Title: The minimal extension of the SM and the neutrino oscillation data

Author: Francisco del Aguila, Janusz Gluza, Marek Zrałek

Citation style: Del Aguila Francisco, Gluza Janusz, Zrałek Marek. (1999). The minimal extension of the SM and the neutrino oscillation data. "Acta Physica Polonica. B" (1999, no. 11, s. 3139-3147).

Vol. 30 (1999) ACTA PHYSICA POLONICA B No 11

THE MINIMAL EXTENSION OF THE SM AND THE NEUTRINO OSCILLATION DATA* **

F. DEL ÁGUILA

Dpto. de Física Teórica y del Cosmos
Universidad de Granada, 18071 Granada, Spain

J. GŁUZA

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland
DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany

AND M. ZRAŁEK

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

(Received October 26, 1999)

We study the simplest Standard Model extension with only one extra right-handed neutrino. In this case there are two massless $m_{1,2}$ and two massive $m_{3,4}$ neutrinos, and in principle both solar and atmospheric anomalies can be described in two different scenarios, $m_3 \ll m_4$ (scheme I) and $m_3 \simeq m_4$ (scheme II). However, neither bi-maximal mixing nor the dark matter problem are explained in this minimal extension. Only scheme II can accommodate simultaneously maximal mixing for atmospheric neutrinos and the small mixing angle MSW solution for the solar anomaly. This scenario can be tested in the BOREXINO experiment.

PACS numbers: 14.60.Pq, 26.65.+t, 95.85.Ry

* Presented by J. Gluza at the XXIII International School of Theoretical Physics "Recent Developments in Theory of Fundamental Interactions", Ustroń, Poland, September 15–22, 1999.

** Work supported in part by Polish Committee for Scientific Research under Grants Nos. 2P03B08414, 2P03B04215 and by CICYT under Contract number AEN96-1672. J.G. would like to thank also the Alexander von Humboldt-Stiftung for fellowship.

1. Introduction

The discovery of atmospheric muon neutrino oscillations by the large Superkamiokande detector [1] implies that neutrinos are massive particles. This experiment has also strengthened the interpretation of the solar neutrino problem in terms of oscillation phenomena [2]. The results of atmospheric neutrino experiments can be explained by $\nu_\mu \to \nu_\tau$ oscillations with [3]

$$\delta m^2_{\rm atm} \sim (1.5 - 6) \cdot 10^{-3} \;\; {\rm eV}^2 \quad {\rm and} \quad A_{\rm atm} \sim 0.82 - 1.0, \tag{1}$$

whereas solar neutrino experiments can be interpreted as a result of the $\nu_e \to \nu_x \, (x = \mu, \tau)$ transition [4] with

$$\delta m^2_{\rm sun} \sim (0.5 - 0.8) \cdot 10^{-10} \;\; {\rm eV}^2, \;\; A_{\rm sun} \sim (0.72 - 0.95) \tag{2}$$

in the case of vacuum oscillation (VO),

$$\delta m^2_{\rm sun} \sim (0.5 - 1) \cdot 10^{-5} \;\; {\rm eV}^2, \;\; A_{\rm sun} \sim (2 - 10) \cdot 10^{-3} \tag{3}$$

in the case of small mixing angle (SMA) MSW transition [5], and

$$\delta m^2_{\rm sun} \sim (0.16 - 4) \cdot 10^{-4} \;\; {\rm eV}^2, \;\; A_{\rm sun} \sim (0.65 - 1.0) \tag{4}$$

in the case of large mixing angle (LMA) MSW transition. Finally, let us also mention that the LSND data can be accommodated if [6]

$$\delta m^2_{\rm LSND} \sim (0.2 - 2) \;\; {\rm eV}^2 \; {\rm and} \; A_{\rm LSND} \sim (0.3 - 4) \cdot 10^{-2}. \tag{5}$$

There is a vast literature exploring models of neutrino oscillations which can accommodate only two (atmospheric + solar) or all three (atmospheric + solar + LSND) anomalies. Most of them try to understand why atmospheric and solar neutrino oscillations require near maximal mixing (Eqs. (1) and (2)). Both are possible in the context of three nearly–degenerate neutrinos or in see-saw models with a neutrino mass hierarchy [7]. Scenarios with additional sterile neutrino(s) where all three anomalies can be explained have been also investigated [8].

