{16}O$. For these systems the experimental data exist for wide energy range starting from below Coulomb barrier region up to several times the Coulomb barrier energy.

In the model presented there are the following parameters: the critical distance R_{cr} and parameters of the nuclear potential $V_N(r)$. In last decade the double folded M3Y Satchler–Love potential [7] was used for both elastic scattering and fusion calculations with satisfactory results [7-9]. This lead us to use this potential with the suitable renormalizating factor N_S as a nuclear part of $V_{eff}(r,l)$. This reduces the number of fitted parameters of the model to two: R_{cr} and N_S, which must be extracted from the comparison of the calculated $\sigma_{fus}(E)$ with the experimental data.

The fitting procedure was divided into two parts: energy region below and above the Coulomb barrier. It was found that the "best-fit" values of the reduced critical radius r_{cr} ($R_{cr} = r_{cr}(A_1^{1/3} + A_2^{1/3})$, where A_1 and A_2 are mass numbers of interacting nuclei) and N_S do not depend on the energy of colliding ions. The results of r_{cr}, R_{cr} and N_S are presented in Table I. The calculated fusion excitation function $\sigma_{fus}(E)$ together with the experimental points are plotted in Figs 2 and 3 for the above and below Coulomb barrier energy regions, respectively. In all presented cases the good agreement between theory and experiment was obtained in the full measured energy range. Particularly in the most troublesome region for BPM *i.e.*, for the energies below the Coulomb barrier we acquired excellent agreement with experimental data. The oscillations which occur in $\sigma_{fus}(E)$ for systems of identical, spin=0 nuclei, *i.e.*, $^{12}C + ^{12}C$ and $^{16}O + ^{16}O$ are also reproduced by our model. The origin of the diffraction structure of $\sigma_{fus}(E)$ is the "quantisation" of the angular momentum in units of 2h [18,19]. It is distinctly seen for the above-Coulomb barrier energies (see Fig. 2) because the difference between two succeeding $\sigma_{fus}(l, E)$ is considerable. For sub-barrier energies the major part of $\sigma_{fus}(E)$ comes from $\sigma_{fus}(l = 0, E)$ what generates that $\sigma_{fus}(E)$ is a smooth function of energy.

TABLE I

The renormalizating factor N_S of Satchler–Love M3Y potential and r_{cr}, R_{cr} values for collisions between 1*p*-shell nuclei

Reaction	$Z_1 * Z_2$	N_S	r_{cr}[fm]	R_{cr} [fm]	Refs. for exp. data below Coulomb barrier	Refs. for exp. data above Coulomb barrier
$^{12}C + ^{12}C$	36	0.90	0.93	4.25	[10]	[11]
$^{14}N + ^{12}C$	42	1.05	0.95	4.46	[12]	[13]
$^{12}C + ^{16}O$	48	0.99	0.98	4.71	[14]	[15]
$^{16}O + ^{16}O$	64	1.16	1.04	5.24	[16]	[17]

Haider and Malik [3] reproduced the general trend of the energy dependence of the fusion cross section in the sub-barrier region but the absolute magnitudes of their $\sigma_{fus}(E)$ values are unpredicted. They can improve the sub-barrier results by changing the "matching" radius R_n but then the fits in the above-barrier region become poorer.

It should be noted that Descouvemont *et al.* [4] obtained satisfactory description of the experimental $\sigma_{fus}(E)$ for collisions between 1*p*-shell nuclei, apart from the $^{16}O + ^{16}O$ system for above Coulomb barrier energies, but they used the shallow nuclear potential. The depth of their Saxon–Woods shape nuclear potential was about 15 MeV. However from recent papers it is known that it should be much deeper. Kondo *et al.* [20] suggest that only the deep ($\sim$ 300 MeV) nuclear potential is able to reproduce the resonant structure of the elastic scattering excitation function for $^{16}O + ^{16}O$

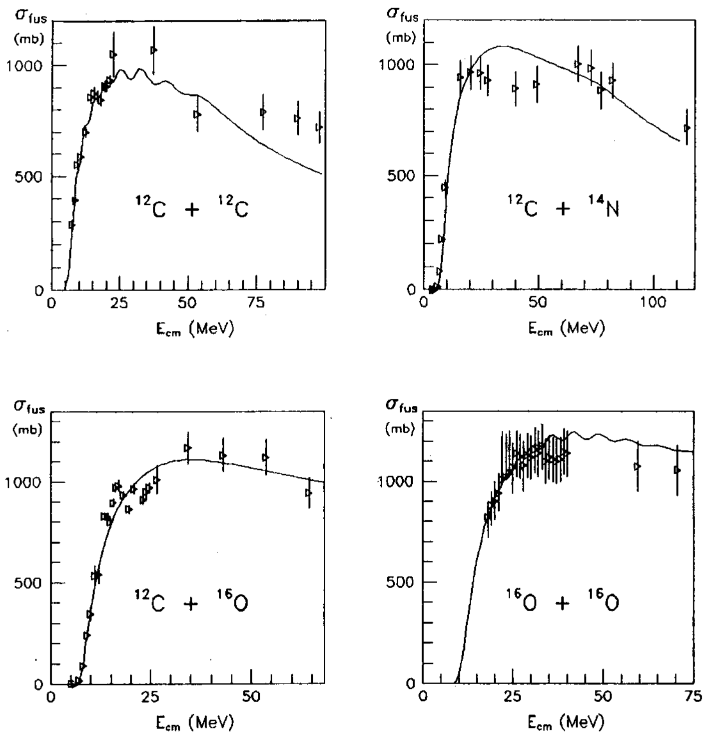

Fig. 2. Fusion cross sections $\sigma_{fus}(E)$ for the energies above the Coulomb barrier for collisions between 1p-shell nuclei. The lines represent $\sigma_{fus}(E)$ obtained with Eq. (2) with parameters from Table I. For experimental data refs. see Table I.

