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Citation style: Jarczyk L., Maciuk B., Siemaszko Marek, Zipper Wiktor. (1976). The optical model and distorted-wave analysis of cross-sections for the scattering of the 24-28 MeV alpha particles from 28Si. "Acta Physica Polonica. B" (1976, no. 7, s. 531-539).

THE OPTICAL MODEL AND DISTORTED-WAVE ANALYSIS OF CROSS-SECTIONS FOR THE SCATTERING OF THE 24—28 MeV ALPHA PARTICLES FROM ^{28}Si

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(Received May 21, 1975)

Elastic and inelastic alpha particle scattering from ^{28}Si has been studied at the bombarding energies of 24, 25, 26, 26.5, 27 and 28 MeV. The analysis was carried out with the optical model and distorted wave method. The possibility of resolving some of the optical model ambiguities, and excitation mechanisms of the 1.78 MeV (2^+) and 4.61 MeV (4^+) states in ^{28}Si are discussed.

1. Introduction

The interaction of alpha particles with nuclei is still not well understood [1, 2]. In particular, difficulties arise in the scattering on $A = 4n$ nuclei [3]. It seems that a simultaneous analysis of the elastic and inelastic processes permits solving some of these problems.

Our analysis is based on the experimental data for the elastic and inelastic scattering of alpha particles on ^{28}Si nuclei, obtained with the Cracow U-120 cyclotron. The angular distributions for the elastic [4] and inelastic [5] scattering leading to the first two excited states in ^{28}Si were measured for the alpha bombarding energies of 24, 25, 26, 26.5, 27 and 28 MeV in the angular range from 25° to 179° (in lab system).

The elastic process was described by means of the optical model and the inelastic one by the distorted wave theory.

The possibility of resolving some of the optical model ambiguities was studied.

2. Optical model analysis

The analysis was performed with the standard four parameter optical model potential, with the Saxon-Woods type geometrical factor:

$$V(r) = -(U+iW)\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1} + V_{\mathrm{Coul}}. \tag{1}$$

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The calculations were made with the MAGALI [6] and MARYLKA [7] automatic optical model codes. The optical model parameters were found by minimizing the standard χ^2 function in order to obtain the best description of the experimental data.

The range of variation of the parameters was restricted as follows:

$$40\ \text{MeV} \leqslant U \leqslant 300\ \text{MeV}; \quad 10\ \text{MeV} \leqslant W \leqslant 30\ \text{MeV}; \quad 1.1\ \text{fm} \leqslant r_0 \leqslant 1.7\ \text{fm};$$

$$0.4 \leqslant a \leqslant 0.8\ \text{fm}.$$

In the first step a grid was made with the aim of obtaining all the parameter sets. Next the minima were found in automatic searches. It should be stressed that the whole angular range of the experimental data was included in the analysis. We found ten parameter sets, which can be divided into two groups according to the discrete geometrical ambiguity previously reported [8].

TABLE I

Optical model parameters with a "large radius" for the elastic scattering of 26 MeV alpha particles on ^{28}Si nuclei, and the extracted β_2 parameters

No.	U [MeV]	W [MeV]	r [fm]	a [fm]	$\|\beta_2\|$
A1	54.733	9.439	1.603	0.606	0.34±0.02
A2	84.954	11.542	1.620	0.577	0.29±0.02
A3	122.699	14.196	1.583	0.504	0.27±0.02
A4	165.912	16.945	1.552	0.487	0.28±0.02
A5	213.485	19.707	1.532	0.474	0.27±0.02

TABLE II

Optical model parameters with a "small radius" for the elastic scattering of 26 MeV alpha particles on ^{28}Si nuclei, and the extracted β_2 parameters

No.	U [MeV]	W [MeV]	r [fm]	a [fm]	$\|\beta_2\|$
A1	67.769	14.649	1.303	0.841	0.46±0.03
A2	110.321	17.325	1.259	0.824	0.45±0.03
A3	141.498	21.742	1.117	0.813	0.46±0.03
A4	230.752	24.660	1.177	0.699	0.47±0.04
A5	298.596	27.148	1.153	0.655	0.38±0.03

The results obtained for the 26 MeV alpha particles are presented in Tables I and II as an example. The best fits are shown in Figs 1a and 1b. Figs 2a and 2b show the energy dependence of the optical model parameters U and W only. It can be seen that no parameter set can reproduce the angular distributions adequately.

