Limit Analysis for Historical Masonry Bridge with CFRP Reinforcements

Antonio Gesualdo[1], Mariano Modano[1], Antonino Iannuzzo[1], Michela Monaco[2]

[1] University of Naples *Federico II*, Department of Structures for Engineering & Architecture, - Naples, Italy

[2] Second University of Naples, Department of Architecture and Industrial Design *L. Vanvitelli*, Aversa, Caserta, Italy

Email: **gesualdo@unina.it**, **michela.monaco@unina2.it**

Web: **www.dist.unina.it/index_en.htm**, **www.architettura.unina2.it/ENG**

Summary

The paper deals with the collapse behavior of a historical masonry arch bridge subject to combined seismic loads, by means of the limit analysis and shakedown theorems. The assessment of the collapse loads and shakedown multiplier have been analyzed using lower bound theorems of the limit analysis and shakedown theory. The retrofitting consists in the application of CFRP strips on the extrados of the arch with a significant increase of the collapses and shakedown multipliers. The results of a FEM analysis have been achieved with ANSYS code involving the non-linear material behavior and the structural role of the spandrel walls and filling.

Keywords

CFRP-Reinforcement, Masonry Arch Bridge, Limit Analysis, Shakedown, FEM Analysis

Introduction

Masonry arch bridges are a common road engineering structure in Italy, as well as other parts of Europe and United States. Some of the most ancient masonry bridges are concentrated in Italy. These ancient bridges are an integral part of the Italian transportation system and they also represent the great engineering accomplishments and heritage of the people that have lived in Italy for centuries. Recent seismic activity in the world underscores the importance of using an advanced set of strengthening techniques on these masonry structures, through the analytical evaluation of collapse and shakedown states. The collapse analysis of these structures is a matter of utmost importance, both to protect human life, and to preserve these historical structures[1].

For every structure, seismic actions can be modeled by means of appropriate horizontal and vertical force distributions, with intensity dependent on proper oscillation periods. It is important to take into account suitable solutions for any construction type involving the seismic action in the completed state, based on the contribution of the complete set of significant proper vibrational modes, frequencies and design seismic spectra. The results of the analyses carried out in this study (see Figure

1), indicate that if only the first vibration mode is taken into account, a significant error of 70% occurs, leading to underestimated seismic actions and an inadequate retrofit design.

