Title: Light neutrino propagation in matter without heavy neutrino decoupling

Author: Francisco del Aguila, Marek Zrałek

Citation style: Del Aguila Francisco, Zrałek Marek. (2002). Light neutrino propagation in matter without heavy neutrino decoupling. "Acta Physica Polonica. B" (2002, no. 9, s. 2585-2596).

LIGHT NEUTRINO PROPAGATION IN MATTER WITHOUT HEAVY NEUTRINO DECOUPLING

Francisco del Águila

Departamento de Física Teórica y del Cosmos
and Centro Andaluz de Física de Partículas Elementales (CAFPE)
Universidad de Granada 18071 Granada, Spain
e-mail: faguila@ugr.es

and Marek Zrałek

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland
e-mail: zralek@us.edu.pl

(Received July 26, 2002)

Dedicated to Stefan Pokorski on his 60th birthday

We review the propagation of light neutrinos in matter assuming that their mixing with heavy neutrinos is close to present experimental limits. The phenomenological implications of the non-unitarity of the light neutrino mixing matrix for neutrino oscillations are discussed. In particular we show that the resonance effect in neutrino propagation in matter persists, but for slightly modified values of the parameters and with the maximum reduced by a small amount proportional to the mixing between light and heavy neutrinos squared.

PACS numbers: 12.15.Ff, 14.60.St, 26.65.+t, 95.85.Ry

1. Introduction

There is convincing evidence for neutrino masses and mixing, being at least three light neutrinos with masses $\lesssim 2.2$ eV [1,2]. In fact LEP has measured the number of light standard neutrinos $\mathrm{N}_\nu = 2.994 \pm 0.012$, excluding new ones with masses below $\sim M_Z/2$. Light neutrinos with small couplings, sterile, and heavy ones are not ruled out, although there are astrophysical and cosmological constraints on their masses, nature and decay lifetimes [3].

In order to describe their interactions it is usually assumed that the mixing of the three light standard neutrinos is given by a unitary matrix, and then that their mixing with heavy neutrinos (and the lack of unitarity) is negligible. In practice this is the case for see-saw models [4]. Indeed, if for the sake of discussion we assume only one light neutrino, the mass matrix

$$\begin{pmatrix} 0 & v \\ v & M \end{pmatrix} \tag{1}$$

requires a very heavy Majorana mass, typically of the order of the unification scale $M \sim 10^{15}$ GeV, to generate light masses $m \sim v^2/M \sim$ eV, with $v \sim 250$ GeV the electroweak vacuum expectation value. As a consequence the mixing between light and heavy neutrinos $v/M \sim 10^{-25}$, and thus completely negligible. The numerical problem can be improved introducing a small Yukawa coupling λ ($v \to \lambda v$ everywhere), but not evaded. However, one can write down models where the light masses and mixings are not correlated, allowing in principle for observable non-decoupling effects proportional to the mixing between light and heavy neutrinos. In particular in the general 2×2 case

$$\begin{pmatrix} a & \lambda v \\ \lambda v & M \end{pmatrix} \tag{2}$$

the light mass $m \sim a - \lambda^2 v^2/M$ can vanish and at the same time the mixing $\sim \lambda v/M$ can be relatively large if we fine tune a. This scenario can be made more natural adding new degrees of freedom. For example, one can write models with two heavy neutrinos N and N' per family and an effective approximate symmetry $L_\nu + L_N - L_{N'}$ implying a mass matrix of the form

$$\begin{pmatrix} 0 & 0 & \lambda v \\ 0 & 0 & M \\ \lambda v & M & 0 \end{pmatrix}, \tag{3}$$

where a large singlet vacuum expectation value M gives a Dirac mass to the heavy neutrinos, whereas the light neutrino is massless and the mixing between the light and heavy neutrinos $\lambda v/M$ arbitrary. This is similar to the light neutrino mass matrix texture obtained imposing the lepton number symmetry $L_{\nu_e} - L_{\nu_\mu} - L_{\nu_\tau}$ [5]. Eq. (3) generalises to three families trivially but leaves three massless neutrinos. If we want to give them a small mass, we can introduce a Majorana mass $m' \ll M$ for the heavy neutrino N, violating the approximate symmetry and inducing a light neutrino mass $m \sim m'\lambda^2 v^2/M^2$. An alternative way is to assume that there exists a much heavier Majorana fermion which through the see-saw mechanism gives a very small mass to the light neutrino, violating also the approximate symmetry (up-left entry), and mixes very little. At any rate, it seems necessary in order to have small enough neutrino masses and at the same time a relatively large mixing between light and heavy neutrinos, that both have different origin.