Here we study the simplest extension of the Standard Model (SM) with a single right-handed neutrino (RH1 model). Since the Higgs sector is not touched, the neutrino mass matrix has four parameters. This simple matrix has two zero eigenvalues and we are not able to explain all three anomalies. Four different masses are needed to do that. So, we put the permanently unsettled LSND result aside and investigate within this model in full detail solar and atmospheric anomalies. The diagonalization and mixing matrix obtained here can be used as a first step for diagonalizing two (RH2) and three (RH3) right-handed neutrino SM extensions, where a full description of the neutrino data will be possible [9].

2. Model with one right-handed neutrino singlet

In the SM with one Higgs doublet and one extra right-handed neutrino singlet ν_{1R} the neutrino mass matrix has the form

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & c \\ a & b & c & M \end{pmatrix} \tag{6}$$

in the basis $(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \overline{\nu}^c_{1L})$. In this case CP is conserved [10] and all parameters can be assumed to be real and positive $(a, b, c, M \geq 0)$. The matrix M_ν is diagonalized

$$U^T M_\nu U = \mathrm{diag}\,(0, 0, m_3, m_4) \tag{7}$$

by the unitary transformation

$$U = \begin{bmatrix} s, & c\cos\theta, & ic\sin\theta\cos\zeta, & c\sin\theta\sin\zeta \\ -c, & s\cos\theta, & is\sin\theta\cos\zeta, & s\sin\theta\sin\zeta \\ 0, & -\sin\theta, & i\cos\theta\cos\zeta, & \cos\theta\sin\zeta \\ 0, & 0, & -i\sin\zeta, & \cos\zeta \end{bmatrix}, \tag{8}$$

where

$$m_{3,4} = \frac{1}{2}M\left[\sqrt{1 + 4\left(\frac{\Lambda}{M}\right)^2} \mp 1\right], \tag{9}$$

$$\sin\theta = \frac{\lambda}{\Lambda}, \quad \cos\theta = \frac{c}{\Lambda}, \tag{10}$$

$$s \equiv \sin\varphi = \frac{b}{\lambda}, \quad c \equiv \cos\varphi = \frac{a}{\lambda}, \tag{11}$$

$$\sin\zeta = \sqrt{\frac{m_3}{m_3 + m_4}}, \quad \cos\zeta = \sqrt{\frac{m_4}{m_3 + m_4}}, \tag{12}$$

$$\lambda = \sqrt{a^2 + b^2}, \quad \Lambda = \sqrt{a^2 + b^2 + c^2}. \tag{13}$$

The two massive neutrinos have opposite CP parities and the non-zero masses depend only on M and Λ/M. If $\Lambda \ll M$, the traditional see-saw mechanism works. This case, with M greater than 1 GeV or even greater than M_Z (heavy neutrino singlet), has been discussed in [11–13]. m_4 is then $\sim M$. However, we are not interested in such a case since we need much smaller $m_{3,4}$ masses to be able to explain simultaneously the small mass squared splittings dictated by solar and atmospheric neutrino data.

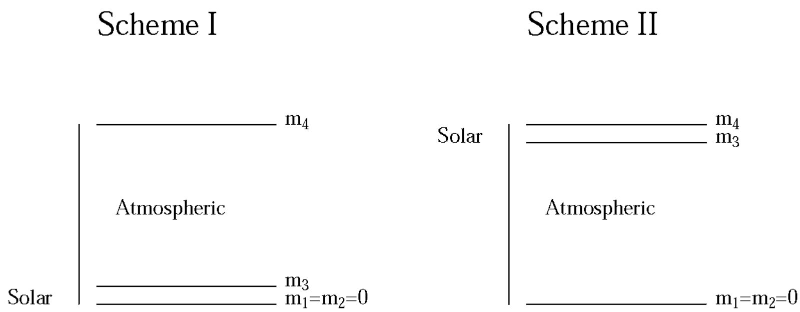

Fig. 1. Two possible neutrino mass spectra which can describe the oscillation data in the RH1 model.

