Ramseyfication and structural realism

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ABSTRACT: Structural Realism (SSR), as embodied in the Ramsey-sentence H^* of a theory H, is defended against the view that H^* reduces to a trivial statement about the cardinality of the domain of H, a view which arises from ignoring the central role of observation within science. Putnam's theses are examined and shown to support rather than undermine SSR. Finally: in view of its synthetic character, *applied mathematics* must enter into the formulation of H^* and hence be shown to be finitely axiomatisable; this is done in the Appendix, which is the most important part of the paper.

Keywords: metaphysics, realism, structuralism, refutability, Ramsey-sentence, Finite Axiomatization.

1. Difficulties facing Realism

Structural Scientific Realism —henceforth referred to as SSR— is an epistemological position whose origins go back to the philosophical insights of Pierre Duhem, Henri Poincaré and Bertrand Russell. SSR flows from two basic theses.

[a] First, consider the characterisation of Realism as the belief in the existence of a mind-independent reality. This definition is unsatisfactory because it presupposes a schism between the mind on the one hand, and that which it tries to grasp on the other. By metaphysical realism, I shall understand the thesis that there exists a structured and undivided reality of which the mind is part; moreover, that far from imposing their own order on things, our mental operations are in their turn governed by the fixed laws which describe the workings of Nature. Hence our brain, or conscious activity, evolves according to the very laws it sets out to capture. As shown by Planck and Popper, this thesis entails that a total explanation of everything in the universe can never be achieved by the human mind: on pain of vicious circularity, the brain is in principle incapable of predicting its own future states. Far from being incompatible with metaphysical realism, this negative result could be regarded as one of its welcome consequences. [M. Planck 1965b. Also K.R. Popper 1982, Chapter 3].

Methodological realism consists of the further assumption that the structure of reality is, at least in part, intelligible to the mind. It is therefore reasonable for scientists to aim at grasping that reality. They may of course choose not to do so and content themselves with constructing phenomenological laws.

Henceforth referred to as SR, scientific realism adds the thesis that successful theories, i.e. those unified systems which explain the data without *ad hoc* assumptions, are approximately true. That is: such systems reflect, in one way or another, the order of things as they are in themselves; successful hypotheses thus model, and in some sense 'correspond' to the real structure of the world. Note that SR is committed neither to the correspondence nor even to Tarski's semantic theory of truth. There is no reason why the way in which a hypothesis mirrors reality should be the usual term-by-term mapping described by traditional semantics. The structure of a scientific system can be held to reflect reality without each component of the system necessarily referring to a separate item of that reality. This is in effect the structural-realist stance which we have denoted by SSR. In its defence, we can say that our usual semantics is inspired by a common-sense ontology which posits discrete *intentional* objects. But such an ontology is not necessarily that of the *external* world; for common-sense tends to assimilate the transcendent to the intentional sphere, and hence posit well-circumscribed entities subsumed by various predicates. This need not however be the way Nature in which operates. External objects are known exclusively by description, i.e. through the mediation of a network of theories. So we cannot directly ascertain what kind of correspondence might obtain between our successful hypotheses and the transcendent world. Only idealists like Berkeley and Husserl felt certain about the nature of such a correspondence, for idealism postulates only *immanent entities*. By contrast, the way in which realist theories latch on to the world constitutes a *problem* for which there exist only conjectural solutions. Thus SR should not be made to stand and fall with any specific theory of truth; all it asserts is that highly corroborated theories reflect the ontological order in a way which scientists may —or may not— choose to specify. There is no reason why realists should not adopt a fallibilist attitude as regards both the truth-value and the mode of reference of their conjectures. Note that Duhem aptly compared scientific concepts to the flat pieces of the armour which never exactly fits a knight's body. These concepts have a metaphorical rather than a strictly referential function. Poincaré's rider should nonetheless never be forgotten: the meanings and referents of the observational terms must be fixed in advance.

Regarding the three positions outlined above, let us note that each is stronger than the one preceding it.

[b] There is secondly the realisation that we are in principle unable to ascertain the truth-values of 'objective' relations, i.e. of predicates connecting terms some of which refer to transcendent entities; for of these we have no knowledge by acquaintance, which is the only kind of knowledge enabling us effectively to decide whether these entities possess given properties or bear to one another certain relations. The only knowledge of these objects that we might conceivably attain is through a nexus of conjectures which, if consistent and empirically well-supported, provides a so called 'system of implicit definitions' of its domain of discourse. In other words, our conjectures delimit classes of models only one of which might be the real world.

2. The Ramsey-sentence

A physical theory Σ is usually presented as the consequence class of finitely many (proper) axioms. The conjunction of the latter can therefore be written as a single formula $G(Q_1, \ldots, Q_n, T_1, \ldots, T_m)$, where the Q_i's denote the observational, and the

T_j's the theoretical predicates of **Σ**. (**Q** and **T** will be written for the two sequences $Q_1, \dots, Q_n$ and $T_1, \dots, T_m$ respectively). Only in the case of the predicates Q_i's do we have a direct access to the referents of their arguments and are thus in a position to decide whether these referents are actually subsumed by the Q_i's. As for the T_j's, all we can legitimately claim to know about them is that they bear to each other and to the 'observables' **Q** the second-order relation $G(\mathbf{Q}, \mathbf{T})$. According to SSR, the maximum amount of scientific knowledge we can legitimately lay claim to is encapsulated by the so-called Ramsey sentence of $G(\mathbf{Q}, \mathbf{T})$, i.e. by $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$; where **t** stands for the m-tuple $(t_1, \dots, t_m)$, and $(\exists \mathbf{t})$ for $(\exists t_1, \dots, t_m)$.

Prima facie, this conclusion seems highly counter-intuitive. For example: according to classical science, physical reality consists of impenetrable atoms; where these ultimate constituents of the observable universe were held to be bits of matter possessing mass and electric charge. One had in mind a concrete picture of mass and charge densities represented by two functions $\mu(\underline{x}, t)$ and $\rho(\underline{x}, t)$, and of a velocity field $\underline{V} = (v_1(\underline{x}, t), v_2(\underline{x}, t), v_3(\underline{x}, t))$; where t denotes the time, and $\underline{x}$ the spatial coordinates. The quantity $\rho(\underline{x}, t)$ is however clearly apprehended only by description, i.e. by means of some theoretical predicate $R(\underline{x}, t, r)$, where: r represents the function $\rho(\underline{x}, t)$ and $(\forall \underline{x})(\forall t)(\exists! r)\ R(\underline{x}, t, r)$ is a consequence of our system. R must therefore be defined in terms of, or else figure among $T_1, \dots, T_m$. Hence everything we might claim to know about the charge, mass and velocity distributions is theoretical; so it must already have been packed into the formula $G(\mathbf{Q}, \mathbf{T})$ and will consequently occur within the scope of $(\exists \mathbf{t})$ in $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$.

Let us now address the problems posed by the logico-mathematical principles of a scientific system. It is well-known that classical logic consists of infinitely many axioms subsumed by various schemes. But since every model of $G(\mathbf{Q}, \mathbf{T})$ automatically satisfies the logical schemes, the latter need not occur explicitly in $G(\mathbf{Q}, \mathbf{T})$. A serious problem however arises in connection with the strictly mathematical axioms. As long as the latter were taken to be analytic, they could be regarded as part of the logical background. But mathematics has long been acknowledged to be synthetic, so that its postulates should figure explicitly in $G(\mathbf{Q}, \mathbf{T})$. Basic mathematical concepts like those of class and class-membership should therefore be quantified over as we pass from $G(\mathbf{Q}, \mathbf{T})$ to the Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$. A major difficulty stems from the fact that mathematics is normally articulated within the framework of Zermelo-Fraenkel set theory (ZFC). The latter possesses infinitely many axioms while $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ is intended to be a finite formula of the object-language. It is therefore essential for us to show that mathematics, or rather that part of it which is used within the sciences, is finitely axiomatisable. This can actually be established by using Gödelian methods [See Appendices 1 and 2. Also K. Gödel 1940].

Having the class-theoretic machinery at one's disposal offers considerable advantages. For example: a finite number of individual names $a_1, \dots, a_m, u_1, \dots, u_s$ can be added to our primitive vocabulary in such a way that, for each $j = 1, 2, \dots, m$, the symbol a_j stands for the extension of the predicate T_j. In this way set-theory can be brought to bear on all classes of physical, as well as of mathematical entities; so we end up with a system of the form $\mathbf{G}(k, \in, a_1, \dots, a_m, u_1, \dots, u_s, Q_1, \dots, Q_n)$ which incorporates applied mathematics and whose Ramsey-sentence is

$$(\exists k, \in)(\exists \varnothing, a_1, \dots, a_m, u_1, \dots, u_s)\ \mathbf{G}(k, \in, a_1, \dots, a_m, u_1, \dots, u_s, Q_1, \dots, Q_n)$$

where: the monadic predicate k and the dyadic relation $\in$ are defined as follows:

$$k(x) \equiv [x \text{ is a class}];\ (x \in y) \equiv [x \text{ is a member of the class } y].$$

Note moreover that $(\exists k, \in)$ represents the only second-order quantifications which need to be carried out in order to obtain the Ramsey-sentence $(\exists k, \in)(\exists a_1, \dots, a_m, u_1, \dots, u_s)\ \mathbf{G}(k, \in, a_1, \dots, a_m, u_1, \dots, u_s, Q_1, \dots, Q_n)$. Should we consider the notions of *class* (k) and of *class-membership* ($\in$) as logical constants, hence as *not* to be quantified over, then the Ramsey-sentence would reduce to the first-order formula

$$(\exists a_1, \dots, a_m, u_1, \dots, u_s)\ \mathbf{G}(k, \in, a_1, \dots, a_m, u_1, \dots, u_s, Q_1, \dots, Q_n).$$

It should however be noted that the conclusions drawn in the rest of this paper in no way depend on any extra *assumptions* concerning the first-order character of the Ramsey-sentence; for the latter anyway turns out, in most of the examples adduced below, to be *logically equivalent* to a first-order formula.