Models with extra dimensions can do the job [6,7]. A neutral fermion living in the bulk can reduce to a massless right-handed neutrino plus a tower of heavy Kaluza–Klein modes. Then as pointed out in Ref. [7] after the electroweak symmetry breaking the new fermions can mix with a standard neutrino and give a massless mode with a relatively large mixing $\sim \lambda v R$, where R is the compactification radius. In this case the truncation of the Kaluza–Klein tower can also generate a tiny neutrino mass $\sim \lambda^2 v^2 / M_{\rm s}$, with $M_{\rm s}$ the mass scale of the underlying (string) theory where the infinite Kaluza–Klein tower is truncated.

If one assumes a relatively large departure of the unitary mixing among light neutrinos, one must wonder about possibly large contributions to rare leptonic processes, *e.g.* $\mu \to e\gamma, \mu \to ee\bar{e}, Z \to e\bar{\mu}, \ldots$. As no such decays have been observed, relatively stringent bounds on the mixing between light and heavy neutrinos and the heavy masses can be derived [8]. In the following independently of their origin we discuss the effects of non-decoupled heavy neutrinos in light neutrino physics, in particular in neutrino oscillations.

New contributions to processes involving only the known fermions as initial and final states are typically proportional to the square of the mixing between light and heavy neutrinos, and then small and difficult to observe. This makes processes forbidden in the absence of such a mixing particularly interesting. Prime examples are the lepton number violating processes involving charged leptons and CP violating neutrino oscillations. If the angle mixing the electron and tau neutrinos is not small but negligible, no CP violating neutrino oscillation is observable if the light neutrino mixing matrix is unitary. This does not need to be the case if the light neutrinos mix with heavy ones making the light neutrino mixing matrix non-unitary [9]. Hereafter we will discuss this possibility following closely Ref. [10] but sticking mainly to the eigenmass basis description of neutrino oscillations. In Section 2 we introduce the neutrino bases convenient for describing neutrino propagation in matter [11], which we review in Section 3. In Section 4 we study the case of two neutrinos propagating in unpolarised, isotropic and neutral matter, and in Section 5 we calculate the corrections to the resonance effect in neutrino oscillations. Section 6 is devoted to conclusions.

The mixing with heavy neutrinos implies the loss of unitarity of the Maki–Nakagawa–Sakata (MNS) mixing matrix [12] describing the charged current interactions. The same happens if the observed charged leptons mix with new heavy ones [13]. The phenomenological consequences are also similar. Both cases are explicit examples of the Standard Model (SM) extensions parametrised in Ref. [14]. Present limits on rare processes postpone any observation of these effects in neutrino oscillations to ν factory experiments [15].

2. Neutrino eigenstates

Let us assume that there are three light active, $n_{\rm s}$ light sterile and $n_{\rm R}$ heavy neutrinos. So the mass matrix has dimension $n = 3 + n_{\rm s} + n_{\rm R}$, being diagonalised by a unitary matrix

$$U_\nu^{\rm T} M U_\nu = (M_\nu)_{\rm diag} \equiv {\rm diag}(m_1 m_2 ... m_{3+n_{\rm s}} M_1 ... M_{n_{\rm R}}) \,, \tag{4}$$

where

$$U_\nu = \begin{pmatrix} \mathcal{U} & \mathcal{V} \\ \mathcal{V}' & \mathcal{U}' \end{pmatrix} , \tag{5}$$

with the $(3+n_{\rm s})\times(3+n_{\rm s})$ matrix $\mathcal{U}$ ($n_{\rm R}\times n_{\rm R}$ matrix $\mathcal{U}'$) describing the mixing among the light (heavy) neutrinos and the matrices $\mathcal{V}$ and $\mathcal{V}'$ parametrising the mixing between the light and heavy neutrinos. Thus, the flavour eigenstates are linear combinations of the mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} (U_\nu^*)_{\alpha i} |\nu_i\rangle = \sum_{i=1}^{3+n_s} \mathcal{U}_{\alpha i}^* |\nu_i\rangle + \sum_{i=3+n_s+1}^{3+n_s+n_{\rm R}} \mathcal{V}_{\alpha i}^* |\nu_i\rangle \,, \tag{6}$$