Two different scenarios are possible in this simple model, scheme I, where $m_3 \ll m_4$ and scheme II with $m_3 \simeq m_4$ (Fig.(1)). Since two masses are zero, the absolute scale of the neutrino mass spectrum is constrained and $m_{3,4}$ fixed, in contrast with the general case where all neutrinos can be massive [14, 15]. Eq. (1) requires

$$0.038 \text{ eV} \leq m_4 \leq 0.078 \text{ eV} \tag{14}$$

in both schemes. Once $m_{1,2,4}$ are determined, m_3 is fixed by δm^2_{sun} (Eqs. (2)–(4)). Hence, we are really interested in quite small M (Λ) values ranging in the milielectronvolt scale. Let us note that these masses do not solve the dark matter problem. Two further remarks are necessary. First, the smallness of the neutrino masses compared to other known particles implies no problem with non-oscillation experiments. For example,the number of neutrino species measured by LEP1 is predicted by the model to be $N_\nu = 3$ (all four neutrinos can be produced in Z^0 decays) [11–13]. Second, the Heidelberg-Moscow limit on the effective neutrino mass, $\langle m_\nu \rangle_{ee} \equiv U^2_{ei}\, m_i \leq 0.2$ eV from the non-observation of the neutrinoless double beta decay [16], is automatically fulfilled, as $\langle m_\nu \rangle_{ee}$ is equal to the element (1,1) of M_ν in Eq. (6), which is equal to zero.

3. Oscillation probabilities and study of the model

Let us apply the probability of the flavour changing $\alpha \to \beta$ neutrino transition in vacuum, which is a function of the traveling distance L,

$$P_{\alpha\to\beta}(L) = \delta_{\alpha\beta} - \sum_{a>b} \left(4R^{ab}_{\alpha\beta} \sin^2 \Delta_{ab} - 2I^{ab}_{\alpha\beta} \sin 2\Delta_{ab} \right) , \tag{15}$$

where

$$R^{ab}_{\alpha\beta} = \text{Re}\left[U_{\alpha a} U_{\beta b} U^*_{\alpha b} U^*_{\beta a}\right] , \qquad I^{ab}_{\alpha\beta} = \text{Im}\left[U_{\alpha a} U_{\beta b} U^*_{\alpha b} U^*_{\beta a}\right] ,$$

and

$$\Delta_{ab} = 1.27\delta m^2_{ab}\left(\mathrm{eV}^2\right)\frac{L\,(\mathrm{km})}{E\,(\mathrm{GeV})},$$

to both mass spectra.

3.1. Scheme I

In this case the oscillation probability reads

$$P_{\alpha\to\beta} \simeq \delta_{\alpha\beta} - \left(4(R^{41}_{\alpha\beta} + R^{42}_{\alpha\beta} + R^{43}_{\alpha\beta})\sin^2\Delta_{\mathrm{atm}} + 4(R^{31}_{\alpha\beta} + R^{32}_{\alpha\beta})\sin^2\Delta_{\mathrm{sun}}\right), \tag{16}$$

where

$$\Delta_{\mathrm{atm}} \simeq \Delta_{43} \simeq \Delta_{41} = \Delta_{42} \quad \text{and} \quad \Delta_{\mathrm{sun}} \simeq \Delta_{31} = \Delta_{32}.$$

For $L = L_{\mathrm{atm}} = \left(20 \div 10^4\right)$ km, $\Delta_{\mathrm{atm}} \gg \Delta_{\mathrm{sun}}$ and the second oscillation term in Eq. (16) has no time to develop. The oscillation of atmospheric neutrinos is then described by

$$P_{\alpha\to\beta}\,(L_{\mathrm{atm}}) \simeq \delta_{\alpha\beta} - 4(R^{41}_{\alpha\beta} + R^{42}_{\alpha\beta} + R^{43}_{\alpha\beta})\sin^2\Delta_{\mathrm{atm}}. \tag{17}$$

On the other hand, at the solar distance scale $L = L_{\mathrm{solar}} \sim 10^8$ km the first oscillation term is averaged, $\sin^2\Delta_{\mathrm{atm}} \to \frac{1}{2}$, and the flavour changing probability is

$$P_{\alpha\to\beta}\,(L_{\mathrm{solar}}) \simeq \delta_{\alpha\beta} - 2(R^{41}_{\alpha\beta} + R^{42}_{\alpha\beta} + R^{43}_{\alpha\beta}) - 4(R^{31}_{\alpha\beta} + R^{32}_{\alpha\beta})\sin^2\Delta_{\mathrm{sun}}. \tag{18}$$