However, the parameter sets with a "large radius" give a good description of the data up to 90° in LAB, while those with a "small radius" lead to better reproduction of

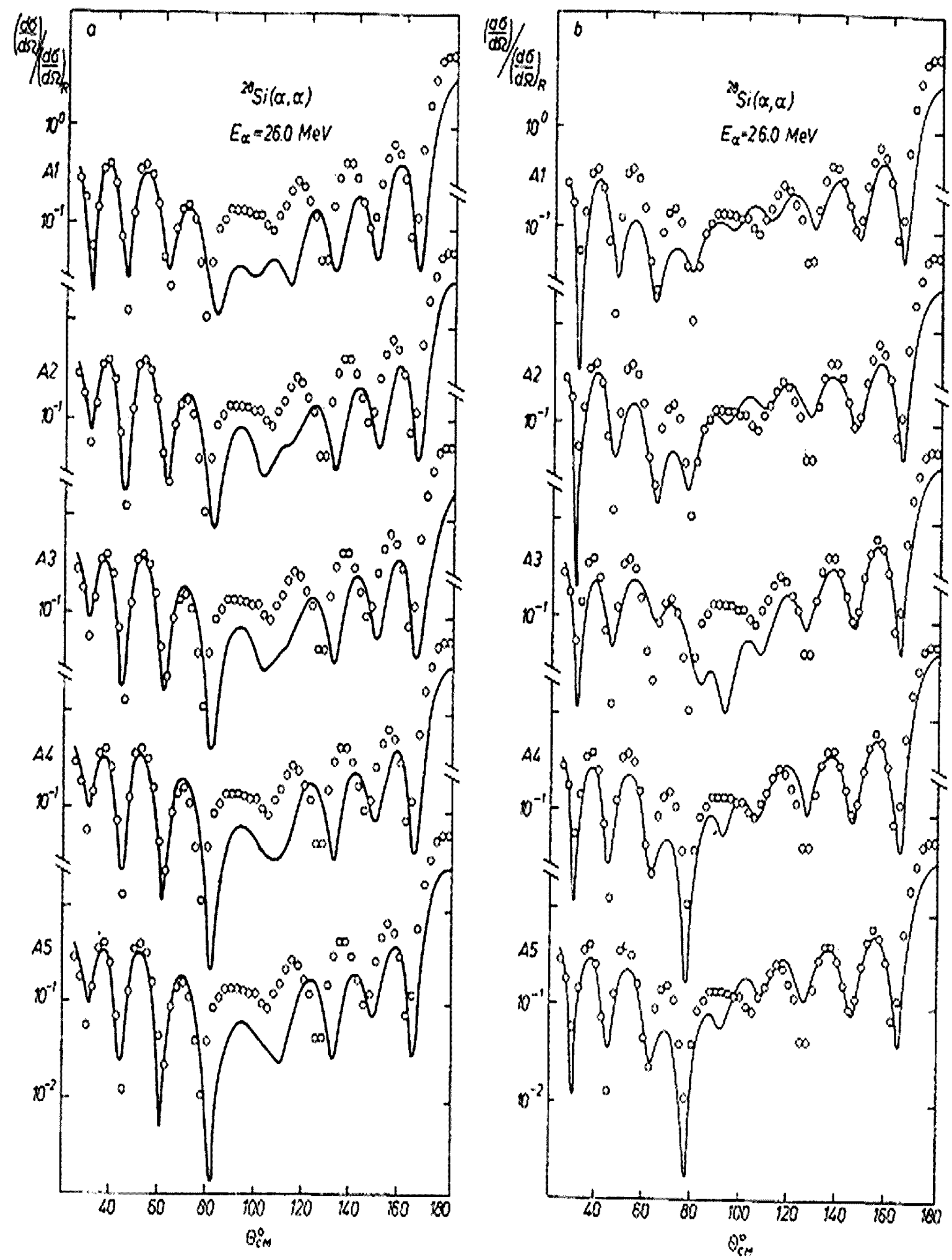

Fig. 1. Angular distributions for the elastic scattering of 26 MeV alpha particles. Full lines indicate the optical model fits with a — "large radius" parameter (Table I) and b — "small radius" parameter (Table II)