system. Furthermore a unique deep potential for this system was found by analyzing nuclear rainbow data [21]. Also Wada and Horiuchi [5], by using the resonating group method, reported that the equivalent local interaction for light heavy-ion systems like ^{16}O + ^{16}O is deep. The deep nuclear potential is consistent with the Pauli principle because it produces wave functions with proper number of radial nodes. It should be stressed here that Satchler–Love M3Y potential, used in this paper, fulfils this condition very well.

It was assumed that the fusion occurs when the critical distance R_{cr} is reached ($R_{cr} = r_{cr}(A_1^{1/3} + A_2^{1/3})$), so it is expected that the value of the reduced critical radius r_{cr} should be close to the overlap nuclear matter half density radius $r_{1/2}$ which has the standard value 1.0 ± 0.07 fm [22]. The obtained value of r_{cr} satisfies this prediction very well (see Table I).

The renormalizing factor N_S of Satchler–Love M3Y potential for fusion takes the values from 0.90 for ^{12}C + ^{12}C system to 1.16 for ^{16}O + ^{16}O

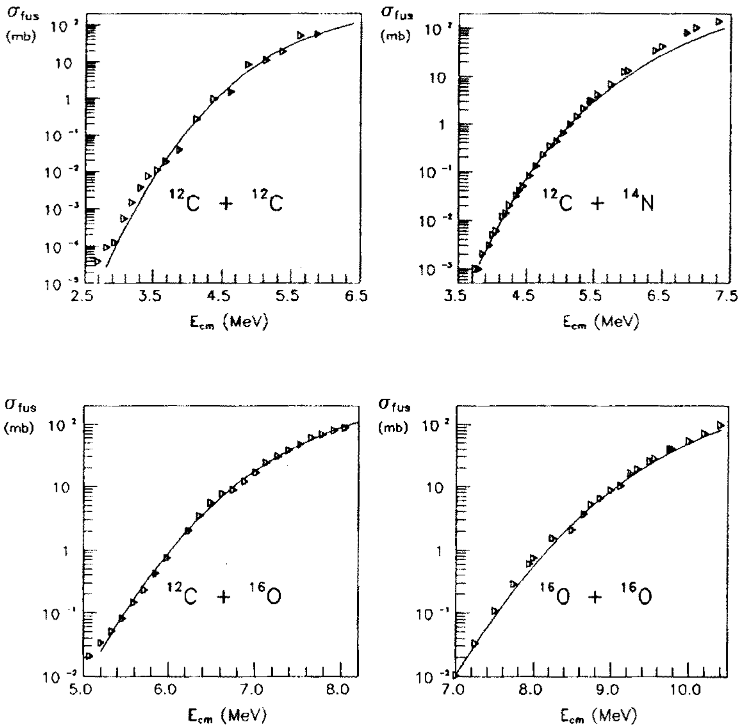

Fig. 3. Fusion cross sections $\sigma_{fus}(E)$ for the energies below the Coulomb barrier for collisions between 1p-shell nuclei. For details see Fig. 2

system (see Table I). It is consistent with N_S value obtained by Satchler and Love from elastic scattering data analysis. They reported [7] the $N_S = 1.11 \pm 0.13$ as a result of fitting procedure for considerable amount of combinations of colliding ions.

4. Conclusions

In this paper the description of the fusion cross section $\sigma_{fus}(E)$ for some combinations of 1p-shell ions is presented. For $^{12}C + ^{12}C$, $^{14}N + ^{12}C$, $^{12}C + ^{16}O$ and $^{16}O + ^{16}O$ systems the calculations of the $\sigma_{fus}(E)$ were performed for the full measured energy range including both below Coulomb barrier energies and energies up to several times Coulomb barrier energy. The BPM with matrix method for the calculations of transmission coefficients through real potential barrier was used. As a nuclear interaction the Satchler–Love double folded M3Y potential was taken.

The remarkable agreement between the theoretical results and experimental data was obtained for the full measured energy range. It was found that the values of two parameters of the model *i.e.*, the reduced critical radius r_{cr} and renormalizating factor N_S of nuclear potential do not depend on the energy of colliding ions and are very similar for all systems taken into account. The found of the fusion value of $N_S \simeq 1.0$ is consistent with Satchler and Love result obtained from elastic scattering analysis. Also the value of the second parameter of the model r_{cr} close to 1.0 fm is related to the overlap nuclear matter half density radius.

The results of this work show that the same nuclear potential, used as a real part of the optical model potential, can reproduce satisfactory the elastic scattering [7] and also describes the energy behaviour of the fusion cross section in the full measured energy range within one-dimensional BPM.

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