3. Newman's Criticism of Russell's Position

Several authors have recently expressed major objections to SSR; objections *so* major that they raise questions about the very "possibility" of structural realism. For any proposition H, let H^* henceforth denote the Ramsey-sentence of H. Thus:

$$(G(\mathbf{Q}, \mathbf{T}))^* =_{\text{Def}} (\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}).$$

The objections are, essentially, as follows:

(i) SSR leads to the view that the cognitive content of any scientific theory H is fully captured by H^*.

(ii) But as was allegedly first shown by the mathematician M.H.A. Newman —responding to Russell's version of structural realism— a theory's Ramsey-sentence is satisfiable in a "trivial" way, supposedly imposing no more than a cardinality constraint on our domain of discourse [M.H.A. Newman 1928].

(iii) However, scientific theories clearly do much more than impose constraints on the size of the universe.

(iv) So structural realism is committed to an entirely untenable account of the cognitive content of scientific theories, namely that this content is captured by the theory's Ramsey-sentence, and is itself untenable.

Newman's argument was reintroduced into the philosophy of science by Demopoulos and Friedman —and via their article, it has attracted a good deal of attention in the past few years [W. Demopoulos & M. Friedman 1985].

I shall however show that despite appearing to be cogent, Newman's objections owe their *superficial* reasonableness to an easily corrigible oversight on Russell's part. After rectifying Russell's error, it will be seen that Newman's argument in no way threatens structural realism —though consideration of this argument helps towards clarifying important aspects of SSR.

4. Ramseyfication and "Trivialisation"

According to Demopoulos and Friedman, Newman showed that any view which identifies the cognitive content of a scientific theory with that of its Ramsey-sentence faces "insurmountable difficulties"; and there is no denying that in his [1927] Russell unfortunately spoke of a *purely* structural description of reality being *inferred* from perceptual results. The fault lies not so much with the inductive overtones of the verb 'infer', invalid though any such inference might be; as with the implicit assumption that once the inference is carried out, an exclusively structural account is obtained in which no observational terms occur. After being used during the inferential process, all observations are supposedly shed, thus yielding a structural proposition $H(T_1, \dots, T_m)$ in which all the predicates $T_1, \dots, T_m$ are theoretical. The proposition $H(T_1, \dots, T_m)$ will henceforth be assumed to be a first-order sentence. Ramseyfying, we obtain $(\exists t_1, t_2, \dots, t_m)\ H(t_1, \dots, t_m)$; where $(\exists t_1, t_2, \dots, t_m)\ H(t_1, \dots, t_m)$ expresses a mere consistency condition for $H(T_1, \dots, T_m)$; i.e. it means that $H(T_1, \dots, T_m)$ possesses at least one model, a result guaranteed by the Löwenheim-Skolem theorem.

In conformity with his overall philosophical position, Russell could however have adopted an alternative, more precisely: a hypothetico-deductivist, approach. He could have put forward a mixed first-order proposition $G(Q_1, \dots, Q_n, T_1, \dots, T_m)$, where the observational terms $Q_1, \dots, Q_n$ figure alongside the theoretical predicates $T_1, \dots, T_m$. Being empirically falsifiable, the corresponding Ramsey-sentence, namely $(\exists t_1, t_2, \dots, t_m)\ G(Q_1, \dots, Q_n, t_1, \dots, t_m)$, would then have been seen to be far from trivially satisfiable. Note however that this defence of Russell's thesis is predicated on a division between "theoretical" and "observational" terms; where the latter are taken to be understood in advance of any physical theory, i.e. directly and without any involvement of description. This means that those "antecedently understood" predicates retain their meaning (that is, in extension) from interpretation to interpretation. But if our basic notions are all "objective", i.e. transcendent and hence known exclusively by description, then we are bound to end up with an observationally untestable system of the form $H(T_1, \dots, T_m)$.

Such an extreme structuralist position is exposed to two twin dangers, those of scepticism and of utter triviality. Every scientific system like $H(T_1, \dots, T_m)$ is supposed to contain ordinary arithmetic and to be both consistent and axiomatisable. $H(T_1, \dots, T_m)$ must therefore be incomplete; so that, for at least one sentence B, neither B nor $\neg B$ follows logically from $H(T_1, \dots, T_m)$. B, or alternatively $\neg B$, can therefore be consistently added to $H(T_1, \dots, T_m)$, thus yielding two new incompatible systems. Continuing in this way, it can be shown that there exist any number of internally consistent, but mutually incompatible theories each one of which contains $H(T_1, \dots, \dots, T_m)$; and it would be impossible to give any epistemological preference to one of these systems over any other. Thus we end up with the sort of scepticism which turns the undeniable technological progress of the sciences into an ongoing miracle.

As for the danger of triviality: as already explained, if structural realism did imply that *all* that can be "known" is structure, then it would entail that scientific hypotheses are indistinguishable from (consistent) mathematical systems and hence face insurmountable difficulties. In fact however, the view is that while *theoretical* entities are known only via description, observational entities are known both by ostension and by description; for it should be noted that in the global description $G(Q_1, \dots, Q_n, T_1, \dots, T_m)$, the theoretical T's are indissolubly entangled with the observational Q's.

Nowadays, it is generally held that there is no absolute or sharp distinction between theoretical and observational predicates. But Russell (along with Poincaré, Grover Maxwell and Ayer) would —rightly— identify the observational predicates with those decidable "phenomenologically" on the basis perception [see Grover Maxwell 1971]. Here however, we need not commit ourselves to any particular account of the "observational basis" of science. But however exactly the distinction is understood, no serious version of structural realism can get going without some such distinction. If *all* the predicates of a scientific theory are taken to be interpreted only within the context of the claims made by the theory, if, that is, none is taken to be anchored in experience independently of our attempted descriptions of the universe, then the constraints imposed by the Ramsey-sentence (or indeed by the original theory itself) would be hopelessly weak. For the theory would then be indistinguishable from a formal mathematical system, where the latter is —properly— subject to a mere consistency condition. The criticism of those who object to structural realism would then reduce to the ungainsayable truism that a consistent formal theory constitutes an "implicit definition" of its primitive concepts; i.e. it merely determines a class of models [see D. Lewis 1970].

It should immediately be granted that Demopoulos and Friedman seem to be aware of this possible counter-objection. So they entered the caveat that only what the Ramsey-sentence asserts *over and above its observational content* is reducible to logic or to mathematics; more precisely: to a consistency or to a cardinality constraint. This "over and above" however proves to be essentially indefinable; for on the one hand, the Ramsey-sentence does not normally follow from its empirical basis, i.e. from the set of true and empirically decidable, hence *singular* sentences. If, on the other hand, all the —generally undecidable— "empirical generalisations" were included in the observational content of a theory, then the Ramsey-sentence might well turn out to be one of them; in which case Demopoulos's and Friedman's thesis collapses into the trivial claim that the Ramsey-sentence follows from itself.

Consider the following —admittedly simplistic— example. Let

$$A \equiv [(\forall x)\ (F(x) \rightarrow T(x)) \wedge (\forall y)\ (T(y) \rightarrow K(y))],$$

where T is a theoretical notion, while both F and K are observational concepts. Thus

$$A^* \equiv (\exists t)\ [(\forall x)\ (F(x) \rightarrow t(x)) \wedge (\forall y)\ (t(y) \rightarrow K(y))].$$

It is obvious that $\vdash [A^* \leftrightarrow (\forall x)\ (F(x) \rightarrow K(x))]$. Thus A^* is provably equivalent to a first-order sentence. Let $\{a_j : j \in \Delta\}$ be a set of distinct names for all the observable entities in the world. Δ could therefore be infinite. Assume that $F(a_j) \wedge K(a_j)$ has been ascertained as true for all $j \in \Delta$. If we are to believe Demopoulos and Friedman, what the Ramsey-sentence asserts beyond $\{F(a_j) \wedge K(a_j): j \in \Delta\}$ is merely tautological. This claim can be easily refuted as follows. First note that the *logical* difference between two statements Y and X—i.e. the weakest formula which, taken in conjunction with X, entails Y— is $(\neg X \vee Y)$ or $(X \rightarrow Y)$; but we are here dealing, not with the difference between A^* and a single statement, but with that between A^* and the infinite set of reports $\{F(a_j) \wedge K(a_j): j \in \Delta\}$. For the sake of argument, let us nonetheless suppose that the infinite conjunction $\bigwedge\{F(a_j) \wedge K(a_j): j \in \Delta\}$ is syntactically well-formed. Demopoulos and Friedman's thesis is tantamount to the claim that $\bigwedge\{F(a_j) \wedge K(a_j): j \in \Delta\} \rightarrow A^*$ is a logical truth; i.e. that

$$\{F(a_j) \wedge K(a_j): j \in \Delta\} \vdash (\forall x)\ (F(x) \rightarrow K(x)).$$

That this inference is invalid can most easily be established by considering any domain $\Delta'' \supset \Delta$. Δ'' clearly yields a model of $\{F(a_j) \wedge K(a_j): j \in \Delta\}$ which falsifies $(\forall x)\ (F(x) \rightarrow K(x))$.