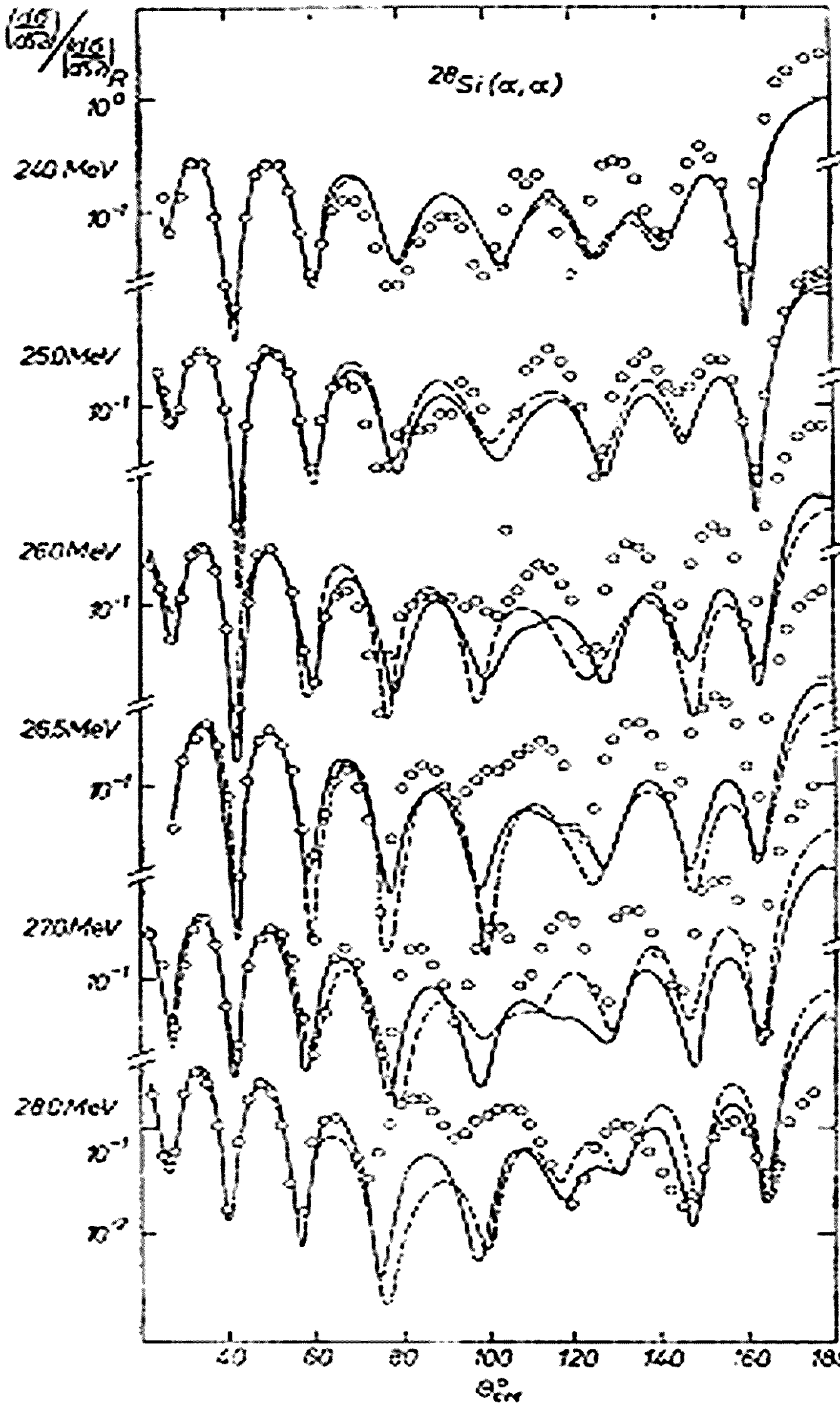

Fig. 3. Experimental angular distributions of the elastic scattering of alpha particles at the energies from 24 to 28 MeV compared with the optical model angular distributions corresponding to averaged optical model parameters (full lines), and the parameters which gave the best forward angles data description (dashed lines)

