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PhD Thesis

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Neutrino oscillations beyond the Standard Model in the future beta–beam experiments

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To my wife Izabela.

Abstract

In the present work we study the possible effects of physics beyond the Standard Model in future neutrino oscillation experiments, where these leptons will be produced in beta decays of accelerated radioactive ions — the so–called beta–beam. Therefore, first we set the limits on the parameters describing new scalar, vector and tensor currents. Then, we use the statistical operator to describe the state of antineutrinos produced in the Fermi and Gamow–Teller nuclear beta decays as such state will be mixed in general. Next, we consider the antineutrinos oscillations in the vacuum and their detection through quasielastic scattering on free protons. Finally, we give a numerical estimate of the possible size of the influence of exotic vector currents on the number of detected antineutrinos, produced in helium-6 decays.

Contents

Introduction

In science, we are always open to surprises, and this is also the case for neutrinos. They were introduced by W. Pauli[1] in 1930 to save the energy conservation principle and fermion/boson statistics in the β–decay. It then took 26 years after which neutrinos were finally proven to exist in an experiment led by C. L. Cowan and F. Reines [2]. Soon after that these illusive leptons participated in another β–decay reaction, where the parity symmetry was found to be broken (in the experiment led by C. S. Wu [3], after the paper by T. D. Lee and C. N. Yang [4], where the parity non–conservation was discussed). This was why neutrinos helped establish the structure of the Standard Model, which has already been enlarged to accommodate the tiny masses of these leptons as neutrinos oscillate. In the present work, we examine whether we should be prepared for another enlargement of the Standard Model in the context of future neutrino oscillation experiments, where these particles will be produced from beta decays of accelerated radioactive ions — the so–called beta–beam [5].

In order to achieve our aim we will first study the neutron decay in the context of a general interaction Hamiltonian, which consists not only of a vector, but also of scalar and tensor currents. In principle we would like to find up–to–date limits on parameters describing such exotic interactions. These considerations will be presented in Chapter 1.

The main part of our work is presented in Chapter 2. After a brief experimental introduction about the beta–beam concept in Section 2.1 we move to main, more theoretical studies. The general interaction Hamiltonian includes left and right neutrino fields. In such case, also in the massless limit, it is possible to produce relativistic antineutrinos not only with the Standard Model positive helicity but also with the negative one. The state of antineutrinos

[1] In his famous letter Pauli originally called the new particle a *neutron* and later Fermi renamed it to *neutrino*. The English version of the letter can be found *e.g.* in Ref. [1].

produced in the process described by such general interaction will be mixed in general and we have to use a statistical operator to describe it. This will be covered in Section 2.2, where we consider the production of antineutrinos in the Fermi and Gamow–Teller beta decays of radioactive ions and evolution of the state of these illusive leptons on their way to a distant detector. Next, in Section 2.3 we describe detection of antineutrinos through their quasielastic scattering on protons. Finally, in Section 2.4 we finish our discussion with a brief numerical analysis of the possible influence of nonstandard vector currents on the number of detected antineutrinos produced in helium-6 decays. At the end we give a brief summary.

Chapter 1

New Physics in neutron decay

1.1 General Hamiltonian

At the beginning of our considerations we will parametrise the physics beyond the Standard Model (SM) by using the general, Lorentz invariant, derivative–free, four fermion contact interaction Hamiltonian. The first such parametrisation of New Physics (NP) for neutron beta decay, which includes parity violating terms, was introduced by Lee and Yang [4] in the context of symmetry breaking. The interaction proposed by Lee and Yang contains effective field operators for nucleons (neutron and proton), which makes it very convenient for studying low energy phenomena such as neutron beta decay. However, such parametrisation is not sufficient for our later studies of high energy scattering of antineutrinos. Therefore, here we will use the Hamiltonian similar to that introduced in Ref. [6] (where the chiral fields of quarks and leptons appear), that in the basis in which mass matrix of charged leptons is diagonal has the form

$$\mathcal{H} = 4\sum_{i=1}^{3}\sum_{\ell=e,\mu,\tau}\sum_{k,l=L,R}\Big\{a_{kl}\,U_{\ell i}^{k}\,\bar{\ell}\gamma_\mu P_k\nu_i\,\bar{u}\gamma^\mu P_l d + A_{kl}\,U_{\ell i}^{k}\,\bar{\ell}P_k\nu_i\,\bar{u}P_l d + \alpha_{kk}\,U_{\ell i}^{k}\,\bar{\ell}\frac{\sigma_{\mu\nu}}{\sqrt{2}}P_k\nu_i\,\bar{u}\frac{\sigma^{\mu\nu}}{\sqrt{2}}P_k d\Big\} + \text{H.c.}, \qquad (1.1)$$

where $P_L = \frac{1}{2}(1-\gamma_5)$, $P_R = \frac{1}{2}(1+\gamma_5)$ and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ (the metric and gamma matrices are the same as *e.g.* in Ref. [7]). Moreover, u, d, ℓ, ν_i are the field operators for up and down quarks as well as for charged leptons ℓ

and the i-th neutrino with a certain mass. The U^L and U^R are 3×3 unitary mixing matrices for the left and right–handed neutrinos, respectively. We assume that a_{kl}, A_{kl}, α_{kk} for $k, l = L, R$ are real parameters. The SM is restored when $a_{kl} = 0$, $A_{kl} = 0$, $\alpha_{kk} = 0$ for $k, l = L, R$ except $a_{LL} = a_{LL}^{SM} = V_{ud} G_F / \sqrt{2}$, where G_F is the usual Fermi constant and V_{ud} is the element of the Cabibbo–Kobayashi–Maskawa mixing matrix. In the SM the U^L is the Pontecorvo–Maki–Nakagawa–Sakata matrix. The a_{kl}, A_{kl}, α_{kk} parameters are not dimensionless. Therefore, we can factorise out the a_{LL} in Eq. (1.1) and use the following ratios ($k, l = L, R$)

$$\hat{a}_{kl} = \frac{a_{kl}}{a_{LL}}, \quad \hat{A}_{kl} = \frac{A_{kl}}{a_{LL}}, \quad \hat{\alpha}_{kk} = \frac{\alpha_{kk}}{a_{LL}}. \tag{1.2}$$

The only dimensional parameter that remains is a_{LL}. For our later purpose let us rewrite the Hamiltonian in Eq. (1.1) as

$$\mathcal{H} = G_\beta \left(\mathcal{H}_{VA} + \mathcal{H}_{SP} + \mathcal{H}_T \right), \tag{1.3}$$

where

$$G_\beta = a_{LL} \left(1 + \hat{a}_{LR} \right). \tag{1.4}$$

The $\mathcal{H}_{VA}$ is given by

$$\begin{aligned} \mathcal{H}_{VA} = \sum_{i=1}^{3} \sum_{\ell=e,\mu,\tau} \Big\{ & U_{\ell i}^{L} \bar{\ell} \gamma_\mu \left(1 - \gamma_5\right) \nu_i \bar{u} \gamma^\mu \left(1 - v_a \gamma_5\right) d \\ & + U_{\ell i}^{R} \bar{\ell} \gamma_\mu \left(1 + \gamma_5\right) \nu_i \bar{u} \gamma^\mu \left(v_+ + v_- v_a \gamma_5\right) d \Big\} + \text{H.c.}, \end{aligned} \tag{1.5}$$

where

$$v_a = \frac{1 - \hat{a}_{LR}}{1 + \hat{a}_{LR}}, \quad v_\pm = \frac{\hat{a}_{RR} \pm \hat{a}_{RL}}{1 \pm \hat{a}_{LR}}. \tag{1.6}$$

While the $\mathcal{H}_{SP}$ and $\mathcal{H}_T$ are

$$\mathcal{H}_{SP} = 4 \sum_{i=1}^{3} \sum_{\ell=e,\mu,\tau} \sum_{k,l=L,R} \left\{ s_{kl} \, U_{\ell i}^{k} \, \bar{\ell} P_k \nu_i \, \bar{u} P_l d \right\} + \text{H.c.}, \tag{1.7}$$

$$\mathcal{H}_T = 4 \sum_{i=1}^{3} \sum_{\ell=e,\mu,\tau} \sum_{k=L,R} \left\{ t_{kk} \, U_{\ell i}^{k} \, \bar{\ell} \frac{\sigma_{\mu\nu}}{\sqrt{2}} P_k \nu_i \, \bar{u} \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_k d \right\} + \text{H.c.}, \tag{1.8}$$

where for $k, l = L, R$ we have

$$s_{kl} = \frac{\hat{A}_{kl}}{1 + \hat{a}_{LR}}, \quad t_{kk} = \frac{\hat{\alpha}_{kk}}{1 + \hat{a}_{LR}}. \tag{1.9}$$

1.2 Structure of the nucleon

General parametrisation. In order to calculate amplitudes for the free neutron beta decay (and for scattering of antineutrinos on free protons as we will see later)

$$n \to p + e^- + \bar{\nu}_i \,, \tag{1.10}$$

we need to know the following matrix elements

$$\begin{aligned}
&\langle p(p_p, \lambda_p)|\bar{u}(x) O_i d(x)|n(p_n, \lambda_n)\rangle = \\
&\bar{\Psi}_p(x, p_p, \lambda_p) H_i(p_p, p_n) \Psi_n(x, p_n, \lambda_n) \,,
\end{aligned} \tag{1.11}$$

where $i = S,\ P,\ V,\ A,\ T$, while $|p(p_p, \lambda_p)\rangle$, $|n(p_n, \lambda_n)\rangle$ are proton and neutron states with corresponding four-momenta p_p, p_n and helicities λ_p, λ_n. The operators O_i are given by

$$O_S = 1 \,, \ O_P = \gamma_5 \,, \ O_V = \gamma_\mu \,, \ O_A = \gamma_\mu \gamma_5 \,, \ O_T = \sigma_{\mu\nu} \,. \tag{1.12}$$

The free proton and neutron wave functions are ($j = p,\ n$)

$$\Psi_j(x, p_j, \lambda_j) = u_j(p_j, \lambda_j) e^{-ip_j x} \,, \tag{1.13}$$

where $u_{n,p}(p_{n,p}, \lambda_{n,p})$ are ordinary Dirac bispinors. The quantities $H_i(p_p, p_n)$ $= H_i(P, q) \equiv H_i$, where $P = p_n + p_p$ and $q = p_n - p_p$, can be parametrised similarly as in Ref. [8]:

$$H_S = g_S(q^2) \,, \tag{1.14a}$$

$$H_P = g_P(q^2)\, \gamma_5 \,, \tag{1.14b}$$

$$H_V = F_1(q^2)\, \gamma_\mu + \frac{iF_2(q^2)}{2m_N}\, \sigma_{\mu\nu} q^\nu + \frac{F_3(q^2)}{m_N}\, q_\mu \,, \tag{1.14c}$$

$$H_A = G_A(q^2)\, \gamma_\mu \gamma_5 + \frac{G_P(q^2)}{m_N}\, q_\mu \gamma_5 + \frac{G_3(q^2)}{m_N}\, P_\mu \gamma_5 \,, \tag{1.14d}$$

$$\begin{aligned}
H_T = {} & g_T(q^2)\, \sigma_{\mu\nu} + \frac{i g_T^{(1)}(q^2)}{m_N}\, (q_\mu \gamma_\nu - q_\nu \gamma_\mu) \\
& + \frac{i g_T^{(2)}(q^2)}{m_N^2}\, (q_\mu P_\nu - q_\nu P_\mu) \\
& + \frac{i g_T^{(3)}(q^2)}{m_N}\, (\gamma_\mu q^\alpha \gamma_\alpha \gamma_\nu - \gamma_\nu q^\alpha \gamma_\alpha \gamma_\mu) \,,
\end{aligned} \tag{1.14e}$$

where m_N is a scaling parameter such that all form factors $F_{1,2,3}(q^2)$, $G_{A,P,3}(q^2)$, $g_{S,P,T}(q^2)$, $g_T^{(1,2,3)}(q^2)$ are dimensionless. The m_N is taken as the average nucleon mass $m_N = (m_n + m_p)/2$ with m_n and m_p being neutron and proton masses, respectively. All form factors are real functions as strong interactions are time reversal invariant [8].

The corresponding matrix elements needed for scattering of antineutrinos on free protons ($\ell = e, \mu, \tau$)

$$\bar{\nu}_i + p \to \ell^+ + n \tag{1.15}$$

can be obtained through the relation

$$\begin{aligned} &\langle n(p_n, \lambda_n)|\bar{d}(x) O_i u(x)|p(p_p, \lambda_p)\rangle = \\ &\delta_i \langle p(p_p, \lambda_p)|\bar{u}(x) O_i d(x)|n(p_n, \lambda_n)\rangle^* , \end{aligned} \tag{1.16}$$

where $\delta_{S,V,A,T} = 1$ and $\delta_P = -1$.

