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The quark strange star in the enlarged Nambu–Jona-Lasinio model

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Abstract. The strange quark star is investigated within the enlarged $SU(3)$ Nambu–Jona-Lasinio model. The stable quark star can exist until a maximal configuration with $\rho_m = 3.1 \times 10^{15}$ g cm^{-3} with $M_m = 1.61\ M_\odot$ and $R_m = 8.74$ km is reached. Strange quarks appear for density above $\rho_c = 9.84$ g cm^{-3} for the quark star with radius $R_c = 8.003$ km and $M_c = 0.77\ M_\odot$. A comparison of quark star properties obtained in the quark mean-field approach to a neutron star model constructed within the relativistic mean-field theory is presented.

1. Introduction

At sufficiently high density, the transition to deconfined strange quark matter is widely expected. Chiral symmetry is spontaneously broken in the QCD vacuum. Lattice QCD simulations at nonzero temperature T and zero baryon chemical potential μ_B indicate that chiral symmetry is restored above a temperature $T \sim 150$ MeV [1]. The Nambu–Jona-Lasinio (NJL) [2] model is an effective theory which is believed to be related to QCD at low energies, when one has integrated out the gluon fields. The NJL model might yield reasonable results in the density range where confinement is no longer crucial but chiral symmetry as a symmetry of full QCD remains important. The NJL model has proved to be very successful in the description of the spontaneous breakdown of chiral symmetry exhibited by the true (nonperturbative) QCD vacuum. This model has been extensively used over the past few years not only to describe hadron properties [3] (see for reviews [4, 5]) and phase transitions in dense matter [6]–[8], but also to describe the quark strange stars [9]–[13]. The detailed properties of the quark phase in compact stars have been a topic of recent interest [14] (for a review see [15, 16]).

Quark strange stars are astrophysical compact objects which are entirely made of deconfined u, d, s quark matter (*strange matter*) staying in β-equilibrium. The possible existence of strange

stars is a direct consequence of the conjecture [17] that strange matter may be the absolute ground state of strongly interacting matter.

The three-flavour NJL model has been discussed by many authors; see e.g. [18]. For the quark phase we follow Buballa and Oertel [19] in using the three-flavour version of the NJL model.

The aim of this paper is to investigate a strange quark star within the enlarged $SU(3)$ NJL model. The comparison of quark star properties obtained in the quark mean-field (QMF) approach to a neutron star model constructed in framework of the relativistic mean-field (RMF) theory will be made. This paper is organized as follows. In section 2 we present general properties of the NJL model in the mean-field approach based on the Feynman–Bogoliubov inequality for the free energy of the system. In this section the equation of state (EoS) employed is calculated for NJL model. The EoS is used then to determine the equilibrium configurations of the quark star in section 3. Finally, in section 4 the main implications of the results are summarized.

2. Nambu–Jona-Lasinio model

The NJL [2] model has been widely used for describing hadron properties [20] and the chiral phase transition [21]. The enlarged [22] simplest version of the model is given by the Lagrangian

$$L = \bar{q}(\mathrm{i}\gamma^\mu \partial_\mu - m_0)q + \tfrac{1}{2}G_\mathrm{s} \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}\lambda^a \mathrm{i}\gamma_5 q)^2] - 2K \prod_{f=\{\mathrm{u,d,s}\}} (\bar{q}_f q_f)$$
$$- \tfrac{1}{2}G_\mathrm{v} \sum_{a=0}^{8} [(\bar{q}\gamma^\mu \lambda^a q)(\bar{q}\gamma_\mu \lambda^a q) + (\bar{q}\gamma^\mu \gamma_5 \lambda^a q)(\bar{q}\gamma_\mu \gamma_5 \lambda^a q)]$$
$$+ \mathrm{i} \sum_{f=1}^{2} \overline{L_f}\gamma^\mu \partial_\mu L_f - \sum_{f=1}^{2} m_f \overline{L}_f L_f + B_0. \tag{1}$$

The first term contains the free kinetic part, including the current quark $q_f = \{q_\mathrm{u}, q_\mathrm{d}, q_\mathrm{s}\}$ masses m_0 which break explicitly the chiral symmetry of the Lagrangian, and the term representing the free relativistic leptons $L_f = \{\mathrm{e}^-, \mu^-\}$. The fermion fields are composed of quarks and leptons (electrons, muons). Here q denotes a quark field with three flavours, u, d and s, and three colours. We restrict ourselves to the isospin $SU(2)$ unbroken-symmetry case, $m_0^\mathrm{u} = m_0^\mathrm{d}$; thus

$$m_0 = m_{0,f}\delta_{f,f'} = \begin{pmatrix} m_{0,\mathrm{u}} & & \\ & m_{0,\mathrm{d}} & \\ & & m_{0,\mathrm{s}} \end{pmatrix}. \tag{2}$$

The generators of the $u(3)$ algebra $\lambda^a = \{\lambda^0 = \sqrt{2/3}I, \lambda^i\}$ (where I is an identity matrix, λ^i are Gell-Mann matrices of the $su(3)$ algebra) obey $\mathrm{Tr}(\lambda^a \lambda^b) = 2\delta_{ab}$. Due to this normalization of this algebra the coupling constants G_s and G_v can be redefined and written as $\bar{G}_\mathrm{s} = (2/3)G_\mathrm{s}, \bar{G}_\mathrm{v} = (2/3)G_\mathrm{v}$.

The NJL model is nonrenormalizable; thus it is not defined until a regularization procedure has been specified. This cut-off limits the validity of the model to momenta well below the cut-off. In most of our calculations we will adopt the parameters presented in table 1. With Λ, G_s specified above, chiral symmetry is spontaneously broken in vacuum.

Table 1. Parameter sets of the NJL models.

	NJL $(su(2))$ [21]	NJL (I) [19]	NJL (II) [22]	ENJL [22]
$m_{\mathrm{u}} = m_{\mathrm{d}}$	5.5 MeV	5.5 MeV	5.5 MeV	3.61 MeV
m_{s}	0	140.7 MeV	132.9 MeV	88.0 MeV
Λ	631 MeV	602.3 MeV	631.4 MeV	750.0 MeV
$\bar{G}_{\mathrm{s}}\Lambda^2$	2.19	3.67	3.67	3.624
$K\Lambda^5$	0	12.36	9.40	9.40
$\bar{G}_{\mathrm{v}}\Lambda^2$	0	0	0	3.842

The model contains eight parameters of the standard NJL model (the current mass $m_{0,\mathrm{u}}$, $m_{0,\mathrm{s}}$ of the light and strange quarks, the coupling constants G_{s}, determinant coupling K and the momentum cut-off Λ) and the additional constant G_{v} ($G_{\mathrm{v}} = x_{\mathrm{v}}G_{\mathrm{s}}$, $x_{\mathrm{v}} = 1.06$ [12]). In the quark massless limit the system has a $U(3)_L \times U(3)_R$ chiral symmetry. The system has following global currents: the baryon current

$$J_B^\nu = \tfrac{1}{3}\overline{q}\lambda^0\gamma^\nu q \tag{3}$$

and the isospin current which exists only in the asymmetric matter:

$$J_3^\nu = \tfrac{1}{2}\overline{q}\gamma^\nu\lambda^3 q, \qquad J_8^\nu = \tfrac{1}{2}\overline{q}\gamma^\nu\lambda^8 q. \tag{4}$$