Fig. 7

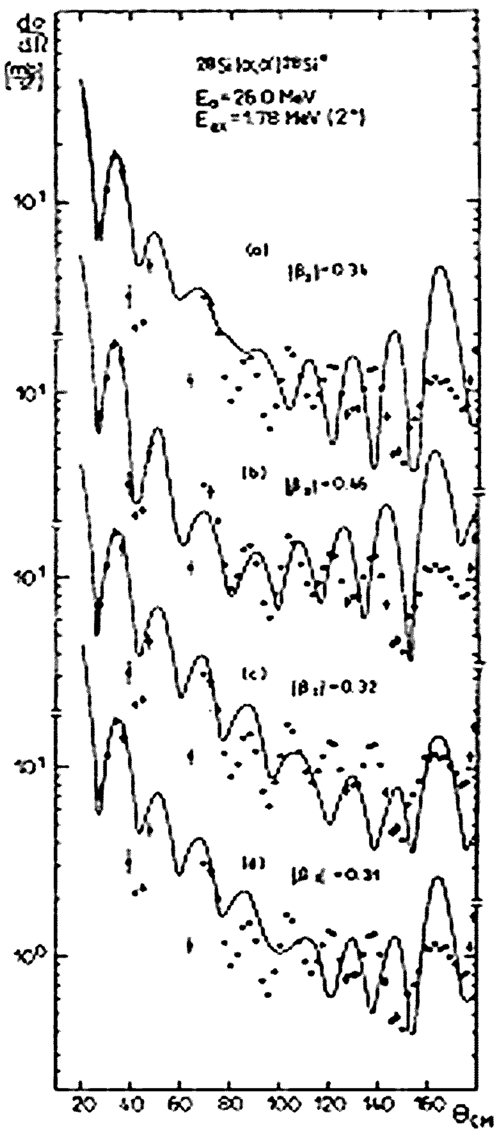

Fig. 8

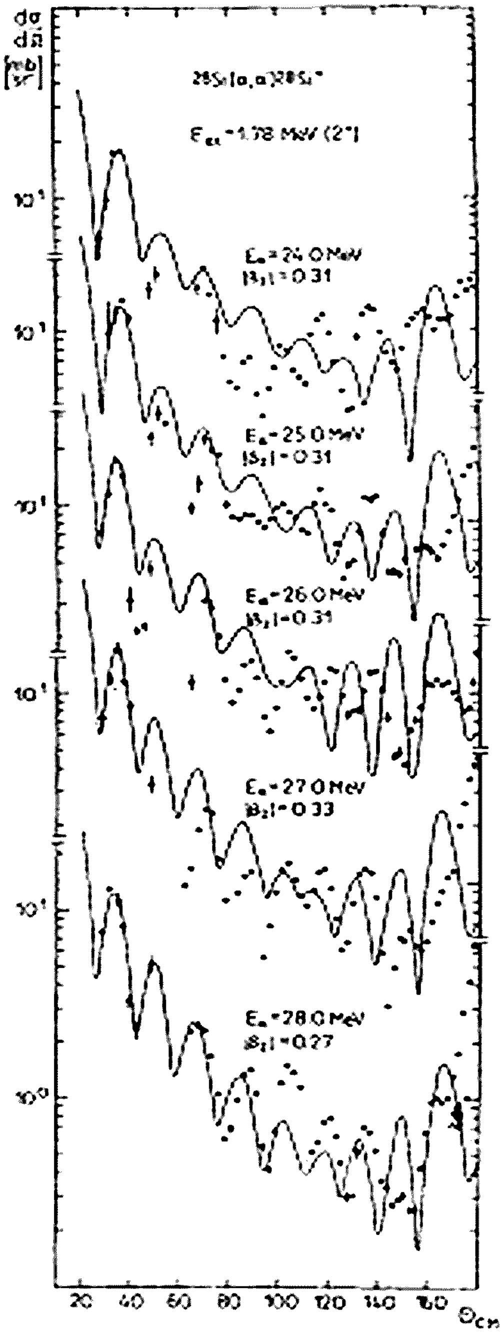

Fig. 7. Comparison of the inelastic alpha particle scattering data (E_α = 26 MeV) leading to the 1.78 MeV (2^+) state in ^{28}Si with the DWBA angular distributions. a — optical model parameters with a "large radius" (Table I, set A1), b — optical model parameters with a "small radius" (Table II, set A1), c — averaged optical model parameters (same in the entrance and exit channels), d — averaged optical model parameters (exit channel parameters U and W changed according to lower alpha particle energy)

