Shape Optimization for Dense Gas Flows in Turbine Cascades

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1 Introduction

In recent years, great attention has been paid to a class of fluids of the retrograde type (i.e. fluids that superheat when expanded), known as the Bethe–Zel'dovich–Thompson (BZT) fluids, which exhibit in the vapor phase, above the upper saturation curve, a region of negative values of the Fundamental Derivative of Gasdynamics $\Gamma := 1 + \frac{\rho}{a}\left(\frac{\partial a}{\partial \rho}\right)_s$, with ρ the fluid density, a the sound speed, and s the entropy. In the transonic and supersonic regimes, this leads to nonclassical gasdynamic behaviors, such as expansion shocks and mixed waves. Moreover, flow discontinuities with jump conditions in the vicinity of the $\Gamma = 0$ contour have necessarily limited strength, producing losses (entropy rise) one order of magnitude lower than usual [2]. The interested reader may refer to [3] for a review of the complex dynamics of BZT fluids.

An appealing application of BZT fluids is efficiency enhancement for Organic Rankine Cycles (ORCs). ORCs' working fluids are heavy organic compounds with large heat capacities: interestingly, several of these fluids possess BZT properties. One major source of losses in ORC turbines is wave drag, since they usually operate in the transonic/supersonic regime: the use of a BZT fluid could avoid shock formation and, ideally, allow isentropic turbine expansion. Unfortunately, simply utilizing a BZT working fluid is not sufficient to maximize the reduction in losses. Operating the turbine cascade at a pressure and temperature near the thermodynamic region where BZT effects appear is also necessary. On the other hand, this region, called the *inversion zone*, has a quite limited extent. As a consequence, a reduction in the cascade pressure ratio is required to operate the turbine entirely within this zone. Now, it is known from thermodynamic theory that a too small pressure ratio leads to poor global thermal cycle efficiency. Thus, the development of BZT ORCs needs finding a reasonable tradeoff between two opposite requirements: on the one hand, turbine expansion must happen as close as possible to the inversion

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zone, on the other, turbine pressure ratio must be sufficiently high for achieving high global cycle efficiency and power output. Previous studies about dense gas flows in turbine cascades [4] have shown that, for moderately high turbine pressure ratios, the use of a BZT working fluid may allow efficiency improvements up to 3% over air for properly chosen operating conditions. For higher pressure ratios, efficiency improvements with respect to a perfect gas progressively disappear, whereas BZT fluids remain advantageous compared with other working fluids typically employed in Rankine cycles, such as steam or toluene. Increasing the inlet pressure and Fundamental Derivative along an isentrope crossing the inversion zone, efficiency tends initially to increase and reach an optimum. At optimum conditions, the average value of Γ is less than 1 at the wall, and the sound speed increases with dropping pressure. This limits Mach number growth during flow expansion and reduces shock strength. For higher inlet values of Γ, Γ_{inl}, efficiency drops again. The higher the cascade pressure ratio, the higher the Γ_{inl} value corresponding to peak efficiency, and the quicker efficiency drops when moving away from optimum conditions. Note however, that efficiency improvements observed in previous studies were simply due to the special nature of the working fluids, since the blade shapes considered were typical gas turbine blade sections, not specifically adapted for dense gas flows. The objective of the present study is finding optimal blade shapes for BZT ORC turbines, providing high efficiency over a large range of operating conditions and working with high cascade pressure ratios.

2 Governing Equations, Flow Solver and Optimizer

In the present study, dense gas flows are modeled by the compressible Euler equations for single-phase, non-reacting flows, completed by Martin-Hou realistic equation of state [5]. The governing equations are discretized by a finite volume method for structured multiblock grids, using a third-order accurate centered spatial approximation [6]. The solution is advanced in time using a four-stage Runge-Kutta scheme with local time-stepping and implicit residual smoothing. The solver has been previously validated for a variety of perfect and dense gas flow simulations.

The flow solver is coupled with a multi-objective genetic algorithm (MOGA). Genetic algorithms (GA) have proved their interest with respect to gradient-based methods because of their high flexibility and their ability to find global optima of multi-modal problem. The MOGA applied in this study is the Non-Dominated Sorting Algorithm proposed by Srinivas and Deb [7]. For single objective problems, the algorithm uses an elitism strategy to ensure that the best individuals survive when algorithm evolves. For multi-objective problems, a Pareto-based genetic algorithm is applied. The MOGA has been previously validated for the optimization of airfoil shapes in dense gas flows [8]. In order to select a proper starting population for the genetic algorithm, the representation of the design space is previously investigated through a Design of Experiment (DOE) procedure using a Sobol sequence technique.

