"Does God Play Dice with Corrugations?": Environmental effects on Growth

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ABSTRACT

Corrugation growth has perplexed many researchers for several decades with remaining challenges including it's reliable prediction in the field. In the present paper, the effect of environmental variations on corrugation growth are investigated using field measurements and mechanics-based modelling. Statistically significant relationships between average daily rainfall, humidity and the growth of rail corrugation were investigated using meterological and railway site field monitoring of a metropolitan network test site with a recurring rail corrugation of about 95mm wavelength. Corrugation growth rate (G_r) was determined by systematically measuring the longitudinal rail profile with a Corrugation Analysis Trolley (CAT) on the 240m radius, narrow gauge, concrete-sleepered curve. Both roughness generated and weld initiated profile growths were investigated. The weather data was obtained from records held by the Bureau of Meteorology. The modelling is developed to provide insight and mechanics based analysis of the field corrugation growth under changes in environmental conditions and vehicle speed variability. Results show a strong correlation between variations in rainfall and corrugation growth that is consistent with changes in steady state wear. Changes in contact patch geometry and the damage mechanism as the corrugation amplitude grows are shown to have substantial but slower effects on growth. The results are used to explain observed changes in corrugation growth rate under variable speed control at the same site.

Keywords: rolling contact mechanics, wear modelling, traction, corrugations, environmental effects

1 INTRODUCTION

The growth of corrugations have intrigued and perplexed many engineers and researchers for several decades. This has motivated ongoing research into understanding, simulating and possibly controlling the phenomenon (see reviews of [1] and [2]). Although much is known now, one of the remaining challenges is the reliable prediction of corrugation growth in the field using mechanics-based modelling. In particular, although many corrugation models and laboratory testrigs have been developed and utilised to provide important insight into the phenomena, reliable prediction of the growth rate of corrugations in the field remains an enigma. This is perhaps not surprising considering that the field offers conditions that are substantially uncontrolled as compared to simulation and laboratory environments. Despite these difficulties, useful field measurements and observations have been performed such as described in [2-6]. For instance field results pertaining to the steady wear parameter indicate that (steady) wear resistant rail is also more resistant to unsteady wear or corrugations. Field results pertaining to the application of friction modifiers [2,4,5] in reducing the occurrence of rail corrugations, verify the importance of changes in friction coefficient and stick-slip behaviour on corrugation growth.

Although these previous studies have provided very useful observations, the environment effects of varying rainfall and humidity as well as contact profile have not been carefully monitored and quantified under field conditions. Indeed, perhaps the lack of reliable, repeatable, field corrugation growth results is due to these random environmental effects; a question that prompted the light-hearted title of this paper. In response, the importance of the environmental effects of rainfall, humidity as well as variations in vehicle speed, contact geometry and the damage mechanism on corrugation growth were determined using field measurements and mechanics modelling of the corrugation growth process on a metropolitan field site.

2 BACKGROUND THEORY – CORRUGATION GROWTH UNDER FIELD VARIATIONS

An analytical model for corrugation growth prediction developed previously by the authors [15,16,24] was utilised for insight and mechanics based analysis of the field data taking into account the effects of environmental conditions, contact patch widening, deformation variations and vehicle pass speed variation/distribution. The conceptual block diagram for the corrugation formation model is depicted in Fig. 1 (based on [2,3,4,8 etc]), where a feedback process occurs over multiple wheelset passes due to the interaction of vehicle/track vibration dynamics I, contact mechanics II and a damage/wear mechanism III, initiated by an initial longitudinal rail profile irregularity.