with $\alpha = 1, 2, 3$ standing for e, μ, τ, respectively. In the charged lepton mass eigenstate basis the first three rows of U_ν parametrise the charged and neutral current interactions, the corresponding Lagrangians being

$$L_{\rm CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{n} \bar{l}_\alpha \gamma^\mu (1-\gamma_5) (U_\nu)_{\alpha i} \nu_i W_\mu^- + \text{h.c.} \tag{7}$$

and

$$\begin{aligned} L_{\rm NC} \;=\; \frac{e}{4\sin\theta_W\cos\theta_W} & \left\{ \sum_{i,j=1}^{n} \bar{\nu}_i \gamma^\mu (1-\gamma_5)\, \Omega_{ij} \nu_j \right. \\ & \left. + \, 2 \sum_{f=e,p,n} \bar{f} \gamma^\mu \left[T_{3f} (1-\gamma_5) - 2 Q_f \sin^2\theta_W \right] f \right\} Z_\mu \,, \end{aligned} \tag{8}$$

where T_{3f} and Q_f are the third component of the weak isospin and the charge of the fermion f, respectively, and $\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (U_\nu)_{\alpha i}^* (U_\nu)_{\alpha j}$. The non-observation of SM deviations (except for neutrino oscillations) bounds the new interactions. Universality sets limits on the diagonal elements of

$$\omega_{\alpha\beta} \equiv (\mathcal{V}\mathcal{V}^\dagger)_{\alpha\beta} = \delta_{\alpha\beta} - \left(\mathcal{U}\mathcal{U}^\dagger\right)_{\alpha\beta} \,, \tag{9}$$

and the off-diagonal ones are mainly constrained by the non-observation of the lepton number violating processes $\mu \to e\gamma, \mu \to ee\bar{e}, Z \to e\bar{\mu}, \ldots$ [8]

$$\omega_{ee} < 0.0054\,,\ \omega_{\mu\mu} < 0.0096\,,\ \omega_{\tau\tau} < 0.016\,,\ |\omega_{e\mu}| < 0.0001\,,\ |\omega_{\mu\tau}| < 0.01 \tag{10}$$

(assuming no model dependent cancellation).

Future experiments will improve these bounds or detect new effects. In neutrino oscillations with low energy production and detection processes and heavy neutrinos not propagating large distances the effective flavour states are obtained truncating Eq. (6)

$$|\tilde{\nu}_\alpha\rangle = \lambda_\alpha^{-1} \sum_{i=1}^{3+n_s} \mathcal{U}_{\alpha i}^* |\nu_i\rangle \equiv \sum_{i=1}^{3+n_s} \tilde{\mathcal{U}}_{\alpha i}^* |\nu_i\rangle\,, \tag{11}$$

where we have also conventionally included the normalisation factor $\lambda_\alpha = \sum_{i=1}^{3+n_s} |\mathcal{U}_{\alpha i}|^2 = \sqrt{1-\omega_{\alpha\alpha}}$. These states do not need to be orthogonal

$$\langle\tilde{\nu}_\alpha|\tilde{\nu}_\beta\rangle = (\lambda_\alpha\lambda_\beta)^{-1}\left(\delta_{\alpha\beta} - \omega_{\alpha\beta}\right)\,, \tag{12}$$

reading in the flavour basis

$$|\tilde{\nu}_\alpha\rangle = \lambda_\alpha^{-1}\left(|\nu_\alpha\rangle - \sum_{\beta=1}^{3+n_s} \omega_{\beta\alpha}|\nu_\beta\rangle + \sum_{\beta=3+n_s+1}^{n} \left(\mathcal{V}'\mathcal{U}^\dagger\right)_{\beta\alpha} |\nu_\beta\rangle\right)\,. \tag{13}$$