3 Results

The MOGA optimizer, coupled with the flow solver and a structured mesh generator is applied to shape optimization of turbine cascades. Blade shape is parametrized using Bezier polynomials, starting from the baseline profile VKI LS-59 [9]. Sixteen control points are imposed to parametrize the profile. Globally the optimization problem depends on twenty-five variables. Geometric constraints are imposed on the maximum blade thickness and on trailing edge thickness (normalized with the blade chord) that may vary within $\pm 10\%$ of the corresponding non-dimensional thickness for the baseline geometry. The turbine cascade is operated with an inlet flow angle of $30°$ and a pressure ratio of 1.82 corresponding, for a perfect diatomic gas, to sonic isentropic exit conditions. For dense gas flows, inlet thermodynamic conditions, i.e. the thermodynamic operation point, should be also specified. All the computations presented in the following are performed on C-grids composed by 192×16 cells, with average non-dimensional height of the closest cells to the wall approximately equal to 5×10^{-3}. This mesh refinement provides a reasonable tradeoff between accuracy and computational cost. Since the objective function chosen for optimization runs is cascade efficiency (defined as real-to-ideal static enthalpy drop), preliminary computations have been performed, for the baseline configuration, to investigate the sensitivity of this parameter to mesh refinement. Results computed on finer grids composed by 382×32 (first cell height 10^{-3}) at several flow conditions show that the magnitude of cascade efficiency tend to increase of about $2\nabla \cdot \mathbf{3}\%$ when refining the grid, but trends of behavior (e.g. efficiency dependency on inlet conditions) are well conserved.

Firstly, a single-objective shape optimization for a diatomic perfect gas (PFG) has been performed in order to maximize turbine efficiency. The initial population of the genetic algorithm is selected after a preliminary DOE over 100 individuals. After about 20 generations the mean and maximum value of the objective function in the population reach approximately the same value, indicating that the GA has converged to a population of almost identical (optimal) individuals. The solution for the baseline blade is characterized by an oblique shock at about 70% the chord and a second shock attached to the trailing edge (Fig. 2a); computed efficiency is 93.2%. Figure 2b shows the solution after shape optimization: the oblique shock is almost suppressed, the trailing edge shock is weaker with respect to the reference case and efficiency grows to 96.2%. Figures 2c,d display Mach number and pressure (normalized by inlet stagnation conditions) distributions at the wall. For the optimal configuration the maximum Mach number at the wall and, consequently, shock strength, is noticeably reduced compared to the baseline configuration; consequently, wave drag lowers and cascade efficiency improves.

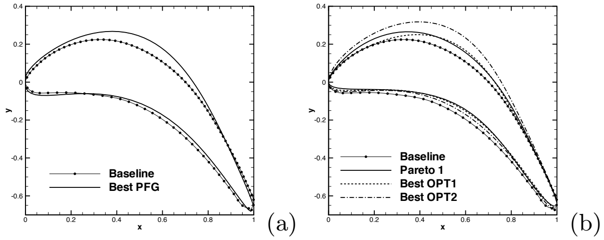

Fig. 1. Optimal shapes for perfect (a) and dense (b) gas flow.

The optimal individual (Fig. 1a) has a thicker leading edge, thinner trailing edge, and greater camber than the baseline configuration.

Then, computations are performed with the BZT fluorocarbon PP10 as the working fluid. Three optimization runs are undertaken. In the first and second run, a single objective function is maximized, i.e. cascade efficiency at fixed operating conditions $p_{inl}/p_c = 1.00$, $\rho_{inl}/\rho_c = 0.752$, $\Gamma_{inl} = 0.416$ (optimization point OPT1) and $p_{inl}/p_c = 1.10$, $\rho_{inl}/\rho_c = 1.09$, $\Gamma_{inl} = 1.91$ (optimization point OPT2), respectively. Subscript c indicates critical point values. Both operation points lay on an isentrope crossing the inversion zone, respectively at lower and higher pressure than peak efficiency conditions. In the third run, both objectives (i.e. efficiencies at OPT1 and at OPT2) are simultaneously maximized. Here again, a DOE is preliminarily run in order

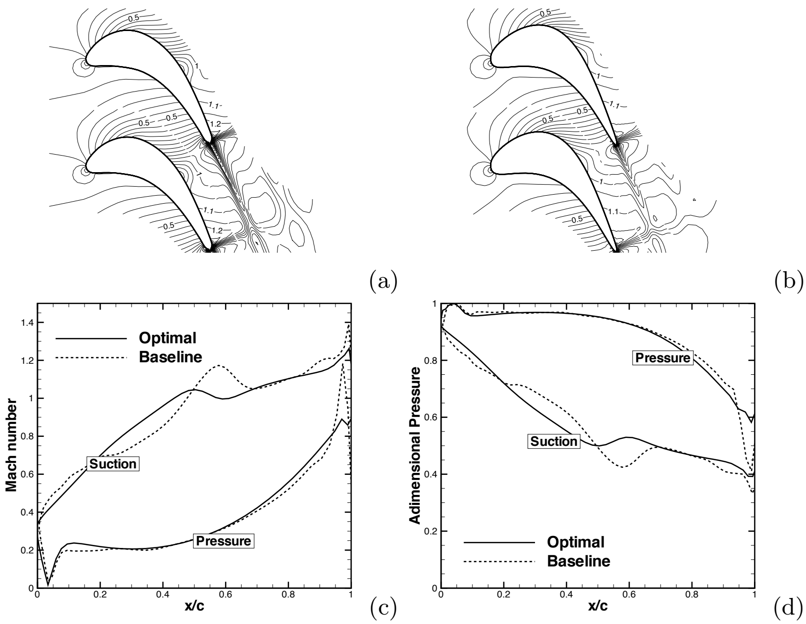

Fig. 2. PFG flow, isoMach lines: a) baseline; b) optimized. c,d) Wall distributions.

