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Determination of the Linear Frequency Response of Single Pipelines using Persistent Transient Excitation: a Numerical Investigation

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ABSTRACT

The linear frequency response of a fluid-filled pipeline extracted by fluid transient waves can be used to detect leaks in pipelines. This research conducts numerical analysis on how to accurately determine the linear frequency response diagram (FRD) of single pipelines using persistent transient pressure signals. Two types of persistent signals, the maximum-length binary sequence (MLBS) and the inverse-repeat sequence (IRS), are compared in terms of the accuracy in estimating the linear response of a pipeline at resonant frequencies. The IRS is found to be more appropriate for identifying the linear portion of the FRD of a pipeline, since its antisymmetric property can supress part of the nonlinear response. Numerical simulations are conducted to verify the findings.

Keywords: fluid transient waves; inverse-repeat sequence; linear frequency response diagram; non-invasive leak detection; pipeline; pseudo-random binary sequence; water hammer.

1 Introduction

The need to test pipelines (water, oil or gas, for example) has resulted in a significant body of research into ways that pipes can be tested without violating the integrity of the system. One promising technique involves using fluid transients injected into the pipeline system and analysing the resulting frequency response diagram (FRD), which is the plot of the frequency response function (FRF) of the system. For any specific pipeline system, the FRD is unique and its characteristics are determined by the properties of the pipeline, including length, friction, boundary conditions and wave speed. The FRD can be used, therefore, to diagnose the condition of the pipeline.

In the last decade, several FRD-based pipeline leak detection techniques (Mpesha *et al.* 2001, Lee *et al.* 2005, Sattar and Chaudhry 2008, Gong *et al.* 2012) have been developed. The accuracy of the FRD estimation has become increasingly important, and is especially reliant on information that indicates the system linear dynamics at the fundamental frequency of the pipeline system and corresponding odd harmonics (Lee *et al.* 2005, Gong *et al.* 2012).

In earlier studies, the sine-stepping technique was commonly used to extract the FRD of a pipeline system. The sine-stepping technique uses single frequency sine oscillatory signals as the input, and this frequency is adjusted to cover the range of frequencies required (Chaudhry 1987). This technique is widely used in numerical modelling with the transfer matrix method to determine the theoretical FRD of a pipeline system (Lee *et al.* 2005, Sattar and Chaudhry 2008). However, performing a sine-stepping is undesirable in real world applications. The system needs to be excited at various different frequencies sequentially, and the measurement can only be taken after a steady-oscillatory flow has been established. As a result, to cover a sufficient number of frequencies, the whole procedure for extracting one FRD can take a number of hours. Furthermore, the final FRD is prone to errors, because the influence of noise can vary throughout independent tests for different frequencies.

An alternative to the sine-stepping technique is to use single step or pulse signals as the excitation, where the FRF can be extracted through a single operation within a few minutes. Mpesha *et al.* (2002) used a step transient (generated by closing or opening a valve) to excite a single pipeline, and the FRD was obtained by a simple fast Fourier transform of the measured head or discharge. Lee *et al.* (2003) analysed the limitation of the step transient-based technique, and a few years later, Lee *et al.* (2006) performed laboratory experiments on pipeline FRD measurement and leak detection using a sharp pulse transient and a correlation-based FRF determination algorithm. A major challenge for the discrete step or pulse excitation is that the bandwidth is usually limited so that the signal-to-noise ratio (SNR) is low for high frequency components. The step or pulse wave is usually not as sharp as desired attributable to the limitation in the manoeuvrability of real signal generation devices (e.g. a side-discharge valve).

One solution to overcome the limitations of the discrete transient excitation while keeping the efficiency of FRD extraction is to use persistent signals as the excitation. Lee *et al.* (2008) developed a customised side-discharge valve that can generate persistent maximum-length (pseudo-random) binary sequence (MLBS)-based transient pressure waves into a laboratory pipeline system [referred to as pseudo-random binary sequences (PRBS) in that paper]. The MLBS is a type of wide bandwidth and periodic signal, so that multiple frequency responses of a pipeline can be determined simultaneously and the SNR can be increased by synchronous averaging of the response periods. However, the magnitude of the input signal must be carefully selected and relatively small, since large valve perturbations can lead to significant nonlinear responses (Lee *et al.* 2002, 2003, 2005).

In this technical note, studies are conducted on the nonlinear response of a pipeline during transient events, and the selection of persistent transient excitation signals with the aim of minimising the effect of nonlinearities on the FRD. It is found that another type of PRBS, namely the inverse repeat sequence (IRS), is better than the MLBS in extracting the linear frequency response of a fluid-filled pipeline. Numerical simulations are conducted to compare the MLBS and the IRS in terms of the accuracy in determining the linear frequency response of a pipeline.

2 Nonlinearities of a pipeline system and linearisation in FRD-based leak detection techniques

In this research, the nonlinear behaviour of a pipeline system is taken into account in the selection of transient excitation. Similar to most systems, in reality a pipeline system has nonlinearities that will be reflected in the extracted FRD. However, only the linear portion of the frequency response of a pipeline system is desired since all existing FRD-based leak detection techniques are based on linear theory. As a result, the nonlinear response in the FRD will introduce error in the leak detection analysis. The nonlinearities of a pipeline and the linearisation in FRD-based leak detection techniques are analysed in the following sections.