The isospin symmetry. If we neglect the difference between up and down quark masses then the QCD Lagrangian is invariant under the isospin symmetry of the form

$$\begin{pmatrix} u(x) \\ d(x) \end{pmatrix} \to \exp\left(-i\boldsymbol{\theta} \cdot \boldsymbol{\sigma}/2\right) \begin{pmatrix} u(x) \\ d(x) \end{pmatrix} , \tag{1.17}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ are real parameters and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are Pauli matrices. If we further set $m_n = m_p$ in $\Psi_{n,p}(x, p_{n,p}, \lambda_{n,p})$ we can show that

$$F_3(q^2) \equiv 0 , \quad G_3(q^2) \equiv 0 , \quad g_T^{(3)}(q^2) \equiv 0 . \tag{1.18}$$

In fact, assuming $m_n = m_p$ results in the isospin symmetry at the nucleon level[1] of the analogical form as given in Eq. (1.17) when the quark fields are substituted with the corresponding fields for nucleons. In the further text we will refer jointly to both these symmetries as simply the isospin symmetry.

Form factors in the SM. Let us explore the possible values of the form factors in the SM. The isospin symmetry allows us to relate the Fermi $F_1(q^2)$ and Dirac $F_2(q^2)$ form factors to the electromagnetic nucleon form factors (see *e.g.* Ref. [7] for a derivation) expressed conventionally in terms of the

[1]This leads to the Conserved Vector Current hypothesis (see *e.g.* Ref. [9]) as $\partial_\mu \left(\bar{\Psi}_p(x, p_p, \lambda_p)\gamma^\mu \Psi_n(x, p_n, \lambda_n)\right) = 0$ for $m_n = m_p$, independent of the particular values of $\lambda_{n,p}$.

electric G_E^j and magnetic G_M^j Sachs form factors [10, 11] of proton $j = p$ and neutron $j = n$ leading to

$$F_1(Q^2) = \frac{[G_E^p(Q^2) - G_E^n(Q^2)] - \frac{Q^2}{4m_N^2}[G_M^p(Q^2) - G_M^n(Q^2)]}{1 + \frac{Q^2}{4m_N^2}}, \tag{1.19a}$$

$$F_2(Q^2) = \frac{[G_M^p(Q^2) - G_M^n(Q^2)] - [G_E^p(Q^2) - G_E^n(Q^2)]}{1 + \frac{Q^2}{4m_N^2}}, \tag{1.19b}$$

where $Q^2 = -q^2$. The Sachs form factors can be parametrised in a simple way as (see *e.g.* Ref. [12], a more sophisticated form can be found *e.g.* in Ref. [13]):

$$G_E^p(Q^2) = G_D(Q^2)\,, \tag{1.20a}$$

$$G_E^n(Q^2) = 0\,, \tag{1.20b}$$

$$G_M^p(Q^2) = \frac{\mu_p}{\mu_N} G_D(Q^2)\,, \tag{1.20c}$$

$$G_M^n(Q^2) = \frac{\mu_n}{\mu_N} G_D(Q^2)\,, \tag{1.20d}$$

where $\mu_p \approx 2.793\,\mu_N$ and $\mu_n \approx -1.913\,\mu_N$ are proton and neutron magnetic moments with μ_N being the nuclear magneton, and

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}\,, \tag{1.21}$$

with $M_V \approx 0.84\,\mathrm{GeV}$ that is taken from experiment (fitted from electron scattering data) and we take its value as *e.g.* in Ref. [12].

The axial form factor is usually taken in the form of

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}\,. \tag{1.22}$$

The $G_A(Q^2)$ form factor has two parameters: the g_A and the M_A. The g_A can be taken *e.g.* from free neutron decay data (as we will see in the next chapter) since for this decay the four–momentum transfer q as well as $q^2 = -Q^2$ are small and then $G_A(Q^2 \approx 0) = g_A$. Given the value of the g_A, the M_A can be fitted from neutrino quasielastic scattering, where q and q^2 are not negligible. For $g_A = 1.2673$ we have $M_A = 1.026 \pm 0.021\,\mathrm{GeV}$ from Ref. [12].

The pseudoscalar form factor $G_P(Q^2)$ can be related to $G_A(Q^2)$ as follows (see *e.g.* Ref. [12])

$$G_P(Q^2) = G_A(Q^2)\frac{2m_N^2}{m_\pi^2 + Q^2}\,, \tag{1.23}$$

where m_π is the charged pion mass.

Let us briefly mention that the QCD lattice calculations of the g_A give rather a broad range of expectations for a value of this quantity [14] $1.1 < g_A < 1.34$ and we have to rely on the experimental value that can be very different when New Physics is taken into account in fits as we will see later. Lattice calculations provide as with the estimates of experimentally unknown values of $g_{S,T} = g_{S,T}(q^2 \approx 0)$ giving [14] $g_S = 0.8 \pm 0.4$ and $g_T = 1.05 \pm 0.35$.

1.3 Limits on parameters describing New Physics

We would like to find the limits of the NP parameters of the general Hamiltonian given at the beginning. The goal is to obtain the differential decay width for neutron beta decay and express it in terms of the so–called correlation coefficients. Given the experimental values for those coefficients we will perform least squares analysis and find constraints on the parameters of the interest. Finally, we briefly compare obtained limits with those from other fields like nuclear and pion decays. The results of such analysis were already published (Refs. [15, 16]).

1.3.1 Correlation coefficients

The general formula for the differential decay width for the decay as given in Eq. (1.10) in the case of initially polarised neutrons is given by

$$d\Gamma_i = \frac{1}{2m_n} \sum_{\lambda_\nu, \lambda_e, \lambda_p} \sum_{\lambda_n, \lambda_n'} \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3 2E_\nu} \frac{d^3\boldsymbol{p}_e}{(2\pi)^3 2E_e} \frac{d^3\boldsymbol{p}_p}{(2\pi)^3 2E_p}$$
$$\times (2\pi)^4 \delta^{(4)}(p_n - p_p - p_e - p_\nu) \left[A_{i;\lambda_n} \, \rho^n_{\lambda_n, \lambda_n} \, A^*_{i;\lambda_n} \right], \tag{1.24}$$

where $p_\alpha = (E_\alpha, \boldsymbol{p}_\alpha)$ and λ_α denote the four-momentum and the helicity of the respective particle α, $A_{i;\lambda_n} \equiv A_{i,\lambda_n;\lambda_\nu,\lambda_p,\lambda_e}(\boldsymbol{p}_\nu, \boldsymbol{p}_n, \boldsymbol{p}_p, \boldsymbol{p}_e)$ is the amplitude for the decay process (1.10) calculated using the Hamiltonian (1.1) and nucleon matrix elements as given in Eqs. (1.14). The

$$\rho^n = \frac{1}{2} \left(I + \boldsymbol{\sigma} \cdot \boldsymbol{\lambda}_n \right) \tag{1.25}$$

describes the initial polarisation of the neutron with $\boldsymbol{\lambda}_n$ being the neutron polarisation vector. Calculations of the differential decay width in Eq. (1.24) were done in the neutron rest frame and we neglected neutrino masses in kinematics as well as all terms proportional to $m_e/m_{n,p}$ (m_e denotes the electron mass) and to $|\boldsymbol{p}_i|/m_j$ for $i = \nu, e, p$ and $j = n, p$ (in particular $E_p = m_p\sqrt{1+\boldsymbol{p}_p^2/m_p^2} \approx m_p$). Under such approximations and taking into account that in the neutron beta decay the four momentum transfer is small the tensors given in Eqs. (1.14) simplify to ($i = S, V, A, T$)

$$H_i \to H_i^0 = g_i\, O_i\,, \tag{1.26}$$

where O_i are given in Eq. (1.12), $g_V = F_1(q^2 \approx 0)$, $g_A = G_A(q^2 \approx 0)$, $g_{S,T} = g_{S,T}(q^2 \approx 0)$. There is no term containing $g_P(q^2 \approx 0)$ since

$$\bar{u}_p^0(\lambda_p)\gamma_5 u_n^0(\lambda_n) \approx 0 \tag{1.27}$$

independent of the particular values of $\lambda_{n,p}$, where

$$u_{n,p}^0(\lambda_{n,p}) \equiv u_{n,p}(|\boldsymbol{p}_{n,p}|/m_{n,p} \approx 0, \lambda_{n,p})\,. \tag{1.28}$$

In order to be consistent with our later derivations we assume $g_V = 1$ (although in the decay under consideration we cannot set $m_p = m_n$ as required for perfect isospin symmetry to hold) and we will limit ourselves to the case of three light neutrinos as already indicated in the Hamiltonian (1.1). These simplifications with respect to Refs. [15, 16] do not affect the results presented in these papers. Then, after summing over antineutrino states $\sum_{i=1}^3 d\Gamma_i = d\Gamma$ we obtain (in analogy to Ref. [17])

$$\begin{aligned}\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{|\boldsymbol{p}_e| E_e E_\nu^2}{(2\pi)^5} G_\beta \xi \bigg\{ & 1 + a\frac{\boldsymbol{p}_e\cdot\boldsymbol{p}_\nu}{E_e E_\nu} + b\frac{m_e}{E_e} \\ & + \boldsymbol{\lambda}_n \cdot \left[A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e\times\boldsymbol{p}_\nu}{E_e E_\nu}\right]\bigg\},\end{aligned} \tag{1.29}$$

where B has the form of

$$B = B_0 + b_\nu \frac{m_e}{E_e}\,, \tag{1.30}$$

Ω_e and Ω_ν are the solid angles of electron and antineutrino emission and $E_\nu = m_n - m_p - E_e$. The D correlation coefficient we mention here only for completeness, since $D \equiv 0$ for real a_{kl}, A_{kl}, α_{kk} where k, $l = L$, R (and because we neglected QED corrections — see Ref. [18], from experiments [19] $D = (-1.2\pm2.0)\times10^{-4}$). The formulas for the correlation coefficients a, b, A, B as well as for the factor ξ are given in the Appendix A as functions of the λ, V_{RL}, V_{RR}, s_L, s_R, T_{LL}, T_{RR} parameters (compare with Refs. [6, 15, 16, 20]) defined as ($k = L$, R)

$$\lambda = g_A v_a\,, \quad v_\pm \equiv V_{RR} \pm V_{RL}\,, \tag{1.31a}$$

$$s_k = g_S(s_{kL} + s_{kR})\,, \quad T_{kk} = g_T t_{kk}\,, \tag{1.31b}$$

where v_a, $v_\pm$ are defined in Eqs. (1.6), while s_{kL}, s_{kR} and t_{kk} are given by Eqs. (1.9). From the definition of $v_\pm$ we have

$$V_{RL} = \frac{\hat{a}_{RL} - \hat{a}_{LR}\hat{a}_{RR}}{1 - \hat{a}_{LR}^2}, \quad V_{RR} = \frac{\hat{a}_{RR} - \hat{a}_{LR}\hat{a}_{RL}}{1 - \hat{a}_{LR}^2}. \tag{1.32}$$

1.3.2 Least Squares Analysis

In the SM $b \equiv 0$ as well as $b_\nu \equiv 0$ and unfortunately experimentalists analyse their data assuming[2] $b \equiv 0$ and $b_\nu \equiv 0$. Therefore, we limit ourselves to the cases of parameter combinations that give us $b \equiv 0$ and $b_\nu \equiv 0$. In particularly we have to set $s_L \equiv 0$ and $T_{LL} \equiv 0$. The limits on s_L and T_{LL} will be given later.

The $\chi^2 \equiv \chi^2(\lambda, V_{RL}, V_{RR}, s_R, T_{RR})$, which was minimized with the fit procedure, is of the form

$$\chi^2 = \sum_i \left[\frac{a_i - a}{\delta a_i}\right]^2 + \sum_j \left[\frac{A_j - A}{\delta A_j}\right]^2 + \sum_k \left[\frac{B_k - B}{\delta B_k}\right]^2, \tag{1.33}$$

where a_i, A_j, B_k denote the central values of the respective decay parameters in a certain experiment and δa_i, δA_j, δB_k denote the corresponding errors. In the Table 1.1 we present our data selection (as given in Ref. [16], compare with Ref. [15]) that is based on that of the PDG [19] with the following changes (i) we used the corrected value for measurement in Ref. [26] given in Ref. [24], (ii) we added new measurements of A parameter given in Refs. [24, 25] and dropped older measurements of this decay parameter given in Refs. [28, 29, 30] as they are poorly consistent with the newer ones and finally (iii) we used only the most precise measurements of a and B ($\delta a_i/a_i \leq 6\%$ and $\delta B_k/B_k \leq 2\%$). In the cases when statistical and systematic errors were reported separately we added these two errors in quadrature. For asymmetric errors we took the larger of the reported errors.