The conserved baryon and isospin charges are given by the relations

$$Q_i = \tfrac{1}{3}\int \mathrm{d}^3x\, q^+\lambda^i q, \qquad i = \{0,3,8\}$$

which are connected to commuting Cartan algebra. The physical system is defined by the thermodynamic potential [23]

$$\Omega = -kT\ln\mathrm{Tr}(\mathrm{e}^{-\beta(H-\mu^i Q_i)}) \tag{5}$$

where H stands for the Hamiltonian. It is more convenient to use chemical potentials connected to the quark flavour f in such a way that $\mu^i Q_i = \sum_f \mu_f \bar{Q}_f$:

$$\mu_{\mathrm{u}} = \mu^0 + \mu^3 + \frac{1}{\sqrt{3}}\mu^8 \qquad \mu_{\mathrm{d}} = \mu^0 - \mu^3 + \frac{1}{\sqrt{3}}\mu^8 \qquad \mu_{\mathrm{s}} = \mu_{\mathrm{s}}^0 - \frac{2}{\sqrt{3}}\mu^8. \tag{6}$$

Quarks and electrons are in β-equilibrium which can be described as a relation among their chemical potentials:

$$\mu_{\mathrm{d}} = \mu_{\mathrm{u}} + \mu_{\mathrm{e}} = \mu_{\mathrm{s}} \qquad \mu_\mu = \mu_{\mathrm{e}}$$

where μ_{u}, μ_{d}, μ_{s} and μ_{e}, μ_μ stand for quarks and lepton chemical potentials, respectively. These conditions mean that matter is in equilibrium with respect to the weak interactions. If the electron Fermi energy is high enough (greater than the muon mass) in the neutron star matter, muons start to appear as a result of the following reactions:

$$\mathrm{d} \rightarrow \mathrm{u} + \mathrm{e}^- + \overline{\nu}_{\mathrm{e}} \qquad \mathrm{s} \rightarrow \mathrm{u} + \mu^- + \overline{\nu}_\mu.$$

The neutron chemical potential is

$$\mu_{\mathrm{n}} \equiv \mu_{\mathrm{u}} + 2\mu_{\mathrm{d}}.$$

In a pure quark state the star should to be charge neutral. This gives us an additional constraint on the chemical potentials:

$$\tfrac{2}{3}n_{\mathrm{u}} - \tfrac{1}{3}n_{\mathrm{d}} - \tfrac{1}{3}n_{\mathrm{s}} - n_{\mathrm{e}} - n_{\mu} = 0 \tag{7}$$

where n_f ($f \in \mathrm{u,d,s}$), n_{e} are the particle densities of quarks and electrons, respectively. The EoS can now be parametrized by just one parameter, namely the dimensionless u quark Fermi momentum x ($k_{F,\mathrm{u}} = Mx$ ($M = 939$ MeV is the nucleon mass)).

In the mean-field approach the quantum correlations

$$(A - \langle A \rangle)(B - \langle B \rangle) = AB - A\langle B \rangle - B\langle A \rangle + \langle A \rangle \langle B \rangle \sim 0$$

may be neglected. This allows us to replace AB by

$$AB \sim A\langle B \rangle + B\langle A \rangle - \langle A \rangle \langle B \rangle.$$

Using this approximation, the Lagrange function $\mathcal{L}$ may be expressed as

$$\bar{L} = \bar{q}(\mathrm{i}\gamma^{\mu}D_{\mu} - m_0)q + g_{\mathrm{s}}\sum_{a=0}^{8}\sigma^a(\bar{q}\lambda^a q) - \tfrac{1}{2}m_{\mathrm{s}}^2\sigma^a\sigma^a + \tfrac{1}{2}m_{\mathrm{v}}^2 V_{\mu}^a V^{a\mu}$$
$$- 2K\sum_f(\bar{q}_f q_f)\prod_{f'\neq f}\langle \bar{q}_{f'}q_{f'}\rangle + 4K\prod_f\langle \bar{q}_f q_f \rangle \tag{8}$$

where the covariant derivative is given by

$$D_{\mu} = \partial_{\mu} + \tfrac{1}{2}\mathrm{i}g_{\mathrm{v}}V_{\mu}^a\lambda^a. \tag{9}$$

Here, the meson fields first appear as nondynamical variables

$$G_{\mathrm{s}}\langle \bar{q}\lambda^a q \rangle = g_{\mathrm{s}}\sigma^a \tag{10}$$

$$G_{\mathrm{v}}\langle \bar{q}\lambda^a\gamma_{\mu}q \rangle = g_{\mathrm{v}}V_{\mu}^a. \tag{11}$$

This pattern may be extended to the axial mesons:

$$G_{\mathrm{s}}\langle \bar{q}\lambda^a\mathrm{i}\gamma_5 q \rangle = g_{\mathrm{s}}\phi^a \tag{12}$$

$$G_{\mathrm{v}}\langle \bar{q}\lambda^a\gamma_{\mu}\mathrm{i}\gamma_5 q \rangle = g_{\mathrm{v}}A_{\mu}^a. \tag{13}$$

The meson masses are defined as

$$m_{\sigma} = g_{\mathrm{s}}/\sqrt{G_{\mathrm{s}}} \tag{14}$$

$$m_{\mathrm{v}} = g_{\mathrm{v}}/2\sqrt{G_{\mathrm{v}}}. \tag{15}$$

This is a process of bosonization in which the NJL model produces essentially the $u(3)$ linear sigma model as an approximate effective theory for the scalar and pseudoscalar meson sector [24].

In this paper the variational method based on the Feynman–Bogoliubov inequality [25] is incorporated (see more details in [26]):

$$\Omega \leq \Omega_1 = \Omega_0(m_{eff}) + \langle H - H_0 \rangle_0 \tag{16}$$

with the trial Lagrange function described by

$$\mathcal{L}_0(m_{eff}) = \bar{q}(\mathrm{i}\gamma^{\mu}\bar{D}_{\mu} - m_{eff})q \tag{17}$$

and suggested by the mean-field form of the Lagrange function (8). The covariant derivative

$$\bar{D}_{\mu} = \partial_{\mu} + \tfrac{1}{2}\mathrm{i}g_{\mathrm{v}}V_{\mu}^a\lambda^a \tag{18}$$

is limited to the commuting Cartan subalgebra $\lambda^i = \{\lambda^0, \lambda^3, \lambda^8\}$. This approach introduces the fermion interactions with homogeneous boson condensates σ^a, V_μ^a which together with the effective masses m_{eff} will be treated as variational parameters. Ω_0 is the thermodynamic potential of the effectively free-quasiparticle system:

$$\Omega_0(m_{eff}) = E_0 - k_B T \frac{N_q}{2\pi^2} \sum_f \int_0^\Lambda \mathrm{d}k \, k^2 \left(\ln \left(1 + \mathrm{e}^{-\beta\left(\sqrt{k^2+m_{eff,f}^2}-\mu_f\right)} \right) \right.$$
$$\left. + \ln \left(1 + \mathrm{e}^{-\beta\left(\sqrt{k^2+m_{eff,f}^2}+\mu_f\right)} \right) \right). \tag{19}$$

