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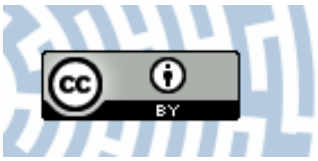

VACUUM INSERTION METHOD FROM CHIRAL $N_c \to \infty$ APPROACH FOR THE $\Delta S = 1$ EFFECTIVE HAMILTONIAN

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We check to what extent the assumptions of the vacuum insertion method, as used in the theory of $K \to \pi\pi$ decays, can be derived from the high N_c approximation to the chiral perturbation theory. We find that, besides the well-known problem of Fierz terms, only the assumption for the K_{l3} formfactor ($f_- = 0$) does not follow. This assumption, however, affects the penguin contributions by less than four per cent and the non-penguin contributions by less than two percent.

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This note is concerned with the matrix elements $< \pi\pi|H_{\text{eff}}|K >$ occurring in the theory of the processes

$$K^0 \to \pi^0\pi^0, \tag{1}$$

$$K^0 \to \pi^+\pi^-, \tag{2}$$

$$K^+ \to \pi^+\pi^0. \tag{3}$$

It is known [1], [2] that, whether one uses the vacuum insertion method (VIM) [3] or whether one uses the leading in $1/N_c$ term of the chiral perturbative expansion (CHNC) [2], one finds very similar results for these matrix elements. We discuss to what extent this fact reflects assumptions common to both approaches and why the differences between the two affect little $< \pi\pi|H_{\text{eff}}|K >$ in the corresponding approximations.

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For the matrix elements of the effective Hamiltonian one uses the approximation

$$< \pi\pi|H_{\text{eff}}|K >= \sum_i c_i(\mu) < \pi\pi|Q_i(\mu)|K > , \tag{4}$$

where the summation extends over all the operators Q_i of dimension six, which can be constructed from the quark fields and have the relevant matrix elements nonzero. One possible choice of these operators [4] is

$$\begin{aligned} Q_1 &= (\bar{s}d)_{V-A}(\bar{u}u)_{V-A} , \\ Q_2 &= (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} , \\ Q_3 &= (\bar{s}d)_{V-A}(\bar{q}q)_{V-A} , \\ Q_4 &= (\bar{s}q)_{V-A}(\bar{q}d)_{V-A} , \\ Q_5 &= (\bar{s}d)_{V-A}(\bar{q}q)_{V+A} , \\ Q_6 &= -8(\bar{s}_L q_R)(\bar{q}_R d_L) , \end{aligned} \tag{5}$$

where the summation over $q = u, d, s$ and over colour in each bracket is implied. One notes the identity

$$Q_4 = Q_2 - Q_1 + Q_3 . \tag{6}$$

Another set [5], which has simpler symmetry properties under SU(3) - flavour, is

$$\begin{aligned} O_1 &= \tfrac{1}{4}Q_1 - \tfrac{1}{4}Q_2 , \\ O_2 &= -\tfrac{1}{4}Q_1 + \tfrac{1}{4}Q_2 + \tfrac{1}{2}Q_3 , \\ O_{34} &= \tfrac{9}{4}Q_1 + \tfrac{3}{2}Q_2 - \tfrac{3}{4}Q_3 , \\ O_5 &= -\tfrac{1}{2N_c}Q_5 + \tfrac{1}{2}Q_6 , \\ O_6 &= \tfrac{1}{4}Q_5 . \end{aligned} \tag{7}$$

Actually in [5] instead of the operator O_{34} two other operators have been introduced:

$$O_{34} = O_3 + 5O_4 , \tag{8}$$

where O_3 corresponds to $\Delta I = \frac{1}{2}$ and O_4 to $\Delta I = \frac{3}{2}$. When the scale μ changes, however, the operators O_3 and O_4 do not mix with other operators (multiplicative renormalization) and have the same anomalous dimensions. Therefore, operator O_{34} evoluates without changing its composition (multiplicative renormalization again) fixed at $\mu = M_W$.