Now, it is straightforward to find the relevant oscillation probabilities. For atmospheric neutrinos we have:

$$P_{\nu_\mu\to\nu_e}\,(L_{\mathrm{atm}}) \simeq \sin^2 2\varphi\sin^4\zeta\sin^4\theta\sin^2\Delta_{\mathrm{atm}}, \tag{19}$$

$$P_{\nu_\mu\to\nu_\tau}\,(L_{\mathrm{atm}}) \simeq \sin^2\varphi\sin^4\zeta\sin^2 2\theta\sin^2\Delta_{\mathrm{atm}}, \tag{20}$$

$$P_{\nu_\mu\to\nu_s}\,(L_{\mathrm{atm}}) \simeq \sin^2\varphi\sin^2 2\zeta\sin^2\theta\sin^2\Delta_{\mathrm{atm}}, \tag{21}$$

and

$$\begin{aligned} &P_{\nu_\mu\to\nu_\mu}\,(L_{\mathrm{atm}}) \simeq 1 - 4\sin^2\varphi\sin^2\zeta\sin^2\theta \\ &\times\left(\cos^2\varphi\sin^2\zeta\sin^2\theta + \sin^2\zeta\cos^2\theta + \cos^2\zeta\right)\sin^2\Delta_{\mathrm{atm}}\,. \end{aligned} \tag{22}$$

Whereas for solar neutrinos the oscillation probabilities are:

$$P_{\nu_e\to\nu_\mu}\,(L_{\mathrm{solar}}) \simeq \sin^2 2\varphi\sin^4\theta\left(\frac{1}{2}\sin^4\zeta + \cos^2\zeta\sin^2\Delta_{\mathrm{sun}}\right), \tag{23}$$

$$P_{\nu_e\to\nu_\tau}\,(L_{\mathrm{solar}}) \simeq \cos^2\varphi\sin^2 2\theta\left(\frac{1}{2}\sin^4\zeta + \cos^2\zeta\sin^2\Delta_{\mathrm{sun}}\right), \tag{24}$$

$$P_{\nu_e\to\nu_s}\,(L_{\mathrm{solar}}) \simeq \frac{1}{2}\cos^2\varphi\sin^2 2\zeta\sin^2\theta, \tag{25}$$

and

$$
\begin{aligned}
P_{\nu_e \to \nu_e}(L_{\rm solar}) \;\simeq\; & 1 - 2\cos^2\varphi \sin^2\theta \\
& \left[\sin^2\zeta \left(\sin^2\varphi \sin^2\zeta \sin^2\theta + \sin^2\zeta \cos^2\theta + \cos^2\zeta\right)\right. \\
+ \;& \left. 2\cos^2\zeta \left(\sin^2\varphi \sin^2\theta + \cos^2\theta\right) \sin^2 \Delta_{\rm sun}\right]. \qquad (26)
\end{aligned}
$$

Since $m_4 \gg m_3$,

$$\sin\zeta \ll \cos\zeta \sim 1. \qquad (27)$$

The oscillation parts of $P_{\nu_e \to \nu_\mu}$ and $P_{\nu_e \to \nu_\tau}$ for solar neutrinos, Eqs. (23) and (24) respectively, are proportional to $\cos^2\zeta$. Depending on the angles φ and θ, the mixing can be large or small, so any solution (Eq. (2), (3) or (4)) is possible. Unfortunately, the probabilities for atmospheric neutrinos (Eqs. (19–22)) are proportional to $\sin^2\zeta$ and thus very small. As a result, it is impossible to explain the observed atmospheric neutrino anomaly in this scheme.

3.2. Scheme II

In this case $m_3 \simeq m_4$ and

$$P_{\alpha \to \beta}(L_{\rm atm}) \simeq \delta_{\alpha\beta} - 4\left(R^{31}_{\alpha\beta} + R^{32}_{\alpha\beta} + R^{41}_{\alpha\beta} + R^{42}_{\alpha\beta}\right) \sin^2 \Delta_{\rm atm} \qquad (28)$$

for atmospheric neutrino oscillations, and

$$P_{\alpha \to \beta}(L_{\rm solar}) \simeq \delta_{\alpha\beta} - 2\left(R^{31}_{\alpha\beta} + R^{32}_{\alpha\beta} + R^{41}_{\alpha\beta} + R^{42}_{\alpha\beta}\right) - 4R^{43}_{\alpha\beta} \sin^2 \Delta_{\rm sun} \quad (29)$$