As an example, let us consider neutrino production in the charged current process $l_\alpha^- X \to \nu_\beta Y$. If the available mass

$$\varDelta m_\beta = \Delta\left(\sqrt{(E_{l_\alpha^-} + E_X - E_Y)^2 - (\vec{p}_{l_\alpha^-} + \vec{p}_X - \vec{p}_Y)^2}\right) \tag{14}$$

is much smaller than the heavy neutrino masses but much larger than the light ones, these will be produced coherently and the amplitude

$$\begin{aligned} A(l_\alpha^- X \to \tilde{\nu}_\beta Y) &= \lambda_\beta^{-1} \sum_{i=1}^{3+n_s} \mathcal{U}_{\beta i} A(l_\alpha^- X \to \nu_i Y) \\ &\simeq \lambda_\beta^{-1} \sum_{i=1}^{3+n_s} \mathcal{U}_{\beta i}\mathcal{U}_{\alpha i}^* A^{\rm SM}(l_\alpha^- X \to \nu_\alpha Y) \\ &= \lambda_\beta^{-1}(\delta_{\beta\alpha} - \omega_{\beta\alpha}) A^{\rm SM}(l_\alpha^- X \to \nu_\alpha Y)\,, \end{aligned} \tag{15}$$

where $A^{\rm SM}(l_\alpha^- X \to \nu_\alpha Y)$ is the SM amplitude for massless neutrinos. In particular

$$\sigma(l_\alpha^- X \to \tilde{\nu}_\alpha Y) \simeq \lambda_\alpha^2 \sigma^{\rm SM}(l_\alpha^- X \to \nu_\alpha Y)\,. \tag{16}$$

3. Neutrino propagation in matter

Similarly to the case of photons the coherent scattering of light neutrinos in a medium modifies their properties. In the first case it gives the index of refraction of light, and for neutrinos it modifies their effective masses changing substantially their oscillation pattern, also showing resonance phenomena eventually [11]. The coherent neutrino scattering is described by a four-fermion Hamiltonian

$$H^{f}_{\rm int}(x) = \frac{G_{\rm F}}{\sqrt{2}} \sum_{i,k=1}^{3+n_s} \sum_{a=V,A} \left[\bar{\nu}_k \Gamma_a \nu_i\right] \left[\bar{f} \Gamma^a \left(g^{ki}_{fa} + \bar{g}^{ki}_{fa} \gamma_5 \right) f\right], \tag{17}$$

where $\Gamma_{V(A)} = \gamma_\mu (\gamma_\mu \gamma_5)$ and f stands for the type of matter, electrons e and nucleons p, n. This Hamiltonian and the couplings g^{ki}_{fa} and $\bar{g}^{ki}_{fa}$ can be calculated from Eqs. (7),(8) [10]. The Feynman diagrams are drawn in Fig. 1.

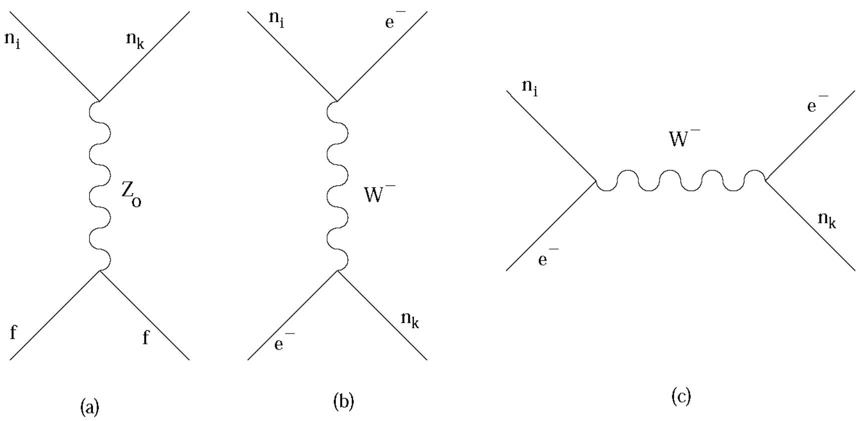

Fig. 1. Feynman diagrams for neutrino scattering in matter. All three diagrams contribute to neutrino-electron scattering $n_i + e^- \to n_k + e^-$ ($f = e$), but only diagram (a) contributes to neutrino–nucleon scattering $n_i + f \to n_k + f$ ($f = p, n$).

$$\begin{aligned}
g^{ki}_{eV} = -\bar{g}^{ki}_{eA} &= \mathcal{U}^*_{ek}\mathcal{U}_{ei} + \rho\Omega_{ki}\left(-\tfrac{1}{2} + 2\sin^2\theta_W\right), \\
\bar{g}^{ki}_{eV} = -g^{ki}_{eA} &= -\mathcal{U}^*_{ek}\mathcal{U}_{ei} + \frac{1}{2}\rho\Omega_{ki}, \\
g^{ki}_{fV} = -\bar{g}^{ki}_{fA} &= \rho\Omega_{ki}\left(T_{3f} - 2Q_f \sin^2\theta_W\right), \\
\bar{g}^{ki}_{fV} = -g^{ki}_{eA} &= -\rho\Omega_{ki} T_{3f}\,,
\end{aligned} \tag{18}$$

where $f = p, n$ and

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}\,, \quad T_{3p} = -T_{3n} = 1/2\,, \quad Q_p = 1\,, \quad Q_n = 0\,. \tag{19}$$