Fig. 8. Comparison of the inelastic alpha particle scattering data for energy E_α = 24–28 MeV leading to the 1.78 MeV (2^+) state in ^{28}Si with DWBA angular distributions. (Averaged optical model parameters, exit channel parameters U and W changed according to lower alpha particle energy)

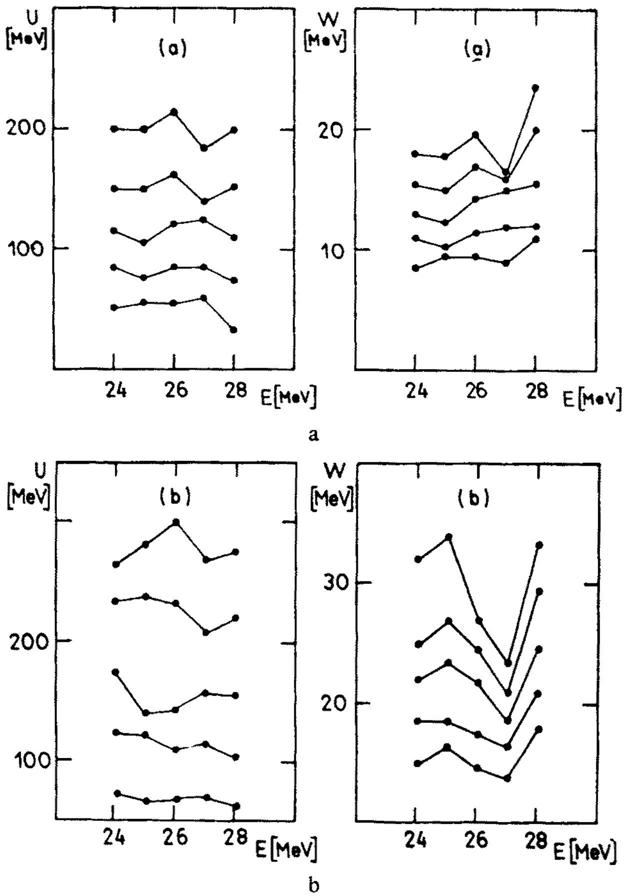

Fig. 2. Energy dependence of the real and imaginary depths of the optical model potentials corresponding to a — "large radius" parameter and b — "small radius" parameter

the back angle region. It should also be noted that the optical model parameters obtained in this way do not change with the bombarding energy monotonically (see Figs 2a and 2b). Next optical model analysis was made once more this time for the forward angles data only as in that region the potential scattering dominates. The analysis, restricted to the forward angles elastic data ($\theta \leqslant 60°$ in CM) yielded five parameter sets, corresponding only to "large radius" parameters. These parameters are listed in Table III.

The analysis of the data of the forward angle differential cross-sections allowed us to derive averaged optical model parameters with the depth of the real part of about 50 MeV. We fixed the geometrical parameters because their energy dependence was insignificant ($r_0 = 1.699$ and $a = 0.505$). The following formulas describe the change of the U and W parameters with the bombarding energy E_α:

$$U(E_\alpha) = 0.067E_\alpha - 52.485 \text{ [MeV]},$$

$$W(E_\alpha) = -0.351E_\alpha + 0.058 \text{ [MeV]}. \tag{2}$$

TABLE III

Optical model parameters for the elastic scattering of 26 MeV alpha particles from the best fits of the forward angle data and the extracted β_2 parameters

No.	U [MeV]	W [MeV]	r [fm]	a [fm]	$\lvert\beta_2\rvert$
A1	49.553	9.428	1.749	0.458	0.32 ± 0.02
A2	82.037	10.589	1.655	0.487	0.27 ± 0.02
A3	106.354	12.703	1.699	0.426	0.29 ± 0.02
A4	147.308	14.136	1.643	0.436	0.27 ± 0.02
A5	190.540	15.986	1.616	0.432	0.26 ± 0.02

The calculated angular distributions obtained for every alpha particle incident energy from averaged optical model parameters indicated above and the parameters from the best forward angle fits were compared with the experimental data in Fig. 3. This analysis, however, did not produce an unique set of optical model parameters, describing the elastic data adequately.