for solar neutrino oscillations. These probabilities reduce for the specific transitions to

$$
\begin{aligned}
P_{\nu_\mu \to \nu_e}(L_{\rm atm}) &\simeq \sin^2 2\varphi \sin^4\theta \sin^2 \Delta_{\rm atm}, & (30)\\
P_{\nu_\mu \to \nu_\tau}(L_{\rm atm}) &\simeq \sin^2\varphi \sin^2 2\theta \sin^2 \Delta_{\rm atm}, & (31)\\
P_{\nu_\mu \to \nu_s}(L_{\rm atm}) &\simeq 0, & (32)\\
P_{\nu_e \to \nu_\mu}(L_{\rm solar}) &\simeq \frac{1}{2}\sin^2 2\varphi \sin^4\theta \left(1 - \frac{1}{2}\sin^2 2\zeta \sin^2 \Delta_{\rm sun}\right), & (33)\\
P_{\nu_e \to \nu_\tau}(L_{\rm solar}) &\simeq \frac{1}{2}\cos^2\varphi \sin^2 2\theta \left(1 - \frac{1}{2}\sin^2 2\zeta \sin^2 \Delta_{\rm sun}\right), & (34)\\
P_{\nu_e \to \nu_s}(L_{\rm solar}) &\simeq \cos^2\varphi \sin^2 2\zeta \sin^2\theta \sin^2 \Delta_{\rm sun} & (35)
\end{aligned}
$$

and

$$P_{\nu_\mu \to \nu_\mu}(L_{\rm atm}) \simeq 1 - \left(\sin^2 2\varphi \sin^4\theta + \sin^2\varphi \sin^2 2\theta\right) \sin^2 \Delta_{\rm atm}, \qquad (36)$$

$$P_{\nu_e\to\nu_e}\left(L_{\text{atm (CHOOZ)}}\right) \simeq 1-\left(\cos^2\varphi\sin^2 2\theta+\sin^2 2\varphi\sin^4\theta\right)\sin^2\varDelta_{\text{atm}}, \tag{37}$$

$$\begin{aligned}P_{\nu_e\to\nu_e}\left(L_{\text{solar}}\right) &\simeq 1-\frac{1}{2}(\sin^2 2\varphi\sin^4\theta+\cos^2\varphi\sin^2 2\theta)\\ &- \cos^4\varphi\sin^2 2\zeta\sin^4\theta\sin^2\varDelta_{\text{sun}}.\end{aligned} \tag{38}$$

Since the non-zero masses are nearly degenerate, the mixing ζ is almost maximal

$$\sin\zeta\simeq\cos\zeta\sim\frac{1}{\sqrt{2}}. \tag{39}$$

The CHOOZ reactor experiment [17] constrains $P_{\nu_e\to\nu_e}$ (Eq. (37)),

$$\cos^2\varphi\sin^2 2\theta+\sin^2 2\varphi\sin^4\theta\le 0.18 \;\text{ for } \delta m^2>0.9\cdot 10^{-3}\text{ eV}^2, \tag{40}$$

and the Superkamiokande experiment constrains $P_{\nu_\mu\to\nu_\mu}$ (Eq. (36)),

$$0.82\le\sin^2 2\varphi\sin^4\theta+\sin^2\varphi\sin^2 2\theta\le 1. \tag{41}$$

Both restrictions are satisfied if $\cos\varphi\sim 0$ and $\sin 2\theta\sim 1$. However, in this case the solar neutrinos do not oscillate (Eq. (38)). This means that bi-maximal mixing for solar and atmospheric neutrinos is not possible in the RH1 model. Although recent Superkamiokande data favour vacuum long-wavelength oscillation of solar neutrinos, this can not be explained with only one extra right-handed neutrino singlet. However, the deficit of solar neutrinos can be also described by the SMA MSW transition (Eq. (3)) and all present observations (without LSND data) can be then accommodated in this minimal SM extension. Indeed the CHOOZ (Eq. (40)) and Superkamiokande (Eq. (41)) constraints are also fulfilled if $\cos\varphi\gg 0$ and $\sin 2\theta\ll 1$. In this case (see Eq. (38))