Then, the interaction Hamiltonian for a ν_i of momentum $\vec{k}$ and helicity λ propagating in matter produces a ν_k with the same momentum and helicity is

$$\begin{aligned} H_{ki}^{\rm int} &= \sum_f \int\limits_{V=1} d^3x \frac{1}{N_f} \sum_{\vec{s}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\vec{p},\vec{s}) \\ &\quad \times \langle \nu_k\ \vec{k}\ \lambda | \langle f\ \vec{p}\ \vec{s} | H_{\rm int}^f(x) | f\ \vec{p}\ \vec{s} \rangle | \nu_i\ \vec{k}\ \lambda \rangle \,, \end{aligned} \tag{20}$$

where $\rho_f(\vec{p},\vec{s})$ is the distribution function for the background fermions of type f, momentum $\vec{p}$ and spin $\vec{s}$ normalised to give the number of fermions per unit volume

$$N_f \equiv \sum_{\vec{s}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\vec{p},\vec{s}) \,. \tag{21}$$

Assuming that the neutrinos are relativistic $\vec{k}^2 \simeq E_{i,k}^2 \gg m_{i,k}^2$ and using Eqs. (17),(18) we can write

$$\begin{aligned} H_{ki}^{\rm int}(\vec{k}\ \lambda=-1) &= -\left[H_{ki}^{\rm int}(\vec{k}\ \lambda=+1) \right]^* \\ &= \sqrt{2} G_{\rm F} \sum_f N_f \left[g_{fV}^{ki} \left(1 - \left\langle \frac{\vec{k}\cdot\vec{p}}{|\vec{k}|E_f} \right\rangle \right) \right. \\ &\quad \left. + \bar{g}_{fV}^{ki} \left(\left\langle \frac{\vec{p}\cdot\vec{s}}{E_f} \right\rangle - m_f \left\langle \frac{\vec{k}\cdot\vec{s}}{|\vec{k}|E_f} \right\rangle - \left\langle \frac{(\vec{k}\cdot\vec{p})(\vec{p}\cdot\vec{s})}{|\vec{k}|(m_f+E_f)} \right\rangle \right) \right] , \end{aligned} \tag{22}$$

where $E_f = \sqrt{m_f^2 + \vec{p}^2}$ and m_f are the energy and mass of the f fermion, respectively,

$$\langle z \rangle \equiv \frac{1}{N_f} \sum_{\vec{s}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\vec{p},\vec{s}) z(\vec{p},\vec{s}) \,, \tag{23}$$

and $\lambda = -1$ $(+1)$ stands for the helicity of the Dirac (antineutrinos) and Majorana neutrinos. This Hamiltonian enters the evolution equation for light neutrinos (expanding to first order in $H^{\rm int}$)

$$\mathrm{i} \frac{d}{dt} \psi_k(\vec{k}\ \lambda, t) = \sum_{i=1}^{3+n_{\rm s}} H_{ki}^{\rm eff} \psi_i(\vec{k}\ \lambda, t) \,, \tag{24}$$

with $\psi_k(\vec{k}\ \lambda, t) = \langle \nu_k\ \vec{k}\ \lambda | \psi(t) \rangle$ and

$$H_{ki}^{\rm eff} = \frac{\Delta m_{i1}^2}{2|\vec{k}|} \delta_{ki} + H_{ki}^{\rm int}(\vec{k}\ \lambda) \,. \tag{25}$$

As usual, we have removed the diagonal pieces of the effective Hamiltonian for they give global unobservable phases in neutrino oscillations. In particular $\Delta m_{i1}^2 \equiv m_i^2 - m_1^2$. With these equations one can evaluate the different probability amplitudes. We apply them to a simple example in next section.