We have already mentioned that some philosophers might choose to consider the "generalisation" $(\forall x)\ (F(x) \rightarrow K(x))$, i.e. the Ramsey sentence A^* itself, as belonging to the empirical basis of the hypothesis A. This would however transgress the canons of even the most liberal version of empiricism; for $(\forall x)\ (F(x) \rightarrow K(x))$ fails to be fully empirically decidable. This is precisely why prominent logical positivists decided to regard synthetic universal statements as expressing inference rules rather than genuine propositions [V. Kraft 1968, pp. 121-137].

5. What cognitive Difference, if any, is there between X and X?*

Summing up what was established above: once a distinction *is* made between the theoretical notions and a fixed set of observational predicates, both Newman's and the Demopoulos-Friedman objections fail.

Let us now go back to the general case of a hypothesis $G(\mathbf{Q}, \mathbf{T})$ and of its Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$. Again note that the observational predicates $\mathbf{Q}$ are *not* quantified over, for they are taken to have referents fixed independently of G. It will easily be proved that the Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ does not merely constrain the cardinality of its models —in fact, it sometimes does less than this, but generally it does much more.

Let us compare the logical strengths of the two formulas

$(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ and $G(\mathbf{Q}, \mathbf{T})$.

First note that —unsurprisingly:

(1) $\vdash [G(\mathbf{Q}, \mathbf{T}) \rightarrow (\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})]$;

i.e. the original theory always entails its Ramsey-sentence.

Consider next the logical difference:

(2) $[(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$

between $G(\mathbf{Q}, \mathbf{T})$ and $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$.

As is well-known, Carnap looked upon (2) as an analytic meaning-postulate. [R. Carnap 1966; Chapter 28, p. 270]. But from a strictly logical viewpoint, his notion of analyticity and his manner of arriving at the conclusion that $(X^* \rightarrow X)$ is an analytic meaning *A*-postulate are highly suspect. He starts by singling out the sequence Q_1, … , Q_n of all observational notions occurring in some scientific hypothesis $G(Q_1, \dots, \dots, Q_n, T_1, \dots, T_m)$. Since n is usually large, he introduces abbreviations by means of so-called *D*-rules. To my mind, these can be expressed only metalinguistically. For example: both 'animal' and 'bird' might stand for conjunctions of certain Q_i's, where the Q_i's defining 'animal' figure in the *definiens* of 'bird'. There is of course nothing wrong with such *D*-rules which are used in order to simplify presentation; but sentences containing words like 'animal' or 'bird' will not be statements expressed in the primitive object-language. They can be *transformed* into such statements through replacing 'animal' and 'bird' by their *definiens* in terms of the Q_i's. Once this is done, then propositions like

(*A*1) *All birds are animals*,

prove to be logically true.

In this sense, there is no harm in calling the statement (*A*1) analytic. Carnap however proposes to circumvent the *D*-rules and directly regard (*A*1) as an analytic *A*-postulate; which clearly entails that sentences like (*A*1) must now belong to the object-language and hence that 'bird' and 'animal' have been added to our primitive vocabulary. But then (*A*1) should be regarded as a synthetic judgement. In other words: once a primitive vocabulary and a list of logical constants have been selected, then one can no longer *decree* what formulas ought to be considered as analytic; for even according to Carnap himself, analyticity must ultimately coincide with logical truth, i.e with truth under all interpretations of the *primitive* descriptive terms.

Anyway, to speak of (*A*1) as of a *meaning*-postulate seems to me to conflate syntax with semantics. Since notions like those of meaning, reference and truth-value are semantic in character, they cannot be adequately rendered by syntax alone; for they involve a reference both to language and to something extra-linguistic. It cannot legitimately be claimed that (*A*1) talks, not only about birds and animals, but also about the meanings of the words 'bird' and 'animal'. Here, Frege's fundamental insight ought to be adhered to: irrespective of the extent to which they might initially have been informed by philosophical considerations, syntactical rules should subsequently fend for themselves. And the best way of avoiding Carnap's ambiguities is to abolish all talk of meaning-postulates; hence to regard (*A*1) as a synthetic axiom expressed in the primitive vocabulary of an enriched object-language. As for the Carnapian concept of a meaning-postulate, it can be retrieved through that of a *metaphysical* proposition —in Popper's sense. The meaning-postulates should be characterised as axioms whose conjunction is metaphysical, i.e. observationally irrefutable. This in no way implies that such conjunctions do no empirical work within the context of a global system; for the omission of a metaphysical component from a scientific hypothesis might well diminish and even destroy the latter's observational content.

Let us now show in what way our conclusions apply to the difference

$$(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})$$

between the theory $G(\mathbf{Q}, \mathbf{T})$ and its Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$. Here, Carnap's claim that this difference is analytic proves untenable; for otherwise, by the very definition given by Carnap, $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})$ ought to be reducible, by means of *D*-postulates, to a logically true formula. This is however far from being normally the case; for the Q_i's and the T_j's can be taken to be primitive predicates belonging to the underlying object-language, thus pre-empting any recourse to *D*-rules. This often enables us to determine a model of $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ which falsifies $G(\mathbf{Q}, \mathbf{T})$ and hence also $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})$. Consider the theory A with its Ramsey-sentence A^* defined above; also any domain in which F, K and T denote the classes F_0, K_0 and T_0 respectively, where $F_0 \subseteq K_0$ but $\mathrm{Not}(F_0 \subseteq T_0)$. This yields a model of A^* under which A is false. A theory is thus essentially stronger than its Ramsey-sentence.

Let us call any proposition containing no theoretical terms an *O*-sentence; and let us define a basic statement as any singular *O*-sentence which is fully empirically decidable. It will be shown that (2) is not only empirically irrefutable and hence metaphysical, but also that it entails no synthetic *O*-sentence. For suppose that

$$\vdash [((\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})) \rightarrow G(\mathbf{Q}, \mathbf{T}))) \rightarrow Z\,],$$

i.e.

$$\vdash [(\neg(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})) \vee G(\mathbf{Q}, \mathbf{T}))) \rightarrow Z\,],$$

where Z does not contains any T_j. Quantifying over $\mathbf{T}$, obtain

$$\vdash [(\exists \mathbf{t})\ (\neg(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \vee G(\mathbf{Q}, \mathbf{t}))) \rightarrow Z\,],$$

i.e.:

$$\vdash [(\neg(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \vee (\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})) \rightarrow Z\,];$$

whence:

$$\vdash Z;$$

i.e. Z is a logical or mathematical truth.

It follows that proposition (2) is not analytic but metaphysical.

To sum up: Carnap's mistake consists in his having conflated 'analytic' with 'metaphysical'; which, given his positivist stance, was almost bound to happen. For according to neopositivism, the absence of an empirically ascertainable content is tantamount to absence of content *tout court*. But this neopositivist dichotomy between the empirical and the analytic, where the latter includes the logical truths, the mathematical principles and all the meaning-postulates, is totally wrong-headed. All we need are Popper's notions of logical truth and of metaphysical proposition. We can legitimately regard the synthetic mathematical principles, which are trivially irrefutable, as metaphysical. By definition, the remaining propositions will be empirically testable and hence scientific; all of which presupposes that we stick to one chosen primitive vocabulary including a list of logical constants. Carnap's meaning-postulates concerning the observational predicates can be replaced by metalinguistic D-rules; i.e. by verbal definitions specifying how certain abbreviations are to be used. Since the D-rules do not belong to the object-language, they are in principle dispensable and hence call for no special treatment. As for the difference $(X^* \rightarrow X)$ between X and its Ramsey-sentence X^*, it does not constitute an analytic meaning-postulate but a metaphysical proposition all of whose consequences are likewise irrefutable.

In his [1999], S. Psillos allowed himself to be misled by Carnap into holding that a structural realist who postulates X^* must also accept, *as merely analytic*, the proposition $(X^* \rightarrow X)$. Such a realist is thus allegedly forced to infer the fully-fledged theory X [pp. 51-69]. When Psillos further claims that the truth of the *interpreted* hypothesis X is an empirical matter, he presupposes that an interpretation, not only of the observational, but also of the theoretical predicates can and must be specified in terms of natural kinds. But this is precisely where the problem lies; for whereas X can in principle be refuted through the falsification of any of its observational consequences, *ascertaining* the truth of X must proceed through the interpretation of its theoretical terms. The latter however refer to the external world; so that, barring what Putnam rightly calls a mystical insight, we are unable to break out of our intentional sphere and fix a semantic relation between our theoretical vocabulary and some *transcendent* reality. Note that the structural realist in no way denies that the whole of a scientific theory is physically interpretable; he simply points to the *epistemological* fact that we are in principle unable effectively to carry out such an integral interpretation. Though capable of giving a syntactic-structuralist account of 'natural kinds', the structural realist refuses to accept, *as analytic*, the implication $(X^* \rightarrow X)$ which he correctly regards as a *synthetic*, albeit *metaphysical* proposition.

Let us now go back to examining, in very general terms, the relationship between the theory $G(\mathbf{Q}, \mathbf{T})$, its Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ and the presence or absence of any observational predicates in $G(\mathbf{Q}, \mathbf{T})$.

We propose first to show that $G(\mathbf{Q}, \mathbf{T})$ and $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ are not only observationally identical but that they furthermore imply the same *O*-sentences. By (1), every consequence of $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ is also trivially entailed by $G(\mathbf{Q}, \mathbf{T})$. Conversely, let $H(\mathbf{Q}, \mathbf{T})$ be any logical consequence of $G(\mathbf{Q}, \mathbf{T})$. That is:

(3) $\vdash G(\mathbf{Q}, \mathbf{T}) \rightarrow H(\mathbf{Q}, \mathbf{T})$.