3. *DWBA analysis*

The differential cross-section for the scattering of a particle with incident momentum $\hbar k_i$ and final momentum $\hbar k_f$, in which the target nucleus is excited from an initial state v_i to a final state v_f is given by:

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i} \sum_{AV} |T_{fi}|^2, \tag{3}$$

where the transition amplitude T_{fi} is given by:

$$T_{fi} = \int d\vec{r}\chi_f^{(-)}(\vec{k}_f\vec{r})\,\langle v_f|V_{eff}|v_i\rangle\chi_i^{(+)}(\vec{k}_i\vec{r}). \tag{4}$$

$\chi^{(\pm)}(\vec{k}\vec{r})$ are the distorted waves which characterize the elastic scattering of a particle on the nucleus before and after the inelastic transition, V_{eff} is the effective interaction potential. In our calculations the distorted wave functions $\chi_i^{(+)}$ and $\chi_f^{(-)}$ were generated by the local optical model potential with the parameters given in the previous section.

The same parameters were used in calculating the matrix elements $\langle v_f|V_{eff}|v_i\rangle$ with the assumption of the collective rotational excitation of the nucleus. The DWBA calculations were performed with the DWUCK program [9] adapted to the ICL 4–50 computer.

The experimental differential cross-section for inelastic scattering is related to the reduced cross-section obtained from DWUCK as follows:

$$\sigma^{exp}(\theta) = \frac{2J_f+1}{2J_i+1}\,\frac{|A_{lsj}|^2}{2s+1}\,\sigma_{lsj}^{DWUCK}(\theta), \tag{5}$$

where J_i and J_f represent the spins of the target nucleus in the ground and excited state, respectively, l is the orbital angular momentum transferred to the target nucleus, s is the spin of the incoming particle, j is the sum of the spin and angular momentum l. For given angular momentum l transferred, the matrix element A_{lsj} can, in the case of rotational excitation, be expressed by the nuclear deformation parameter β_l as follows:

$$\frac{A_{lsj}}{(2s+1)^{1/2}} = \frac{1}{(2l+1)^{1/2}}\beta_l. \tag{6}$$

In the case of inelastic alpha particle scattering on spinless nuclei the differential cross-section calculated from DWUCK can be related to the experimental cross-section by the following expression:

$$\sigma^{\mathrm{exp}}(\theta) = \frac{(2J_f+1)}{(2l+1)}\beta_l^2\sigma^{\mathrm{DWUCK}}(\theta). \tag{7}$$

The quadrupole deformation parameters β_2 are usually derived from comparison of the calculated cross-section $\sigma^{\mathrm{DWUCK}}(\theta)$ with the experimental distribution, according to the relation [7], at a first maximum.

We calculated the DWBA differential cross-sections leading to the first two excited states ($E_{ex} = 1.78$ MeV, $J = 2^+$ and $E_{ex} = 4.61$ MeV, $J = 4^+$) in ^{28}Si for the bombarding energies of: 24, 25, 26, 26.5, 27 and 28 MeV.

The deformation parameters were also calculated for each case.

3.1. The 1.78 MeV (2^+) state

The DWBA angular distributions were calculated with the complex collective (rotational) form-factor including the Coulomb excitation. The distorted waves were generated by the optical model potential with the parameters given in Sec. 2. Figs 4–6 show the calculated DWBA angular distributions for the 26 MeV alpha particles.

Fig. 4 presents a comparison of the DWBA angular distributions calculated for the optical model parameters taken from Table III with the experimental data. The next two figures represent comparison of the experimental and calculated DWBA angular distributions based on the optical model parameters with a "large-radius" (Fig. 5) and a "small-radius" (Fig. 6). Similar quality of the experimental data description was obtained for other alpha particle incident energies. All calculations were performed with the same potential parameters in the entrance and exit channels.

From our analysis clearly follows that of the three groups of parameter sets (see Figs 4–6) those with a shallow real depth give the best description of the experimental data. All others cannot reproduce even the second oscillation maximum.

The same calculations were made including the changes of the optical model parameters in the exit channel for a lower alpha particle energy. All the calculations were performed for the averaged optical model parameters, according to formula (2). However, no appreciable improvement in the quality of the description was observed. Fig. 7 shows a comparison of the calculated DWBA angular distributions (26 MeV case) which were based on possible choices of the optical model parameter sets.