4. Propagation in an unpolarised, isotropic and electrically neutral medium

Let us assume that for each fermion type f matter is unpolarised $\langle \vec{s} \rangle = 0$ and isotropic $\langle \vec{p} \rangle = 0$, and as a whole electrically neutral $N_e = N_p \neq N_n$. In this case the interaction Hamiltonian is momentum and helicity independent

$$\begin{aligned} H_{ki}^{\rm int}(\vec{k}\ \lambda) &= \sqrt{2} G_{\rm F} \left[N_e (g_{eV}^{ki} + g_{pV}^{ki}) + N_n g_{nV}^{ki} \right] \\ &= \sqrt{2} G_{\rm F} \left(N_e \mathcal{U}_{ek}^* \mathcal{U}_{ei} - \frac{1}{2} \rho \Omega_{ki} N_n \right). \end{aligned} \tag{26}$$

For constant density the evolution equation (24) can be easily solved diagonalising the effective (Hermitian) Hamiltonian

$$\begin{aligned} H_{ki}^{\rm eff} &= \frac{1}{2|\vec{k}|} \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & \Delta m_{21}^2 & 0 & 0 & \cdots \\ 0 & 0 & \Delta m_{31}^2 & 0 & \cdots \\ 0 & 0 & 0 & \Delta m_{41}^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{ki} \\ &\quad + \sqrt{2} G_{\rm F} \sum_{\alpha,\beta=1}^{3} \mathcal{U}_{k\alpha}^\dagger \begin{pmatrix} (N_e - \frac{N_n}{2}) & 0 & 0 \\ 0 & -\frac{N_n}{2} & 0 \\ 0 & 0 & -\frac{N_n}{2} \end{pmatrix}_{\alpha\beta} \mathcal{U}_{\beta i} \\ &= \frac{1}{2|\vec{k}|} \sum_{j=1}^{3+n_{\rm s}} W_{kj}^\dagger \tilde{m}_j^2 W_{ji}\,, \end{aligned} \tag{27}$$

where $\tilde{m}_j^2$ are the effective (real) masses and W_{ji} the diagonalising (unitary) matrix giving the effective mass neutrinos as linear combination of the vacuum mass ones. Hence

$$\begin{aligned} A_{\tilde{\nu}_\alpha \to \tilde{\nu}_\beta}(L) &\equiv \langle \tilde{\nu}_\beta(0) | \tilde{\nu}_\alpha(t = L) \rangle \\ &= \lambda_\beta^{-1} \lambda_\alpha^{-1} \sum_{k,j,i=1}^{3+n_{\rm s}} \mathcal{U}_{\beta k} W_{kj}^\dagger {\rm e}^{-i \frac{\tilde{m}_j^2}{2|\vec{k}|} L} W_{ji} \mathcal{U}_{\alpha i}^*\,. \end{aligned} \tag{28}$$

The λ factors result from the normalisation of the effective flavour states in Eq. (11). If we ask for transitions of flavour neutrinos travelling long distances (allowing for heavy neutrinos to decay), these factors must be removed according to Eq. (15)

$$A_{\nu_\alpha\to\nu_\beta}(L) = \lambda_\alpha\lambda_\beta A_{\tilde\nu_\alpha\to\tilde\nu_\beta}(L)\,. \tag{29}$$

For illustration we calculate the probability amplitudes for the case of 2 standard families and 1 heavy neutrino. We can as usual parametrise $\mathcal{U}$ and $\mathcal{V}$ in Eq. (5) with 3 mixing angles and 1 phase

$$\mathcal{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} \end{pmatrix}, \tag{30}$$

$$\mathcal{V} = \begin{pmatrix} s_{13}e^{-i\delta} \\ s_{23}c_{13} \end{pmatrix}, \tag{31}$$

where s_{ij}, c_{ij} stand for $\sin\theta_{ij}, \cos\theta_{ij}$, respectively, and s_{13} and s_{23} are small, with their products being constrained by Eq. (10). (U_ν has the same form as the mixing matrix for three families but now the third row corresponds to the mainly heavy singlet neutrino, and the third column to the corresponding heavy mass eigenstate. The other two phases needed to parametrize U_ν in general are not observable in neutrino oscillations.) We can use the vacuum expressions to learn about the new effects. Indeed, taking W equal to the identity

$$P_{\nu_e\to\nu_e}(L) = |A_{\nu_e\to\nu_e}(L)|^2 = c_{13}^4\left(1 - \sin^2 2\theta_{12}\sin^2\Delta\right), \tag{32}$$

with $\Delta = \frac{\Delta m_{21}^2 L}{4|\vec{k}|}$, and

$$\begin{aligned} P_{\nu_e\to\nu_\mu}(L) &= c_{13}^2 s_{13}^2 s_{23}^2 + \sin 2\theta_{12} c_{13}^2 \left\{ s_{13}s_{23}c_{23}\sin\delta\sin 2\Delta \right. \\ &+ \left. \left[\sin 2\theta_{12}\left(c_{23}^2 - s_{13}^2 s_{23}^2\right) + \cos 2\theta_{12} s_{13}\sin 2\theta_{23}\cos\delta\right]\sin^2\Delta \right\}. \end{aligned} \tag{33}$$