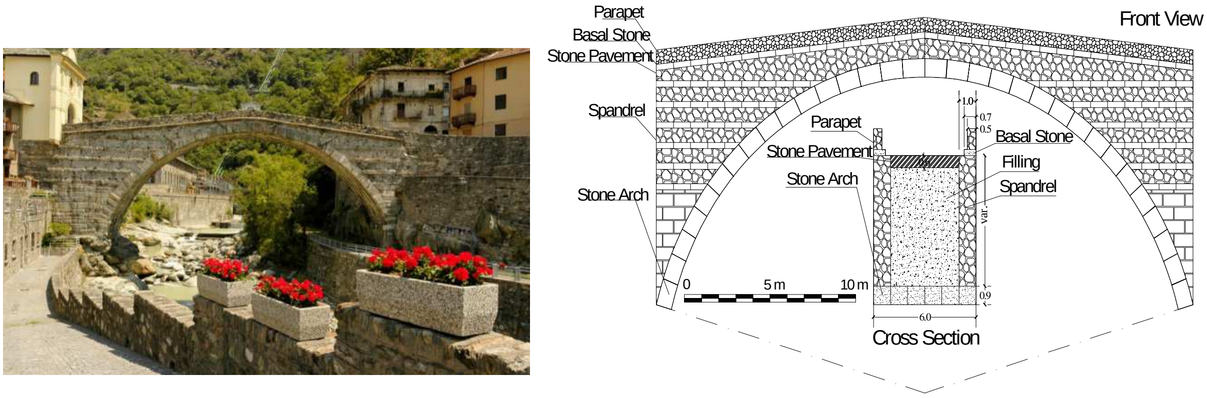

Figure 1: Saint Martin bridge, Saint Martin town, Aosta Valley, Italy

The importance of designing an efficient, effective retrofit for historical masonry arch bridges, that addresses seismic vulnerability concerns, but also maintains architectural beauty of the bridge, is obvious. In the case of analyzed ancient Roman arch bridges, constructed of gneiss, conventional retrofitting techniques, such as intrados reinforcement with steel plates, CFRP (Carbon Fiber Reinforced Polymer) strip or surface concrete coating, are not acceptable because the design would destroy the aesthetic, historical and architectural appeal of the bridge. A retrofit that uses CFRP strips at the extrados for reinforcement, and a bond filling that reduces the risk of delamination addresses both engineering and aesthetic concerns, and is the most suitable retrofitting solution[2]. The thin amount of CFRP required for strengthening is especially important when designing a seismic retrofit for historical bridges, as it minimizes changes to the appearance of the bridge. The application of CFRP composites to masonry structures is less well established, although it has been the subject of research and development in recent years [3]. It has been demonstrated that CFRP can be used to upgrade the structural performance of a variety of masonry elements. The National Research Council in Italy [4] and the American Concrete Institute [5] recognize CFRP for this purpose, and have issued design guidance. Further work is required, however, to apply CFRP strengthening to increase the load capacity of masonry arch bridges. CFRPs were initially proposed for the reinforcement of concrete structures. The use of these materials on masonry structures has been recently studied, both experimentally and analytically. FRP is made of a polymeric matrix with different fibers (glass, carbon, etc.). As a strengthening material, it presents a number of advantages, including high tensile strength, negligible self-weight and corrosion resistance [6].

An important aspect of structural analysis, especially for ultimate safety assessment or design, requires evaluating the maximum load the structure can sustain. The Limit Analysis research field has been the focus of intensive research efforts recently. In general, the most up-to-date formulations are derived within an optimization problem framework, aiming to take advantage of the latest mathematical developments in nonlinear convex programming algorithms. Nevertheless, in spite of the rapid evolution in computer performance, determining accurate collapse load estimates can still present a significant computational effort. Additionally, as is the case in most plasticity problems, the principle of superposition of loads does not hold true in the framework of classical limit analysis. This principle is extremely useful in the treatment of many practical engineering problems and has been widely exploited in the past two centuries on the basis of linearized strain and constitutive laws and linear equilibrium equations for the stress state. In the classic theory of plasticity, owing to both non-uniqueness of the stresses in strains and nonlinearity of the incremental elastic-plastic process until the collapse, superposition cannot be applied. In the framework of classic plasticity, even when limit multipliers and collapse mechanisms associated with different loads independently acting on a solid or structure are known, not much can be inferred on the limit multiplier of the combined loading [7]. Recently, it has been suggested that the useful life of many seismically vulnerable short-span bridges could be extended significantly if these structures were allowed to enter into the inelastic range for a low number of cycles. This limit state is known as the shakedown or incremental collapse limit. Shakedown is a term used to describe structural behavior under large cyclic loads and implies that after repeated applications of a prescribed load history that exceeds the elastic limit, but not the plastic collapse load of the structure; the residual deflections in the structure will stabilize. Residual deflections are the permanent deformation remaining in the structure after the load has been removed. Because yielding has occurred, additional forces, known as residual moments, will be locked into the structure when the loads are removed. It is important to note that shakedown implies some damage to the structure, generally in the form of yielding of main members, and thus may result in a serviceability failure. However, a key feature of shakedown is that once the deflections stabilize, the structure will respond elastically to any additional cycles of the prescribed load history. Because the residual deflections stabilize, shakedown does not result in a structural collapse or ultimate strength failure. If the shakedown limit is exceeded, the structure will keep deflecting more and more, with each successive application of the load and fail by an incremental collapse mode [8].

The solving procedures can be divided into two large categories corresponding to incremental methods and limit analysis. The former is referred to in Castigliano [9], and the latter in the works of Kooharian [10] and Heyman [11], who extended the plastic limit analysis theorems to structural systems with a slight tensile yield material.