We cannot ignore the energy of quantum fluctuations because it depends on the quark effective mass. As fermions give $-(1/2)\hbar\omega$ to the vacuum energy, we get

$$\mathcal{E}_0 = -\frac{N_q}{2\pi^2} \sum_{f=\{u,d,s\}} \int_0^\Lambda \mathrm{d}k \, k^2 \sqrt{k^2 + m_{eff,f}^2} \tag{20}$$

assuming that if $m_{eff} = m_0$, then the energy of quantum fluctuations may be neglected. The effective quark masses entering into the Lagrangian function $L_0(m_{eff})$ of the trial system are calculated from the extremum conditions

$$\frac{\partial \Omega_1}{\partial m_{eff,f}} = 0 \tag{21}$$

which give

$$(m_{eff})_{f,f'} = m_{eff,f}\delta_{f,f'} = m_c\delta_{f,f'} - G_s \sum_{a=0}^{8} \langle \bar{q}_f \lambda^a q_f \rangle_0 \lambda_{f,f'}^a + 2K\delta_{f,f'} \prod_{f'\neq f} \langle \bar{q}_{f'} q_{f'} \rangle_0$$

or

$$m_{eff,f} = m_{c,f} - \bar{G}_s \langle \bar{q}_f q_f \rangle_0 + 2K \prod_{f'\neq f} \langle \bar{q}_{f'} q_{f'} \rangle_0, \tag{22}$$

where

$$\langle \bar{q}_f q_f \rangle_0 = \frac{m_{eff,q} N_q}{\pi^2} \int_0^\Lambda \frac{k^2 \, \mathrm{d}k}{\sqrt{k^2 + m_{eff,f}^2}} \left\{ \frac{1}{\exp\left(\beta\left(\sqrt{k^2+m_{eff,f}^2} - \mu_f\right)\right) + 1} \right.$$
$$\left. + \frac{1}{\exp\left(\beta\left(\sqrt{k^2+m_{eff,f}^2} + \mu_f\right)\right) + 1} - 1 \right\}. \tag{23}$$
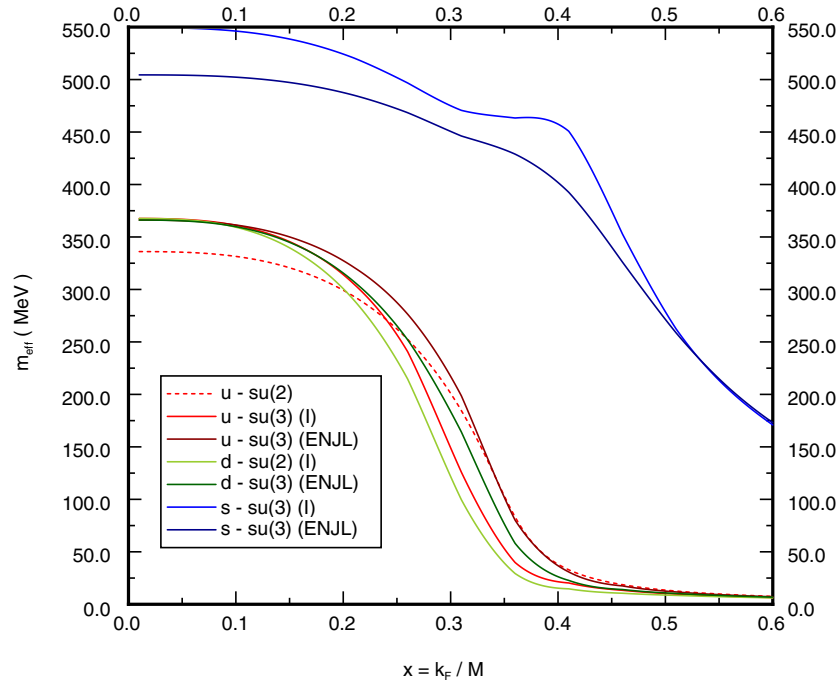
At $T = 0$ we have only

$$\langle \bar{q}_f q_f \rangle_0 = -m_{eff,f} \frac{N_q}{2\pi^2} \int_{k_F}^\Lambda \mathrm{d}k \, \frac{k^2}{\sqrt{k^2 + m_{eff,f}^2}}. \tag{24}$$

In vacuum we get the constituent quarks with mass

$$m_c = m_0 - 2\bar{G}_s \langle \bar{q}q \rangle_{0v} + 2K \prod_{f'=\{u,d,s\}} \langle \bar{q}_{f'} q_{f'} \rangle_{0v} \tag{25}$$

where

$$\langle \bar{q}_f q_f \rangle_{0v} = -m_{v,f} \frac{N_q}{2\pi^2} \int_0^\Lambda \mathrm{d}k \, \frac{k^2}{\sqrt{k^2 + m_{v,f}^2}}. \tag{26}$$

Figure 1. The effective quark masses for different NJL models as a function of the u quark dimensionless Fermi momentum $x = k_{F,\mathrm{u}}/M$ ($M = 939$ MeV is the nucleon mass).

At minimum the effective free energy has the form

$$\Omega_{eff} = \Omega_1|_{min} = \Omega_0(m_{eff}) + B_{eff} \tag{27}$$

with the effective bag constant

$$B_{eff} = \tfrac{1}{2}G_\mathrm{s}\langle\bar{q}\lambda^a q\rangle^2 - 4K \prod_{f=\{\mathrm{u,d,s}\}} \langle\bar{q}_f q_f\rangle - \tfrac{1}{2}G_\mathrm{v}\langle\bar{q}\gamma^\mu\lambda^a q\rangle\langle\bar{q}\gamma_\mu\lambda^a q\rangle - B_0. \tag{28}$$

Quarks as effectively free quasiparticles in vacuum with nonvanishing bag 'constant'. The constant B_0 was chosen in this way to have free massive ($m_{\mathrm{v,u,d}} = 367.61$ MeV, $m_{\mathrm{v,s}} = 549.45$ MeV for the NJL (I) parameters set and $m_{\mathrm{v,u,d}} = 366.13$ MeV, $m_{\mathrm{v,s}} = 504.13$ MeV for the enlarged NJL model). However, in a high-density medium they are less massive (figure 1) but the effective bag constant (figure 2) grows to $B_{eff}^{1/4} \simeq 150$–180 MeV. The frequently encountered case with current quarks and bag constant is valid only in very high-density limit. This is the case when the quark matter phase is being modelled in the context of the MIT bag model [9, 27, 28] as a Fermi gas of u, d and s quarks. In this model the phenomenological bag constant B_{MIT} is introduced to mimic QCD interactions. In the original MIT bag model the bag constant was constant and the value $B = B_c = (154.5 \text{ MeV})^4$ makes the strange matter absolutely stable.

To avoid quantum fluctuations, the meson fields may be redefined to produce the phenomenological sigma field as

$$g_\mathrm{s}\varphi_a = G_\mathrm{s}(\langle\bar{q}\lambda^a q\rangle_0 - \langle\bar{q}\lambda^a q\rangle_{0\mathrm{v}}) \tag{29}$$

so the effective quark mass can be rewritten as

$$m_{eff} = m_c - g_\mathrm{s}\varphi_a\lambda^a. \tag{30}$$
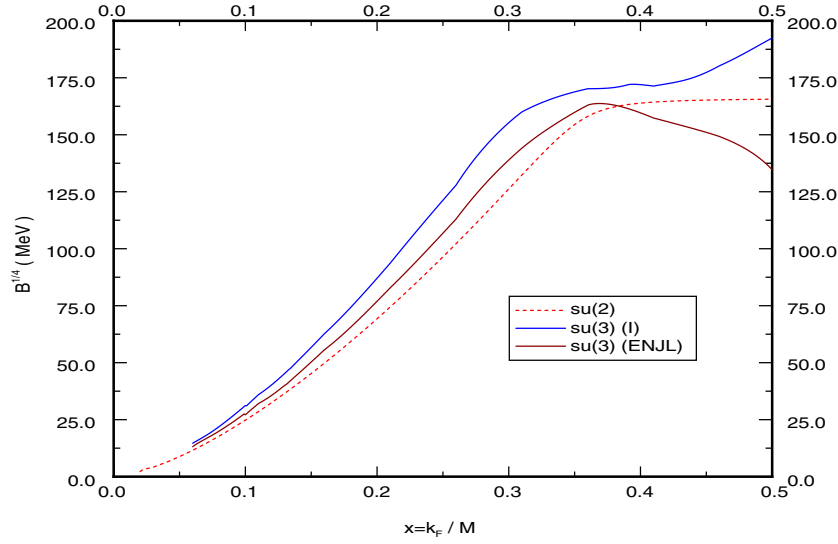
Figure 2. The effective bag constant as a function of the u quark dimensionless Fermi momentum $x = k_{F,\mathrm{u}}/M$ ($M = 939$ MeV is the nucleon mass).