The sum of both probabilities is always smaller than 1. In fact if we also add the probability amplitude for producing the mainly heavy flavour eigenstate $P_{\nu_e\to\nu_N}(L)$ (which one may eventually detect through its decay products [16]), we obtain c_{13}^2, which is smaller than 1 if the electron neutrino mixes with the heavy mass eigenstate, $s_{13}\neq 0$. Besides, there are CP violating effects even with two families (or with three families and a vanishing mixing between the first and third one, or two degenerate light masses)

$$\begin{aligned} \Delta P^{\mathrm{CP}}_{\nu_e\to\nu_\mu}(L) &= P_{\nu_e\to\nu_\mu}(L) - P_{\bar\nu_e\to\bar\nu_\mu}(L) \\ &= c_{13}^2\sin 2\theta_{12}s_{13}\sin 2\theta_{23}\sin\delta\sin 2\Delta\,. \end{aligned} \tag{34}$$

At any rate, all new effects are suppressed by at least the product of two small mixings s_{13} and/or s_{23}, and thus they are bounded by the stringent limits in Eq. (10). Obviously we call the initial neutrino e and the final μ but they stand for any two flavours. In fact the larger effects are expected for $\nu_\mu \to \nu_\tau$ transitions.

5. Resonant oscillation of light neutrinos without heavy neutrino decoupling

The same is true for neutrino oscillations in matter. For example in this case the usual resonant behaviour

$$\sin 2\theta_{\text{eff}} = \frac{\sin 2\theta_{12}}{\sqrt{(\frac{2\sqrt{2}G_{\text{F}}|\vec{k}|N_e}{\Delta m_{21}^2} - \cos 2\theta_{12})^2 + \sin^2 2\theta_{12}}} \tag{35}$$

writes

$$\sin 2\theta_{\text{eff}} = c_{13}^2 \frac{A}{\sqrt{(B - \cos 2\theta_{12})^2 + A^2}} \tag{36}$$

with

$$\begin{aligned} A^2 &= \left[\sin 2\theta_{12} + \frac{\sqrt{2}G_{\text{F}}|\vec{k}|N_n}{\Delta m_{21}^2} s_{13} \sin 2\theta_{23} \cos\delta\right]^2 \\ &\quad + \left(\frac{\sqrt{2}G_{\text{F}}|\vec{k}|N_n}{\Delta m_{21}^2}\right)^2 s_{13}^2 \sin^2 2\theta_{23} \sin^2\delta\,, \\ B &= \frac{\sqrt{2}G_{\text{F}}|\vec{k}|(2N_e - N_n)}{\Delta m_{21}^2} c_{13}^2 + \frac{\sqrt{2}G_{\text{F}}|\vec{k}|N_n}{\Delta m_{21}^2}(c_{23}^2 - s_{13}^2 s_{23}^2)\,. \end{aligned} \tag{37}$$

Thus the form is the same, but the resonance effect corresponds to values of the parameters corrected by amounts again suppressed by at least the product of two small mixings s_{13} and/or s_{23}. The important point is that the maximum $\sin 2\theta_{\text{eff}}$ is not 1 but c_{13}^2 what gives another (difficult) way to measure the mixing between light and heavy neutrinos.

6. Conclusions

Light neutrino masses are so small that mixing between light and heavy neutrinos must have a different origin if it is to be observable. This requires either fine tuning or models with two different heavy scales. Natural SM extensions realizing this scenario are E_6 models with two heavy scales of gauge symmetry breaking. Models with extra dimensions have also typically two such scales, the compactification and the string scale.