As we can see from the expressions listed in the Appendix A the V_{RL}, V_{RR}, s_R, T_{RR} enter quadratically or as mixed terms between pairs of these parameters in the formulas for the correlation coefficients. Therefore, the χ^2 function in Eq. (1.33) has the following symmetry

$$\begin{aligned} &\chi^2(\lambda, V_{RL}, V_{RR}, s_R, T_{RR}) = \\ &\chi^2(\lambda, -V_{RL}, -V_{RR}, -s_R, -T_{RR}). \end{aligned} \tag{1.34}$$

[2]Presently there are no experimental indications for non–zero values of b and b_ν. For the limits on b and b_ν see *e.g.* Ref. [21].

PAR.	VALUE	ERROR	PAPER ID		
a	-0.1054	0.0055	BYRNE	02	[22]
	-0.1017	0.0051	STRATOWA	78	[23]
A	-0.11954	0.00112	MENDENHALL	12	[24]
	-0.11996	0.00058	MUND	12	[25]
	-0.11942	0.00166	LIU	10	[26, 24]
	-0.1189	0.0007	ABELE	02	[27]
B	0.980	0.005	SCHUMANN	07	[31]
	0.967	0.012	KREUZ	05	[32]
	0.9801	0.0046	SEREBROV	98	[33]
	0.9894	0.0083	KUZNETSOV	95	[34]

Table 1.1: The values of correlation coefficients measured in free neutron beta decay. All PAPER ID names are those from Ref. [19] (PDG) except the new ones for Refs. [24, 25].

1.3.3 Results

The λ value.

First, we consider the case when all a_{kl}, A_{kl}, α_{kk} parameters are zero except a_{LL} and a_{LR}. Then, $V_{RL} = 0$, $V_{RR} = 0$, $T_{kk} = 0$, $s_k = 0$ for $k = L, R$ and the only non–zero parameter is $\lambda = g_A v_a$ (as given in Eq. (1.31a), for $a_{LR} \equiv 0$ we have $\lambda = g_A$). In this case the formulas for decay parameters simplify to

$$a = -\frac{\lambda^2 - 1}{3\lambda^2 + 1}, \tag{1.35a}$$

$$A = -\frac{2\lambda(\lambda - 1)}{3\lambda^2 + 1}, \tag{1.35b}$$

$$B = \frac{2\lambda(\lambda + 1)}{3\lambda^2 + 1}. \tag{1.35c}$$

These are the well known SM expressions for $\lambda = g_A$. We performed the one-parameter fit on λ to the data presented in the Table 1.1 and obtained $\chi^2_{min} = 9.542$ (the value of χ^2 at minimum) with

$$\lambda = 1.2755 \pm \begin{cases} 0.0011 \text{ (68.27\% C.L.)}, \\ 0.0018 \text{ (90\% C.L.)}, \\ 0.0022 \text{ (95.45\% C.L.)}. \end{cases} \tag{1.36}$$

The PDG average given in Ref. [19] is[3] $\lambda = 1.2701 \pm 0.0025$, which differs from our result because of different data selection (mainly of the A decay parameter, compare with Ref. [15]).

Many–parameter fits.

Next, we consider the cases of all possible two- and three- parameter combinations giving $b \equiv 0$ and $b_\nu \equiv 0$. These are the cases when V_{RL}, V_{RR} or T_{kk}, s_k parameters can be non–zero. Our results of such many–parameter fits are presented in Figs. 1.1, 1.2 and 1.3. We cannot perform fits where vector parameters are fitted together with scalar or tensor couplings since such combinations would lead to b and b_ν being not identically zero. Because of the lack of space below the plots we describe the results in the following text.

[3] The error was scaled by PDG by 1.9. We changed the sign of the value given in Ref. [19] since in our convention $\lambda > 0$ and we set $g_V = 1$ (the PDG allows this parameter to differ from unity). These simplifications does not affect the overall result on λ presented in Ref. [19].

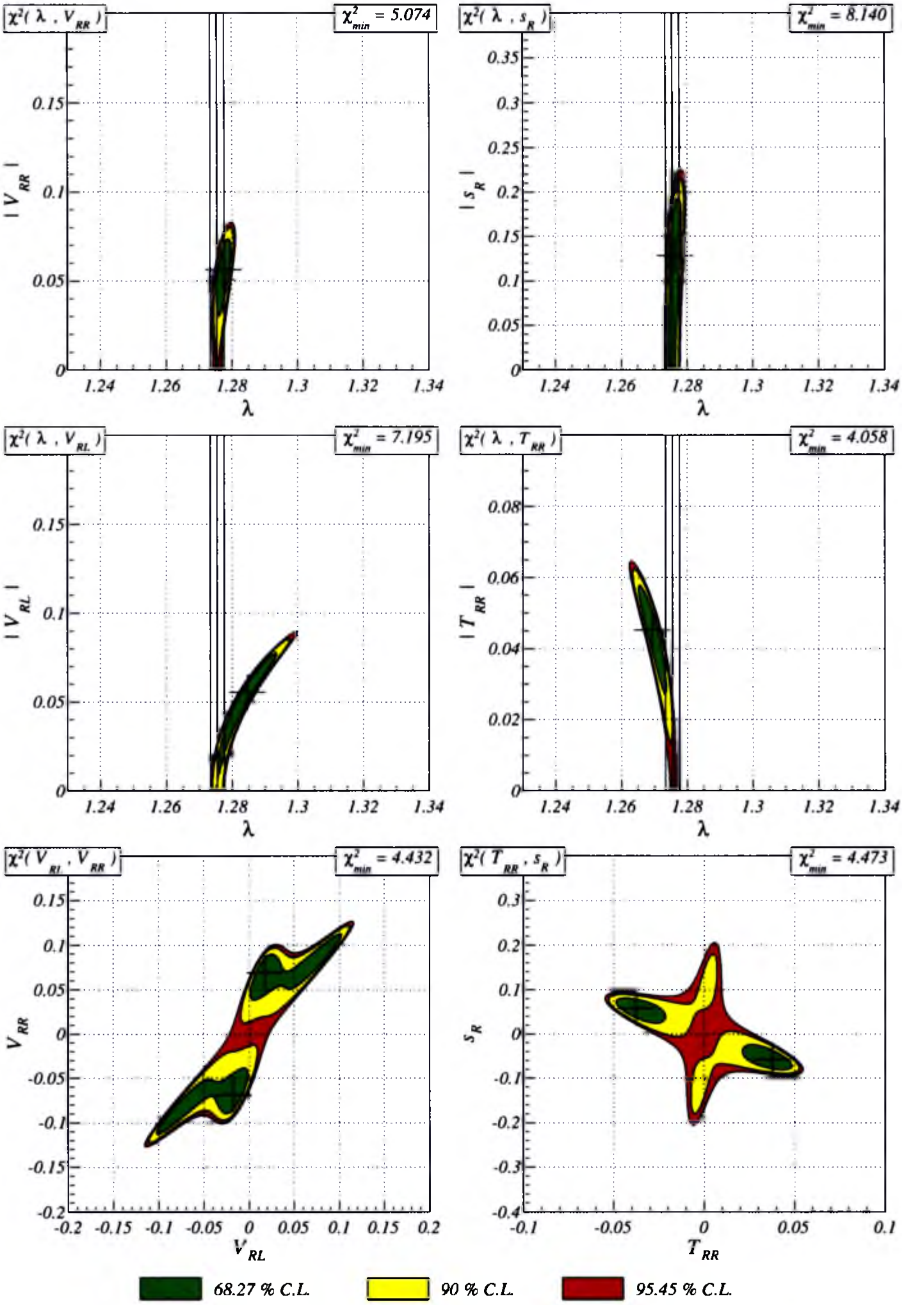

Figure 1.1: The results of the two-parameter fits for the vector, tensor and scalar parameters.

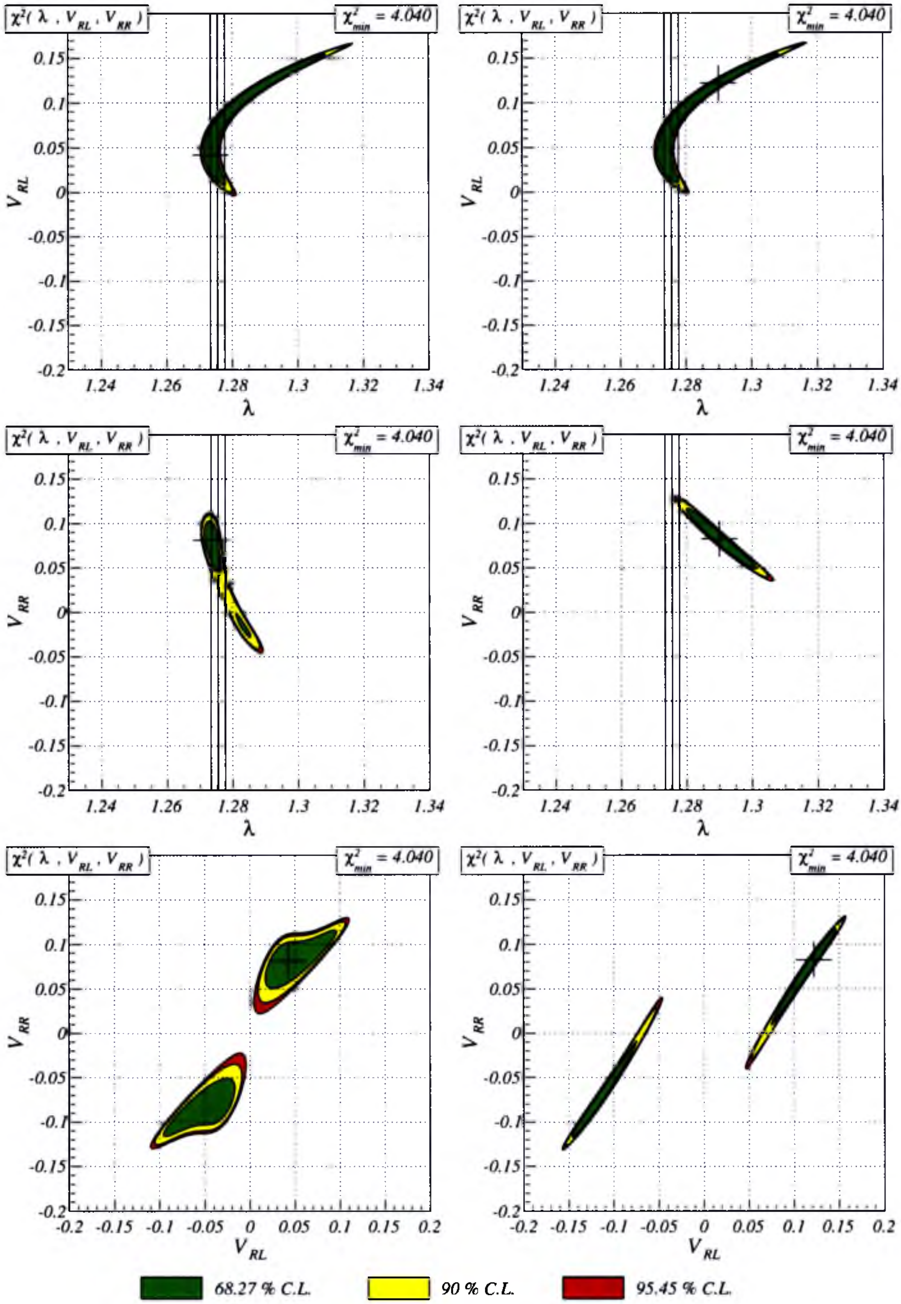

Figure 1.2: The result of the three-parameter fit for the vector parameters.