$$A_{\text{sun}}\simeq\cos^4\varphi\sin^4\theta \tag{42}$$

satisfies Eq. (3). For example, $\cos^2\varphi=0.17$ and $\sin^2\theta=0.35$ fulfill Eqs. (40) and (41), implying

$$A_{\text{sun}}=0.0035 \tag{43}$$

which lies within the SMA MSW limits. The mixing angles determine the mixing matrix in Eq.(8)

$$\begin{aligned}\nu_e &= +0.91\nu_1+0.33\nu_2+i0.17\nu_3+0.17\nu_4,\\ \nu_\mu &= -0.41\nu_1+0.73\nu_2+i0.38\nu_3+0.38\nu_4,\\ \nu_\tau &= -0.59\nu_2+i0.57\nu_3+0.57\nu_4,\\ \nu_s &= -i0.71\nu_3+0.71\nu_4,\end{aligned} \tag{44}$$

and Eqs. (1) and (3) are fulfilled by the neutrino masses

$$m_3 = 0.05477 \text{ eV}, \quad m_4 = 0.05482 \text{ eV}. \tag{45}$$

These eigenvectors and eigenvalues are obtained from the M_ν entries (Eq. (6))

$$a = 0.013376 \text{ eV}, b = 0.02953 \text{ eV}, c = 0.04418 \text{ eV}, M = 5 \cdot 10^{-5} \text{ eV}. \tag{46}$$

In this model, contrary to what happens in the popular see-saw mechanism, the right-handed Majorana mass term M is much smaller than the Dirac masses a, b, c.

4. Conclusions

The RH1 model seems to be too simple to explain the observed neutrino anomalies. The popular bi-maximal solution for the atmospheric and solar anomalies can not be realized in this model, neither the dark matter problem can be solved. Although not favoured, only the small mixing angle MSW transition for solar neutrinos and the maximal neutrino mixing oscillation solution for atmospheric neutrinos can be accommodated. The model which is the simplest SM extension, will be definitively excluded if the favoured 'just so' mechanism for solar neutrinos persists.

REFERENCES

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81** (1998) 1562; Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. **B436**, 33 (1998); Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Phys. Lett.* **B433**, 9 (1998).

[2] See *e.g.* the collection of talks in: S.M. Bilenky *et al.*, "Summary of the NOW'98 Phenomenology Working Group", hep-ph/9906251.

[3] See [1]; T. Kajita, talk given at the International Conference *Neutrino '98*, Takayama, Japan, June 1998; K. Scholberg, [SuperKamiokande Collaboration], hep-ex/9905016.

[4] M.B. Smy, hep-ex/9903034; J.N. Bahcall, P.I. Krastev, A. Yu. Smirnov, *Phys. Rev.* **D58**, 096016 (1998).

[5] L. Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); *Phys. Rev.* **D20**, 2634 (1979); S.P. Mikheyev, A. Y. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1986); *Nuovo Cimento* **9C**, 17 (1986).

[6] C. Athanassopoulos *et al.*, *Phys. Rev. Lett.* **77**, 3082(1996); *Phys. Rev.* **C54**, 268 (1996); *Phys. Rev.* **C58**, 2489 (1998); *Phys. Rev. Lett.* **81**, 1774 (1998); S.J. Yellin, hep-ex/9902012.

[7] N. Hata, P. Langacker, *Phys. Rev.* **D56**, 6107 (1997); J.N. Bahcall, P.I. Krastev, A.Y. Smirnov in [4]; G.L. Fogli, E. Lisi, D. Montanino, *Astropart. Phys.* **9**, 119 (1998); V. Castellani *et al.*, *Phys. Rep.* **281**, 309 (1997); S.M. Bilenky, C.Giunti, hep-ph/9802201; H. Minakata, O. Yasuda, *Nucl. Phys.* **B523**, 597 (1998); *Phys. Rev.* **D56**, 1692 (1997); F. Vissani, hep-ph/9708483; G.L. Fogli *et al.*, *Phys. Lett.* **B425**, 341 (1998); *Phys. Rev.* **D55**, 4385 (1997); R. Foot, R.R. Volkas, O. Yasuda, *Phys. Lett.* **B433**, 82 (1998); *Phys. Lett.* **B421**, 245 (1998); O. Yasuda, *Phys. Rev.* **D58**, 091301 (1998); G.L. Fogli, E. Lisi, A. Marrone, *Phys. Rev.* **D57**, 5839 (1998); C. Giunti, C.W. Kim, M. Monteno, *Nucl. Phys.* **B521**, 3 (1998); Y. Oyama, *Phys. Rev.* **D57**, 6594 (1998); V. Barger, T.J. Weiler, K. Whisnant, *Phys. Lett.* **B440**, 1 (1998).