Quantifying over **T**, infer:

(4) $\vdash (\forall\mathbf{t})\ [G(\mathbf{Q}, \mathbf{t}) \rightarrow H(\mathbf{Q}, \mathbf{t})]$.

An immediate consequence of (4) is:

(5) $\vdash [(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow (\exists\mathbf{t})\ H(\mathbf{Q}, \mathbf{t})]$.

Note first that (5) is in general much weaker than (4); secondly that, in (3), we could have generalised over *all* the predicates Q_i *and* T_j. We refrained from doing so precisely because the Q_i's are observational, hence possess fixed accessible meanings; so that quantifying over them —as Newman seems to have done— would involve a loss of valuable empirical information. Two importantly different cases must now be considered.

[a] *H* contains no occurrences of any T_j's and is therefore an *O*-sentence. Thus we can write $H'(\mathbf{Q})$ for $H(\mathbf{Q}, \mathbf{T})$. In this case, (4) is actually equivalent to:

(6) $\vdash [(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow H'(\mathbf{Q})]$.

(3) and (6) tells us that $G(\mathbf{Q}, \mathbf{T})$ and its Ramsey-sentence $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ have the same *O*-consequences and are *a fortiori* observationally identical. Should $H'(\mathbf{Q})$ turn out to be a basic statement, then (6) would enable us, through the experimental verification of $\neg H'(\mathbf{Q})$, to refute $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$. Referring to the example given in Section 1: both *A* and *A** could be refuted by a basic statement of the form $F(a) \wedge \neg K(a)$.

Thus the Ramsey-sentence of every scientific theory has empirical import and does not convey information *only* about the cardinality of its domain of discourse. In fact, the difference $[(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$ between a fully-fledged scientific hypothesis $G(\mathbf{Q}, \mathbf{T})$ and its Ramsey-sentence $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ fits exactly Popper's definition of a metaphysical thesis. It might *prima facie* be thought that Popper held a proposition *M* to be metaphysical if and only if *M* is *a priori* in the sense of being independent of experience, i.e. of being neither provable nor falsifiable by observation. But Popper insisted only on the empirical irrefutability condition, thus conceding that *M* might be experimentally verifiable. $[(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$ illustrates this possibility. We have seen that $[(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$ entails no synthetic *O*-statement and is *a fortiori* untestable. $[(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$ could however be experimentally verified through the empirical refutation of $(\exists\mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$, which was shown to be observationally equivalent to the scientific theory $G(\mathbf{Q}, \mathbf{T})$. Thus Carnap ended up by regarding an *empirically verifiable* statement, namely $[(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow G(\mathbf{Q}, \mathbf{T})]$, as analytic and hence, according to the Logical Positivists themselves, as devoid of sense. This is why there is to my mind no doubt as to the superiority of Popper's trichotomy into scientific, metaphysical and logically true propositions, over Carnap's excluded middle between analytic and empirical statements.

The question now arises as to whether some (non-empirical) reason could make us accept the theory $G(\mathbf{Q}, \mathbf{T})$ over and above the logically weaker Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$. The answer to this question will be shown to be decidedly negative.

According to structuralism, there exist no experimental, theoretical or even aesthetic reason which would make us opt for $G(\mathbf{Q}, \mathbf{T})$ in preference to the logically weaker Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$: first, we have seen that $G(\mathbf{Q}, \mathbf{T})$ and $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ have exactly the same *O*-consequences; secondly, no unity or simplicity criteria could possibly demarcate between $G(\mathbf{Q}, \mathbf{t})$ and $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$; for apart from the existential quantifier $(\exists \mathbf{t})$, the two formulas have essentially the same syntactical structure. [In the case of the propositions A and A^* above, A^* even proves to be simpler than A.]

[b] Let us now turn to the second special case of (3) where $H(\mathbf{Q}, \mathbf{T})$ is assumed to contain no occurrence of any observational predicates Q_i. We can thus write $H''(\mathbf{T})$ for $H(\mathbf{Q}, \mathbf{T})$, so that (5) reduces to the statement:

$$(6)\ \vdash [(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow (\exists \mathbf{t})\ H''(\mathbf{t})].$$

$H''(\mathbf{T})$ could e.g. be the sentence $(\exists \mathbf{q})\ G(\mathbf{q}, \mathbf{T})$, so that, by (6):

$$(7)\ \vdash [(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t}) \rightarrow (\exists \mathbf{t})(\exists \mathbf{q})\ G(\mathbf{q}, \mathbf{t})], \text{ which is a logical truth.}$$

This important special subcase arises if all the terms contained in $G(\mathbf{Q}, \mathbf{T})$ are regarded as theory-laden and hence as essentially theoretical. I hold this 'modern' view to be deeply mistaken; for it would mislead us into quantifying over *all* the predicates occurring in $G(\mathbf{Q}, \mathbf{T})$, thus yielding not $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ but $(\exists \mathbf{t})(\exists \mathbf{q})\ G(\mathbf{q}, \mathbf{t})$ as the Ramsey-sentence of $G(\mathbf{Q}, \mathbf{T})$. Whatever the case may be, it is clear that $(\exists \mathbf{t})\ H''(\mathbf{t})$ might well express a tautology or a logical contradiction.

So far, no information about the cardinality of our domain of discourse has been conveyed. Such information will be forthcoming only in the case of finite domains; for if $H''(\mathbf{T})$ is first-order and has one infinite model, then $H''(\mathbf{T})$, and hence also $(\exists \mathbf{t})\ H''(\mathbf{t})$, will possess infinite models of arbitrarily large cardinalities [by the 'upward' Löwenheim-Skolem theorem]. As for the finite case, it arises if $H''(\mathbf{T})$ were e.g. a formula of the form:

$$[(\forall x)\ T_1(x) \wedge (\exists x_1, x_2, \ldots, x_r)\ ((x_1 \neq x_2) \wedge (x_1 \neq x_3) \wedge \ldots$$

$$\ldots \wedge (x_{r-1} \neq x_r) \wedge (\forall x)\ (T_1(x) \leftrightarrow ((x = x_1) \vee (x = x_2) \vee \ldots \vee (x = x_r))))],$$

where r is a specific natural number. $(\exists t_1)\ H''(t_1)$ would then be equivalent to the assertion that the domain of individuals consists of exactly r members.

To sum up: the Ramsey-sentence $(\exists \mathbf{t})\ G(\mathbf{Q}, \mathbf{t})$ reduces to $(\exists \mathbf{t})\ H''(\mathbf{t})$ only if $n = 0$; i.e. only if there exist no observational predicates at all; or alternatively: if no distinction is made between the observational and the theoretical terms. This is however far from being the position of prominent structural realists like Poincaré, Duhem and Russell. The structural realist is committed only to regarding our knowledge of the theoretical predicates $T_1, \ldots, T_m$ as wholly conveyed by the nexus $G(Q_1, \ldots, Q_n, T_1, \ldots, T_m)$; i.e. to holding that $T_1, \ldots, T_m$ are known exclusively by description, namely through their mutual relations and the way —described by $G(Q_1, \ldots, Q_n, T_1, \ldots, \ldots, T_m)$— in which they link up with the observational predicates $Q_1, \ldots, Q_n$. As for the latter, they are known *both* by description, *via* $G(Q_1, \ldots, Q_n, T_1, \ldots, T_m)$, and by acquaintance, i.e. phenomenologically or ostensively. In the last analysis, the Demopoulos-Friedman objection rests on the unwarranted assumption that the structural realist is bound to deny the observational/theoretical distinction.

6. Putnam's model-theoretic arguments

At first sight, Putnam's results constitute the most powerful argument against scientific realism —whether classical or structural. Putnam showed that given *any* epistemically ideal theory T_I, we have *no* reason not to regard T_I *as being true* [see H. Putnam 1983]. We shall moreover prove that given certain cardinality constraints, T_I possesses a 'standard model' in the world, i.e. a model in which all the observational notions have their commonly accepted meanings. This would entail that the Ramsey-sentence T_I^* of T_I *is true simpliciter*.

In order to assess the relevance and scope of Putnam's theorem, it is necessary briefly to examine his proof and its presuppositions. We shall then discuss whether structural realism is really undermined by Putnam's results.