Thus, the effective quark mean-field (QMF) theory appears. The Lagrange function in the mean-field approximation may be written in the following form:

$$\mathcal{L}_{QMF} = \bar{q}(\mathrm{i}\gamma^\mu D_\mu - m_c)q + \tfrac{1}{2}\partial_\mu\varphi_a\partial^\mu\varphi_a - \tfrac{1}{2}m_\sigma^2\varphi_a\varphi_a - \tfrac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} + \tfrac{1}{2}m_\mathrm{v}^2 V_\mu^a V^{a\mu}. \tag{31}$$

Unfortunately, the vacuum quantum fluctuations are missed, but meson fields gain the dynamical character. Restricting ourselves to just the $u(2) \times u(1)$ subalgebra ($a = \{0, 1, 2, 3, 8\}$) case we have the simplest version of the QMF theory. Defining a new basis with τ^a, $a = \{0, 1, 2, 3, 4\}$ as

$$\tau^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \tau^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & 0 \end{pmatrix} \qquad \text{for } \tau^4 = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{32}$$

($i = \{1, 2, 3\}$) the meson fields may be decomposed as follows:

$$\varphi = \varphi_a\tau^a = \sigma\tau^0 + \delta_i\tau^i + \sigma_\mathrm{s}\tau^4 \qquad \text{and} \qquad V_\mu = V_\mu^a\lambda^a = \omega_\mu\tau^0 + \omega_{\mathrm{s},\mu}\tau^4 + b_i\tau^i.$$

Now the new meson fields are denoted by

$$\omega_\mu = \sqrt{\frac{2}{3}}V_\mu^0 + \frac{1}{\sqrt{2}}V_\mu^8,$$

$$\omega_{\mathrm{s},\mu} = \sqrt{\frac{1}{3}}V_\mu^0 - V_\mu^8,$$

and $b_\mu^i = V_\mu^i$ (with $i = \{1, 2, 3\}$), respectively. The simplest $u(2)$ version ($\sigma_\mathrm{s} = 0$, $\omega_{\mathrm{s},\mu} = 0$) has the Lagrange density function with the following form:

$$\begin{aligned}
\mathcal{L}_{QMF} = {} & \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \tfrac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \tfrac{1}{2}M_\omega^2\omega_\mu\omega^\mu + \tfrac{1}{2}\partial_\mu\delta_i\partial^\mu\delta_i - \tfrac{1}{2}m_\delta^2\delta_i^2 \\
& - \tfrac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \tfrac{1}{2}M_\rho^2 b_\mu^a b^{a\mu} + \bar{q}(\mathrm{i}\gamma^\mu D_\mu - m_c)q + g_\sigma^q\sigma\bar{q}q + g_\delta^q\delta_i\bar{q}\tau^i q. \tag{33}
\end{aligned}$$

The field tensors $R_{\mu\nu}^a$, $\Omega_{\mu\nu}$ and the covariant derivative D_μ are given by

$$R_{\mu\nu}^a = \partial_\mu b_\nu^a - \partial_\nu b_\mu^a + g_\rho\varepsilon_{abc}b_\mu^b b_\nu^c, \tag{34}$$

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{35}$$

$$D_\mu = \partial_\mu + \tfrac{1}{2}\mathrm{i}g_\rho b_\mu^a \tau^a + \tfrac{1}{2}\mathrm{i}g_\omega \omega_\mu. \tag{36}$$

The $\delta = 0$ limit gives the simplest version of the QMF model.

Some years ago, Guichon proposed an interesting model concerning the change of the nucleon properties in nuclear matter (the quark–meson coupling (QMC) model) [30]. The model construction mimics the RMF theory, where the scalar (σ) and the vector meson (ω) fields couple not with nucleons but directly with quarks. The quark mass has to change from its bare current mass due to the coupling to the σ meson. More recently, Shen and Toki [31] have proposed a new version of the QMC model, where the interaction takes place between constituent quarks and mesons. They refer the model as the quark mean-field model (QMF). In this work we shall also investigate the quark matter within the QMF theory using parameters coming from the enlarged Nambu–Jona-Lasinio (ENJL) model. Enlargement of the NJL model is based on inclusion of vector mesons while the QMF model includes vector mesons at the beginning.

Here the QMF model is somewhat generalized by the inclusion of the isovector $\delta(a_0(980))$ meson. It splits u and d masses (or proton and neutron masses in the case of the RMF approach [35, 36]). Both δ^i- and b_μ^i-mesons may be neglected in the case of symmetric nuclear matter. Their role in the asymmetric nuclear matter of the neutron star is significant and is a subject of current interest to us.

The QMF model is more flexible. The $SU(3)$ symmetry restricts $g_\sigma^q = \sqrt{2/3}g_s$, $g_\omega^q = g_\omega$, $g_\rho^q = g_\rho$ to $g_\rho = g_\omega = g_v$ and $m_\rho = m_\omega = m_v$.

The dependence of the effective quarks mass $m_{F,f}$ (or $\delta_f = m_{F,f}/M$) on the dimensionless Fermi momentum $x = k_{F,\mathrm{u}}/M$ is presented in figure 1.

There is no quark confinement in the NJL and QMF models. There is no mechanism (except for in the NJL solvable model [13]) to prevent hadrons from decaying into free constituent quarks. Free constituent quarks will produce nearly the same density and pressure as free nucleons: $3m_{\mathrm{v,u,d}} \sim M$. Without any mechanism of confinement the quark star for small densities will have properties very similar to those of neutron stars (the case $x_\mathrm{v} > 0.65$ in paper [12]) or even white dwarfs. However, this is a rather unphysical artefact. It is natural to assume that quarks are not allowed to propagate over the distance $\lambda \sim m_{eff}^{-1}$. In this language the confinement mechanism introduces the infrared cut-off λ [40]. The quark confinement mechanism in the form of the harmonic oscillator potential [31] may give the nucleon mass $M = M(\sigma) = M - g_{N\sigma}\sigma + \cdots$ and generate the RMF approach.

RMF theory [32] is very useful in describing nuclear matter and finite nuclei. Recent theoretical studies show that the properties of nuclear matter can be described nicely in terms of the RMF theory. Properties of the neutron star in this model have also been examined [9, 10, 26, 33, 34].