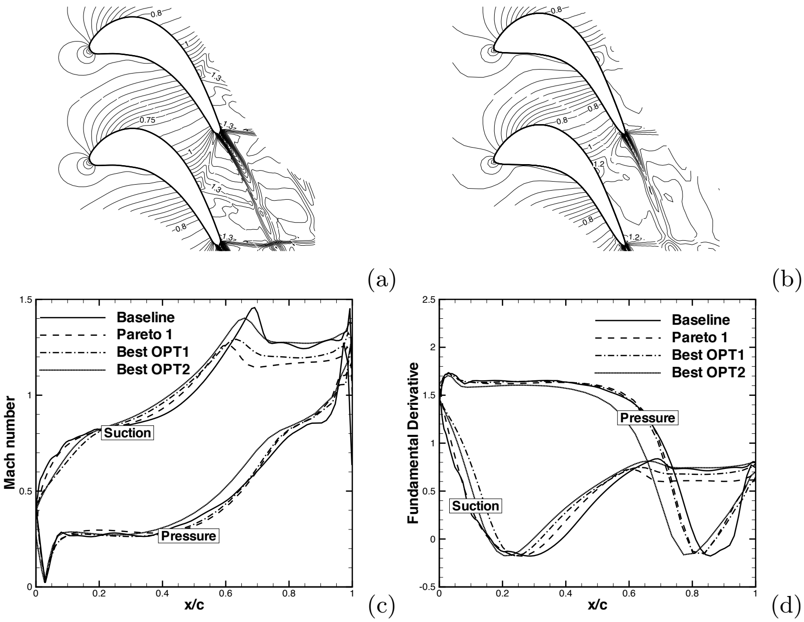

Fig. 3. DG flow, isoMach lines: a) baseline; b) optimized. c,d) Wall distributions.

to properly initialize the population of the GA. Figure 1b shows optimal individuals for operating conditions OPT1 and OPT2 and an individual selected on the Pareto front resulting from the two-point optimization. The baseline profile is also represented. For each configuration, a parametric study of cascade efficiency at off-design conditions has been performed. In addition to operation points OPT1 and OPT2, two test points corresponding to operating conditions $p_{inl}/p_c = 1.02$, $\rho_{inl}/\rho_c = 0.813$, $\Gamma_{inl} = 0.886$ (TEST1) and $p_{inl}/p_c = 1.05$, $\rho_{inl}/\rho_c = 0.944$, $\Gamma_{inl} = 1.62$ (TEST2) are considered. Results are summarized in Table 1. Note that optimal individuals provide an improvement of $1\nabla \cdot \mathbf{2}\%$ only over the baseline at optimization conditions. However, improvements up to 7% are obtained at conditions TEST1 and TEST2, which lay close to peak efficiency conditions for the baseline cascade. The best overall results are obtained for the individual derived from the two-point optimization ("Pareto 1"). Note that its shape is quite similar to that of the optimal individual for perfect gas flow. Figures 3a,b show iso-Mach lines for the baseline configuration and for individual "Pareto 1" at operating condition TEST2. Figures 3c,d show the Mach number and Γ distributions at the wall for the best individuals from single-objective optimizations, the "Pareto 1" individual and the baseline blade. Here again the mechanism leading to efficiency improvement is a reduction in the maximum Mach number of the flow and hence of shock strength: major gains come from a significant weakening of the trailing edge shock for the optimized configuration. This is related to lower values of Γ at the rear part of the suction side, causing slower growth of the Mach number when the flow re-expands downstream of the first shock.

Table 1. Efficiencies (%) for optimal blade shapes at several operating conditions.

	OPT1	TEST1	TEST2	OPT2
Γ_{inl}	0.416	0.886	1.62	1.91
Best OPT1	91.25	95.74	97.20	89.71
Best OPT2	89.67	95.45	96.08	90.22
Pareto 1	91.00	96.36	97.47	89.75
VKI LS-59	88.66	90.72	90.80	88.77

4 Final remarks

Shape optimization for flows of perfect and dense gases in turbine cascades has been achieved by means of a multi-objective genetic algorithm. For perfect gas flows, shape optimization allows efficiency improvements of about 3% over the baseline configuration. For dense gas flows, proper optimization starting from the same baseline geometry leads to efficiency gains up to 7%. Multipoint optimization allows improving performance over a large range of thermodynamic operating conditions. For the high cascade pressure ratio considered in this study, BZT effects play a minor role in efficiency improvement. The use of properly designed turbine cascades working with somewhat lower pressure ratios could allow higher efficiency improvements due to BZT effects, opening the door to the development of BZT turbines for Organic Rankine Cycles.

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