In CHNC an explicit calculation shows that the nonzero matrix elements of the operators $Q_{i\neq 4}$ can be rewritten as follows

$$
\begin{aligned}
&< \pi^0\pi^0|Q_1|K^0 > = 2 < \pi^0|(\bar{s}d)_{V-A}|K^0 >< \pi^0|(\bar{u}u)_{V-A})|0 >,\\
&< \pi^+\pi^-|Q_2|K^0 > =< \pi^-|(\bar{s}u)_{V-A}|K^0 >< \pi^+|(\bar{u}d)_{V-A})|0 >,\\
&< \pi^+\pi^0|Q_1|K^+ > =< \pi^+|(\bar{s}d)_{V-A}|K^+ >< \pi^0|(\bar{u}u)_{V-A})|0 >,\\
&< \pi^+\pi^0|Q_2|K^+ > =< \pi^0|(\bar{s}u)_{V-A}|K^+ >< \pi^+|(\bar{u}d)_{V-A})|0 >,\\
&< \pi^+\pi^-|Q_6|K^0 > = -8[< \pi^-|(\bar{s}_L u_R)|K^0 >< \pi^+|(\bar{u}_R u_L)|0 >\\
&\qquad\qquad + < \pi^+\pi^-|(\bar{d}_R d_L)|0 >< 0|(\bar{s}_L d_R)|K^0 >],\\
&< \pi^0\pi^0|Q_6|K^0 > = -8[< 0|(\bar{s}_R d_R)|K^0 >< \pi^0\pi^0|(\bar{d}_R d_L)|0 >\\
&\qquad\qquad + < \pi^0|(\bar{s}_L d_R)|K^0 >< \pi^0|(\bar{d}_R d_L)|0 >]. \qquad (9)
\end{aligned}
$$

The factor two in the first formula corresponds to two possible allotments of the π^0's. In VIM the products on the right hand sides contribute to the matrix elements on the left hand sides by definition, but then additional "Fierz terms"

$$
\begin{aligned}
&\Delta < \pi\pi|Q_1|K > = \tfrac{1}{N_c} < \pi\pi|Q_2|K >,\\
&\Delta < \pi\pi|Q_2|K > = \tfrac{1}{N_c} < \pi\pi|Q_1|K >,\\
&\Delta < \pi\pi|Q_3|K > = \tfrac{1}{N_c} < \pi\pi|Q_4|K >,\\
&\Delta < \pi\pi|Q_4|K > = \tfrac{1}{N_c} < \pi\pi|Q_3|K >,\\
&\Delta < \pi\pi|Q_5|K > = \tfrac{1}{N_c} < \pi\pi|Q_6|K >,\\
&\Delta < \pi\pi|Q_6|K > = \tfrac{1}{N_c} < \pi\pi|Q_5|K >, \qquad (10)
\end{aligned}
$$

are added. These terms are of course negligible in the $N_c \to \infty$ limit. For N_c finite the matrix elements on the right-hand-sides are interpreted as in (9). Moreover, for $i = 1,2,3,4$ one could expect in the vacuum insertion method terms of the form

$$
< \pi\pi|(\bar{q}q)_{V-A}|0 >< 0|(\bar{q}q)_{V-A}|K >, \qquad (11)
$$

where the flavour indices of the quarks have not been explicitly written. These terms vanish, however, because $< 0|j_L|K > \sim p_K = p_{\pi_1} + p_{\pi_2}$, while only the vector current contributes to $< \pi\pi|j_L|0 >$. Thus the product is proportional to the divergence of the vector current, which is negligible. In CHNC $< \pi\pi|j_L|0 > \sim p_{\pi_1} - p_{\pi_2}$, which also makes the product vanish.

The six matrix elements occurring in (9) and (10) can be reduced to two by using the Eckart–Wigner theorem. The currents $-(\bar{s}d)_{V-A}$ and $(\bar{s}u)_{V-A}$ form an isospin doublet. Thus

$$
\begin{aligned}
-\sqrt{2} < \pi^0|\bar{s}d|K^0 > &=< \pi^-|\bar{s}u|K^0 >\\
&= - < \pi^+|\bar{s}d|K^+ >= \sqrt{2} < \pi^0|\bar{s}u|K^+ >, \qquad (12)
\end{aligned}
$$

where the subscripts $V-A$ have been omitted. Since $\bar{q}q$ is an isosinglet, we can define

$$Y = < \pi^+\pi^-|Q_6|K^0 > = < \pi^0\pi^0|Q_6|K^0 > . \tag{13}$$

Here following [2] we have chosen the sign of the $|\pi^0\pi^0>$ component of the isosinglet opposite to that given by the Condon–Shortley convention. The currents $\bar{u}d, (\bar{u}u - \bar{d}d)/\sqrt{2}, \bar{d}u$ form an isotriplet. Thus

$$\sqrt{2} < \pi^0|(\bar{u}u|0 > = - < \pi^+|\bar{u}d|0 > . \tag{14}$$

Combining this with (12), using (9) and keeping in mind the phase convention for the two π^0 state we can define

$$\begin{aligned} X &= < \pi^+\pi^-|Q_2|K^0 > = \sqrt{2} < \pi^+\pi^0|Q_1|K^+ > \\ &= \sqrt{2} < \pi^+\pi^0|Q_2|K^+ > = - < \pi^0\pi^0|Q_1|K^0 > . \end{aligned} \tag{15}$$

These reductions are common to VIM and CHNC.