Its extrapolation to large charge asymmetry is of considerable interest in nuclear astrophysics and particularly in constructing a neutron star model where extreme conditions of isospin are realized. The construction of neutron star model is based on various realistic equations of state and results in a general picture of neutron star interiors. Thus the proper form of the EoS is essential in determining neutron star properties such as the mass range, mass–radius relation and the crust thickness. However, a complete and more realistic description of a neutron star requires taking into consideration not only the interior region of a neutron star but also the remaining layers, namely the inner and outer crust and the surface. The Lagrangian of the RMF theory which helps towards the construction of a neutron star model contains baryon and meson
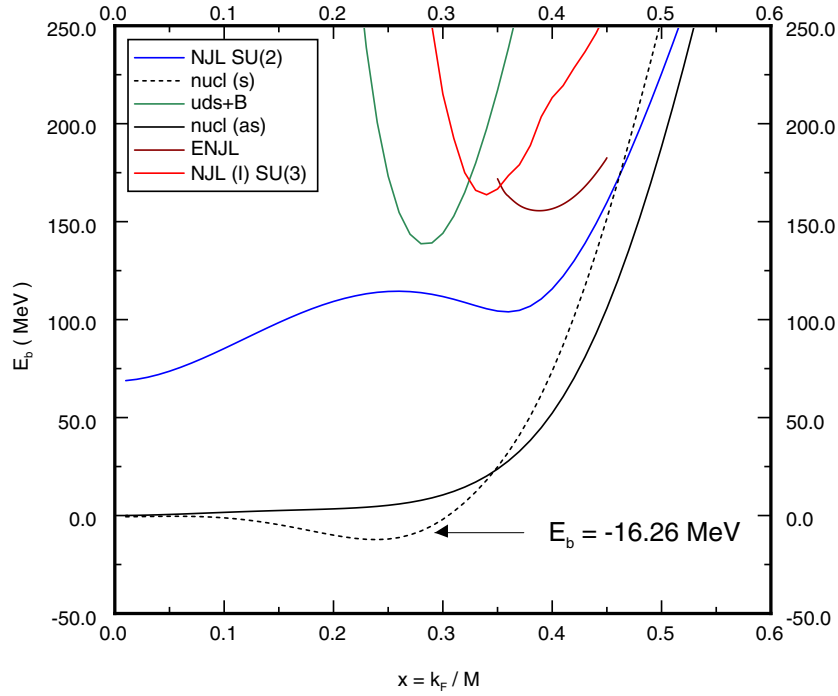
Figure 3. The binding energy E_b for the quark and nuclear system. The pure quark system (u, s, d quarks with the bag constant B) and QMF have nonrealistic behaviour for small densities as $B \neq 0$. For comparison, the symmetric nuclear matter (the RMF approach with TM1 [39] parametrization) result is presented.

degrees of freedom and, as input quantities, coupling constants of the mesons and parameters of the potential $U(\sigma)$ which are determined from nuclear matter properties:

$$\mathcal{L}_{QMF} = \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \tfrac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \tfrac{1}{2}M_\omega^2\omega_\mu\omega^\mu$$
$$- \tfrac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \tfrac{1}{2}M_\rho^2 b_\mu^a b^{a\mu} + \tfrac{1}{4}c_3(\omega_\mu\omega^\mu)^2 + \overline{\psi}(\mathrm{i}\gamma^\mu D_\mu - M)\psi + g_{N\sigma}\sigma\overline{\psi}\psi. \quad (37)$$

Now ψ describes nucleons and $g_{N\sigma} = 3g_s^q$, $g_{N\omega} = 3g_\omega^q$ and $g_{N\rho} = g_\rho^q$.

The potential function $U(\sigma)$ may possesses a polynomial form introduced by Boguta and Bodmer [37] in order to get a correct value for the compressibility K of nuclear matter at saturation density:

$$U(\sigma) = \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4. \quad (38)$$

Different parameter sets give different forms of the EoS in the high-density region above saturation. In this paper the TM1 [39] parameter set was exploited. The RMF model has its own phenomenology, but its parameters should be connected to the enlarged NJL model. The TM1 parametrization suggests that $x_{\mathrm{v}} = 0.65$.

The ρ-meson plays decisive role in accounting for the asymmetry energy of nuclear matter; thus its inclusion in a theory of neutron star matter is essential. Also the proton number density is determined by this meson. The results for the binding energy are presented in figure 3. It shows that the nucleon or quark matter in β-equilibrium has a larger energy per particle than symmetric nuclear matter. For neither parameter set are these matters self-bounded. Figure 3 depicts the binding energy for different models. It reproduces the standard results for symmetric nuclear

matter too. The asymmetric matter is less bound than the symmetric matter. In this model we are dealing with the electrically neutral neutron or quark star matter being in β-equilibrium. Therefore the imposed constrains, namely the charge neutrality and β-equilibrium, imply the presence of leptons.

3. The quark star properties

To calculate the properties of the quark star we need the energy–momentum tensor. The energy–momentum tensor can be calculated taking the quantum statistical average

$$\bar{T}_{\mu\nu} = \langle T_{\mu\nu} \rangle, \tag{39}$$

where

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}. \tag{40}$$

In the case of the fermion fields it is more convenient to use the reper field e_μ^a defined as follows: $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$ where η_{ab} is the flat Minkowski space-time matrix. Then

$$T_{\mu\nu} = e_\mu^a \frac{\partial \mathcal{L}}{\partial e^{a\nu}} - g_{\mu\nu}\mathcal{L}. \tag{41}$$

We define the density of energy and pressure by means of the energy–momentum tensor

$$\bar{T}_{\mu\nu} = (P + \epsilon)u_\mu u_\nu - P g_{\mu\nu} = \begin{pmatrix} \epsilon = c^2\rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \tag{42}$$

where u_μ is a unit vector ($u_\mu u^\mu = 1$). Both ϵ and P depend on the quark chemical potential μ or Fermi momentum x_f. The fermion (quarks and leptons) contributions to the energy and pressure are
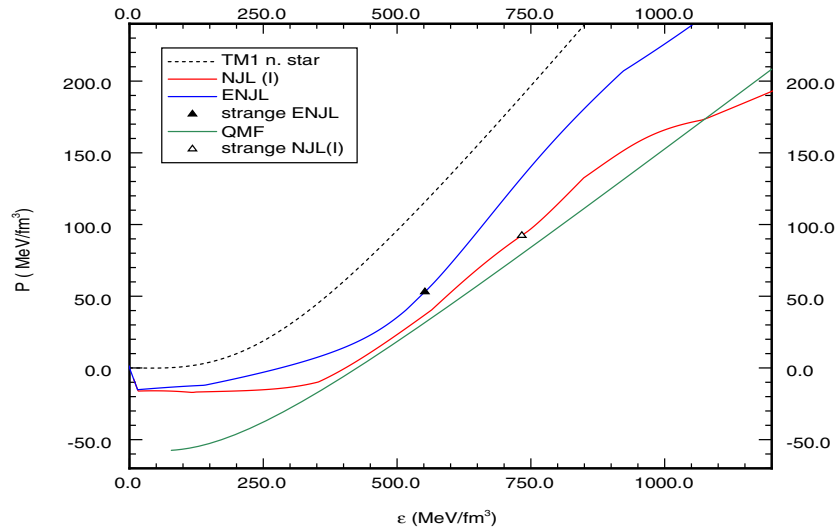
$$\epsilon_F = \sum_{f=\{u,d,s\}} \epsilon_0 \chi_B(x_f, T) + \sum_{f=\{e,\mu\}} \epsilon_0 \chi_L(x_f, T) \tag{43}$$

$$P_F = \sum_{f=\{u,d,s\}} P_0 \phi(x_f, T) + \sum_{f=\{e,\mu\}} P_0 \phi(x_f, T). \tag{44}$$