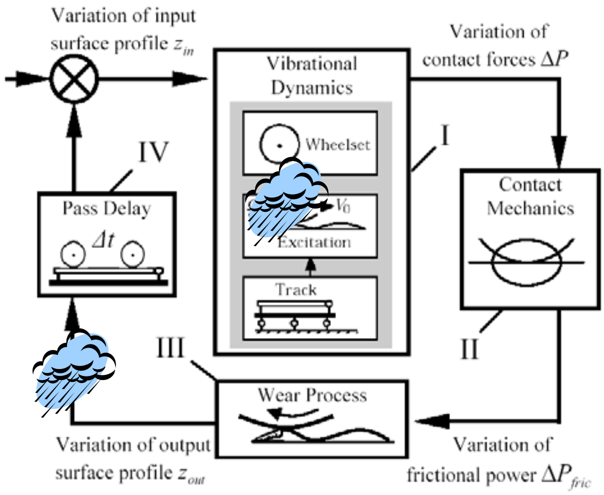

Figure 1. Feedback model for corrugation formation (based on [15]).

The model has been used for considering either vertical or lateral vibration modes, interacting with longitudinal or lateral contact forces. In such a way, both straight track [24,13] or cornering conditions [15] may be modelled. In the present case, cornering conditions, dominated by vertical vibrations interacting with lateral traction, slip and wear variations, are modelled based on evidence from previous modelling and field investigations at the same site [15]. The contact mechanics model is based on quasistatic microslip and considers small linear variations about nominal non-linear operating conditions of which the sensitivity coefficients may be derived from any quasistatic creep theory (ie [20,21,25]). Note that typical high slip cornering conditions can be modelled based on variations about a nominal full sliding creep curve condition. The wear model is based on assuming that the rate of wear is proportional to the frictional power dissipated. Due to computational efficiency, it allows for an investigation into measurable reductions in growth rate that may be achieved by using different pass speed distributions, for a range of initial track profiles and changing frictional/wear conditions over many vehicle passages. The field condition variations in contact patch geometry, rainfall, humidity would primarily affect parameters associated with the wheel/rail rolling contact mechanics and wear on the rail surfaces (as indicated in figure 1).

Essentially, the conceptual closed loop feedback system for corrugation growth may be represented by a recursive equation of grouped railway parameters of the form [15,16,24],

$$\frac{z_{n+1}}{z_n} = K_b \frac{1}{1+k_c\left(R_r(\omega)+R_w(\omega)\right)} + 1 \qquad (1)$$

where z_n is the nth pass longitudinal rail profile, K_b is a grouped parameter representing the sensitivity of wear variations to wheel/rail contact deflection variations, k_c is the contact stiffness, and R_r and R_w are the vertical rail and wheel receptance functions of frequency ω. K_b encapsulates the contact mechanics and wear process parameters and is defined by,

$$K_b = \Delta z_o C_{\xi P} k_c / P_o \,, \quad \Delta z_o = \frac{k_o Q \xi}{2 b \rho} \qquad (2)$$

where Δz_o is the steady state wear (uncorrugated rail longitudinal profile wear per pass), $C_{\xi P,}$ is the sensitivity of lateral creep to normal force fluctuations, P_o is the steady state normal force, Q is the lateral traction force, ξ is the creep, b is the contact patch width and ρ is the rail density and k_0 is the wear coefficient. Details of the derivation of these results have been provided in [15,16,24]. Further, under the assumptions that; 1. one mode of vibration of damping constant ζ_i dominates the response of the combined wheel/rail system and 2. lateral frictional power variations dominate in cornering, the growth of corrugations can be shown to be given by [15,24],

$$\left|\frac{Z_{n+1}}{Z_n}\right| = 1 + G_{r_i} \,, \quad G_{r_i} \approx K_b \left(1 + \frac{K_{ci}}{4\zeta_i (1+\zeta_i)}\right). \qquad (3)$$

The accumulated wear Zn can be summed up over multiple passes as shown subsequently in equations (7) and (8), however the growth rate parameter G_{r_i} can be determined without performing this process. In particular, the corrugation growth equation is in the form of compounding interest where G_{r_i} is the approximate exponential growth rate parameter for mode/frequency *i*, equivalent to an interest rate per vehicle passage. The quotient term involving the parameters K_{ci} and ζ_i is a measure of the growth contribution from the vehicle/track corrugation mode vibration and will be determined by the characteristic properties of the structural dynamics. K_{ci} is a grouped parameter, representing the modal sensitivity of the wheel/rail relative displacement to a change in input longitudinal profile. It is proportional to the ratio of contact stiffness to the corrugation modal stiffness of mode *i*. More details are provided in [24]. This simplified but efficient model doesnot include the effects of contact filtering for short pitch corrugations.