Putnam denotes by S the set of all observable entities. Thus $S \subseteq W$, where 'W' denotes the whole universe. Since the results of all measurements are signified by rational numbers, S can be taken to be enumerable. A set of empirical predicates $\{\pi_i : i \in \Omega\}$ is also posited, where, for each $i \in \Omega$ and for all $\alpha, \beta, \ldots, \gamma$ in S, $\pi_i(\alpha, \beta, \ldots, \gamma)$ is supposed to be ascertainably true or false. An epistemically ideal theory T_I is then defined as any consistent first-order system which possesses a set of terms $\{t_\alpha : \alpha \in S\}$ such that $T_I \vdash (t_\alpha \neq t_\beta)$ for any two distinct elements α and β of S. For each $i \in \Omega$, T_I is furthermore required to possess a predicate P_i which has the same number of argument-places as π_i and is such that, for all $\alpha, \beta, \ldots, \gamma$ in S, we have:

$$T_I \vdash P_i(t_\alpha, t_\beta, \ldots, t_\gamma) \text{ if } \pi_i(\alpha, \beta, \ldots, \gamma) \text{ is true, and}$$

$$T_I \vdash \neg P_i(t_\alpha, t_\beta, \ldots, t_\gamma) \text{ if } \pi_i(\alpha, \beta, \ldots, \gamma) \text{ is false.}$$

Let J be any model of T_I. Put:

$$(8)\quad \tau_\alpha =_{\text{Def}} J(t_\alpha) \quad \text{for all } \alpha \in S.$$

It can easily be shown that $\pi_i(\alpha, \beta, \dots, \gamma) \equiv J(P_i)(\tau_\alpha, \tau_\beta, \dots, \tau_\gamma)$ for all $\alpha, \beta, \dots, \gamma$ in S. Thus suppose that $\pi_i(\alpha, \beta, \dots, \gamma)$ is true. By hypothesis,

$$T_I \vdash P_i(t_\alpha, t_\beta, \dots, t_\gamma).$$

Since J is a model of T_I, then $J(P_i)(J(t_\alpha), J(t_\beta), \dots, J(t_\gamma))$, i.e. $J(P_i)(\tau_\alpha, \tau_\beta, \dots, \tau_\gamma)$, is true. And if $\pi_i(\alpha, \beta, \dots, \gamma)$ is false, then again by hypothesis,

$$T_I \vdash \neg P_i(t_\alpha, t_\beta, \dots, t_\gamma).$$

hence $J(\neg P_i(t_\alpha, t_\beta, \dots, t_\gamma))$ is true; that is: $J(P_i(t_\alpha, t_\beta, \dots, t_\gamma))$, which is identical with $J(P_i)(\tau_\alpha, \tau_\beta, \dots, \tau_\gamma)$, is false. Thus:

$$(9)\ \ J(P_i)(\tau_\alpha, \tau_\beta, \dots, \tau_\gamma) \equiv \pi_i(\alpha, \beta, \dots, \gamma), \text{ for all } \alpha, \beta, \dots, \gamma \text{ in } S.$$

J will be said to be *standard* if, for all $\alpha \in S$, we have: $J(t_\alpha) = \alpha$, i.e. if $\tau_\alpha = \alpha$.
(9) entails that in a standard model:

$$(10)\ \ J(P_i)(\alpha, \beta, \dots, \gamma) \equiv \pi_i(\alpha, \beta, \dots, \gamma), \text{ for all } \alpha, \beta, \dots, \gamma \text{ in } S;$$

i.e., for all i in Ω, $J(P_i)$ coincides with π_i on S.

Although epistemically ideal systems are almost impossible to construct in practice, it will be assumed, if only for the sake of argument, that at least one such theory is achievable.

Putnam established that every epistemically ideal T_I has a standard model; for by the Löwenheim-Skolem theorem, T_I has at least one (countable) model, J say. Letting D be the domain of J, we have, by (8):

$$\{\tau_\alpha: \alpha \in S\} = \{J(t_\alpha): \alpha \in S\} \subseteq D.$$

Put: $\Delta = \{\tau_\alpha: \alpha \in S\}$ and $\Delta' = D \backslash \Delta = D \backslash \{\tau_\alpha: \alpha \in S\}$. Thus:

$$(11)\ \ D = \Delta \cup \Delta' = \{\tau_\alpha : \alpha \in S\} \cup \Delta'.$$

Hence $|D| = |\Delta| + |\Delta'| = |\{\tau_\alpha: \alpha \in S\}| + |\Delta'|$.

Since $T_I \vdash t_\alpha \neq t_\beta$ for $\alpha \neq \beta$ then $J(t_\alpha) \neq J(t_\beta)$ since J is a model of T_I. Hence:

$$(12)\ \ \tau_\alpha \neq \tau_\beta \text{ for } \alpha \neq \beta.$$

Therefore: $|\Delta| = |\{\tau_\alpha: \alpha \in S\}| = |S|$. By the above:

$$(13)\ \ |D| = |\Delta| + |\Delta'| = |S| + |\Delta'|.$$

Note that if S is taken to be infinite, then D, and hence every model J of T_I will be infinite. Now define the one-one map f as follows:

$$(14)\ \ f(\tau_\alpha) = \alpha \text{ for all } \alpha \in S;$$

and for any $\mu \in \Delta'$, define $f(\mu)$ in any way consistent with the requirement that the resulting f be a bijection whose domain is the whole of D.

Now determine the interpretation J_0 as follows. The domain of J_0 will be the set:

$$(15)\quad D_0 =_{\mathrm{Def}} \mathrm{S} \cup f[\Delta'] = \{f(\tau_\alpha): \alpha \in S\} \cup \{f(\mu): \mu \in \Delta'\}.$$

For any function letter (or name) g, let:

$$(16)\quad J_0(g)(\xi, \eta, \ldots, \zeta) =_{\mathrm{Def}} f(J(g)(f^{-1}(\xi), f^{-1}(\eta), \ldots, f^{-1}(\zeta))),$$

for any $\xi, \eta, \ldots, \zeta$ in D_0.

Finally, for any predicate R, define:

$$(17)\quad J_0(R)(\xi, \eta, \ldots, \zeta) \equiv_{\mathrm{Def}} J(R)(f^{-1}(\xi), f^{-1}(\eta), \ldots, f^{-1}(\zeta)),$$

for all $\xi, \eta, \ldots, \zeta$ in D_0.

As is well-known, (16) entails:

$$(18)\quad J_0(u) = f(J(u)),$$

for all terms u. Trivially: since J is a model of T_I, then so is J_0 (for J_0 can be considered as obtained by relabelling the elements of J). J_0 is furthermore a standard model, since:

$$(19)\quad J_0(t_\alpha) = f(J(t_\alpha)) \text{ [by (18)] } =_{\mathrm{Def}} f(\tau_\alpha) =_{\mathrm{Def}} \alpha \text{ [by (14)]}.$$

This result can be strengthened by imposing constraints on the cardinalities of various sets. Let us henceforth assume that the world W is infinite and that T_I has at least one infinite model; then by the 'upward' Löwenheim-Skolem theorem, T_I has a model having cardinality $|W|$. In the above, it can therefore be assumed that $|D| = |W|$. Let us moreover suppose that $|W| > |S|$; which is not an unreasonable assumption, since S is countable while the set of all physical events has at least the power of the continuum. Thus:

$$(20)\quad W = S \cup S', \text{ where } S' =_{\mathrm{Def}} (W \setminus S).$$

Hence $|W| = |S| + |S'|$.

Since W is infinite and $|W| > |S|$, it follows that:

$$(21)\quad |W| = |S'|.$$

Thus $|W| = |D| = |S| + |\Delta'|$ [by (13)].

Since $|W|$ is infinite and greater than $|S|$,

$$(22)\quad |W| = |\Delta'|;$$

hence $|S'| = |\mathrm{W}| = |\Delta'|$; and so:

$$(23)\quad W = S \cup S';$$

$D = \{\tau_\alpha: \alpha \in S\} \cup \Delta'$; where $|S'| = |W| = |D| = |\Delta'|$.

Let ψ be any one-one map of Δ' onto S'. Going back to (14), we see that f can be defined as follows:

(24) $f(\tau_\alpha) =_{\mathrm{Def}} \alpha$

for all $\alpha \in S$; and for any $\mu \in \Delta'$, let $f(\mu) =_{\mathrm{Def}} \psi(\mu)$.

By (23) and (24), f maps $\{\tau_\alpha: \alpha \in S\} \cup \Delta' = \Delta \cup \Delta' = D$ one-one onto $S \cup S' = W$. Thus W = [Range of f] $=_{\mathrm{Def}}$ D_0. By the above, J_0 is a standard model of T_I in the world W; which clearly entails that the Ramsey-sentence $T_I{}^*$ of T_I is standardly true in W, or *true simpliciter*. The question now is: should the structural realist be worried about these results? We shall show that the answer is decidedly negative.

Putnam in effect provides a partial answer to a question posed by Einstein in his *Weltbild*; namely: is a scientific hypothesis which accounts for *all* the facts uniquely determined by the latter? [A. Einstein 1934, pp. 175-180]. Putnam has effectively shown that truth —rather than uniqueness— might well be attainable. More precisely: given the consistency of an epistemically ideal theory T_I, we have no empirical reason not to hold the Ramsey-sentence $T_I{}^*$ of T_I to be true. Given further assumptions about the cardinalities of the world W and of the models of T_I, we can even assert the truth of $T_I{}^*$. Qua structural realists, we would thus know that the structure described by T_I is realised in the world. In principle, a problem arises only if two ideal theories T_I and H_I were such that $T_I{}^*$ and $H_I{}^*$ are logically incompatible; but Jane English has proved that this is never the case if —as is tacitly assumed— an *epistemically ideal* theory entails no false observational statement (i.e. no false O-sentence) [Jane English 1980]. It is therefore as unexceptionable for a structural realist to claim that the two structures displayed by T_I and by H_I are realised in W as it is for a mathematician to accept that the same set can exhibit both an algebraic and a topological structure.