The fact that effective quark mass $m_{eff,f} = \delta_f M$ depends on fermion concentration (or quark chemical potential μ_f) must now be included in $\chi(x_f, T)$ and $\phi(x_f, T)$:

$$\chi(x, T) = \frac{3}{\pi^2} \int_\lambda^{\Lambda/M} dz\, z^2 \sqrt{z^2 + \delta^2(x)} \left\{ \frac{1}{\exp\left(\left(\sqrt{\delta^2(x) + z^2} - \mu'\right)/\tau\right) + 1} \right.$$

$$\left. + \frac{1}{\exp\left(\left(\sqrt{\delta^2(x) + z^2} + \mu'\right)/\tau\right) + 1} \right\}, \tag{45}$$

$$\phi(x, T) = \frac{1}{\pi^2} \int_\lambda^{\Lambda/M} \frac{z^4\, dz}{\sqrt{z^2 + \delta^2(x)}} \left\{ \frac{1}{\exp\left(\left(\sqrt{\delta^2(x) + z^2} - \mu'\right)/\tau\right) + 1} \right.$$

$$\left. + \frac{1}{\exp\left(\left(\sqrt{\delta^2(x) + z^2} + \mu'\right)/\tau\right) + 1} \right\} \tag{46}$$

Figure 4. The form of the EoS for quark (NJL (I), enlarged NJL and QMF model) matter and the nucleon one (TM1 [39]). The blue colouring ($n_B^s = 3.94 n_B^0$) indicates the strange (s) quark appearance in the strange star.

where $\tau = (k_B T)/M$,

$$\mu' = \mu/M = \sqrt{\delta^2(x) + x^2} \tag{47}$$

and

$$x = k/M \tag{48}$$

for each flavour f. As in [33], we have introduced in (47), (48) the dimensionless 'Fermi' momentum even at finite temperature, which exactly corresponds to the Fermi momentum at zero temperature. To avoid free-quark contributions to the EoS coming from small densities, the infrared cut-off $\lambda = \delta$ [40] was introduced. The case $\lambda = 0$ with the NJL (I) parameter set nicely reproduces the result of the paper [12].

The parametric dependence on μ (or x_f) defines the EoS. The various equations of state for different parameter sets are presented in figure 4. The binding energies

$$E_b = \rho/n_B - Mc^2$$

for the bulk nuclear (n_B is the baryon number density, $n_B = (n_p + n_n)$) and quark matter ($n_B = (n_u + n_d + n_s)/3$) are presented in figure 3.

The metric is static, spherically symmetric and asymptotically flat:

$$g_{\mu\nu} = \begin{pmatrix} e^{\nu(r)} & 0 & 0 & 0 \\ 0 & -e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix} \tag{49}$$

(where $\nu(r)$ and $\lambda(r)$ are functions of a radius r). The Einstein equations (in the isotropic case) lead to the standard Tolman–Oppenheimer–Volkoff ('OTV') equations [41]. The equations describing masses and radii of quark stars are determined by the proper form of the EoS. The
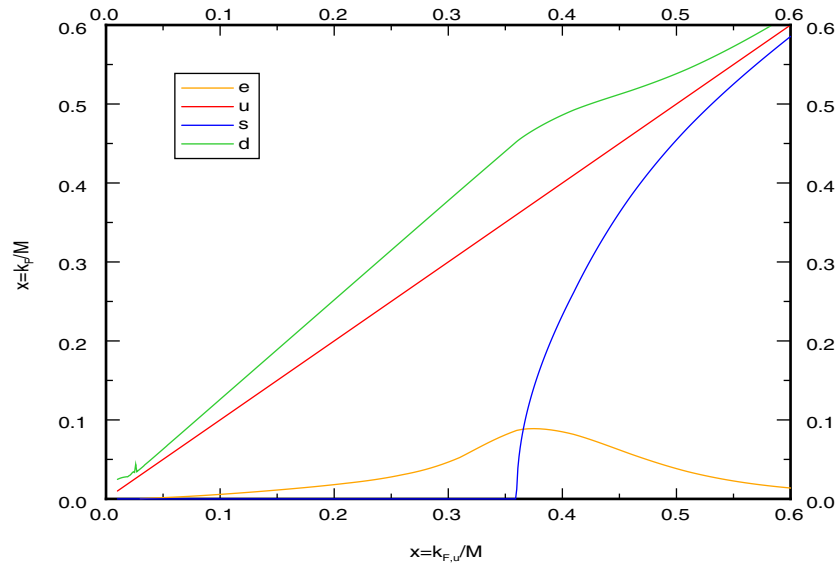
Figure 5. The electron and quark dimensionless Fermi momenta as functions of the u quark one ($x = k_{F,\mathrm{u}}/M$, $M = 939$ MeV is the nucleon mass). The muon distribution is not visible on this scale.

form obtained for the EoS is the basis for calculating macroscopic properties of the star. In order to construct the mass–radius relation for a given form of the EoS the OTV equations have to be solved:

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{G}{r^2}\left(\rho(r) + \frac{P(r)}{c^2}\right)\frac{(m(r) + (4\pi/c^2)P(r)r^3)}{(1 - 2Gm(r)/c^2r)} \tag{50}$$

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r). \tag{51}$$

The continuity condition for the energy–momentum tensor $T^{\mu\nu}_{;\nu} = 0$ defines the connection between the gravitational potential $\nu(r)$ (49) and the pressure and density profiles $P(r)$ and $\rho(r)$:

$$\frac{\mathrm{d}\nu(r)}{\mathrm{d}r} = -\frac{2}{P(r) + c^2\rho(r)}\frac{\mathrm{d}P(r)}{\mathrm{d}r}. \tag{52}$$

Equation (51) determines the function $\lambda(r)$:

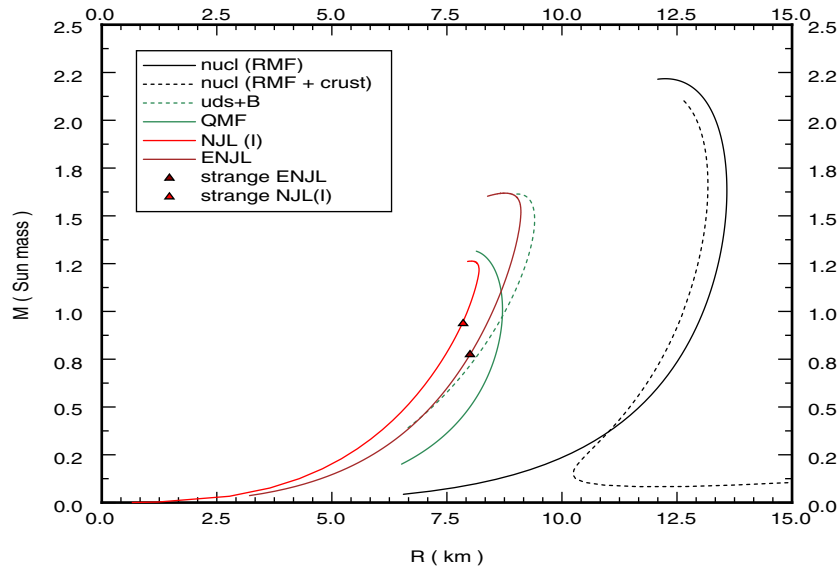$$\mathrm{e}^{-\lambda(r)} = 1 - \frac{2Gm(r)}{r}.$$