For realistic railway parameters, K_b, K_{ci} and ζ_i are always positive valued. It is noted that an exponential growth rate is expected only to occur initially when changes in environmental, vehicle passing speed, contact patch geometry and wear/deformation conditions are small. However, like interest rates, it is a useful measure of local/present growth and of slow and fast variations from nominal corrugation growth conditions.

When a randomly distributed sequence of pass speeds is introduced of distribution *p(x)* the expected amplitude ratio of the transfer function becomes [15],

$$\frac{Z_{n+1}}{Z_n} = exp \int_{-\infty}^{\infty} ln \left| K_b \frac{1}{1 + k_c\left(R_r(V/\lambda) + R_w(V/\lambda)\right)} + 1 \right| p(x) dx \quad (4)$$

where multiple wheel passes at varying speeds are considered by varying the frequency as a ratio of speed *V* to fixed wavelength, λ. Note the contact mechanics and wear parameter K_b and hence the steady state wear Δz_o is unaffected by speed variations.

2.1 Mechanisms of environmental effects on corrugation growth rate

The field condition variations in contact patch geometry, rainfall and humidity would primarily affect parameters associated with the wheel/rail rolling contact mechanics II and wear on the rail surfaces III. In particular, Equations (1-4) highlight that the growth rate parameter G_r is directly proportional to the contact mechanics and wear parameter, K_b. Therefore mechanisms for the action of environmental effects are expected to act through this parameter. In particular, by inspection of equation (4) changes in rainfall and humidity may be expected to cause changes in corrugation growth rate G_r via four options; (a) the wear coefficient, k_0, (b) the steady state traction, Q, (c) the steady state creep, ξ_0, and/or (d) the sensitivity of creep to normal force fluctuations, $C_{\xi P}$. These are discussed sequentially in the following:

(a) The wear coefficient, $k_{0,}$ is likely to change as interfacial lubricants are known to change the wear behavior of steel in rolling contact. For instance, recent experimental results [26,27] with liquid friction modifiers at fixed frictional power showed that the wear coefficient was drastically reduced. Under fixed frictional power and accounting for associated changes in the creep curve, the experimentally measured wear coefficient was seen to reduce by up to 5-10 times. Water due to rainfall or relative humidity condensate will act in a similar fashion, lowering steady and dynamic wear rates. Further experimental results are required to confirm this effect.

(b) The steady state traction, Q, will be expected to change under different environmental conditions via creep law changes. In particular, the effect of wet running conditions on the creep curve is well discussed in the literature; see for example [21]. The generic effect of a wet contact can be seen in Figure 2.

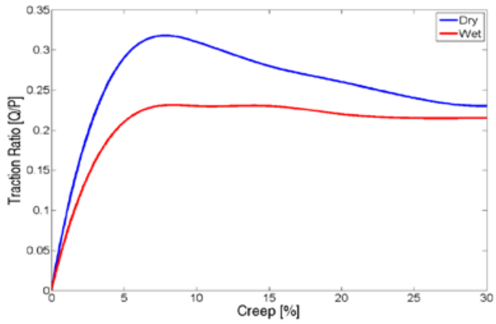

Figure 2. Wet and Dry measured creep curves [reproduced from [21]]

In steady cornering the wheelset will typically take up a fixed angle of attack with respect to the rail, generating large lateral creep. It can be seen that the traction force given by the contact for a fixed creep (arising from a fixed angle of attack) will be higher for a dry contact than a wet contact (approximately 1.5 times higher for saturated creep). Through equation (2) this can be seen to give rise to larger steady state wear in dry conditions than wet.