The more demanding structural realist can go further than this, but only by appealing to a metaphysical principle similar to the one invoked by both Poincaré and Einstein. The latter put it as follows: the world must be taken to be the realisation of what is mathematically simplest. As for Poincaré, he held the degree of unity of a hypothesis to be an index of its truth-likeness. Note that despite being incapable of *epistemologically* distinguishing between a theory and its Ramsey-sentence, i.e. between T_I and $T_I{}^*$, we can certainly adjudicate between T_I and H_I, or rather between $T_I{}^*$ and $H_I{}^*$. We might e.g. come to the conclusion that $T_I{}^*$ is more unified than $H_I{}^*$; given the principle just stated, only $T_I{}^*$ will be taken to reflect —if only approximately— the real structure of W. Thus, only in the case of $T_I{}^*$ do we allow ourselves to interpret *realistically* the second-order existential quantifications over the theoretical predicates. According to Poincaré, we would otherwise be attributing 'too great a role to chance'; i.e. the excess 'organic compactness' of $T_I{}^*$ over $H_I{}^*$ would have to be regarded as an accidental feature of $T_I{}^*$; and we know that theoretical physicists of Poincaré's or of Einstein's temper of mind are most reluctant to regard such accidents as ultimately inexplicable.

The above considerations also show that a structural realist can account for the notion of *natural kind* without abandoning his syntactic-structuralist stance. For the best guide as to what kinds are natural is the degree of unity of a theory like T_I, or of any highly successful hypothesis T in which such kinds play a central role. In other words: natural kinds are precisely those signified by the predicate and function symbols which give T its most coherent formulation. Let us repeat that both the empirical content and the so-called 'degree of internal perfection' of a theory T coincide with those of its Ramsey-sentence T^*; so there is nothing to choose between T and T^*. Note also that a most ardent defender of natural kinds, namely David Lewis, grants that those kinds are natural which are talked about by our best theories. [D. Lewis 1970, p. 428]. But what are these 'best' theories? They are clearly those highly unified hypotheses which save the facts without *ad hoc* assumptions. Thus, we do not possess a prior insight into natural kinds on which we then base our conjectures; we rather start by formulating empirically adequate hypotheses *and then* select, from among them, the most unified theory, T say. T^* will then express exactly what we want to assert; namely that the theoretical predicates quantified over in T^* *denote the natural kinds which are really actualised in the world.* In this sense, the Keplerian (near-) ellipses, as opposed to the complex curves which would have to be postulated by any Tychonic reformulation of Kepler's laws, really exist. Similarly: let E be Einstein's gravitational theory and let N denote any of its 'Euclidean reformulations'. N is known to be empirically equivalent to E, at any rate within a large region of the spatio-temporal manifold. Scientists —who normally are demanding realists— do not look upon the unity displayed by the structure of E, as opposed to the *ad hoc* character of N, as an accidental feature of E. They hold that E describes reality more adequately than does N.

Let us finally add that no ideal theory like T_I has so far been put forward; and it seems doubtful that such a theory could ever be achieved *in practice*. So science must content itself with provisional hypotheses, or rather with the latter's Ramsey-sentences, which might at best be compatible with all *known* experimental results. It is precisely in these situations that SSR proves its mettle. Consider two such hypotheses —call them T_1 and T_2— which are empirically identical. Thus both T_1 and T_2 could subsequently be refuted —by the same facts. Yet T_2 —but not T_1— might possess a structure which points, or heuristically leads to a more powerful theory, S say; where S supersedes both T_1 and T_2 empirically. S might e.g. explain some of the results which undermined T_1 and hence also T_2; yet only the structure of T_2 survives —possibly in a modified form— in S. As an example, take the case where: $S \equiv$ Quantum Mechanics; $T_1 \equiv$ Newtonian Dynamics; and $T_2 \equiv$ (Classical) Hamiltonian Dynamics. This example is instructive, first because it does not rely on hindsight: long before the birth of Quantum Physics, Poincaré had noted that despite their strict observational equivalence, T_1 and T_2 have very different structures and hence make different statements about the world. Secondly: as we go over from T_2 to S, only the mathematical structure, as typified by the form of the Hamiltonian function, is preserved; even the logical types of this function's arguments are altered: whereas the arguments of the classical Hamiltonian are functions of the time-variable, those of its quantum-mechanical correlate are operators, i.e. functions of functions, hence higher-order entities. No example illustrates more clearly the essential role played by the notion of structure in the development of fundamental science.

It might be maintained that structural *realism* does not deserve its name. This, of course, is a purely terminological and hence irrefutable objection. It can nonetheless be pointed out that the most prominent advocates of structural realism, namely Duhem, Poincaré and Russell were intuitive realists; they instinctively believed in the existence of a structured and mind-independent reality which the scientist tries to capture; but their —ungainsayable— insight was that our knowledge of the mode of reference of theoretical terms is as problematic as that of the truth of scientific hypotheses. According to these authors as well as to a modern realist like E. MacMullin, we are entitled to regard some of our concepts as *metaphors*, which nonetheless model the external world; in which case there is no longer any question of a strict *term-by-term* reference to anything at all, whether it be intended or not [E. MacMullin 1984].

Appendix 1: Informal Presentation of the Problem

[I] Despite all efforts made to reduce it to Logic, Mathematics remains a synthetic discipline. Moreover, basic notions like those of class [k] and of class-membership [$\in$] can hardly be claimed to be known by acquaintance. Hence k and $\in$ are theoretical predicates. The conjunction **M** of all mathematical postulates must therefore form an explicit part of every physical system **G**; so that the predicates k & $\in$ ought to be quantified over in the Ramsey-sentence of **G**.

[II] As usually formulated, **M** consists of an infinite number of axioms, namely those of **ZFC**; whereas **G** is taken to be a finite (first-order) formula.

[III] Hence some method of finitely axiomatising **M** must be found. This has already been achieved —for pure mathematics— by Gödel in his [1940]. Gödel's basic idea is as follows. In **ZFC**, consider e.g. the Replacement Scheme, which subsumes a denumerable infinity of axioms; namely:

$$(\forall u)\ [(\forall x)(\exists^* y)\ \mathbf{B}(x,y) \rightarrow (\exists v)(\forall y)\ ((y \in v) \leftrightarrow (\exists x \in u)\ \mathbf{B}(x,y))];$$

where $\mathbf{B}(x,y)$ is an arbitrary wff of **ZFC**.

Gödel's method was to introduce the notion k of 'class' where: every set was to constitute a special kind of class; $\{\langle x,y\rangle : \mathbf{B}(x,y)\}$ would moreover always be a class, though not necessarily a set; so the above scheme could be replaced by the single sentence:

$$(\forall u, B)\ [(\forall x, y, z)\ (((\langle x,y\rangle \in B) \wedge (\langle x, z\rangle \in B)) \rightarrow (y = z)) \rightarrow$$

$$\rightarrow (\exists v)(\forall y)\ ((y \in v) \leftrightarrow (\exists x \in u)\ (\langle x,y\rangle \in B))]$$

where lower-case variables range over sets, and capital letters over classes.

[IV] Note that in $\mathbf{B}(x, y)$ only sets are quantified over. In order to guarantee that the extension, in x and y, of every $\mathbf{B}(x, y)$ is a class, Gödel had to start by postulating that the extension of every atomic wff is a class. He had then to insure that the basic logical operations [negation, conjunction and existential quantification *over sets*] lead from classes to classes. The formation rules of the formulas of **ZFC** were consequently mirrored in Gödel's axiom system.

[V] My main problem was to extend Gödel's finite axiomatisation methods in order to cover the kind of mathematics used in more comprehensive systems; these have a richer ontology which normally includes *Urelemente*, i.e. entities which are non-classes. Furthermore: over and above the predicates k and $\in$, other primitive notions have to be considered, e.g. those represented by $T_1, \dots, T_m$ and $Q_1, \dots, Q_n$. Let 'element' stand for: set or *Urelement.*

$T_1, \dots, T_m$ will be construed as classes of tuples, which enables us to bring set theory to bear on all theoretical entities. Thus, for each T_j, an individual name a_j can be introduced, where: a_j denotes the class of all p_j-tuples of elements satisfying T_j, p_j being the number of argument-places of T_j. As for the observational predicates $Q_1, \dots, Q_n$, since they are not to be quantified over, they ought to be regarded as primitive [For otherwise, a basic statement like $Q(d)$ would have to be interpreted as $(d \in Q)$; Q however represents an observational concept, while $\in$ is a theoretical notion which must be quantified over in the Ramsey-sentence. There would be no basic statements free from theoretical predicates, which is a highly undesirable consequence].

[VI] Let α, β, γ be any (primitive) individual names. Having formulated the usual axioms of set theory (using class language), the next step is to make sure that all the atomic wff's are extensional; i.e. that when considered as propositional functions in some, or in all of their element variables, the following formulas have classes as extensions:

$(x = y)$; $(x = x)$; $(x = \alpha)$; $k(x)$; $(x \in y)$; $(x \in x)$; $(x \in \alpha)$; $(\alpha \in x)$ and

$Q_m(t_1, \dots, t_p)$ for each $m = 1, \dots, n$; where every t_i is either a name or a variable.

[VII] We also have to require: corresponding to negation, that every class possesses a complement; corresponding to conjunction, that any 2 classes possess an intersection and, corresponding to the existential quantifier, that

$$(\forall x)\ [k(x) \to (\exists b)\ (k(b) \wedge (\forall z)\ ((z \in b) \leftrightarrow (e(z) \wedge (\exists y)\ (e(y) \wedge (<y, z> \in x)))))]$$

should hold. (See [13]-[15] below).

[VIII] We also need to insure that n-tuples of elements can be permuted at will.

Gödel showed that in his system only 3 axioms sufficed; where the latter deal with 2- and 3-tuples. These are Axioms [24]-[26] in Appendix 2. *I failed however to get by without the introduction of a fourth axiom, namely* [27]*, which refers to 4-tuples. The reason is probably that Gödel dealt with at most binary relations, whereas I had to consider arbitrary numbers of argu-*

ments. Axiom [27] *might of course prove redundant; but this would not affect any of my philosophical theses.*

The main point of the axioms below is to insure that every normal propositional function, i.e. every formula H which involves quantifications only over element-variables, has, with respect to its free element-variables, a class as its extension.