Now we have solved the OTV equation, the pressure $P(r)$, mass $m(r)$ and density profile $\rho(r)$ are obtained. To obtain the total radius R of the star, the fulfilment of the condition $P(R) = 0$ is necessary. Introducing the dimensionless variable ξ, which is connected with the star radius r by the relation $r = a\xi$, enables us to define the functions $P(r)$, $\rho(r)$ and $m(r)$:

$$\rho(r) = \rho_0 \chi(x(\xi)) \tag{53}$$
$$P(r) = P_0 \varphi(x(\xi)) \tag{54}$$
$$m(r) = M_\odot u(\xi) \tag{55}$$

Figure 6. The mass–radius $M(R)$ dependence for the quark star (NJL (I), ENJL ($x_v = 1.06$, the solid brown curve, and $x_v = 0.65$, the solid violet curve), QMF model and pure u, d, s matter (QMC)). To compare this relation to the neutron star in the RMF approach, it is presented as the black curve (TM1—solid curve) for pure npe nuclear matter and (TM1 + Bonn + Negele + Vautherin—dotted curve) with a crust.

in terms of ξ. Dimensionless functions defined as

$$\alpha_0 = \frac{GM_\odot \rho_c}{P_0 a}, \qquad \beta_0 = 3\frac{M_s}{M_\odot}, \qquad M_s = \tfrac{4}{3}\pi \rho_0 a^3 \tag{56}$$

are needed to obtain the OTV equation of the following form:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\xi} = -\alpha_0(\chi(x(\xi)) + \varphi(x(\xi)))\frac{u(\xi) + \beta_0 \varphi(x(\xi))\xi^3}{\xi^2(1 - (r_g/a)u(\xi)/\xi)} \tag{57}$$

$$\frac{\mathrm{d}u}{\mathrm{d}\xi} = \beta_0 \chi(x(\xi))\xi^2 \tag{58}$$

with r_g being the gravitational radius. The equations (57), (58) are easy integrated numerically. These are equations for dimensionless mass $u(\xi) = m(r)/M_\odot$ up to dimensionless radius ξ and the u quark dimensionless Fermi momentum $x = k_{F,u}/M$. Knowing the variable x, all star properties can be calculated. Quark and electron dimensionless Fermi momenta dependences on x are presented in figure 5.

Both nuclear and quark matter, being in β-equilibrium, are not bound (figure 3). Quark matter at moderate densities is bound, due to the presence of the bag constant B which acts as a negative pressure $P = -B + \cdots$ (figure 4). Higher-density matter is bound only by gravity. The gravitational binding energy of the star is defined as
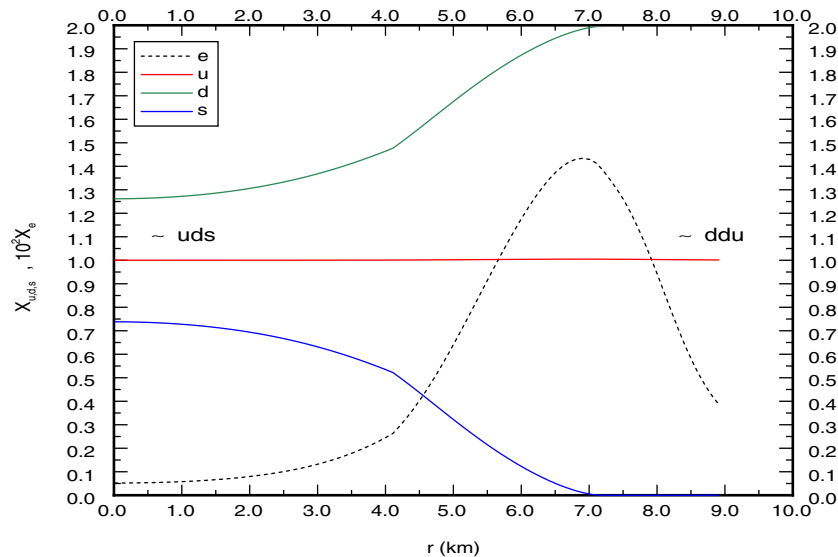
$$E_{b,g} = (M_p - m(R))c^2 \tag{59}$$

where

$$M_p = 4\pi \int_0^R \mathrm{d}r\, r^2 \left(1 - \frac{2Gm(r)}{c^2 r}\right)^{-1/2} \rho(r) \tag{60}$$
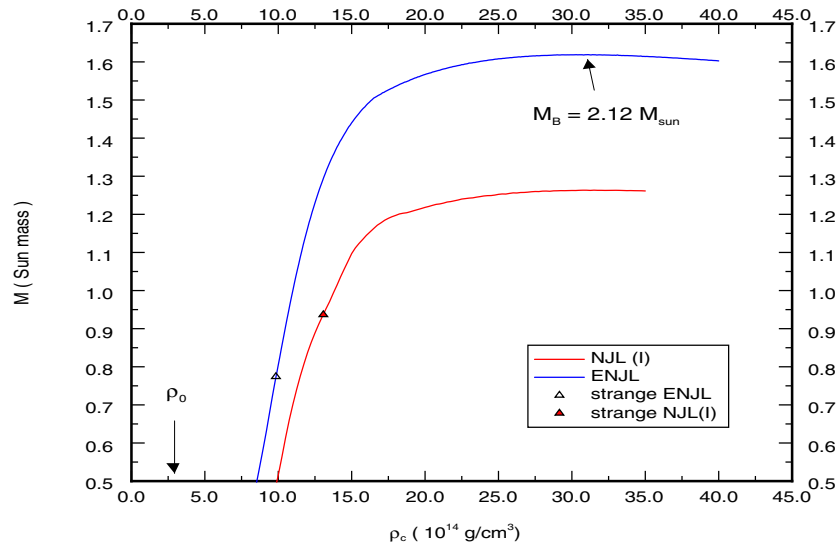
Figure 7. The dependence of the quark star mass $M(\rho_c)$ on the its central density ρ_c.

Figure 8. The profile for the effective quark Fermi momentum of the maximal quark star (ENJL model) with the central density $\rho_c = 3.11 \times 10^{15}$ g cm^{-3}.

is the proper star mass.

Strange quark stars in the NJL model are rather small in comparison to neutron ones. To allow comparison of these strange stars to neutron star models obtained in the RMF approach, the mass–radius relations are also presented in figure 6. (The black solid line (TM1) for pure npe nuclear matter and the dotted line for a star with a crust (TM1 + Bonn + Negele + Vautherin [42]).) The smaller size of the quark star is due to the fact that the pressure (figure 4) reaches zero for higher densities ($n_B^m \sim 0.26$ fm$^{-3} = 1.75 n_B^0$) than for the symmetric
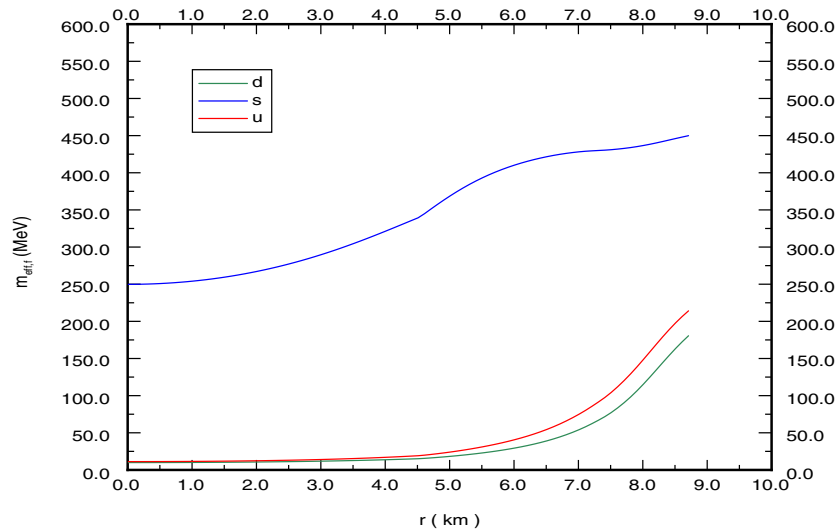
Figure 9. The quark effective mass profile inside the maximal quark star (ENJL model) with the central density $\rho_c = 3.11 \times 10^{15} \ \text{g cm}^{-3}$.