(c) and (d) Changes to the creep curve will also give different equilibrium cornering conditions, which will have effects on both the steady state creep ξ_0 and the sensitivity of creep to normal force fluctuations $C_{\xi P}$. The likely effect of these for a bogie in cornering is not obvious and requires more detailed field data augmented with accurate vehicle dynamics simulations to quantify. This is beyond the scope of the present paper.

3 FIELD SITE AND METHODS

The field site investigated is a section of suburban track prone to corrugations occurring on a curve with a 242 metre radius, a recommended speed of 50 km/h and a cant of 56 mm. The traffic is composed of 3 and 6 carriage Electrical Multiple Unit (IMU/SMU) trains, half of which stop at the previous station and half that run express with a recommended speed of 60 km/h. Corrugations were measured to have a dominant wavelength in the 90-110mm range, as detailed in the following. As part of an ongoing study into corrugation growth at various sites around Australia (see [23]) measurements of the longitudinal rail profile at this site have been recorded over a period of 4 years using a Corrugation Analysis Trolley [2].

3.1 Environmental Parameters Measurement

Rainfall and humidity were the two environmental parameters chosen for investigation as they have been highlighted in previous research as influencing steady wear and contact mechanics [28]. Rainfall data was collected from Bureau of Meteorology records for the ‘Brisbane’ station which was assumed to be indicative (on average) of the amount of rain at the suburban Brisbane site some 17km away. The cumulative rainfall observed between each CAT test was divided by the number of days in that period, to give an average rainfall per day for each interval. This was compared against variations in the corrugation growth rate parameter G_r and steady state wear Δz for comparison.

3.2 Rail Profile Measurement and Analysis

CAT measurements of longitudinal rail profile were taken semi-periodically (averaging approximately every 2 months) between initial grinding in September 2006 and final grinding in May 2009. These CAT measurements were used to obtain the corrugation growth over the entire test section as well as that initiated by a weld over the entire period. 3rd Octave filtering techniques were utilised on raw CAT output files of longitudinal profile to identify corrugation wavelength as shown in Figure 3.

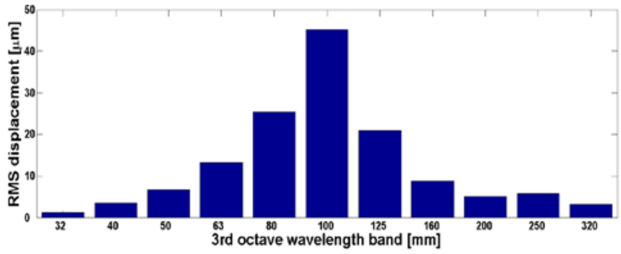

Figure 3. 3rd Octave graph of profile from the test section for 1,053,256 passes

The corrugation growth was then obtained by plotting the magnitude of the RMS displacement of the dominant corrugation 100mm band for each measurement against the total number of driven wheel passes over that interval.

As described in section 2, exponential corrugation growth is theoretically expected due to the feedback nature of the corrugation growth mechanism for static rail conditions , i.e.

$$Z_n = Z_0 e^{G_r n} \qquad (5)$$

where Z_n is the nth pass profile amplitude and G_r is the corrugation growth rate parameter. A local corrugation growth rate is defined to capture the variation in G_r due to variations in environmental parameters, contact geometry, the damage mechanism etc over time. The local corrugation growth rate between the m^{th} and n^{th} passes can then be determined by looking at the ratio of corrugation amplitudes between successive measurements, i.e.

$$G_r \approx \frac{log\left(\frac{Z_n}{Z_m}\right)}{n-m} \qquad (6)$$

Corrugation growth was also captured as initiated by a weld for comparison using an analysis of CAT measurements near a thermite weld calibrated to less regular miniprof measurements. In particular, the rate of peak to peak weld growth and steady state wear on the test site was estimated by observing the growth of the 'dip' with a wavelength of about 80mm around the heat affected zone (HAZ) of a thermite weld. The methodology is based on the impulse response of the corrugation growth rate transfer function,

$$\frac{Z_n(\omega)}{Z_0(\omega)} = \prod_{n=1}^{N}\left(\frac{\Delta z_0 C_{\xi P}}{P_0}\frac{k_c}{1+k_c\left(R_r(x_n\omega)+R_w(x_n\omega)\right)}+1\right) \quad (7)$$

where Z_n is the Fourier transform of the longitudinal rail profile on the n^{th} pass, and x_n is the ratio of the current pass speed to the mean speed. If the impulse response is taken, the total response is given by [17]

$$\frac{Z_{n(\omega)}}{Z_0(\omega)} = (1+K_b)^N(1+higher\ order\ terms) \qquad (8)$$