Independent of its origin one may wonder about the phenomenological implications of having heavy neutrinos with relatively large mixing with the SM ones. This case does not exhibit the cancellations present in the SM with only three massive light neutrinos but the departure from the SM predictions is bounded to be small, in fact smaller than the limits quoted in Eq. (10). These bounds result from charged lepton processes highly suppressed in the SM. New heavy neutrinos manifest in these transitions through their interchange in loops; whereas in neutrino processes they show up at tree level. In any case it can be proven that the corrections involve at least two powers of the small mixing between light and heavy neutrinos. No such new effects have been observed, the required precision for their detection demanding improved measurements of rare charged lepton processes or neutrino experiments at a ν factory. In this case the main signature is the observation of CP violation together with no mixing between the first and third families. Other effects which are corrections to SM processes like the sum of probabilities not adding to 1 or modified resonance effects will be difficult to discriminate. At any rate the best place to look for is in μ and τ processes not involving e because present limits are less stringent. Besides their masses are larger and it is generally believed that mixing effects have some kind of scaling with them, favouring the observation of SM departures in heavy flavours.

We thank J. Gluza and J. Santiago for useful comments. The work was supported in part by the Polish State Committee for Scientific Research (KBN) under Grant 2P03B05418, MCYT under contract FPA2000-1558, Junta de Andalucía group FQM 101 and the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 Physics at Colliders.

REFERENCES

[1] D.E. Groom, F. James, R. Cousins, *Eur. Phys. J.* **C15**, 191 (2000).

[2] For a recent review on neutrino physics M.C. González-García, Y. Nir, `hep-ph/0202058`; see also S.M. Bilenky, C. Giunti, W. Grimus, *Prog. Part. Nucl. Phys.* **43**, 1 (1999); E.Kh. Akhmedov, in Trieste 1999, Particle physics, p. 103, `hep-ph/0001264`; J.W.F. Valle, `hep-ph/0205216`.

[3] R.N. Mohapatra, P.B. Pal, *Massive Neutrinos in Physics and Astrophysics, Lectures Notes in Physics*, Vol.41, World Scientific, Singapore 1991.

[4] M. Gell-Mann, P. Ramond, R. Slansky, in *Supersymmetry*, eds. D. Freedman and P. van Nieuwenhuizen North Holland, Amsterdam 1979, p. 315; T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto, KEK, Tsukuba,

Japan (1979); R. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980); *Phys. Rev.* **D23**, 165 (1981).

[5] R. Barbieri *et al.*, *J. High Energy Phys.* **9812**, 017 (1998); G. Altarelli, R. Feruglio, *Phys. Rep.* **320**, 295 (1999).

[6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, J. March-Russell, *Phys. Rev.* **D65**, 024032 (2002); Y. Grossman, M. Neubert, *Phys. Lett.* **B474**, 361 (2000).

[7] K.R. Dienes, E. Dudas, T. Gherghetta, *Nucl. Phys.* **B557**, 25 (1999).

[8] F. del Aguila, M.J. Bowick, *Nucl. Phys.* **B224**, 107 (1983); P. Langacker, D. London, *Phys. Rev.* **D38**, 907 (1988); E. Nardi, E. Roulet, D. Tommasini, *Phys. Lett.* **B327**, 319 (1994); D. Tommasini, G. Barenboim, J. Bernabeu, C. Jarlskog, *Nucl. Phys.* **B444**, 451 (1995); S. Bergmann, A. Kagan, *Nucl. Phys.* **B538**, 368 (1999); J.I. Illana, T. Riemann, *Phys. Rev.* **D63**, 053004 (2001).

[9] F. del Aquila, M. Zrałek, *Nucl. Phys.* **B447**, 211 (1995); J.A. Aquilar-Saavedra, G.C. Branco, *Phys. Rev.* **D62**, 096009 (2000).

[10] B. Bekman *et al.*, `hep-ph/0207015` and references therein; see also M. Czakon, J. Gluza, M. Zrałek, *Acta Phys. Pol.* **B32**, 3735 (2001).

[11] L. Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); S.P. Mikheev, A.Y. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1985) [*Yad. Fiz.* **42**, 1441 (1985)].

[12] Z. Maki, M. Nakagawa, S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).

[13] G.C. Branco *et al.*, University of Granada preprint, UG-FT-139/02.

[14] M.C. González-García, Y. Grossman, A. Gusso, Y. Nir, *Phys. Rev.* **D64**, 096006 (2001).

[15] M.M. Alsharo'a *et al.*, `hep-ex/0207031`.

[16] F. del Aguila, E. Laermann, P.M. Zerwas, *Nucl. Phys.* **B297**, 1 (1988).