nuclear matter ($n_B^0 \sim 0.15 \ \text{fm}^{-3}$). At the surface, such a star has higher density than saturated nuclear matter. This makes the star smaller and denser. The same situation arises in the QMF approach when the bag constant is included. In the NJL model the effective bag constant has dynamical origin (equation (28)). Its main contributions come from quantum vacuum fluctuations and scalar mesons. The strange quark s appears only for densities that are high enough ($n_B > n_B^s = 3.94 n_B^0$). For smaller densities the quark star is built only from u and d quarks. It is important to stress that even then the quantum vacuum fluctuations come from all flavours including s quarks and all antiparticles. There are no s quarks for small densities in the strange quark star—only its quantum fluctuations. In the QMF approach there are no quantum fluctuations at all. This represents a significant difference between the NJL and QMF approaches. Another one is that the bag constant in the QMF model must be added 'by hand'. This makes the QMF approach unreliable for smaller densities (figure 4).

The gravitational binding energy of a strange quark star (QMF approach) and that of a neutron (RMF) star are presented in figure 10. The arrow shows a possible transition from the unstable neutron star to the strange one with conservation of the baryon number $M_B = M_\odot \ (M_B = m_\text{n} c^2 N_B)$.

In the enlarged NJL model [22], vector mesons are included. Their contributions to the effective bag constant are positive (figure 2). This means that a strange quark star possesses a slightly bigger radius and mass than in the NJL (I) model.

However, a maximum stable strange quark star is obtained for $\rho_2 = 3.1 \times 10^{15} \ \text{g cm}^{-3}$ and has the following parameters: $M = 1.61 \ M_\odot$ and $R = 8.74$ km. The baryon number for this star is the same as in the case of a pure neutron star with $M_B = 2.126 \ M_\odot$. Below the density $\rho_s = 3.94 \rho_0 \ (\rho_0 = 2.5 \times 10^{14} \ \text{g cm}^{-3})$, there are no strange quarks and quark stars. The stable stars are those with $\mathrm{d}M/\mathrm{d}\rho_c > 0$ [38] (figure 7). The gravitational binding energy for a strange quark is lower than the neutron one for $\rho > 1.6 \times 10^{15} \ \text{g cm}^{-3}$. The $M(\rho)$ dependence for the quark star is presented in figure 7. For the quark star with the maximal central density $\rho_c = 3.11 \times 10^{15} \ \text{g cm}^{-3}$, the star profile is presented in figure 8. Quark and electron mass
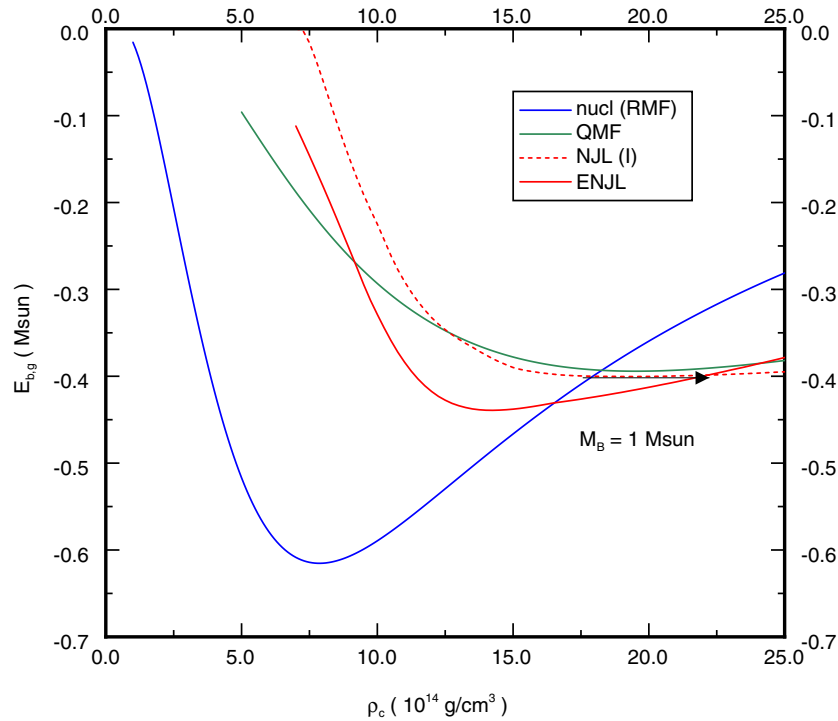
Figure 10. The gravitational binding energy of the quark strange (ENJL and QMF approach) and neutron (RMF) stars. The arrow shows a possible transition from the unstable neutron star to the strange one with conservation of the baryon number $M_B = m_n c^2 N_B$.

distributions inside the star are presented in figure 8. The quark partial fractions defined as

$$X_f = \frac{n_f}{(n_u + n_d + n_s)} = \frac{3n_f}{n_B},$$

where $f = (u, d, s)$, are presented in figure 5. The quark and electron partial fraction distribution inside the star is presented in figure 8.

4. Conclusions

The properties of the strange quark star in the bag model with $B = B_c$ and the current quark masses ($m_u = m_d = 0$, $m_s = 150\,\text{MeV}$) are presented in [9]. The star model based on the QMC is very similar (the dotted green curve in figure 6) and close to that based on the ENJL model parametrized by the TM1 parameter set ($x_v = 0.65$, the solid violet curve 6). The properties of strange stars with quark masses changing continuously from the constituent quark masses to the small current (see figure 1) are presented in [43]. All these stars are more compact than neutron stars (see the figure 6) and are similar to those of the NJL (I) model.

In this paper the enlarged NJL model is used to construct the EoS and properties of the strange quark star. The stable strange quark star exists from the minimal central density up to the maximal one $\rho_2 = 3.1 \times 10^{15}\,\text{g cm}^{-3}$, which gives the following star parameters: $M = 1.61\,M_\odot$ and $R = 8.74\,\text{km}$. Its baryon number is the same as for the pure neutron star with $M_B = 2.126\,M_\odot$.

A very similar strange star—but less compact—is obtained in the solvable NJL model [13]. The gravitational binding energy for a strange quark star is lower than the neutron one for densities $\rho > 1.6 \times 10^{15}$ g cm^{-3}. The conversion of an unstable neutron star into a strange star is an exciting subject which may help to explain the gamma-ray-burst enigma [44].

Similarly to the QMF model, the enlarged NJL one includes the coupling to vector mesons. This is crucial for the quark star properties. Also the quark s mass is important. The mass of an s quark is also relevant because its smaller mass causes strange quarks to appear for lower densities. Nonzero strangeness of the matter gives, as a result, a strange star. It is fascinating that the neutron and strange star properties are strictly connected to the inner structures of nuclei and nucleons.

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