In the time domain the impulse area will grow proportionally to $(1+K_b)^N$ as detailed in [17].

The peak to trough size of the dip was determined for each CAT measurement over the entire grinding cycle. A representative measurement is shown in figure 4.

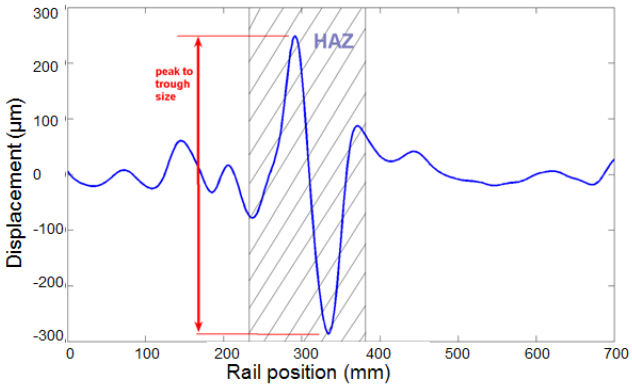

Figure 4. CAT profile (30-300mm filtered) for heat affected zone (HAZ) in thermite weld at test site

Note the filtering range 30-300mm was used based on perceived reliability from the moving accelerometer based device (the CAT). The resulting weld growth can then be determined as a function of driven wheel passes (as shown latter in figure 5). This also provides a measurement from which the contact mechanics and wear parameter K_b and the steady state wear Δz_o can be determined directly using equations (2) and (8). According to the theory presented in section 2, these parameters should be immune to variations in the pass speed distribution so this effect can be isolated.

3.3 Isolating Environmental Variations in Corrugation Growth

Exponential growth of corrugations is expected only to occur when the effect of changes in the quasistatic conditions such as contact patch geometry and wear/plastic deformation conditions are small. In the field measurements these effects slowly change the steady local corrugation growth rate G_r over time as the amplitude increases. In particular, widening of the contact patch occurs due to conforming wear and/or deformation of wheel and rail surfaces over many wheelset passages. In addition as the corrugations become larger in amplitude, the dynamic forces increase in level to conditions under which plastic deformation occurs in addition to wear ie the damage mechanism changes. Changes in the damage mechanism may also be due to non-uniform distribution of wear in relation to the corrugation profile. These effects need to be taken into account when analysing the field data to isolate the effect of environmental variations which occur on a much smaller time scale. Experimental evidence [27] suggests that the broadening of the lateral contact patch width and changes in the damage mechanism on steady state wear, over a number of wheelset passes may be described by,

$$\Delta z_0 = \frac{k_0 Q_0 \xi_0}{2\rho b_0 (n+n_0)^a} \qquad (9)$$

Since corrugation growth rate is proportional to the steady state wear (eqns 1-3) the reduction in steady state wear is equal to the reduction in corrugation growth rate suggesting a local growth rate function of the same form. Hence a high pass filter of the form of equation (9) has been applied to the local corrugation growth data to isolate the effects of environmental fluctuations occurring on a much smaller timescale (ie order of 10^3 wheelset passages as compared to 10^6).

4 RESULTS AND DISCUSSIONS

The methods described in section 3 were applied to obtain and analyse the field data to identify correlations between environmental, steady state wear and corrugation growth variations as detailed in the following.

Corrugation growth history was obtained first using the CAT measured RMS displacement of the 100mm band at multiple intervals over 2.6yrs. For comparison, the growth in steady state wear was also obtained using the methods described in section 3 as shown in Figure 5.