Let me now give a more formal treatment of these guidelines.

Appendix 2: The System Σ

A finite list of axioms will be given for a Gödel-Bernays type of class-theory which will be part of an arbitrary scientific system $\mathbf{G}(k, \in, \underline{h}, \varnothing, a_1, \dots, a_m, Q_1, \dots, Q_n)$ having $Q_1, \dots, Q_n$ as observational predicates. $a_1, \dots, a_m$ are individual class names for the extensions of the theoretical predicates $T_1, \dots, T_m$ respectively.

(i) Adopt the Gödel-Bernays finitely axiomatised set theory S with its two basic notions of class $[k(x) \equiv (x \text{ is a class})]$ and class-membership $[(x \in y) \equiv (x \text{ is a member of the class } y)]$.

(ii) Extend S by adding to its vocabulary the name $\varnothing$ and the binary function letter $\underline{h}$. [$\varnothing$ denotes the empty set; and, for any two (element) terms t_1 and t_2, $\underline{h}(t_1, t_2)$ will denote the set whose only elements are t_1 and t_2].

(iii) For each theoretical term T_j, enter a class name a_j where: p_j being the number of argument-places of T_j, a_j will denote the class of p_j-tuples subsumed by T_j.

Also add a finite number of element names $u_1, u_2, \dots, u_s$ to the basic vocabulary; the latter will therefore contain a *finite* number of individual names, namely: $\varnothing, a_1, \dots, a_m, u_1, \dots, u_s$. The primitive predicates $Q_1, \dots, Q_n$ will finally be adjoined to our primitive vocabulary.

(iv) *Abbreviations*

The basic logical connectives will be: $\neg$ (negation), $\wedge$ (conjunction) and $\exists$ (existential quantification). All the other connectives are taken to be mere abbreviations. We shall sometimes write $A \Rightarrow B$ and $A \Leftrightarrow B$ for $\vdash (A \rightarrow B)$ and $\vdash (A \leftrightarrow B)$ respectively.

We shall also make use of the following abbreviations:

$$e(t) =_{\text{Def}} (\exists y)\ [t \in y];$$

$$s(t) =_{\text{Def}} [e(t) \wedge k(t)] =_{\text{Def}} [(\exists y)\ (t \in y) \wedge k(t)]$$

(*Intuitively*: $e(t) \equiv (t$ is a set or an *Urelement*));

$$\{t_1, t_2\} =_{\text{Def}} \underline{h}(t_1, t_2);$$

$$\{t\} =_{\text{Def}} \{t, t\} =_{\text{Def}} \underline{h}(t, t);$$

$$<t> =_{\text{Def}} t;$$

$$<t_1, t_2> =_{\text{Def}} \{\{t_1, t_1\}, \{t_1, t_2\}\} =_{\text{Def}} \underline{h}(\underline{h}(t_1, t_1), \underline{h}(t_1, t_2));$$

then, by meta-induction: $<t_1, t_2, \ldots, t_q> =_{\mathrm{Def}} <t_1,<t_2, \ldots, t_q>>$.

Thus, by Logic alone: $\vdash [(t_1 \in t_2) \rightarrow e(t_1)]$; $\vdash [s(t) \rightarrow e(t)]$; $\vdash [s(t) \rightarrow k(t)]$. Also:

$$Un(t) =_{\mathrm{Def}} [k(t) \wedge (\forall x, y, z)\ ((e(x) \wedge e(y) \wedge e(z) \wedge (<x, z> \in t) \wedge$$
$$\wedge (<y, z> \in t)) \rightarrow (x = y)];$$

where t, t_1, t_2 are three arbitrary terms. By Logic: $\vdash [Un(t) \rightarrow k(t)]$.

(v) *Axioms*: The (finitely many) mathematical axioms of our system, which will henceforth be denoted by Σ, are as follows (note that these axioms are not logically independent of each other):

[1] $k(a_i)$; and $e(u_j)$ for all $i = 1, \ldots, m$ & all $j = 1, \ldots, s$

Thus, by definition, $\vdash (\exists y)\ (u_j \in y)$.

[2] $(\forall x, y)\ [(x \in y) \rightarrow k(y))]$

(*This proposition follows from the meaning of 'class'.*)

[3] $(\forall x, y)[(k(x) \wedge k(y) \wedge (\forall z)\ (e(z) \rightarrow ((z \in x) \leftrightarrow (z \in y))) \rightarrow (x = y)]$

(*This the postulate of the extensionality of all classes.*)

By (iv): $\vdash (\forall x, y)\ [(k(x) \wedge k(y) \wedge (\forall z)\ ((z \in x) \leftrightarrow (z \in y))) \rightarrow (x = y)]$.

[4] $[e(\emptyset) \wedge k(\emptyset) \wedge (\forall z)\ (e(z) \rightarrow (z \notin \emptyset))]$

(*Existence of the empty set.*)

By (iv): $\vdash [e(\emptyset) \wedge k(\emptyset) \wedge (\forall z)\ (z \notin \emptyset)]$.

[5] $(\forall x, y)\ [e(\{x, y\}) \wedge k(\{x, y\}) \wedge (\forall z)\ (e(z) \rightarrow ((z \in \{x, y\}) \leftrightarrow ((z = x) \vee (z = y)))))]$

(*Pairing Axiom.*)

Once again, by virtue of (iv):

$\vdash (\forall x, y)\ [e(\{x, y\}) \wedge k(\{x, y\}) \wedge (\forall z)\ ((z \in \{x, y\}) \leftrightarrow (e(z) \wedge ((z = x) \vee (z = y)))))]$.

By Logic:

$\vdash (\forall x, y)\ [e(\{x, y\}) \wedge k(\{x, y\}) \wedge (\forall z)\ ((z \in \{x, y\}) \leftrightarrow ((e(x) \wedge (z = x)) \vee (e(y) \wedge (z = y)))))]$.

We can in fact dispense with $\emptyset$ and with $\underline{h}$ as primitive symbols through replacing [4] and [5] by the following axioms [4'] and [5']:

[4'] $(\exists x)\ [e(x) \wedge k(x) \wedge (\forall z)\ (e(z) \rightarrow (z \notin x))]$

I.e.: $(\exists x)\ [s(x) \wedge (\forall z)(e(z) \rightarrow (z \notin x))]$;

whence, by (iv): $\vdash (\exists x)\ [s(x) \wedge (\forall z)\ (z \notin x)]$.

[5'] $(\forall x, y)(\exists v)\ [e(v) \wedge k(v) \wedge (\forall z)\ (e(z) \to ((z \in v) \leftrightarrow ((z = x) \vee (z = y))))]$

By (iv): $\vdash (\forall x, y)(\exists v)\ [e(v) \wedge k(v) \wedge (\forall z)\ ((z \in v) \leftrightarrow (e(z) \wedge ((z = x) \vee (z = y))))]$.

By Logic:

$\vdash (\forall x, y)(\exists v)\ [e(v) \wedge k(v) \wedge (\forall z)\ ((z \in v) \leftrightarrow ((e(x) \wedge (z = x)) \vee (e(y) \wedge (z = y))))]$.

That is: $\vdash (\forall x, y)(\exists v)\ [s(v) \wedge (\forall z)\ ((z \in v) \leftrightarrow ((e(x) \wedge (z = x)) \vee (e(y) \wedge (z = y))))]$.

Thus $\varnothing$ and $\underline{h}(t_1, t_2)$, i.e. $\{t_1, t_2\}$, and hence also $<t_1, t_2>$, can be regarded as incomplete, and hence eliminable symbols.

[6] $(\forall x, b)\ [(e(x) \wedge k(x) \wedge Un(b) \wedge k(b)) \to (\exists y)\ (e(y) \wedge k(y) \wedge (\forall z)\ (e(z) \to ((z \in y) \leftrightarrow$

$\leftrightarrow (\exists v)\ (e(v) \wedge (v \in x) \wedge (<z, v> \in b))))]$

(*Axiom of Replacement.*)
That is:

$(\forall x, b)\ [(s(x) \wedge Un(b) \wedge k(b)) \to (\exists y)\ (s(y) \wedge (\forall z)(e(z) \to ((z \in y) \leftrightarrow$

$\leftrightarrow (\exists v)\ (e(v) \wedge (v \in x) \wedge (<z, v> \in b))))]$.

By (iv):

$\vdash (\forall x, b)\ [(s(x) \wedge Un(b)) \to (\exists y)\ (s(y) \wedge (\forall z)\ ((z \in y) \leftrightarrow (e(z) \wedge (\exists v)\ (e(v) \wedge$

$\wedge (v \in x) \wedge (<z, v> \in b))))]$.

[7] $(\forall x, b)\ [(s(x) \wedge k(b)) \to (\exists y)\ (s(y) \wedge (\forall z)\ (e(z) \to ((z \in y) \leftrightarrow ((z \in x) \wedge (z \in b)))))]$

(*Axiom of Separation.*)

By (iv): $\vdash (\forall x, b)\ [(s(x) \wedge k(b)) \to (\exists y)\ (s(y) \wedge (\forall z)\ ((z \in y) \leftrightarrow ((z \in x) \wedge (z \in b))))]$.