The matrix element X is calculated as follows. The matrix elements (14) are expressed in both approaches by the pion decay constant

$$< \pi^+|(\bar{u}d)_{V-A}|0 > = -i\sqrt{2}F_\pi p_\pi , \tag{16}$$

where $F_\pi = 93MeV$ is the physical pion decay constant. For the matrix elements (12) the general formula involves two formfactors:

$$< \pi^-|j_L|K^0 > = (p_K + p_\pi)f_+ + (p_K - p_\pi)f_- . \tag{17}$$

In VIM one puts $f_+ = 1$ and $f_- = 0$, while in CHNC the calculation yields $f_+ = 1$, but

$$f_- = \frac{m_K^2 - m_\pi^2}{\Lambda_\chi^2} \approx \tfrac{1}{4} . \tag{18}$$

The experimental estimates [6] are $f_-/f_+ = -0.35 \pm 0.15$ for K^+ and $f_-/f_+ = -0.11 \pm 0.09$ for K_L^0. In VIM, therefore,

$$X = -i\sqrt{2}F_\pi(m_K^2 - m_\pi^2) , \tag{19}$$

while in CHNC

$$X = -i\sqrt{2}F_\pi(m_K^2 - m_\pi^2)\left(1 + \frac{m_\pi^2}{\Lambda_\chi}\right) , \tag{20}$$

which differs from the previous formula by less than two per cent.

The evaluation of the matrix elements of operator Q_6 is based on the following identities

$$(p_K - p_\pi) < \pi^-|(\bar{s}u)_{V-A}|K^0> = 2(m_s - m_d) < \pi^-|(\bar{s}_R u_L)|K^0>, \quad (21)$$

$$p_\pi < \pi^+|(\bar{u}d)_{V-A}|0> = 2(m_u + m_d) < \pi^+|\bar{u}_R d_L)|0>, \quad (22)$$

$$m_\pi^2(1 + \tfrac{m_K^2}{\Lambda_\chi^2}) = 2(m_u + m_d) < \pi^+\pi^-|(\bar{d}_R d_L)|0>, \quad (23)$$

$$< 0|(\bar{s}d)_{V-A}|K^0> = i\sqrt{2}F_K p_K, \quad (24)$$

$$p_K < 0|(\bar{s}d)_{V-A}|K^0> = 2(m_s + m_d) < 0|(\bar{s}_L d_R)|K^0>, \quad (25)$$

which hold in both VIM and CHNC provided Λ_σ from VIM is identified with Λ_χ from CHNC. Substituting these formulae into (9) one finds using $f_- = 0$

$$Y = -i\sqrt{2}f_\pi r^2 \frac{m_K^2 - m_\pi^2/2}{\Lambda_\chi^2}, \quad (26)$$

or using (18)

$$Y = -i\sqrt{2}f_\pi r^2 \frac{m_K^2 - m_\pi^2}{\Lambda_\chi^2}. \quad (27)$$

The two expressions for Y differ by less than four per cent. Here

$$r = \frac{2m_{K^0}^2}{m_s + m_d} = \frac{2m_\pi^2}{m_u + m_d}. \quad (28)$$

In order to reproduce the phases from [2] one has to absorb the phase factor $-i$ into e.g. the state $|K>$. The difference between formulae (26) and (27) again follows from the different assumptions about the formfactor f_- in the two approaches.

To summarize: comparing the derivations of the matrix elements for kaon decays $< \pi\pi|Q_i|K >$ in CHNC and VIM (with the Fierz terms rejected) one finds that not only the final results are similar, but also most of the assumptions of VIM can be derived in the framework of CHNC. The only relevant difference in assumptions is that about the formfactor f_-. The corresponding contributions, however, are multiplied by m_π^2 and therefore numerically the two sets of formulae are almost equivalent. VIM could of course be updated by inserting the experimental value of f_-. Since, however, the ratio of the $\Delta I = 1/2$ to the $\Delta I = 3/2$ amplitudes for $K \to \pi\pi$ decays is underestimated by about a factor of four in the approach described here, this small correction does not seem very important. Let us also note that the Fierz terms, omitted in the leading term of CHNC, reappear in the next approximation of this method.

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