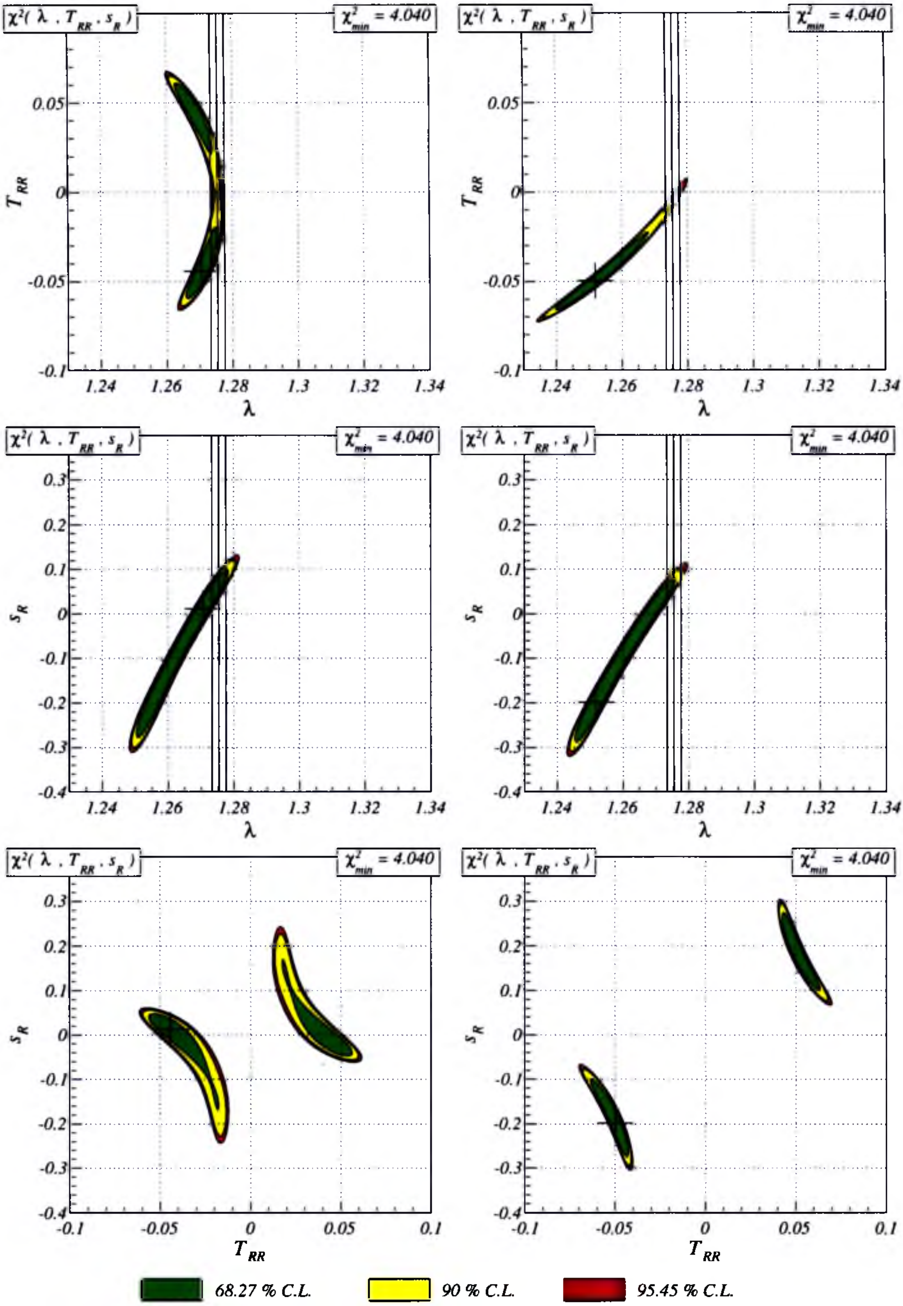

Figure 1.3: The result of the three-parameter fit for the tensor and scalar parameters.

General remarks. In all figures we present the fitted parameters as arguments of the χ^2 function in the left upper corner of each plot, while the remaining ones are set to 0 and λ, if not fitted, is set to its central value given in Eq. (1.36). The cross gives the position of the χ^2 minimum, while the solid vertical lines mark area the 95.45% C.L. interval on λ in Eq. (1.36).

Two-parameter fits. In the case of two-parameter fits presented in Fig. 1.1 there are always 2 equivalent minima since the χ^2 function has the symmetry given in Eq. (1.34).

Three-parameter fits. In the case of the three-parameter fits presented in Figs. 1.2 and 1.3 we show the two-dimensional slices through the corresponding three-dimensional χ^2 volume with planes that include the χ^2 minimum point and are parallel to the respective planes spanned on the main axes in the corresponding parameter space. In the case of these three-parameter fits there are $2 \times 2 = 4$ equivalent minima. It is so because the minimization procedure found 2 equivalent minima corresponding to the different values of λ, that we call $\lambda_{1,2}$, and for each such value of λ there are 2 sets of V_{RL}, V_{RR} or s_R, T_{RR} parameters from χ^2 symmetry in Eq. (1.34). Therefore, we show the respective slices of the three-dimensional χ^2 volume only in 2 equivalent minima that corresponds to different values of λ. The slices belonging to λ_1 are grouped in one column, separated from slices belonging to λ_2. The plots in the remaining two minima can be easily obtained through appropriate symmetries according to Eq. (1.34)

1.3.4 Limits from other low–energy probes

Let us now briefly summarise the constraints form other sources than free neutron decay and compare them with our limits. All constraints presented below are at the 90% C.L.

Left–handed neutrino couplings. Let us first start from the following definition as given in Ref. [6]

$$a_{LL} = a_{LL}^{SM} + a'_{LL} \,. \tag{1.37}$$

Then, the strongest limit on $(a'_{LL} + a_{LR})/a_{LL}^{SM}$ when all other NP parameters (a_{RL}, a_{RR}, A_{kl}, α_{kk} for k, $l = L$, R) are zero comes from the unitarity of the

CKM matrix as claimed in Ref. [6] and recently in Ref. [35]. The constraints given in Ref. [35] translated to the parametrisation used in Ref. [6] are (see also the discussion in Refs. [6, 36])

$$\frac{a'_{LL} + a_{LR}}{a_{LL}^{SM}} < 5 \times 10^{-4} . \tag{1.38}$$

As stated in Ref. [14] the strongest limit on s_L comes from superallowed Fermi nuclear transitions analysed in Ref. [36]. The corresponding limit on s_L (when all other NP parameters are zero) is[4] [14]

$$-1.0 \times 10^{-3} < s_L < 3.2 \times 10^{-3} . \tag{1.39}$$

Similarly, in Ref. [14] the authors conclude that the strongest limit on tensor interactions comes from the radiative pion decay $\pi^+ \to e^+ + \nu_e + \gamma$. The corresponding limit given in Ref. [14] in the parametrisation used in Ref. [6] takes the form

$$-2.2 \times 10^{-3} < \frac{\alpha_{LL}}{a_{LL}^{SM}} < 2.72 \times 10^{-3} \tag{1.40}$$

and all other NP parameters are zero.

Right–handed neutrino couplings. Finally, let us remark that it is possible to obtain stronger limits than ours when nuclear decays are included in the analysis. In particular from the joint analysis of neutron and nuclear decays the limits on vector parameters are [6]

$$|\hat{a}_{RR}| < 6.3 \times 10^{-2} \tag{1.41}$$

when all other parameters are zero (including $\hat{a}_{RL} \equiv 0$) and

$$|\hat{a}_{RL}| < 3.7 \times 10^{-2} \tag{1.42}$$

if $\hat{a}_{RL}$ is the only non–zero parameter. Similarly, if only α_{RR} contributes then from nuclear decays the limits are [6]

$$\left|\frac{\alpha_{RR}}{a_{LL}^{SM}}\right| < 4 \times 10^{-2} . \tag{1.43}$$

Finally, the nuclear decays of ^{32}Ar give the limit on $\left|(A_{RR} - A_{RL})/a_{LL}^{SM}\right|$ at the level of 0.1 as stated in Ref. [6].

[4]The s_L parameter used in Ref. [14] is roughly the same as in the present work, while Ref. [36] uses a different parametrisation.

Chapter 2

Neutrinos from beta–beams

2.1 The beta–beam concept

The idea of beta–beam was first introduced by P. Zucchelli [5]. The concept includes production of a large number of radioactive ions, their acceleration and circulation in a properly shaped storage ring, where the ion β-decays produce a huge flux of ν_e or $\bar{\nu}_e$ (without contamination of other neutrino flavours — contrary to the case of muon or pion decays). These (anti)neutrinos are sent to a distant detector and the oscillation process can be observed.

A good candidate, as suggested in Ref. [5], for a production site is CERN with its PS and SPS accelerator system that allow to boost ions up to $\gamma = 150$. The storage ring could have two straight sections having $2 \times 36\%$ of its total length of 6880 m — matching roughly the SPS circumference — see Fig. 2.1.

The right choice of an radioactive nuclei is a key issue. The candidates are characterized by (i) their half–life times $T_{1/2}$ and (ii) through the so–called Q_β–values, that are approximately the maximum energies of the emitted (anti)neutrinos in the rest frame of decaying nuclei. Therefore, the ions should decay quick enough to have many (anti)neutrinos at a given time but not too fast in order to accelerate and store them in a large number. The distance L to the detector determines the Lorentz factor γ for the particular value of Q_β in order to observe a maximal oscillation signal.

Zucchelli suggested: ^{6}He as the $\bar{\nu}_e$ emitter with $T_{1/2} \approx 0.81\,\mathrm{s}$, $Q_\beta \approx 3.51\,\mathrm{MeV}$ and ^{18}Ne as the ν_e emitter with $T_{1/2} \approx 1.67\,\mathrm{s}$, $Q_\beta \approx 3.41\,\mathrm{MeV}$ (main decay fraction). The average (anti)neutrino energies after the Lorentz boost with $\gamma = 150$ are then $\langle E_\nu \rangle \approx 581\,\mathrm{MeV}$ for ^{6}He and $\langle E_\nu \rangle \approx 558\,\mathrm{MeV}$

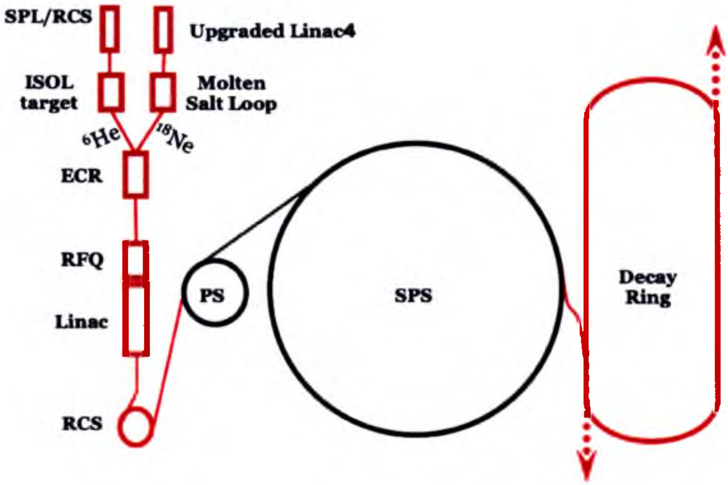

Figure 2.1: Schematic illustration of the beta–beam concept. Picture taken from Ref. [37] — see a detailed description therein. The left part illustrates the ion production facilities, the middle corresponds to the PS and SPS accelerator system in CERN and on the right part the storage ring is illustrated.

for ^{18}Ne. These energies will roughly match 1$^{\text{st}}$ $\nu_e \to \nu_\mu$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) oscillation maximum at Fréjus Underground Laboratory — 130 km from CERN, opening a window for CP-violation measurements, when production infrastructure is combined with new water Čerenkov detector (see *e.g.* Refs. [38, 40]).

Since the original proposal [5] was published many other studies have been carried out[1] that are reviewed *e.g.* in Refs [39, 40, 41]. In particular studies were done for $\gamma = 100$ (see *e.g.* Ref. [41]) since then the average neutrino (antineutrino) energies better match the $\nu_e \to \nu_\mu$ ($\bar{\nu}_e \to \bar{\nu}_\mu$) oscillation maximum for $L = 130$ km (at the cost of lower detection cross sections). In present work we will focus on the antineutrinos produced from ^{6}He decays (since the predicted flux in this case is the biggest among all considered ions) within original scenario with $\gamma = 100$.

[1]Also new ions have been proposed (see Ref. [42]): ^{8}Li as the $\bar{\nu}_e$ emitter with $T_{1/2} \approx 0.83$ s, $Q_\beta \approx 12.96$ MeV and ^{8}B as the ν_e emitter with $T_{1/2} \approx 0.77$ s, $Q_\beta \approx 13.92$ MeV. These ions allow to study neutrino oscillations at larger distance (like CERN–Gran Sasso) within present SPS technical abilities. Let us however mention that the profit of higher cross sections (because of higher energies) does not overcome the lower flux in the larger distance (see *e.g.* Refs. [40]). See also studies for the United States site [43].

2.2 State of produced antineutrinos

The general Hamiltonian presented at the beginning of our considerations includes not only left but also right neutrino fields and both of these fields can have different mixing matrices. Therefore, the state of antineutrinos produced in the decay of radioactive ions will not be pure in general and we have to describe it by the statistical operator. The considerations will be carried out first in the center–of–mass (CM) frame that is the rest frame of the decaying nucleus and then we transform the statistical operator to the laboratory (LAB) frame, where the ions move in the decay ring. Finally, we consider an evolution of the state of the produced antineutrinos on their way to a distant detector.