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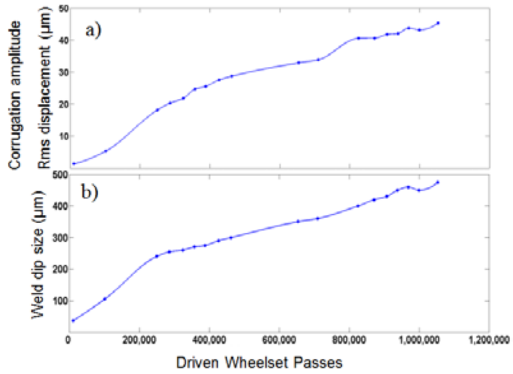

Figure 5. Growth in a) 100mm band 3rd Octave of measured longitudinal rail profile at test section and b) weld/HAZ dip size.

Figure 5a) depicts an initial exponential like growth in corrugations that is retarded slowly over time. The similarity in the trend of growth is clear when comparing Figure 5 a) and b) and confirms that corrugation growth initiated from initial roughness is similar that initiated from a weld. On a smaller time scale there is some evidence of local variations in these trends. To investigate these more directly, the local corrugation and steady state wear growth rates were measured as described in section 3.2 and are shown in Figure 6.

The most obvious trend in Figure 6 a) and b) is the progressive decrease in corrugation growth rate throughout the growth cycle due to the effects of contact patch widening and deformation changes with time. To take this into account, a trend-line based on contact patch widening was fitted to the experimentally derived growth rate by assuming the power law form of equation (9) with a = 1.64. The similarity of the trends in Figure 6 a) and b) highlight a proportional dependence of corrugation growth rate G_r to the contact mechanics and wear, K_b, and steady state wear, Δz_o, parameters as theoretically defined in equations (1-3) and in section 2.

To isolate the environmental effects on corrugation growth, measured variations in the growth rate from this trend-line of equation (9) were determined and compared against measured variations in humidity and rainfall as shown in Figure 7. In particular, the variations are highlighted by plotting the local growth rate as a ratio of the smooth trendline. Figure 7 visually shows an approximate inverse (or negative) correlation between variations in the environmental factors (rainfall and relative humidity) and corrugation growth rate, respectively.

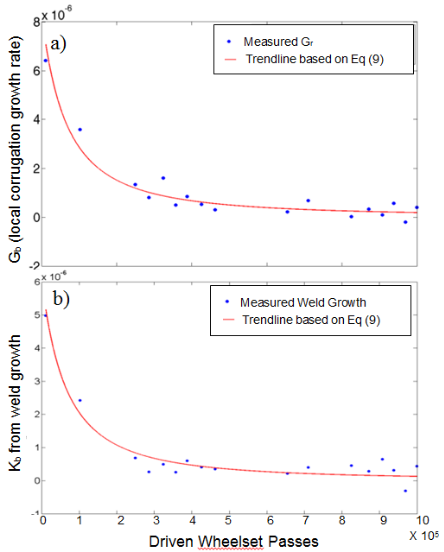

Figure 6. Histories of a) Local corrugation growth rate in the 100mm band and b) contact mechanics and wear parameter (proportional to steady state wear) estimated from weld growth; as functions of wheelset passes.