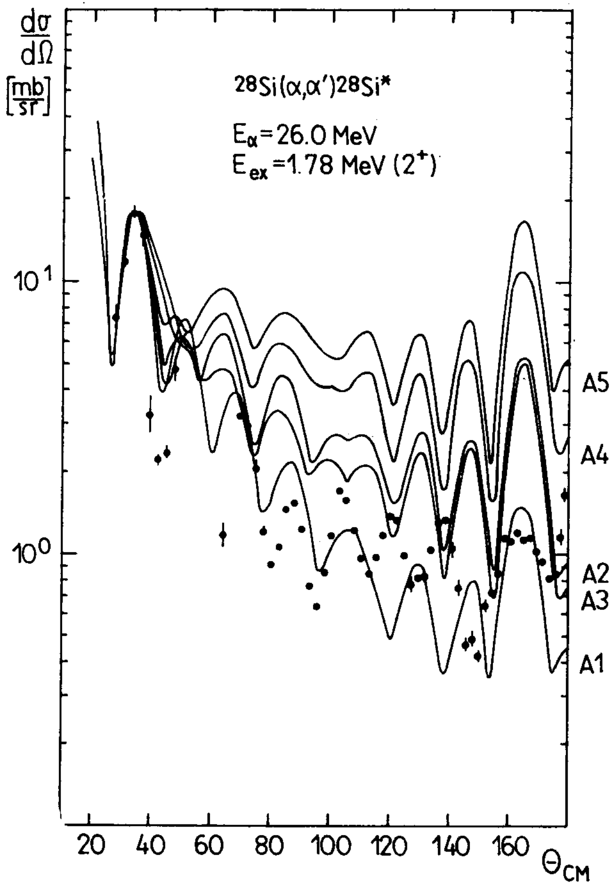

Fig. 4

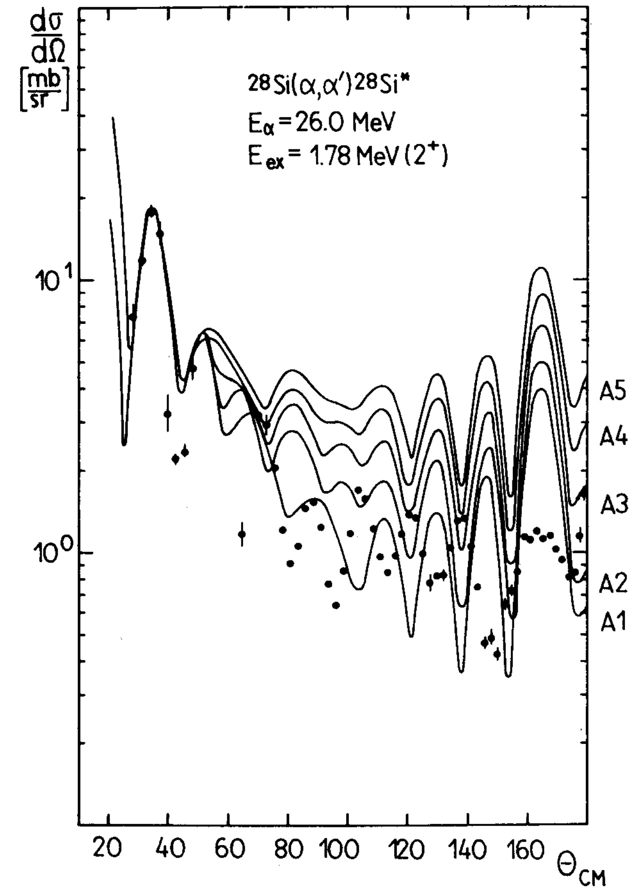

Fig. 5

Fig. 4. Comparison of the experimental data with the DWBA calculated angular distributions of the 26 MeV inelastic alpha particle scattering leading to the 1.78 MeV (2^+) excited states of ^{28}Si nucleus. Optical model parameters taken from Table III