2.2.1 Statistical operator in the CM frame

First, we would like to put the reaction of our interest

$$ {}^6_2\mathrm{He}^{+2} \to {}^6_3\mathrm{Li}^{+3} + e^- + \bar{\nu}_i \tag{2.1}$$

in a more general context — as an example of the process of the type

$$ X \to Y + e^- + \bar{\nu}_i \, , \tag{2.2}$$

where X and Y denote the initial and final nuclear states, respectively. We are interested only in those antineutrinos that (after appropriate Lorentz boost to the LAB frame) would reach the detector. The statistical operator describing the state of such antineutrinos in the CM frame produced in the process (2.2) may be written as[2]

$$ \rho = \sum_{i,k=1,2,3} \sum_{\lambda,\delta=\pm 1} \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu \frac{d\rho_{\lambda,i;\delta,k}}{dE_\nu d\Omega_\nu} |\bar{\nu}_i(\lambda, \mathbf{p}_\nu)\rangle\langle\bar{\nu}_k(\delta, \mathbf{p}_\nu)| \, , \tag{2.3}$$

where Ω_ν^{max} defines the angular size (with respect to the line defined by the respective straight section of the decay ring) of the detector in the CM frame, λ and δ denote the helicities of the respective antineutrino mass states i and k, while E_ν^{max} is the maximum energy of the emitted antineutrinos. The

[2] As through all this thesis also here we assume that neutrinos have definite values of momenta and energies. In the wave packet approach in general there will be also non–zero off–diagonal matrix elements of the statistical operator under consideration in the momentum–energy basis (see for *e.g.* the definition of the matrix elements in Ref. [44] and the discussion in Refs. [45, 46, 47]).

non–zero matrix elements of the corresponding statistical operator (the so–called density matrix elements) in the case of unpolarised parent nuclei and without measuring the polarisations of the electrons and the daughter nuclei are obtained from[3] (compare with Refs. [44]–[47])

$$d\rho_{\lambda,i;\lambda,k} = \frac{1}{N} \sum_{\lambda_X,\lambda_Y,\lambda_e} \frac{d^3\boldsymbol{p}_Y}{(2\pi)^3 2E_Y} \frac{d^3\boldsymbol{p}_e}{(2\pi)^3 2E_e} \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3 2E_\nu} \\ \times (2\pi)^4 \delta^{(4)}(p_X - p_Y - p_e - p_\nu) A_{\lambda,i} A^*_{\lambda,k} \,, \tag{2.4}$$

where $p_\alpha = (E_\alpha, \boldsymbol{p}_\alpha)$ and λ_α denote the four-momentum and the helicity of the respective particle α, $A_{\lambda,i} \equiv A_{\lambda,i;\lambda_X,\lambda_Y,\lambda_e}(\boldsymbol{p}_\nu, \boldsymbol{p}_X, \boldsymbol{p}_Y, \boldsymbol{p}_e)$ is the amplitude for the decay process (2.2) calculated using the Hamiltonian (1.1). As before, we neglected the effect of neutrino masses in kinematics, so $d^3\boldsymbol{p}_\nu = E_\nu^2 dE_\nu d\Omega_\nu$. The N is such that the density matrix is properly normalised[4]

$$\sum_{i=1}^{3} \sum_{\lambda=\pm1} \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu \frac{d\rho_{\lambda,i;\lambda,i}}{dE_\nu d\Omega_\nu} = 1 \,. \tag{2.5}$$

We will not show herein the details of the calculations of the nuclear matrix elements as such calculations were presented in the literature (*e.g.* in Refs. [48, 49, 50]) and we are more interested in the discussion of the density matrix. Therefore, basing on Refs. [48, 49, 50], we will give only a brief overview of such calculations pointing out the most important aspects and then give the final result. First, we recall the free neutron beta decay. Under the approximations given in Sec. 1.3.1 (in particular $|\boldsymbol{p}_{n,p}|/m_{n,p} \approx 0$) the amplitude for the free neutron beta decay consist the following terms $\bar{u}_p^0(\lambda_p) O_i \bar{u}_p^0(\lambda_p)$ for $i = S, V, A, T$ (and $\bar{u}_p^0(\lambda_p) O_P \bar{u}_p^0(\lambda_p) \approx 0$), where O_i are defined in Eq. (1.12) and $u_{n,p}^0(\lambda_{n,p}) \equiv u_{n,p}(|\boldsymbol{p}_{n,p}|/m_{n,p} \approx 0, \lambda_{n,p})$ are simply equal to $(j = n, p)$

$$u_j^0(\lambda_j) = \sqrt{2m_j} \begin{pmatrix} \chi(\lambda_j) \\ 0 \end{pmatrix} \,, \tag{2.6}$$

where

$$\chi(\lambda_j = +1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \,, \quad \chi(\lambda_j = -1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.7}$$

[3]The off–diagonal elements in λ and δ vanish because of the angular momentum conservation.

[4]After diagonalizing the density matrix its elements give the densities of a probability of finding the antineutrino in a particular state.

are the ordinary two component spinors. The terms containing $O_V = \gamma^\mu$ are equal to $\bar{u}_i^0(\lambda_p)\gamma^0\bar{u}_n^0(\lambda_n) = 2\sqrt{m_n m_p}\chi^+(\lambda_p)\chi(\lambda_n)$ and $\bar{u}_i^0(\lambda_p)\gamma^k\bar{u}_n^0(\lambda_n) = 0$ for $k = 1,2,3$, while the terms containing $O_A = \gamma^\mu\gamma^5$ are equal to $\bar{u}_i^0(\lambda_p)\gamma^0\gamma^5\bar{u}_n^0(\lambda_n) = 0$ and $\bar{u}_i^0(\lambda_p)\gamma^k\gamma^5\bar{u}_n^0(\lambda_n) = 2\sqrt{m_n m_p}\chi^+(\lambda_p)\sigma^k\chi(\lambda_n)$, where $k = 1,2,3$ as before. Similarly, from $O_S = 1$ we obtain terms proportional to $\chi^+(\lambda_p)\chi(\lambda_n)$, while from $O_T = \sigma^{\mu\nu}$ we get terms proportional to $\chi^+(\lambda_p)\sigma^k\chi(\lambda_n)$. In the nuclear physics the $\chi(\lambda_{n,p})$ spinors are multiplied by the orbital wave function of the nucleon in the nucleus. This is so if we assume that at the time of the decay the nucleus can be treated as set of non–interacting, non–relativistic particles. Therefore, the total wave function of the initial and final nuclear states is obtained as an antisymmetric product of the individual nucleon wave functions and we sum over all neutrons in the nucleus that can decay. Moreover, we assume that once the leptons are produced they do not interact with the nuclear medium and we can neglect the terms $r|\boldsymbol{q}| \approx 0$, where $\boldsymbol{q}$ is the momentum transfer and r varies from 0 to maximal radius of the nucleus. From this we conclude that there will be no change of the orbital angular momentum in the decay under consideration. Then, the $\chi^+(\lambda_p)\chi(\lambda_n) = \delta_{\lambda_p,\lambda_n}$ terms give rise to the so–called Fermi transitions in which the spin S_X of the initial nucleus is the same as the spin S_Y of the final nucleus. On the other hand, the terms $\chi^+(\lambda_p)\sigma^k\chi(\lambda_n)$ give rise to the so–called Gamow–Teller transitions in which $S_Y - S_X = 0, \pm 1$ except the case when $S_Y = S_X = 0$. The Fermi and Gamow–Teller transitions are commonly called allowed transitions and form a subgroup of all nuclear beta decays.

The non–zero antineutrino density matrix elements calculated in the rest frame of decaying nucleus X after neglecting the momentum transfer from nucleons to leptons as well as QED corrections and the recoil momentum $\boldsymbol{p}_Y$ of the daughter nucleus ($E_Y = m_Y\sqrt{1 + \boldsymbol{p}_Y^2/m_Y^2} \approx m_Y$, where m_Y is the mass of the final nuclear state Y) are

$$\frac{d\rho_{+,i;+,k}}{dE_\nu d\Omega_\nu} = U_{ei}^L \left(U_{ek}^L\right)^* \rho_{+,+}(E_\nu), \quad \frac{d\rho_{-,i;-,k}}{dE_\nu d\Omega_\nu} = U_{ei}^R \left(U_{ek}^R\right)^* \rho_{-,-}(E_\nu) \tag{2.8}$$

with

$$\rho_{+,+}(E_\nu) = E_\nu^2 E_e |\boldsymbol{p}_e| \frac{|M_{GT}|^2 u_L + |M_F|^2 v_L}{|M_{GT}|^2(\bar{u}_L + \bar{u}_R) + |M_F|^2(\bar{v}_L + \bar{v}_R)}, \tag{2.9a}$$

$$\rho_{-,-}(E_\nu) = E_\nu^2 E_e |\boldsymbol{p}_e| \frac{|M_{GT}|^2 u_R + |M_F|^2 v_R}{|M_{GT}|^2(\bar{u}_L + \bar{u}_R) + |M_F|^2(\bar{v}_L + \bar{v}_R)}, \tag{2.9b}$$

where M_F and M_{GT} denote the Fermi and Gamow–Teller reduced (in the sense of the Wigner–Eckart theorem) matrix elements (see *e.g.* Refs. [48, 49]), $E_e = m_X - m_Y - E_\nu = Q_\beta + m_e - E_\nu$ (the m_X denotes the mass of the initial nucleus X) and

$$\bar{w} = \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu E_\nu^2 E_e |\boldsymbol{p}_e| w \tag{2.10}$$

for $w = u_{L,R}, v_{L,R}$ that are given by

$$u_L = 4T_{LL}^2 - 4\lambda T_{LL}\frac{m_e}{E_e} + \lambda^2\,, \tag{2.11a}$$

$$u_R = \lambda^2 (V_{RR} - V_{RL})^2 - 4\lambda T_{RR} (V_{RR} - V_{RL}) \frac{m_e}{E_e} + 4T_{RR}^2\,, \tag{2.11b}$$

$$v_L = s_L^2 + 2s_L\frac{m_e}{E_e} + 1\,, \tag{2.11c}$$

$$v_R = (V_{RR} + V_{RL})^2 + 2s_R (V_{RR} + V_{RL}) \frac{m_e}{E_e} + s_R^2\,. \tag{2.11d}$$

The density matrix elements (2.9a) and (2.9b) can be easily computed for the pure Fermi transitions ($|M_F|^2 \neq 0$, $|M_{GT}|^2 = 0$) and for the pure Gamow–Teller decays ($|M_F|^2 = 0$, $|M_{GT}|^2 \neq 0$), as well as for antineutrinos from free neutron β-decay (setting $|M_{GT}|^2/|M_F|^2 = 3$ — free neutron β-decay can be viewed as a mixed Fermi and Gamow–Teller transition). The decay (2.1) is a canonical example of the pure Gamow–Teller transition, when from spinless ${}^6_2\mathrm{He}$ nucleus the spin one ${}^6_3\mathrm{Li}$ nucleus is formed (see *e.g.* Ref. [50]).

2.2.2 Statistical operator in the LAB frame

The statistical operator in the LAB frame ρ' (the prime denotes the respective quantities in the LAB frame) describing antineutrinos (produced from the ions moving in the decay ring), that reach the detector, can be written in general case as

$$\rho' = \sum_{i,k=1,2,3} \sum_{\lambda,\delta=\pm1} \int_0^{\Omega_\nu'^{max}} d\Omega_\nu' \int_0^{E_\nu'^{max}(\Omega_\nu')} dE_\nu' \frac{d\rho_{\lambda,i;\delta,k}}{dE_\nu' d\Omega_\nu'} |\bar{\nu}_i(\lambda, \boldsymbol{p}_\nu')\rangle\langle\bar{\nu}_k(\delta, \boldsymbol{p}_\nu')|\,, \tag{2.12}$$

where $\Omega_\nu'^{max}$ defines the angular size of the detector in the LAB frame. First, we would like to relate the density matrix elements in the LAB frame with the respective elements in the CM frame. We will further focus on the case described previously, when the parent nuclei were unpolarised and we did not measure the polarisations of the electrons and the daughter nuclei. For simplicity we assume that we are dealing with a cylindrical detector with a radius D, placed at the distance L from the production point and situated such that the z axis defined by the straight sections of the decay ring covers the main axis cylindrical. Then, the azimuthal angle ϕ'_ν in $d\Omega'_\nu = d\phi'_\nu d\cos\theta'_\nu$ is no more interesting, $E'^{max}_\nu(\Omega'_\nu) = E'^{max}_\nu(\cos\theta'_\nu)$ so we can easily integrate over ϕ'_ν and obtain

$$\int_0^{2\pi} d\phi'_\nu \frac{d\rho_{\lambda,i;\lambda,k}}{dE'_\nu d\Omega'_\nu} = 2\pi \frac{d\rho_{\lambda,i;\lambda,k}}{dE'_\nu d\cos\theta'_\nu} . \tag{2.13}$$

Similarly, we can perform analogous integration in the case of the respective matrix elements in the CM frame

$$\int_0^{2\pi} d\phi_\nu \frac{d\rho_{\lambda,i;\lambda,k}}{dE_\nu d\Omega_\nu} = 2\pi \frac{d\rho_{\lambda,i;\lambda,k}}{dE_\nu d\cos\theta_\nu} . \tag{2.14}$$

Then, after the Lorentz transformation (that is characterized by the usual γ factor) along the z axis from the CM frame to the LAB frame we obtain the following relation between density matrix elements in these two frames[5]

$$\frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta'_\nu\, dE'_\nu} = \frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta_\nu\, dE_\nu} |\det J| , \tag{2.15}$$

where

$$\det J = \begin{vmatrix} \frac{\partial E_\nu}{\partial E'_\nu} & \frac{\partial E_\nu}{\partial\cos\theta'_\nu} \\ \frac{\partial\cos\theta_\nu}{\partial E'_\nu} & \frac{\partial\cos\theta_\nu}{\partial\cos\theta'_\nu} \end{vmatrix} = \frac{1}{\gamma - \sqrt{\gamma^2-1}\cos\theta'_\nu} \tag{2.16}$$

with[6]

$$\cos\theta_\nu = \frac{\sqrt{\gamma^2-1} - \gamma\cos\theta'_\nu}{\sqrt{\gamma^2-1}\cos\theta'_\nu - \gamma} , \tag{2.17}$$

$$E_\nu = \left(\gamma - \sqrt{\gamma^2-1}\cos\theta'_\nu\right) E'_\nu . \tag{2.18}$$

Then, we obviously have

$$E'^{max}_\nu(\cos\theta'_\nu) = \frac{E^{max}_\nu}{\gamma - \sqrt{\gamma^2-1}\cos\theta'_\nu} , \tag{2.19}$$

$$\cos\theta'^{max}_\nu = \frac{L}{\sqrt{L^2+D^2}} . \tag{2.20}$$

[5]Since we neglect the neutrino masses in kinematics there will be no Wick rotation of states.