Seismic Limit Analysis Procedure

In the paper the implementation of two fundamental theories has been carried out: the dynamic finite elements method (DFEM) and the associated modal analysis which enable us to evaluate the seismic action employing the response spectra, the static FEM which obtains the elastic solutions of seismic loads and the self-equilibrated stress states of structure, and finally the limit analysis procedure, in order to assess the collapse loads and the shakedown. The DFEM procedure has been implemented in Mathematica. The theorems of Limit Analysis are the milestone of the classical elasto-plastic analysis of structures: the static, the kinematic and the shakedown theorem. The lower bound theorems are implemented in symbolic code Mathematica with a constrained optimization problem, involving the solutions in terms of bending moment-axial force interaction, both in the real case and in the retrofitted one. The application of the limit analysis theorems in the stone arch bridge was discussed in the relevant paper of Kooharian [10]. An important result obtained concerns the shakedown seismic load multiplier that coincides with the minimum collapse load multiplier: the nonlinear material behavior of arch bridge is fully stable.

Outcome For Real Structure

The limit analysis and shakedown lower bound theorems on a stone arch bridge with different seismic load conditions are applied in our study to assess the collapse and shakedown seismic load multiplier. The analysis was led according to the subsequent mechanical properties: Young modulus Ey=400 MPa, Ultimate compression strength σ_{oc}=30 MPa, Ultimate tensile strength σ_{ot} =5 MPa. The nonlinear yield domain (see Figure 2) was determined for the effective arch's rectangular b h cross section with length equal to 6 m and thickness equal to 0.9 m. The upper and lower boundary of the ultimate bending-axial force interaction (M-P) domain are represented in the next relation and sketched in the attached figure:

$$
\begin{cases}
M_{\mathrm{up}} = \dfrac{P^2}{2b(\sigma_{oc}-\sigma_{ot})} - \dfrac{P\,h(\sigma_{oc}+\sigma_{ot})}{2(\sigma_{oc}-\sigma_{ot})} + \dfrac{b\,h^2\sigma_{oc}\,\sigma_{ot}}{2(\sigma_{oc}-\sigma_{ot})} \\[2ex]
M_{low} = -\dfrac{P^2}{2b(\sigma_{oc}-\sigma_{ot})} + \dfrac{P\,h(\sigma_{oc}+\sigma_{ot})}{2(\sigma_{oc}-\sigma_{ot})} - \dfrac{b\,h^2\sigma_{oc}\,\sigma_{ot}}{2(\sigma_{oc}-\sigma_{ot})}
\end{cases}
$$

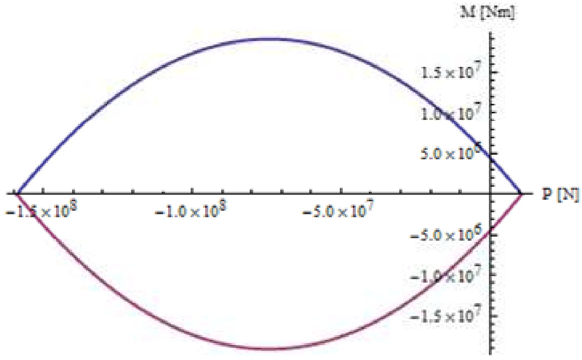

Figure 2: Yield domain in the unreinforced case

The horizontal and vertical seismic actions are determined by calculating the natural periods of the structure with a finite element modal analysis, implemented in Mathematica, and by the design seismic spectra of the Italian Seismic Rule [12] for horizontal and vertical ground accelerations. The seismic actions have great intensity, although the site is a low seismicity zone, since the masses involved are important. After the modal and seismic load analyses for each loads conditions, the constrained optimization problem has been implemented in Mathematica for the determination of the collapse loads. Figure 3 illustrates an outline of the implementation of this algorithm:

- **Bending moment and axial force**

$$\mathtt{Do}\Big[M_{LC_k} = \lambda_k\, M_{ElLC_k} + \sum_{j=1}^{n_{SE}} X_j\, \hat{M}_{X_j},\ \{k,\,1,\,n_{LC}\}\Big];$$

$$\mathtt{Do}\Big[N_{LC_k} = \lambda_k\, N_{ElLC_k} + \sum_{j=1}^{n_{SE}} X_j\, \hat{N}_{X_j},\ \{k,\,1,\,n_{LC}\}\Big];$$