[8] $(\forall x)\ [(s(x) \wedge (\forall y \in x)\ (e(y) \to k(y))) \to (\exists z)\ (s(z) \wedge (\forall v)\ (e(v) \to ((v \in z) \leftrightarrow$

$\leftrightarrow (\exists y)\ (e(y) \wedge (v \in y \in x)))))]$

(*Axiom of the Union.*)
By (iv):

$\vdash (\forall x)\ [(s(x) \wedge (\forall y \in x)\ k(y)) \to (\exists z)\ (s(z) \wedge (\forall v)\ ((v \in z) \leftrightarrow (\exists y)\ (v \in y \in x)))]$.

[9] $(\forall x)\ [s(x) \to (\exists y)\ (s(y) \wedge (\forall z)\ (e(z) \to ((z \in y) \leftrightarrow (k(z) \wedge (\forall v)\ ((e(v) \wedge (v \in z)) \to$

$\to (v \in x)))))]$

(*Power Set Axiom.*)

By (iv) and by definition of $s(t)$:

$\vdash (\forall x)\ [s(x) \rightarrow (\exists y)\ (s(y) \land (\forall z)\ ((z \in y) \leftrightarrow (s(z) \land (\forall v \in z)\ (v \in x))))]$.

[10] $(\exists d)\ [Un(d) \land (\forall x)\ ((e(x) \land (\exists y)\ (e(y) \land (y \in x))) \rightarrow (\exists z)\ (e(z) \land (z \in x) \land (<z, x> \in d))))]$

(*Strong version of the Axiom of Choice.*)
By (iv):

$\vdash (\exists d)\ [Un(d) \land (\forall x)\ ((e(x) \land (\exists y)\ (y \in x)) \rightarrow (\exists z)\ ((z \in x) \land (<z, x> \in d))))]$.

[11] $(\forall b)\ [(\exists x)\ (x \in b) \rightarrow (\exists y)\ ((y \in b) \land \neg(\exists z \in b)\ (z \in y))]$

(*Axiom of Foundation.*)

[12] $(\exists x)\ [s(x) \land (\forall y \in x)\ s(y) \land (\exists y \in x)(\forall z)\ (z \notin y) \land (\forall y \in x)(\exists z \in x)(\forall v)\ ((v \in z) \leftrightarrow$

$\leftrightarrow (e(v) \land ((v \in y) \lor (v = y))))]$

(*Axiom of Infinity.*)
By (iv):

$\vdash (\exists x)\ [s(x) \land (\forall y \in x)\ \mathrm{s}(y) \land (\exists y \in x)(\forall z)\ (z \notin y) \land$

$\land (\forall y \in x)(\exists z \in x)(\forall v)\ ((v \in z) \leftrightarrow ((v \in y) \lor (v = y))))$.

Now we come to the axioms which ensure the extensional character of the basic —logical as well as non-logical— notions.

[13] $(\forall x)\ [k(x) \rightarrow (\exists b)\ (k(b) \land (\forall y)\ ((y \in b) \leftrightarrow (e(y) \land (y \notin x))))]$

(*Extensionality of negation*, i.e. of $\neg$.)

[14] $(\forall x, y)\ [(k(x) \land k(y)) \rightarrow (\exists b)\ (k(b) \land (\forall z)\ ((z \in b) \leftrightarrow ((z \in x) \land (z \in y))))]$

(*Extensionality of the conjunction* $\land$.)

[15] $(\forall x)\ [k(x) \rightarrow (\exists b)\ (k(b) \land (\forall z)\ ((z \in b) \leftrightarrow (e(z) \land (\exists y)\ (e(y) \land (<y, z> \in x)))))]$

(*Extensionality of the existential quantifier* $\exists$.)

[16] $(\forall x, y)\ [(k(x) \land e(y)) \rightarrow (\exists b)\ (k(b) \land (\forall z)\ (e(z) \rightarrow ((z \in b) \leftrightarrow (<z, y> \in x))))]$

By (iv):

$\vdash (\forall x, y)\ [(k(x) \land e(y)) \rightarrow (\exists b)\ (k(b) \land (\forall z)\ ((z \in b) \leftrightarrow (e(z) \land (<z, y> \in x))))]$.

[17] $(\forall x)\ [k(x) \rightarrow (\exists b)\ (k(b) \land (\forall y, z)\ ((e(y) \land e(z)) \rightarrow ((<y, z> \in b) \leftrightarrow (z \in x))))]$

(*Existence of the Cartesian product* $V \times x$, V being the universal class, i.e. the class of all elements.)

[18] $(\exists b)\ [k(b) \wedge (\forall x)\ (e(x) \to (x \in b))]$

(This axiom entails the extensionality of $e(x)$ and of $(x = x)$).

[19] $(\exists b)\ [k(b) \wedge (\forall x, y)\ ((e(x) \wedge e(y)) \to ((<x, y> \in b) \leftrightarrow (x = y)))]$

(*Extensionality of the identity* $(x = y)$.)

[20] $(\exists b)[k(b) \wedge (\forall x)\ (e(x) \to ((x \in b) \leftrightarrow (x \in x)))]$

(*Extensionality of* $(x \in x)$.)

By (iv): $\vdash (\exists b)\ [k(b) \wedge (\forall x)\ ((x \in b) \leftrightarrow (x \in x))]$.

[21] $(\exists b)\ [k(b) \wedge (\forall x, y)\ ((e(x) \wedge e(y)) \to ((<x, y> \in b) \leftrightarrow (x \in y)))]$

(*Extensionality of* $(x \in y)$.)

[22] $(\exists b)\ [k(b) \wedge (\forall x)\ (e(x) \to ((x \in b) \leftrightarrow k(x)))]$

(*Extensionality of* k.)

By (iv): $\vdash (\exists b)\ [k(b) \wedge (\forall x)\ ((x \in b) \leftrightarrow (e(x) \wedge k(x)))]$.

Hence, by definition of s:

$\vdash (\exists b)\ [k(b) \wedge (\forall x)\ ((x \in b) \leftrightarrow s(x))]$

(*Extensionality of* s.)

[23] Consider any primitive descriptive predicate R having, say, r argument-places [R could be one of the observational predicates $Q_1, \dots, Q_n$ of our underlying physical theory].

Choose, and then fix, any $(r + 1)$ distinct variables: $b, x_1, x_2, \dots, x_r$. Let $t_1, t_2, \dots, t_r$ be any sequence of (not necessarily distinct) symbols chosen from among $x_1, x_2, \dots, x_r, \varnothing, a_1, \dots, a_m, u_1, \dots, u_s$. [There exist $(r + m + s + 1)^r$ distinct sequences having this property]. For a given sequence $t_1, t_2, \dots, t_r$, let q be the number of its (distinct) members which are variables, i.e. which figure among $x_1, x_2, \dots, x_r$. Hence $q \le r$.

Let $y_1, y_2, \dots, y_q$ be any sequence of these variables. [A given sequence $t_1, t_2, \dots, t_r$ determines q and consequently $q!$ sequences of the form $y_1, y_2, \dots, y_q$]. For every p such that $0 \le p \le q - 1$, postulate:

$$(\forall y_1, y_2, \dots, y_p)(\exists b)\ [k(b) \wedge (\forall y_{p+1}, y_{p+2}, \dots, y_q)\ ((e(y_{p+1}) \wedge e(y_{p+2}) \wedge \dots \wedge e(y_q)) \to$$
$$\to ((<y_{p+1}, y_{p+2}, \dots, y_q> \in b) \leftrightarrow R(t_1, t_2, \dots, t_r)))].$$

(Here we have a finite number N_R of formulas, where $N_R \le (r + m + s + 1)^r.r!.r$.)

Let us finally go over to the permutation axioms, namely:

[24] $(\forall d)\ [k(d) \to (\exists b)\ (k(b) \wedge (\forall x, y)\ ((e(x) \wedge e(y)) \to ((<x, y> \in d) \leftrightarrow (<y, x> \in b)))]$

(*Axiom of Commutativity*.)

[25] $(\forall d)\ [k(d) \to (\exists b)\ (k(b) \wedge (\forall x, y, z)\ ((e(x) \wedge e(y) \wedge e(z)) \to ((<x, y, z> \in d) \leftrightarrow$

$\leftrightarrow (<z, x, y> \in b)))]$

[26] $(\forall d)\ [k(d) \to (\exists b)\ (k(b) \wedge (\forall x, y, z)\ ((e(x) \wedge e(y) \wedge e(z)) \to ((<x, y, z> \in d) \leftrightarrow$

$\leftrightarrow (<x, z, y> \in b)))]$

[27] $(\forall d)\ [k(d) \to (\exists b)\ (k(b) \wedge (\forall x, y, z, u)\ ((e(x) \wedge e(y) \wedge e(z) \wedge e(u)) \to ((<x, y, z, u> \in d) \leftrightarrow$

$\leftrightarrow (<x, z, y, u> \in b)))]$

We have thus determined a finite number of axioms, whose conjunction will be denoted by $\boldsymbol{M}$. $\boldsymbol{M}$ can be added to the empirical postulates of any scientific theory, thereby yielding the system Σ mentioned above. Σ can therefore be put in the form $\mathbf{G}(k, \in, \underline{h}, \varnothing, a_1, \ldots, a_m, u_1, \ldots, u_s, Q_1, \ldots, Q_n)$, whose Ramsey-sentence is:

$$(\exists k, \in, \underline{h})(\exists \varnothing, a_1, \ldots, a_m, u_1, \ldots, u_s)\ \mathbf{G}(k, \in, \underline{h}, \varnothing, a_1, \ldots, a_m, u_1, \ldots, u_s, Q_1, \ldots, Q_n).$$

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