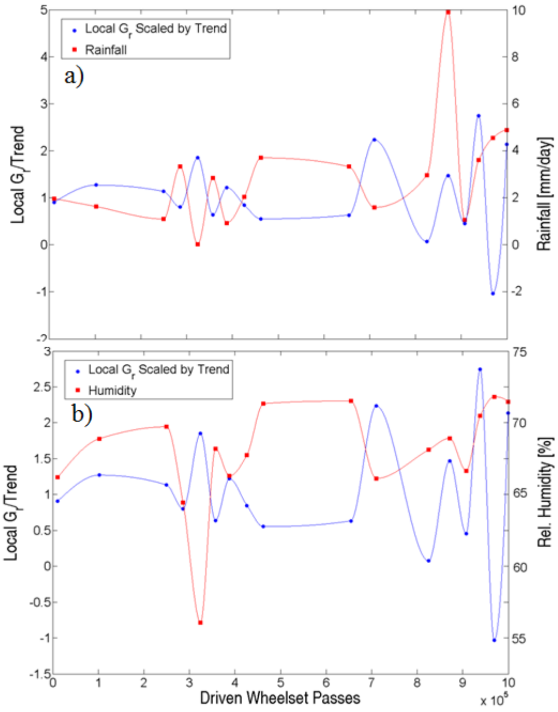

Figure 7. Field measured a) Rainfall and b) relative humidity versus Local corrugation growth rate.

The statistical significance of these correlations were quantified as shown in Figure 8.

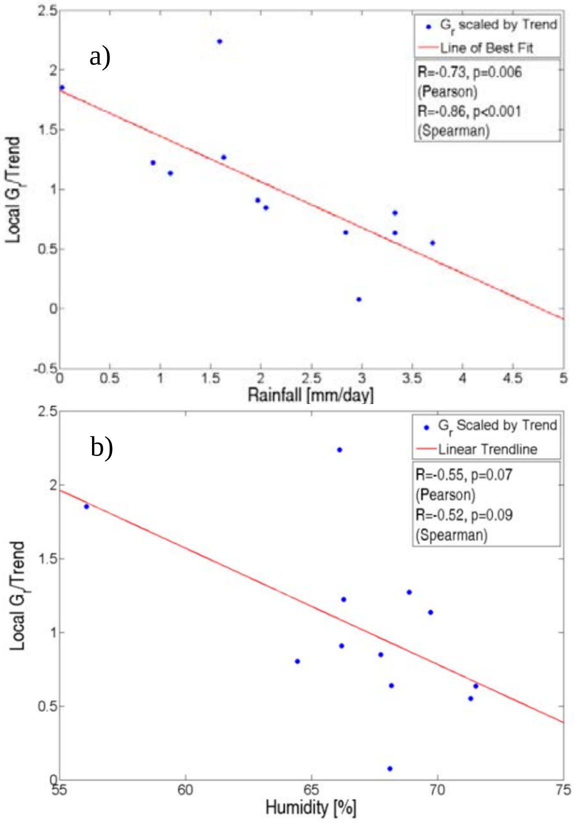

Figure 8. Correlations between Local corrugation growth rate and a) average rainfall and b) relative humidity in measurement interval.

Figure 8 confirms a strong correlation (< 0.1% probability of no correlation based on p test) between variations in rainfall and corrugation growth rate. The correlation with humidity is marginal (< 10% probability of no correlation based on p test). Also humidity and rainfall were shown to be correlated to themselves (R=0.62, p=0.03 for the interval considered), so the correlation from rainfall is considered the primary effect.

Correlations between variations in the environmental factors and steady state wear calculated from the weld growth were also determined but no significant correlations were found for the small time scale variations. This may be due to differences in the contact and wear mechanics in the heat affected zone near the weld. However the strong correlations in trends shown between corrugation and weld growth in Figure 5 and corrugation growth rate and contact mechanics and wear parameter (and steady state wear) in Figure 6 over the long period shown, indicates that these parameters may encapsulate the effects of environmental growth in the long term.

5 IMPLICATIONS FOR CORRUGATION CONTROL

The field results shown in the previous section were applied to a corrugation case study in which both environmental and pass speed variations occurred. In particular, the same site has been the focus of an investigation of pass speed variation on corrugation growth rate (see [27] and [15] for details). In particular, the distribution of passing train speed was controlled such that the average speed standard deviation over the section was controlled and increased from a nominal 4.4 km/h to 7.6 km/h. This speed-based corrugation control system (CCS) was turned on when the corrugation amplitude exceeded approximately 40 microns as shown in Figure 9. During this control phase (CCS ON) the vehicle passing speed standard deviation was increased by 1.7 times, while maintaining the same average speed as the CCS Off condition. The predicted reduction in corrugation growth rate (without considering environmental variations) due to this increase in pass speed distribution was 38% [15]. However, the field measured reduction obtained was approximately 60% as shown in Figure 9 and Table 1.