[6]The inverse relations are

$$\cos\theta'_\nu = \frac{\sqrt{\gamma^2-1} + \gamma\cos\theta_\nu}{\sqrt{\gamma^2-1}\cos\theta_\nu + \gamma} ,$$

$$E'_\nu = \left(\gamma + \sqrt{\gamma^2-1}\cos\theta_\nu\right) E_\nu .$$

For practical reasons we are interested in the limit of very small $\theta_\nu'^{max}$. Then, we can make an approximation

$$E_\nu \approx (\gamma - \sqrt{\gamma^2 - 1})E_\nu' \tag{2.21}$$

as well as

$$E_\nu'^{max}(\cos\theta_\nu') \approx E_\nu'^{max}(\cos(0)) \equiv E_\nu'^{max} = (\gamma + \sqrt{\gamma^2 - 1})E_\nu^{max}\,. \tag{2.22}$$

Then, the statistical operator in the LAB frame ρ' can be written as

$$\rho' \approx \bar\rho = \sum_{i,k=1,2,3}\sum_{\lambda=\pm1}\int_0^{E_\nu'^{max}} dE_\nu' \frac{d\bar\rho_{\lambda,i;\lambda,k}}{dE_\nu'}|\bar\nu_i(\lambda,\boldsymbol{p}_{\nu,z}')\rangle\langle\bar\nu_k(\lambda,\boldsymbol{p}_{\nu,z}')|\,, \tag{2.23}$$

with $\boldsymbol{p}_{\nu,z}' = (0,0,E_\nu')$ and

$$\frac{d\bar\rho_{\lambda,i;\lambda,k}}{dE_\nu'} = 2\pi\int_{\cos\theta_\nu'^{max}}^1 d\cos\theta_\nu' \,\frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta_\nu'\,dE_\nu'}\,. \tag{2.24}$$

The corresponding density matrix elements can easily be calculated and we obtain

$$\begin{aligned}\frac{d\bar\rho_{+,i;+,k}}{dE_\nu'} &= U_{ei}^L\left(U_{ek}^L\right)^* 2\pi\,(1-\cos\theta_\nu^{max}) \\ &\times (\gamma-\sqrt{\gamma^2-1})\rho_{+,+}\Big((\gamma-\sqrt{\gamma^2-1})E_\nu'\Big)\,, \qquad (2.25a)\\ \frac{d\bar\rho_{-,i;-,k}}{dE_\nu'} &= U_{ei}^R\left(U_{ek}^R\right)^* 2\pi\,(1-\cos\theta_\nu^{max}) \\ &\times (\gamma-\sqrt{\gamma^2-1})\rho_{-,-}\Big((\gamma-\sqrt{\gamma^2-1})E_\nu'\Big) \qquad (2.25b)\end{aligned}$$

with $\rho_{\pm,\pm}(E_\nu)$ given in Eqs. (2.9) and (see Eq. (2.17))

$$\cos\theta_\nu^{max} = \frac{\sqrt{\gamma^2-1} - \gamma\cos\theta_\nu'^{max}}{\sqrt{\gamma^2-1}\cos\theta_\nu'^{max} - \gamma}\,. \tag{2.26}$$

The elements in Eqs. (2.25) are independent of the particular value of $\cos\theta_\nu^{max}$ as the factor $2\pi\,(1-\cos\theta_\nu^{max})$ cancels with the corresponding one in the normalisation of the $\rho_{\pm,\pm}(E_\nu)$ in Eqs. (2.9). It can be also checked that the density matrix is properly normalised as

$$\sum_{i=1}^3\sum_{\lambda=\pm1}\int_0^{E_\nu'^{max}} dE_\nu' \frac{d\bar\rho_{\lambda,i;\lambda,i}}{dE_\nu'} = 1\,. \tag{2.27}$$

Let us also define (for the later purpose) the density of the flux of antineutrinos with helicities $\lambda = \pm 1$ and energy E'_ν as

$$j_\lambda(E'_\nu) = \frac{N_P}{\pi D^2} \frac{1 - \cos\theta_\nu^{max}}{2} \sum_{i=1}^{3} \frac{d\bar{\rho}_{\lambda,i;\lambda,i}}{dE'_\nu}, \tag{2.28}$$

where $\cos\theta_\nu^{max}$ is given in Eq. (2.26) and N_P is the number of decays of the radioactive ions per unit time. The $\frac{1}{2}(1 - \cos\theta_\nu^{max})$ describes the fraction of produced antineutrinos that reach the detector (when the decaying nuclei were not polarised). The total density of the flux is obviously given by

$$j(E'_\nu) = \sum_{\lambda=\pm 1} j_\lambda(E'_\nu)\,. \tag{2.29}$$

2.2.3 Evolution of the state

Once the antineutrinos are produced they travel the distance L and after the time T they reach the detector. This evolution can be also described in terms of the statistical operator and leads to the neutrino oscillations [51, 52, 53]. We assume that oscillation length is not very big, so we can consider that antineutrinos are moving practically in a vacuum. Let us first consider a general situation, when the distance to the detector is given by vector $\boldsymbol{L}$ that may not be equal to $\boldsymbol{L} = (0, 0, L)$. Then, the evolution of the statistical operator in Eq. (2.12) can be described as

$$\rho'(\boldsymbol{L}, T) = \mathcal{U}(\boldsymbol{L}, T)\rho' \mathcal{U}^{+}(\boldsymbol{L}, T)\,. \tag{2.30}$$

Since all quantities are in the LAB frame so we will drop the prime in the following text. As we concentrate on the oscillations of antineutrinos in the vacuum the evolution operator is given by

$$\mathcal{U}(\boldsymbol{L}, T) = \exp(-i\mathcal{P}^{\mu} X_{\mu}) \tag{2.31}$$

with $X = (T, \boldsymbol{L})$ and the action of four momentum operator on the antineutrino states is defined as

$$\mathcal{P}^{\mu}|\bar{\nu}_i(\lambda, \boldsymbol{p}_\nu)\rangle = p_i^{\mu}|\bar{\nu}_i(\lambda, \boldsymbol{p}_\nu)\rangle\,, \tag{2.32}$$

where $p_i = (E_i, \boldsymbol{p}_\nu)$ is the four-momentum[7] and have to take non-zero antineutrino masses $m_i \neq 0$ at the moment of calculating the oscillation phases so that $E_i = \sqrt{\boldsymbol{p}_\nu^2 + m_i^2}$.

Let us now consider the situation described in the previous section so that $\boldsymbol{L} = (0, 0, L)$ and $\boldsymbol{p}_\nu = \boldsymbol{p}_{\nu,z} = (0, 0, E_\nu)$. Since neutrinos are nearly massless we take $T \approx L$ and $E_i \approx E_\nu + m_i^2/(2E_\nu)$ so that

$$\bar{\rho}(L) = \sum_{i,k=1,2,3} \sum_{\lambda=\pm 1} \int_0^{E_\nu^{max}} dE_\nu \frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} |\bar{\nu}_i(\lambda, \boldsymbol{p}_{\nu,z})\rangle\langle\bar{\nu}_k(\lambda, \boldsymbol{p}_{\nu,z})|\,, \tag{2.33}$$

where

$$\frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} = \frac{d\bar{\rho}_{\lambda,i;\lambda,k}}{dE_\nu} \exp\left(-i\frac{\Delta m_{ik}^2}{2E_\nu} L\right) \tag{2.34}$$

with $\Delta m_{ik}^2 = m_i^2 - m_k^2$.

[7]This corresponds to the so-called equal momentum approximation, since in general antineutrino states with different masses m_i will have different energies E_i and momenta $\boldsymbol{p}_i$.

2.3 Description of the detection process

2.3.1 Number of detected antineutrinos

The precise description of antineutrinos scattering on nuclei in the detector is a complicated task and it is a subject to the specialised programs — the Monte Carlo event generators approach (see *e.g.* Ref. [54]). However, we would like only to find the size of the possible NP on the beta–beam neutrino phenomena and such details are not so important for us. Therefore, as a detection reaction we will choose the scattering of antineutrinos on free protons as given in Eq. (1.15). The corresponding number of produced leptons $\ell = e, \mu, \tau$ (that is also the respective number of detected antineutrinos in the limit of no background) irrespective of their energy is given by

$$N_\ell = \int_{E_\nu^{th,\ell}}^{E_\nu^{max}} N_\ell(E_\nu) dE_\nu \,, \tag{2.35}$$

where $E_\nu^{th,\ell}$ is the threshold energy of the antineutrino for the production of the lepton ℓ and

$$N_\ell(E_\nu) = n_D t_D \frac{N_P}{\pi D^2} \frac{1-\cos\theta_\nu^{max}}{2} \sigma_{e\to\ell}(E_\nu, L) \tag{2.36}$$

with n_D being the number of scattering centres in the detector (*i.e.* the free protons in our case), t_D is the time period during which measurements are carried out. The N_P, D, $\cos\theta_\nu^{max}$ were defined previously (for Eq. (2.28) — in particular $\cos\theta_\nu^{max}$ denotes the respective quantity in the CM frame as given in Eq. (2.17)), while the $\sigma_{e\to\ell}(E_\nu, L)$ in the LAB frame (defined before, in which the detector is at rest) is given by[8] (compare with Refs. [45, 46])

$$\begin{aligned}\sigma_{e\to\ell}(E_\nu, L) = \frac{1}{4\sqrt{(p_\nu p_p)^2}} \frac{1}{2} \sum_{\lambda,\lambda_p,\lambda_n,\lambda_\ell=\pm 1} \sum_{i,k=1,2,3} \int \frac{d^3\boldsymbol{p}_\ell}{(2\pi)^3 2E_\ell} \int \frac{d^3\boldsymbol{p}_n}{(2\pi)^3 2E_n} \\ \times (2\pi)^4 \delta^{(4)}(p_p + p_\nu - p_n - p_\ell) A_{\lambda,i}^{D,\ell} \frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} \left(A_{\lambda,k}^{D,\ell}\right)^*,\end{aligned} \tag{2.37}$$

where i and k denote the respective antineutrino mass states, $p_\alpha = (E_\alpha, \boldsymbol{p}_\alpha)$ and λ_α mark the four-momentum and the helicity of the particle α, $A_{\lambda,i}^{D,\ell} \equiv A_{\lambda,i,\lambda_n,\lambda_p,\lambda_\ell}^{D,\ell}(\boldsymbol{p}_\nu, \boldsymbol{p}_n, \boldsymbol{p}_p, \boldsymbol{p}_\ell)$ is the amplitude for the scattering process (1.15)

[8] We have averaged over the polarisation of the proton and summed over the polarisations of the final particles.

calculated using the Hamiltonian (1.1) and the corresponding nucleon matrix elements as given through Eq. (1.16). We assume that the target protons in the detector are at rest in our LAB frame, so $p_p = (m_p, \mathbf{0})$.