[8] D.O. Caldwell, R.N. Mohapatra, *Phys. Rev.* **D48**, 3259 (1993); *Phys. Rev.* **D50**, 3477 (1994); J.T. Peltoniemi, J.W.F. Valle, *Nucl. Phys.* **B406**, 409 (1993); J.T. Peltoniemi, D. Tommasini, J.W.F. Valle, *Phys. Lett.* **B298**, 383 (1993); Z.G. Berezhiani, R.B. Mohapatra, *Phys. Rev.* **D52**, 6607 (1995); E. Ma, P. Roy, *Phys. Rev.* **D52**, 4780 (1995); E. Chun, A.S. Joshipura, A.Yu. Smirnov, *Phys. Lett.* **B357**, 608 (1995); *Phys. Rev.* **D54**, 4654 (1996); G.M. Fuller, J.R. Primack, Y.Z. Qian, *Phys. Rev.* **D52** 1288 (1995) ; D. Suematsu, *Phys. Lett.* **B392**, 413 (1997); A.Y. Smirnov, M. Tanimoto, *Phys. Rev.* **D55**, 1665 (1997); H. Minakata, *Phys. Rev.* **D52**, 6630 (1995); J.J. Gomez-Cadenas, G.M. Gonzalez-Garcia, *Z. Phys.* **C71**, 443(1996); V. Barger, T.J. Weiler, K. Whisnant, *Phys. Lett.* **B427**, 97 (1998); S.M. Bilenky, C. Giunti, W. Grimus, *Phys. Rev.* **D58**, 033001 (1998); *Eur. Phys. J.* **C1**, 247 (1998); *Phys. Rev.* **D57**, 1920 (1998); S.M. Bilenky, C. Giunti, W. Grimus, T. Schwetz, *Astropart. Phys.* **11**, 413 (1999); N. Okada, O. Yasuda, *Int. J. Mod. Phys.* **A12**, 3669 (1997); J.M. Gelb, S.P. Rosen, hep-ph/9909293.

[9] F. del Aguila, J. Gluza, M. Zrałek, in preparation.

[10] F. del Aguila, M. Zrałek, *Nucl. Phys.* **B447**, 211 (1995); *Acta Phys. Pol.* **B27**, 971 (1996).

[11] C. Jarlskog, *Phys. Lett.* **B241**, 579 (1990); *Nucl. Phys.* **A518**, 129 (1990).

[12] L.N. Chang, D. Ng, N. Ng, *Phys. Rev.* **D50**, 4589 (1994).

[13] C.O. Escobar *et al.*, *Phys. Rev.* **D47**, R1747 (1993).

[14] V. Barger, T.J. Weiler, K. Whisnant, *Phys. Lett.* **B442**, 255 (1998).

[15] M. Czakon, J. Gluza, M. Zrałek, hep-ph/9906381.

[16] H.V. Klapdor-Kleingrothaus, J. Hellmig, M. Hirsch, *J. Phys.* **G24**, 484 (1998); J. Hellmig, H.V. Klapdor-Kleingrothaus, *Z. Phys.* **A359**, 351 (1997); H.V. Klapdor-Kleingrothaus, M. Hirsch, *Z. Phys.* **A359**, 361 (1997); C.E. Aalseth *et al.*, *Nucl. Phys.* (Proc. Suppl.) **70**, 236 (1999); X. Sarazin (NEMO Coll.) *Nucl. Phys.* (Proc. Suppl.) **70**, 239 (1999); V.D. Ashitkov *et al. Nucl. Phys.* (Proc. Suppl.) **70**, 233 (1999); E. Fiorini, *Phys. Rep.* **307**, 309 (1998).

[17] CHOOZ Collaboration, M. Apollonio *et al.*, *Phys. Lett.* **B420**, 397 (1998).