Fig. 5. Same as in Fig. 4 except for optical model parameters taken from Table I

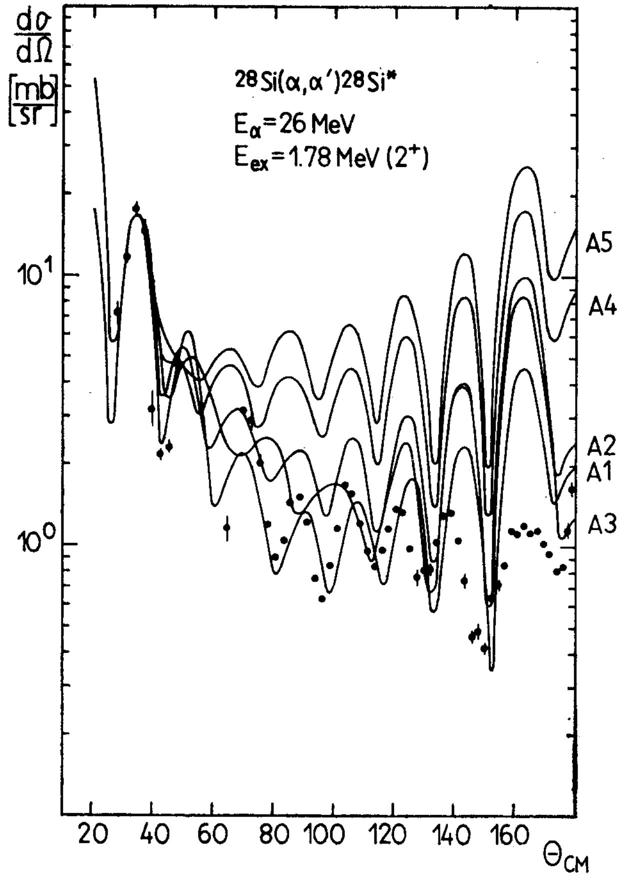

Fig. 6. Same as in Fig. 4 except for optical model parameters taken from Table II

The normalization of the calculated angular distributions allowed us to determine the β_2 deformation parameters for all of the optical model parameter sets. They vary within the range of 0.26 to 0.47. The most right columns in Tables I, II and III list the values of the β_2 parameters corresponding to all groups of the optical model parameters investigated. They do not differ significantly from those found in the literature (Ref. [10–18]).

Examination of the calculated DWBA angular distributions and the deformation parameters permits the following conclusions:

(*i*) the parameter groups with "large" r_0 values and with the real depth close to 50 MeV give an acceptable fit to the data and a slightly larger value of the β_2 parameter, while the other parameter sets show an enhancement of the calculated angular distributions over the experimental points (particularly for larger angles) and too small β_2 parameter;

(*ii*) in the parameter groups with "small" r_0 values only the sets with U close to 70 MeV give acceptable fits to the experimental data (but β_2 parameters are too large); good β_2 values could be extracted from the normalization of the angular distributions calculated with the parameters corresponding to the real depth of the potential close to 210 MeV, but the description of the data in this case is very poor;

(*iii*) in the case of the parameter sets fitting the forward angular distributions our conclusions are similar to those in (*ii*);

(*iv*) the averaged optical model parameters with and without changing the parameters in the exit channel give a comparable quality of data description and do not change the β_2 parameter significantly.

Fig. 8 presents the best DWBA fits for the averaged optical model parameter sets compared with the 2^+ inelastic data for all alpha particle bombarding energies.

3.2. The 4.61 MeV (4^-) state

Several attempts were made to find a satisfactory description of the experimental angular distributions leading to the 4^+ excited state. However, all of the optical model parameter sets used in the calculations could not reproduce the experimental data. Besides, the calculated angular distributions lie much lower than the experimental points in the back angle region.

4. *Conclusions*

The simultaneous analysis of elastic (optical model) and inelastic (DWBA method) angular distributions in the case of the scattering of alpha particles seems to be an useful tool for solving some questions of the alpha particle interaction. Due to the optical model ambiguities such complex analysis also gives additional information. In our case this method has indicated that only the parameter sets with the real depth of about 50 MeV allow us to obtain an acceptable (but still not satisfactory) description of the experimental data (elastic and the first inelastic angular distribution). The only reasonable deformation parameters β_2 were obtained with the "large radius" parameter sets.

The angular distributions leading to the 2^+ state are not reproduced satisfactorily. This suggests that this state has not a purely collective nature and that some admixture of the compound process occurs. On the other hand the 4^- state could not be described on the basis of a simple collective excitation mode, thus it seems that more complicated processes come into play.

We wish to thank Prof. A. Strzałkowski for many useful discussions and for reading the manuscript. We would also like to thank Miss J. Beme for her kind help in preparing the data and for computational assistance.

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