Because, as usual, we also neglected the effect of non–zero antineutrino masses in kinematics, we can factorise the amplitudes $A^{D,\ell}_{\lambda,i}$ in the following way

$$A^{D,\ell}_{\lambda=+1,i} = (U^L_{\ell i})^* \, M^{D,\ell}_{\lambda=+1}\,, \quad A^{D,\ell}_{\lambda=-1,i} = (U^R_{\ell i})^* \, M^{D,\ell}_{\lambda=-1}\,, \tag{2.38}$$

where $M^{D,\ell}_{\lambda} \equiv M^{D,\ell}_{\lambda;\lambda_n,\lambda_p,\lambda_\ell}(\boldsymbol{p}_\nu, \boldsymbol{p}_n, \boldsymbol{p}_p, \boldsymbol{p}_\ell)$ describes the scattering of massless antineutrinos. Thus, we have

$$\begin{aligned} &\sum_{i,k=1,2,3} A^{D,\ell}_{+1,i} \frac{d\bar{\rho}_{+,i;+,k}(L)}{dE_\nu} \left(A^{D,\ell}_{+1,k}\right)^* = \\ &\sum_{i=1}^{3} \frac{d\bar{\rho}_{+,i;+,i}}{dE_\nu} \left|M^{D,\ell}_{+1}\right|^2 P^L_{e\to\ell}(E_\nu, L) \end{aligned} \tag{2.39}$$

and similarly

$$\begin{aligned} &\sum_{i,k=1,2,3} A^{D,\ell}_{-1,i} \frac{d\bar{\rho}_{-,i;-,k}(L)}{dE_\nu} \left(A^{D,\ell}_{-1,k}\right)^* = \\ &\sum_{i=1}^{3} \frac{d\bar{\rho}_{-,i;-,i}}{dE_\nu} \left|M^{D,\ell}_{-1}\right|^2 P^R_{e\to\ell}(E_\nu, L)\,, \end{aligned} \tag{2.40}$$

where $(j = L, R)$

$$P^j_{e\to\ell}(E_\nu, L) = \sum_{i,k=1,2,3} U^j_{ei} U^j_{\ell k} \left(U^j_{ek}\right)^* \left(U^j_{\ell i}\right)^* \exp\!\left(-i\frac{\Delta m^2_{ik}}{2E_\nu} L\right) \tag{2.41}$$

has the form of the usual probability, such that $\sum_{\ell=e,\mu,\tau} P^j_{e\to\ell} = 1$ for $j = L, R$. Therefore, the $N_\ell(E_\nu)$ can be decomposed into

$$N_\ell(E_\nu) = N_{+;\ell}(E_\nu) + N_{-;\ell}(E_\nu)\,, \tag{2.42}$$

where

$$N_{+;\ell}(E_\nu) = n_D t_D j_+(E_\nu) \sigma_{+;\ell}(E_\nu) P^L_{e\to\ell}(E_\nu, L)\,, \tag{2.43}$$

$$N_{-;\ell}(E_\nu) = n_D t_D j_-(E_\nu) \sigma_{-;\ell}(E_\nu) P^R_{e\to\ell}(E_\nu, L) \tag{2.44}$$

with $j_\pm(E_\nu)$ being the density of the flux of antineutrinos defined in Eq. (2.28) and $\sigma_{\lambda;\ell}(E_\nu)$ denoting the cross–section for the production of lepton ℓ when the incoming (massless) antineutrino has helicity $\lambda = \pm 1$.

2.3.2 Cross–sections

Let us now briefly focus on the possible values of the cross–sections. We will work in the limit of the perfect isospin symmetry (see Sec. 1.2). For technical reasons, it is very convenient to express $\sigma_{\pm;\ell}(E_\nu)$ as

$$\sigma_{\pm;\ell}(E_\nu) = \int_{Q^2_{min}(E_\nu)}^{Q^2_{max}(E_\nu)} dQ^2 \frac{d\sigma_{\pm;\ell}(E_\nu)}{dQ^2}, \tag{2.45}$$

where[9]

$$Q^2_{min}(E_\nu) = \frac{2E_\nu^2 m_N - m_N m_\ell^2 - z}{2E_\nu + m_N}, \tag{2.46a}$$

$$Q^2_{max}(E_\nu) = \frac{2E_\nu^2 m_N - m_N m_\ell^2 + z}{2E_\nu + m_N} \tag{2.46b}$$

with

$$z = E_\nu m_\ell^2 - E_\nu \sqrt{(s - m_\ell^2)^2 - 2(s + m_\ell^2)m_N^2 + m_N^4} \tag{2.47}$$

and $s = m_N^2 + 2E_\nu m_N$.

First, we consider the SM case. The formula for the respective differential cross–section can be written in the usual form as (in analogy to Ref. [55])

$$\frac{d\sigma_\ell^{SM}}{dQ^2} = \frac{(G_\beta^{SM})^2 m_N^2}{4\pi E_\nu^2} \left[A - \frac{s-u}{m_N^2} B + \left(\frac{s-u}{m_N^2} \right)^2 C \right], \tag{2.48}$$

where $G_\beta^{SM} = V_{ud} G_F/\sqrt{2}$ is the value of G_β within the SM (see Sec. 1.1), $s - u = 4m_N E_\nu - Q^2 - m_\ell^2$ and

$$A = \frac{m_\ell^2 + Q^2}{m_N^2} \Big\{ (1+r)G_A^2 - (1-r)F_1^2 + r(1-r)F_2^2 + 4rF_1F_2$$
$$- \frac{m_\ell^2}{4m_N^2} \left[(F_1+F_2)^2 + (G_A + 2G_P)^2 - 4(r+1)G_P^2 \right] \Big\}, \tag{2.49a}$$

$$B = 4rG_A(F_1 + F_2), \tag{2.49b}$$

$$C = \frac{1}{4}(G_A^2 + F_1^2 + rF_2^2) \tag{2.49c}$$

with m_ℓ being the mass of the lepton ℓ, $r = Q^2/(4m_N^2)$, $F_{1,2} \equiv F_{1,2}(Q^2)$, $G_{A,P} \equiv G_{A,P}(Q^2)$.

In general we do not know the form factors in the case of NP and therefore we do not know the corresponding cross–sections. For our later purpose let

[9] Recall that we work under the approximation that the target protons in the detector are at rest.

us however briefly discuss the case when only vector currents are present. Then, the formulas for the cross–sections $d\sigma_{\pm;\ell}/dQ^2$ can be obtained form the expression for the SM cross–section through the following substitutions[10]

$$\frac{d\sigma_{+;\ell}}{dQ^2} = \frac{d\sigma_\ell^{SM}}{dQ^2}(G_\beta^{SM} \to G_\beta, G_{A,P} \to v_a G_{A,P})\,, \tag{2.50a}$$

$$\frac{d\sigma_{-;\ell}}{dQ^2} = \frac{d\sigma_\ell^{SM}}{dQ^2}(G_\beta^{SM} \to G_\beta, F_{1,2} \to v_+ F_{1,2}, G_{A,P} \to v_a v_- G_{A,P})\,. \tag{2.50b}$$

Let us stress that in the SM the form factors have parameters fitted from experiments. Therefore, if NP is included the formulas used in the fits change. This implies that the values obtained through such a general analysis can change also. This applies in particular to $G_A(Q^2)$ (and $G_P(Q^2)$ that is related to $G_A(Q^2)$ through Eq. (1.23)) form factor that is fitted from neutrino and antineutrino scattering on nuclei and nucleons (the $F_{1,2}(Q^2)$ are expressed in terms of the electromagnetic form factors fitted from electron scattering data, as already stated in Sec. 1.2).

[10]When we set the NP parameters to 0 then the $d\sigma_{-;\ell}/dQ^2$ vanishes and the $d\sigma_{+;\ell}/dQ^2 = d\sigma_\ell^{SM}/dQ^2$.

2.4 Numerical results

Finally, we would like to estimate numerically how big are the NP effects if the NP parameters satisfy the bounds described in the Section 1.3. For this purpose we inspect the ratio

$$\Delta N_{e\mu}(E_\nu) = 1 - \frac{N_\mu^{NP}(E_\nu)}{N_e^{NP}(E_\nu)} \Big/ \frac{N_\mu^{SM}(E_\nu)}{N_e^{SM}(E_\nu)}, \tag{2.51}$$

where $N_{e,\mu}^{SM,NP}(E_\nu)$ are the corresponding number of events calculated in the case of the SM or NP, respectively. The $\Delta N_{e\mu}(E_\nu)$ ratio is independent of the detector size D and the number of radioactive ions (per unit time) N_P as well as of the values of n_D, t_D, G_β and G_β^{SM}. This can be easily understood as the factors that contain those quantities cancel out between numerators and denominators of the respective ratios. If the NP parameters are close to 0, then the $\Delta N_{e\mu}(E_\nu)$ ratio is also close to 0.

We will focus on the antineutrinos produced in the decays of ^{6}He ions in case when the only non–zero NP parameters are the vector ones $\hat{a}_{ik}$ $(i, k = L, R)$. We will use the SM form factors for the NP cross sections. As discussed previously the parameters of these form factors can change if they are fitted to NP formulas rather than to SM expressions. This is the subject of the values of M_A and g_A as discussed in the Sec. 1.2. Therefore, to make our discussion more clear we assume $\hat{a}_{LR} = 0$ then from Eqs. (1.6) and (1.31) we see that λ simplifies to $\lambda = g_A$ as well as $V_{RL} = \hat{a}_{RL}$, $V_{RR} = \hat{a}_{RR}$. We choose the value $g_A = 1.2755$ as given in Eq. (1.36) as the SM value of this parameter. We assume that for such value of g_A we can set[11] $M_A = 1.026$ as given in the Sec. 1.2. We consider two cases, first we set $\hat{a}_{RL} = 0$ and $\hat{a}_{RR} = 0.06$ and next we set $\hat{a}_{RL} = 0.03$ and $\hat{a}_{RR} = 0$. Both values of $\hat{a}_{RL}$ and $\hat{a}_{RR}$ obey the constrains on those parameters taken for $g_A \approx 1.2755$ (as given in the Sec. 1.3).

The mixing matrices we parametrise through mixing angles and phases $(k = L, R)$

$$U^k = \begin{pmatrix} e^{i\alpha_1^k} & 0 & 0 \\ 0 & e^{i\alpha_2^k} & 0 \\ 0 & 0 & e^{i\alpha_3^k} \end{pmatrix} U(\theta_{12}^k, \theta_{13}^k, \theta_{23}^k, \delta_{CP}^k) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_4^k} & 0 \\ 0 & 0 & e^{i\alpha_5^k} \end{pmatrix}, \tag{2.52}$$

[11]The $M_A = 1.026$ was extracted for $g_A = 1.2673$.

where $U \equiv U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$ we take in the same form as in the PDG [19]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \tag{2.53}$$

with $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. The $\alpha^{L,R}_{1-5}$ phases cancel out in the formulas for $P^{L,R}_{e\to\ell}(E_\nu, L)$, therefore

$$\Delta N_{e\mu}(E_\nu) \equiv \Delta N_{e\mu}(E_\nu; \Theta^L, \Theta^R)\,, \tag{2.54}$$

where $\Theta^k \equiv (\theta^k_{12}, \theta^k_{13}, \theta^k_{23}, \delta^k_{CP})$. We calculate $\Delta N_{e\mu}(E_\nu)$ varying the angles $\theta^L_{ij} \equiv \theta_{ij} \in [0, \pi/2]$ in the SM limits recommended by the PDG [19]

$$\sin^2(2\theta_{12}) = 0.857 \pm 0.024\,, \tag{2.55a}$$

$$\sin^2(2\theta_{23}) > 0.95\,, \tag{2.55b}$$

$$\sin^2(2\theta_{13}) = 0.098 \pm 0.013 \tag{2.55c}$$

and with $\delta^L_{CP} = 0$, while θ^R_{ij} can change in the range $[0, \pi/2]$ and δ^R_{CP} we take from $[0, 2\pi]$. The respective neutrino mass differences we set to the central values of the PDG [19] limits

$$\Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5}\,\mathrm{eV}^2\,, \tag{2.56a}$$

$$|\Delta m^2_{32}| = 0.00232^{+0.00012}_{-0.00008}\,\mathrm{eV}^2 \tag{2.56b}$$

and we choose $\Delta m^2_{32} > 0$ (that corresponds to the so–called normal hierarchy). The results of our analysis for $L = 130\,\mathrm{km}$ and $\gamma = 100$ are presented in Figs. 2.2 and 2.3.

Let us briefly discuss the accuracy in the future beta–beam experiments looking for $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations. The signal in such experiments is the number of detected antimuons. The key issues are residual systematic errors of the signal and the backgrounds. Both these errors were estimated in Ref. [56] (compare also with Ref. [57]) to be not smaller than 2% in the case of a water Čerenkov detector. Our calculations were made for antineutrinos scattering on free protons in the case of perfect detector efficiency and no background. Therefore, if we wish to compare our results with the predicted accuracy we have to include in principle also nuclear effects in oxygen nuclei, which can generate not only additional NP effects, but also uncertainties coming from a modelling the nuclei. The calculated effects of NP are below 0.5% level, but the full answer to the question of the influence of NP in the future beta–beam experiments is still the subject of further investigations.