- **Section ultimate carrying capacity without CFRP reinforcement**

$$M_{up}[Nu\_] := \frac{(Nu - b\,h\,\sigma_{oc})\ (Nu - b\,h\,\sigma_{ot})}{2\,b\ (\sigma_{oc} - \sigma_{ot})};\ M_{low}[Nu\_] := -M_{up}[Nu];$$

$$\mathtt{Do}\big[C_{LC_k} = \mathtt{Table}\big[M_{low}[N_{LC_k}[[i]]] \le M_{LC_k}[[i]] \le M_{up}[N_{LC_k}[[i]]],\ \{i,\,1,\,n_{CS}\}\big],\ \{k,\,1,\,n_{LC}\}\big];$$

- **Section ultimate carrying capacity with CFRP reinforcement**

$$M_{upR}[Nu\_] := \frac{Nu^2}{2\,b\ (\sigma_{oc}-\sigma_{ot})} + \frac{b\,h^2\ \sigma_{oc}\ \sigma_{ot}}{2\ (\sigma_{oc}-\sigma_{ot})} + Nu\left(-\frac{h\,\sigma_{oc}}{2\ (\sigma_{oc}-\sigma_{ot})} - \frac{h\,\sigma_{ot}}{2\ (\sigma_{oc}-\sigma_{ot})}\right);$$

$$M_{lowR}[Nu\_] := \frac{Nu^2}{2\,b\ (\sigma_{oc}-\sigma_{ot})} + \frac{b\,h^2\ \sigma_{oc}\ \sigma_{ot}}{2\,b\ (\sigma_{oc}-\sigma_{ot})} + \frac{h\,s_w\,b_w\ \sigma_{oc}\ \sigma_{otw}}{(\sigma_{oc}-\sigma_{ot})} + \frac{{s_w}^2\,{b_w}^2\ {\sigma_{otw}}^2}{2\,b\ (\sigma_{oc}-\sigma_{ot})} - Nu\left(\frac{h\,\sigma_{oc}}{2\ (\sigma_{oc}-\sigma_{ot})} + \frac{h\,\sigma_{ot}}{2\ (\sigma_{oc}-\sigma_{ot})} + \frac{s_w\,b_w\ \sigma_{otw}}{b\ (\sigma_{oc}-\sigma_{ot})}\right);$$

$$\mathtt{Do}\big[C_{LC_k} = \mathtt{Table}\big[M_{lowR}[N_{LC_k}[[i]]] \le M_{LC_k}[[i]] \le M_{upR}[N_{LC_k}[[i]]],\ \{i,\,1,\,n_{CS}\}\big],\ \{k,\,1,\,n_{LC}\}\big];$$

- **Solution of the constrained optimization problems for the limit analyses**

$$\mathtt{Do}\big[\mathtt{SolOpt}_{LC_k} = \mathtt{N}\big[\mathtt{Maximize}\big[\lambda_k,\ C_{LCk},\ \{\lambda_k,\ X_j,\ \{j,\ n_{SE}\}\}\big]\big],\ \{k,\,n_{LC}\}\big];$$

- **Solution of the constrained optimization problem for shakedown problem**

$$\mathtt{SolOptShakeDown} = \mathtt{N}\big[\mathtt{Maximize}\big[\lambda,\ \{C_{LCk},\ \{k,\ n_{LC}\}\},\ \{\lambda,\ X_j,\ \{j,\ n_{SE}\}\}\big]\big];$$

Figure 3: Static and shakedown theorems of limit analysis in Mathematica

The collapse and shakedown load multipliers for no strengthened structure are listed below:

$$S_{C,LC_1} = 2.17 \quad S_{C,LC_2} = 20.53 \quad S_{C,LC_3} = 2.24 \quad S_{SD} = 2.17$$

and next Figure 4 shows the collapse mechanism associated with each load conditions:

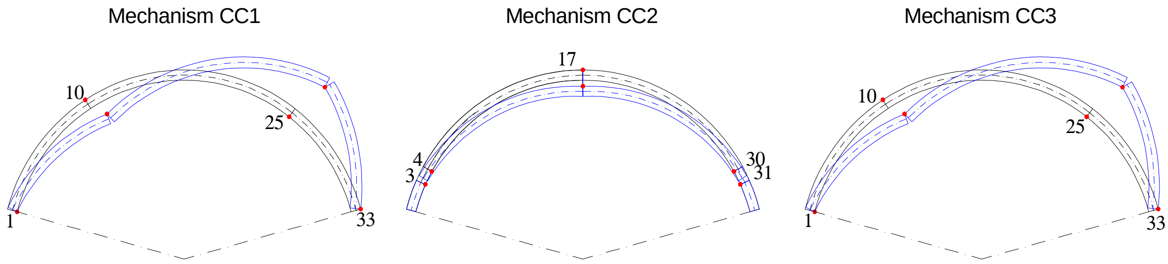

Figure 4: Collapse mechanisms

Outcome For Retrofitted Stone Arch

The same procedure has been employed for the determination of the collapse and shakedown load multipliers in the case of CFRP strengthening application at the extrados of the arch. It has been suggested that the reinforcement consisting of six strips of 0.40 m width at the extrados of the arch and placed at uniform offsets will be effective. For the installation the entire filling must be removed by hand, because equipment vibration may cause damage to the bridge. The filling is then reapplied by hand after installation. The filling creates a bond for the CFRP strips which reduces the risk of the dangerous phenomenon of delamination. As shown in the following pages, through the application of the CFRP strips, a significant increase of the collapse multiplier of 150% for horizontal and combined loads and 340% for vertical loads occur. The nonlinear boundary of the yield domain of the reinforced section is given by the following equations:

$$\begin{cases} M_{upR} = \dfrac{P^2}{2b(\sigma_{oc}-\sigma_{ot})} - \dfrac{P\,h(\sigma_{oc}+\sigma_{ot})}{2(\sigma_{oc}-\sigma_{ot})} + \dfrac{b\,h^2\sigma_{oc}\,\sigma_{ot}}{2(\sigma_{oc}-\sigma_{ot})} \\[2ex] M_{lowR} = \dfrac{P^2}{2b\,(\sigma_{oc}-\sigma_{ot})} - \dfrac{P}{2(\sigma_{oc}-\sigma_{ot})}\left(h\sigma_{oc} + h\sigma_{ot} + 2\dfrac{s_w\,b_w\sigma_{otw}}{b}\right) \\ \qquad + \dfrac{b\,h^2\sigma_{oc}\sigma_{ot}}{2(\sigma_{oc}-\sigma_{ot})} + \dfrac{h\,s_w\,b_w\sigma_{oc}\,\sigma_{otw}}{\sigma_{oc}-\sigma_{ot}} + \dfrac{s_w^2\,b_w^2\sigma_{otw}^2}{2b\,(\sigma_{oc}-\sigma_{ot})} \end{cases}$$

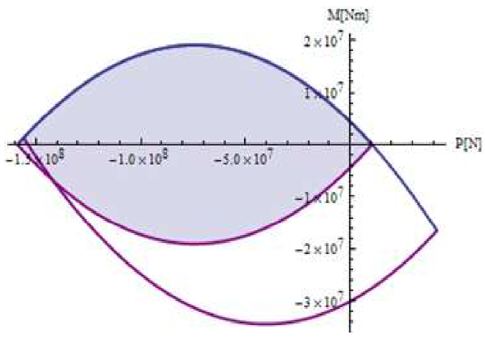

Figure 5:Yield domain with CFRP's strips and comparison with unreinforced case

A representation of the reinforced ultimate domain M-P is provided in Figure 5. By making a comparison with the yield domain of the section without reinforcement, it can be observed that the presence of the CFRP strips enlarges the plastic domain in the tensile zone and in the bending part that involve the compression of the lower fibers and the tensile of the upper ones. The collapse and shakedown load multipliers for retrofitted structure are the following:

$$S^R_{C,LC_1} = 5.49 \quad S^R_{C,LC_2} = 90.66 \quad S^R_{C,LC_3} = 5.63 \quad S^R_{SD} = 5.49$$

which considers the relevant increases of every multiplier; the result ensures the effectiveness of the adopted retrofitting strategy. The correspondent collapse mechanisms are similar to real case (see Figure 4). Furthermore, the values of the collapse multipliers are confirmed by the direct application of the kinematic theorem to every previous collapse mechanisms. A parallel analysis of the St. Martin Bridge, with the variable that the bridge was located in a seismically hazardous zone of Italy, such as Irpinia, Campania, southern Italy. Irpinia is located in one of the most hazardous seismic regions of Italy, at the juncture of the Eurasian and African tectonic plates. In this last case, the yield and shakedown load multipliers, for the structure without and with CFRP reinforcement, are as follows:

$$S_{C,LC_1} = 0.47 \quad S_{C,LC_2} = 2.45 \quad S_{C,LC_3} = 0.61 \quad S_{SD} = 0.47$$

$$S^R_{C,LC_1} = 1.45 \quad S^R_{C,LC_2} = 3.79 \quad S^R_{C,LC_3} = 1.53 \quad S^R_{SD} = 1.45$$

Two natural considerations are initially apparent; the large increase of the multipliers by application of the considered retrofitting technique; second, the reduction of the multipliers in a more dangerous seismic area. These factors may explain the lack of Roman bridge in the area of Southern of Italy. The limit analysis and shakedown lower bound theorems on a stone arch bridge with different seismic load conditions are applied in our study to assess the collapse and shakedown seismic load multiplier.

FEM Analysis with ANSYS

The Drucker-Prager criterion was assumed as failure criterion for all the materials. To evaluate the elastic parameters, the stone masonry has been considered as a material obtained after a homogenization procedure, regarding the assemblage of stone blocks and mortar as a composite medium. The homogenized characteristics have been obtained by means of the classical differential scheme [13].

The method is based on the idea that the composite is constructed explicitly from an initial material (stone) through a series of incremental additions (mortar). Due to the lack of experimental data, the Poisson's ratio was assumed equal to 0.2, although it has been shown that a variation in the Poisson's ratio provides sensible variation in the evaluation of the safety degree [14].

The analysis has been performed for the dead load only and the FEM mesh, involving both solid and tetrahedra elements, has been designed according the scheme picted in figure 6-a, where the four constituents materials are shown too. A total number of 28388 elements and 11068 nodes have been considered. In figure 6-b-c a map of the maximum principal stress (more significant in this case for the barrel vault) is represented. As it can be seen, the maximum value in the arch is 14 N/mm^2, lower than both the stone and mortar strength. This confirms that loss of equilibrium is the major cause of global failure: the material failure is absent, as it has been observed in several collapsed stone block masonry structures, such as Selinunte and Agrigento temples. In these cases the collapsed blocks are in perfect conditions, so that the restoration can be done by means of a simple rebuilding. Moreover, the distribution and intensity of stresses is similar to that obtained considering the spandrel and fill as dead load only, and considering the arch supported at the springing.

Since the value of the safety degree cannot be based on the comparison between the masonry strength and the stress evaluated by means of the F.E.M. analysis, the minimum load multiplier for which the displacements make sense is assumed as the safety degree of the bridge. In the present analysis the safety degree is 9,4. Although the mathematical solution of the problem is possible for higher load factors, the present analysis has been carried on until limited increases of the load multiplier give as a result great increments of the maximum displacement.

The safety degree evaluated considering the spandrel and fill as dead load is about one half of that evaluated in the present analysis and, as it has been shown, a little higher than that one evaluated by means of a limit analysis. The concentration of stresses coincides with the hypothesis of six hinges in the final mechanism of the arch elsewhere presented [15]. The strain distribution is presented in figure

7 a-b: localizations of the plastic strain are noticeable in limited areas of the barrel vault, while the spandrels and the foundations are completely free from plastic deformations. To allow more clarity only the characteristics relative to a quarter of the bridge have been represented in the figures.