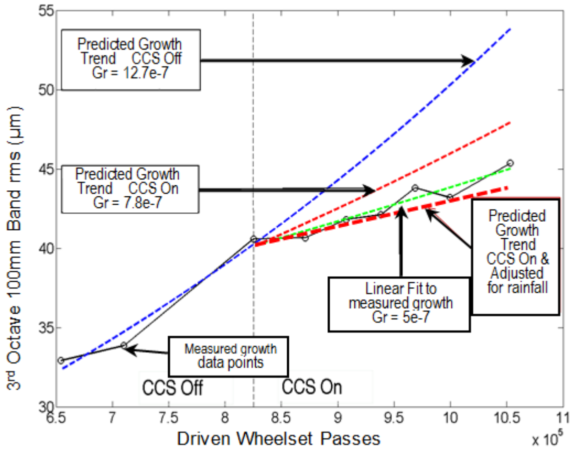

Figure 9. Measured, predicted and adjusted corrugation growth rates for field site test with a Corrugation Control System (CCS).

To account for the difference in predicted and measured performance, measurements of rainfall during this period were performed in order determine if environmental factors had significantly changed. During the control period (CCS ON) the average rainfall increased from 2.1±0.4 mm/day up to 4.1±1.2 mm/day. According to the inverse correlation identified in Figure 7 this higher rainfall will lower the corrugation growth rate. Based on this, the linear trend-line for correlation between corrugation growth and rainfall shown in Figure 8 was used to predict the influence of the increased rainfall in this period on corrugation growth. Note that any negative predictions of growth rate have been set instead to zero in these calculations. These growth rate estimates are outlined in table 1 and shown compared to the observed profile growth in Figure 9.

Table 1. Corrugation Growth rate predictions under different conditions

Condition	Growth rate parameter Gr
Uncontrolled	1.27×10^{-6}
Uncontrolled, rainfall adjusted (-46%)	0.69×10^{-6}
Controlled (-38%)	0.78×10^{-6}
Controlled & rainfall adjusted (-63%)	0.47×10^{-6}
Measured (-60%)	$\mathbf{0.5\times10^{-6}}$

The results in table 1 highlight a close match between the predicted (-63%) and field (-60%) results once environmental effects have been taken into account. Note that the effect of rainfall or vehicle passing speed variation, individually provide a substantial drop in corrugation growth. These results highlight the importance of considering environmental variations on corrugation growth. It is noted that in this case study, the effects of contact patch widening and/or the damage mechanism based on the results in Figure 6, are assumed to be relatively small and slow after the CCS is activated and relatively small as compared to the environmental effects over the subsequent period investigated. However it is recommended to study this more closely in future field investigations.

6 CONCLUSIONS

Environmental effects have been investigated in the field based on insight gained using mechanics-based modelling of the growth-rate of wear-type corrugations. The relationship between average daily rainfall, humidity and the growth of rail corrugation was observed by comparing weather data to results from the monitoring of longitudinal rail profile on a suburban test site in Australia. Corrugation growth rate (G_r) was determined by systematically measuring the longitudinal rail profile with a Corrugation Analysis Trolley (CAT) on a 240m radius, narrow gauge, concrete-sleepered curve on the metropolitan network that was known to suffer from significant short-pitch rail corrugation of about 95mm wavelength. The field results show a strong correlation between variations in rainfall and the growth rate of corrugations in the field.

Results also showed a strong correlation between corrugation growth along the test site and peak to peak weld growth which is predicted via mechanics based modelling. Contact patch widening and changes in the the damage mechanism are shown to be important effects to consider as the amplitude of corrugations grow. The results are then applied to a field case under variations in environmental conditions and pass speed distribution and show that changes in growth rate were substantial and able to be (retrospectively) predicted. The results have important implications for quantifying field corrugation growth and baselining the effect of field corrugation control strategies such as friction modifiers and variable speed control.

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