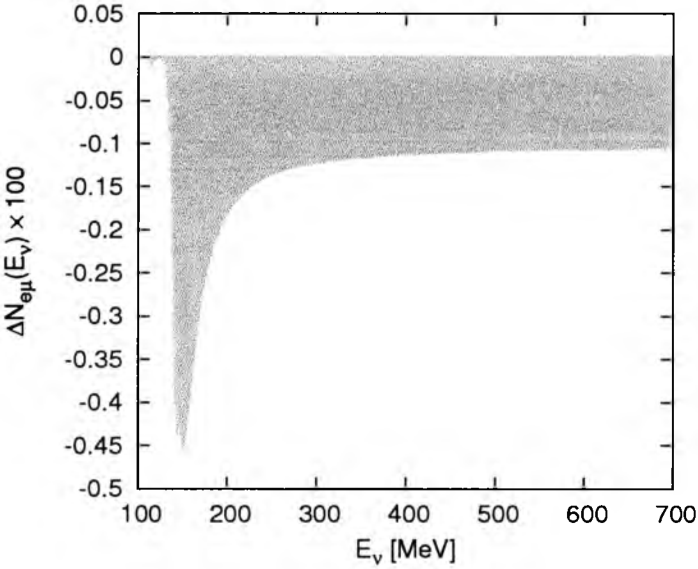

Figure 2.2: The $\Delta N_{e\mu}(E_\nu)$ calculated for $\hat{a}_{RL} = 0$ and $\hat{a}_{RR} = 0.06$ when antineutrinos were produced in the decays of ^{6}He ions accelerated to $\gamma = 100$. The detector is located $L = 130\,\mathrm{km}$ away from the decay ring.

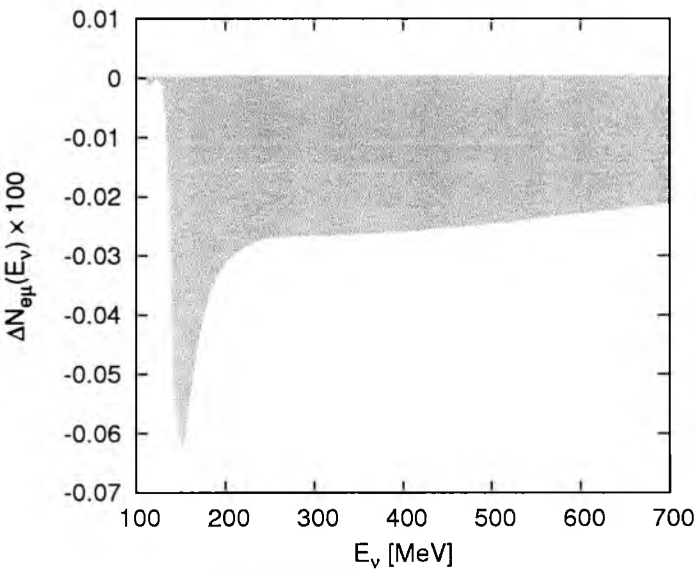

Figure 2.3: The $\Delta N_{e\mu}(E_\nu)$ calculated for $\hat{a}_{RL} = 0.03$ and $\hat{a}_{RR} = 0$ when antineutrinos were produced in the decays of ^{6}He ions accelerated to $\gamma = 100$. The detector is located $L = 130\,\mathrm{km}$ away from the decay ring.

Summary

In the present work we discussed the possible influence of physics beyond the Standard Model in the future neutrino oscillation experiments, where these particles will be produced from beta decay of accelerated radioactive ions. In order to achieve this aim we conducted our research in a few steps.

First, we considered the general, Lorentz invariant, derivative–free, four fermion contact interaction Hamiltonian at the quark–lepton level. Next, we discussed the nucleon matrix elements needed to calculate the amplitudes for antineutrino production and detection processes. We applied these considerations to calculate the so–called correlation coefficients in free neutron beta decay and performed the least squares analysis using the most precise and recent experimental data for these coefficients. In such a way the limits on parameters describing New Physics were found.

After these initial preparations, we studied the antineutrino production, oscillation and detection in the future beta–beam experiments. We used the statistical operator to describe the state of the antineutrinos produced in the Fermi and Gamow–Teller nuclear beta decays, as in general such state is mixed. In particular we found the formulas for the matrix elements of such statistical operator in the rest frame of decaying nucleus and then in the laboratory frame, when the radioactive ions move in the storage ring. Next, we considered the evolution of the state of antineutrinos that leads to oscillations and the detection of these particles in the distant detector. In particular, we gave the formulas of antineutrino scattering on free protons when only vector currents are present.

Finally, we estimated the influence of physics beyond the Standard Model when antineutrinos were produced in helium-6 decays and detected through their scattering on free protons. We considered the situation when only vector currents are present. The obtained deviation of New Physics from the Standard Model signal is below the expected experimental accuracy. However, the complete analysis requires further research, especially the calculation of the nuclear effects in the detection process.

Appendix A

Formulas for correlation coefficients

We would like to present formulas for the correlation coefficients a, b, A, $B = B_0 + b_\nu m_e/E_e$ given in Ref. [16], which are functions of the parameters defined in Eqs. (1.31). The presented formulas agree with those obtained earlier in Ref. [17] after appropriate change of the parametrisation as given in Ref. [6]. Thus, we have

$$\begin{aligned}\xi &= 3\lambda^2\left[(V_{RR}-V_{RL})^2+1\right]\\ &+(V_{RR}+V_{RL})^2+1\\ &+12(T_{RR}^2+T_{LL}^2)+s_R^2+s_L^2\,,\end{aligned}\tag{A.1}$$

$$\begin{aligned}a\xi &= -\lambda^2\left[(V_{RR}-V_{RL})^2+1\right]\\ &+(V_{RR}+V_{RL})^2+1\\ &+4(T_{RR}^2+T_{LL}^2)-s_R^2-s_L^2\,,\end{aligned}\tag{A.2}$$

$$\begin{aligned}b\xi &= -12\lambda\left[T_{RR}(V_{RR}-V_{RL})+T_{LL}\right]\\ &+2\left[s_R(V_{RR}+V_{RL})+s_L\right]\,,\end{aligned}\tag{A.3}$$

$$\begin{aligned}A\xi &= -2\lambda^2\left[1-(V_{RR}-V_{RL})^2\right]\\ &+2\lambda\left[1-(V_{RR}+V_{RL})\,(V_{RR}-V_{RL})\right]\\ &-4\left(2T_{RR}^2+s_RT_{RR}-2T_{LL}^2-s_LT_{LL}\right)\,,\end{aligned}\tag{A.4}$$

$$
\begin{aligned}
B_0\xi = {} & 2\lambda^2\left[1-(V_{RR}-V_{RL})^2\right] \\
& + 2\lambda\left[1-(V_{RR}+V_{RL})(V_{RR}-V_{RL})\right] \\
& - 4\left(2T_{RR}^2 - s_R T_{RR} - 2T_{LL}^2 + s_L T_{LL}\right) ,
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
b_\nu\xi = {} & 2\lambda\left[(4T_{RR}-s_R)(V_{RR}-V_{RL}) - (4T_{LL}-s_L)\right] \\
& + 4\left[T_{RR}(V_{RR}+V_{RL}) - T_{LL}\right] .
\end{aligned}
\tag{A.6}
$$

Bibliography

[1] I. J. R. Aitchison, A. J. G. Hey, *Gauge theories in particle physics: a practical introduction. Vol. 2*, Institute of Physics, 2004

[2] C. L. Cowan *et al.*, Science **124** (1956) 103

[3] C. S. Wu *et al.*, Phys. Rev. **105** (1957) 1413

[4] T. D. Lee, C. N. Yang, Phys. Rev. **104** (1956) 254

[5] P. Zucchelli, Phys. Lett. B **532** (2002) 166

[6] P. Herczeg, Prog. Part. Nucl. Phys. **46** (2001) 413

[7] C. Giunti, C. W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press, 2007

[8] S. Weinberg, Phys. Rev. **112** (1958) 1375

[9] B. P. Feynman, M. Gell-Mann, Phys. Rev. **109** (1958) 193

[10] F. J. Ernst, R. G. Sachs, K. C. Wali, Phys. Rev. **119** (1960) 1105

[11] R. G. Sachs, Phys. Rev. **126** (1962) 2256

[12] V. Bernard, L. Elouadrhiri, U. G. Meissner, J. Phys. G **28** (2002) R1

[13] H. S. Budd, A. Bodek, J. Arrington, `arXiv:0308005`

[14] T. Bhattacharya *et al.*, Phys. Rev. D **85** (2012) 054512

[15] J. Holeczek, M. Ochman, E. Stephan, M. Zralek, Acta Phys. Polon. B **42** (2011) 2493

[16] J. Holeczek, M. Ochman, E. Stephan, M. Zralek, `arXiv:1303.5295v1`

[17] J. D. Jackson, S. B. Treiman, H. W. Wyld, Jr., Phys. Rev. **106** (1957a) 517

[18] J. D. Jackson, S. B. Treiman, H. W. Wyld, Jr., Nucl. Phys. **4** (1957b) 206

[19] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86** (2012) 010001

[20] F. Gluck, I. Joo, J. Last, Nucl. Phys. A **593** (1995) 125

[21] D. Dubbers, M. G. Schmidt, Rev. Mod. Phys. **83** (2011) 1111

[22] J. Byrne *et al.*, J. Phys. G **28** (2002) 1325

[23] C. Stratowa, R. Dobrozemsky, P. Weinzierl, Phys. Rev. D **18** (1978) 3970

[24] M. P. Mendenhall *et al.*, `arXiv:1210.7048`

[25] D. Mund *et al.*, `arXiv:1204.0013`

[26] J. Liu *et al.*, Phys. Rev. Lett. **105** (2010) 181803

[27] H. Abele *et al.*, Phys. Rev. Lett. **88** (2002) 211801

[28] P. Liaud *et al.*, Nucl. Phys. A **612** (1997) 53

[29] B. Erozolimsky, I. Kuznetsov, I. Stepanenko, Y. A. Mostovoi, Phys. Lett. B **412** (1997) 240

[30] P. Bopp *et al.*, Phys. Rev. Lett. **56** (1986) 919

[31] M. Schumann *et al.*, Phys. Rev. Lett. **99** (2007) 191803

[32] M. Kreuz *et al.*, Phys. Lett. B **619** (2005) 263

[33] A. P. Serebrov *et al.*, JETP **86** (1998) 1074

[34] I. A. Kuznetsov *et al.*, Phys. Rev. Lett. **75** (1995) 794

[35] O. Naviliat-Cuncic, M. Gonzalez-Alonso, `arXiv:1304.1759v1`

[36] J. C. Hardy, I. S. Towner, Phys. Rev. C **79** (2009) 055502

[37] C. Hansen, `arXiv:1109.1956`

[38] C. K. Jung, AIP Conf. Proc. **533** (2000) 29

[39] C. Volpe, J. Phys. G **34** (2007) R1

[40] E. Fernandez-Martinez, Nucl. Phys. B **833** (2010) 96

[41] J. Bernabeu *et al.*, `arXiv:1005.3146`

[42] C. Rubbia, A. Ferrari, Y. Kadi, V. Vlachoudis, Nucl. Instrum. Meth. A **568** (2006) 475

[43] A. Jansson, O. Mena, S. J. Parke, N. Saoulidou, Phys. Rev. D **78** (2008) 053002

[44] M. Ochman, R. Szafron, M. Zralek, J. Phys. G **35** (2008) 065003

[45] R. Szafron, M. Zralek, Prog. Part. Nucl. Phys. **64** (2010) 210

[46] M. Ochman, R. Szafron, M. Zralek, Nucl. Phys. Proc. Suppl. **217** (2011) 347

[47] R. Szafron, M. Zralek, Phys. Lett. B **718** (2012) 113

[48] M. Morita, *Beta decay and muon capture*, W. A. Benjamin, 1973

[49] K. Grotz, H. V. Klapdor, *The weak interaction in nuclear, particle, and astrophysics*, Adam Hilger, 1990

[50] W. Greiner, B. Muller, *Gauge Theory of Weak Interactions*, Springer-Verlag, 2000

[51] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33** (1957) 549

[52] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **34** (1958) 247

[53] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28** (1962) 870

[54] T. Abe *et al.*, JINST 4:T05001, 2009, `arXiv:0712.4129`

[55] C. H. Llewellyn Smith, Phys. Rept. 3 (1972) 261

[56] M. Mezzetto, J. Phys. G **29** (2003) 1771

[57] M. Mezzetto, Nucl. Phys. Proc. Suppl. **155** (2006) 214