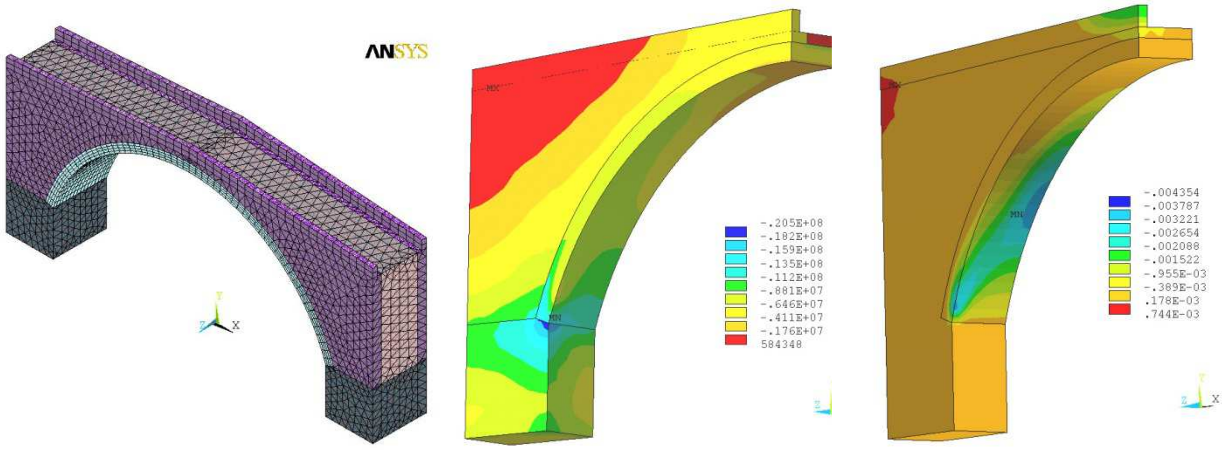

Figure 6 a-b-c: Map of maximum principal stresses [N/m2] and the strain distribution [m/m]

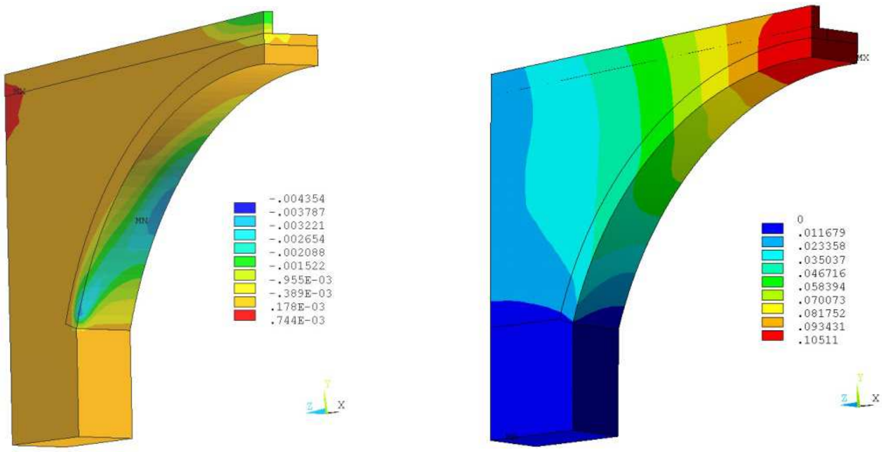

Figure 7 a-b: Maximum plastic principal strain [m/m] and displacement field [m]

Conclusions

This study gives the limit and shakedown load multipliers for seismic load conditions. The analyses are applied on the real structure and on the retrofitted one with a set of CFRP strips. The limit analysis and the shakedown theory endorse the efficacy of the adopted retrofitting. The elastic-plastic behavior of the structures is always stable, otherwise the incremental collapse cannot occur. The results of a F.E.M. analysis can be useful, in case of restoration of a masonry arch, by giving a qualitative map of the “intervention areas”. It must be noted that they are strongly dependent on the exactness of mechanical parameters, which often are difficult to evaluate by experimental analyses, especially in the cases of monuments and historical buildings.

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