Unsteady pipe flow can be described by the governing equations of continuity and motion (Chaudhry 1987):

$$\frac{\partial Q}{\partial x}+\frac{gA}{a^2}\frac{\partial H}{\partial t}=0 \tag{1}$$

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{fQ|Q|}{2gDA^2}=0 \tag{2}$$

where Q is flow, H is head, g represents gravitational acceleration, A is cross-sectional area of the pipe, a is wave speed, D is pipe diameter, f is the Darcy-Weisbach friction factor, x is distance and t is time. Equation (2) is a nonlinear partial differential equation.

In the time domain, Eqs. (1) and (2) can be solved by the method of characteristics (MOC). In the MOC, the two partial differential equations are transformed to two ordinary differential equations that hold along characteristic lines, and are described as the MOC compatibility equations (Chaudhry 1987), which retain the nonlinear component associated with friction. The MOC compatibility equations are integrated and solved in a step by step manner in the time domain. Typically a finite-difference technique (e.g. the trapezoidal rule) is used in the integration process, which is sufficiently accurate when the time steps used in solving these equations are small and the friction losses are adequately approximated by a linear variation across a time step along the characteristic lines (Chaudhry 1987, Wylie and Streeter 1993). Although approximation is introduced, the MOC inherits the nonlinear nature from the original governing equation of motion when pipe friction is included.

FRD-based leak detection techniques (Mpesha *et al.* 2002, Covas *et al.* 2005, Lee *et al.* 2005, Sattar and Chaudhry 2008, Gong *et al.* 2012) are based on the analysis of steady-oscillatory flow, where the pipeline system is excited by periodic pressure fluctuations. For the study of steady-oscillatory flow, the linear theory has to be introduced. It is assumed that a pipeline is a linear system, so that the head H and flow Q are linear superposition of a steady component and an oscillatory component, which can be described as

$$Q = Q_0 + \mathrm{Re}(qe^{j\omega t}) \tag{3}$$

$$H = H_0 + \mathrm{Re}(he^{j\omega t}) \tag{4}$$

where Q_0 and H_0 are the steady components, q and h are the oscillatory components, j represents the imaginary unit, ω denotes the angular frequency and $\mathrm{Re}(\ )$ denotes the real value operator. Substitution of Eqs. (3) and (4) into Eqs. (1) and (2) and subsequent linearisation yields

$$\frac{dq}{dx} + \frac{gA\omega j}{a^2} h = 0 \tag{5}$$

$$\frac{dh}{dx} + \left(R + \frac{\omega j}{gA} \right) q = 0 \tag{6}$$

where R is a linearised resistance term, and it is $fQ_0 / (gDA^2)$ for turbulent flow. Details of the linearisation process can be found in Chaudhry (1987) and Wylie and Streeter (1993). The approximation embedded in the derivation is that q and h are much smaller than Q_0 and H_0.

The linearised equations for oscillatory flow in a pipeline, Eqs. (5) and (6), can be solved by the transfer matrix method (Chaudhry 1987). When friction is neglected, the solution for a pipe is given as

$$\begin{Bmatrix} q \\ h \end{Bmatrix}^{n+1} = \begin{bmatrix} \cos\left(\frac{L\omega}{a}\right) & -j\frac{gA}{a}\sin\left(\frac{L\omega}{a}\right) \\ -j\frac{a}{gA}\sin\left(\frac{L\omega}{a}\right) & \cos\left(\frac{L\omega}{a}\right) \end{bmatrix} \begin{Bmatrix} q \\ h \end{Bmatrix}^{n} \tag{7}$$

where the superscripts $n+1$ and n represent positions in the downstream and upstream ends of the section of pipe, respectively, L is the length of the pipe.

In the analysis of FRD-based leak detection techniques, typically a leak is introduced as the target for detection and a valve is used as the transient generator. Approximation and linearisation are necessary in the derivation of the governing equations for them. The orifice equation (Wylie and Streeter 1993) is used to describe the relationship between the head and flow across the leak or valve. For a leak, it is written as

$$Q_{L0} = C_{Ld} A_L \sqrt{2gH_{L0}} \tag{8}$$

where Q_{L0} is leak discharge, C_{Ld} is discharge coefficient for leak, A_L is area of the leak orifice, and H_{L0} is the head at the leak. Linearising Eq. (8) gives the transfer matrix solution of a leak (Lee *et al.* 2002, 2005):

$$\begin{Bmatrix} q \\ h \end{Bmatrix}^{n+1} = \begin{bmatrix} 1 & -\dfrac{Q_{L0}}{2H_{L0}} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} q \\ h \end{Bmatrix}^{n} \qquad (9)$$

When an oscillating valve is used to excite the pipeline in a sine-stepping manner, it is assumed that the oscillating valve satisfies the orifice equation in steady-oscillatory flow condition. After linearization, the transfer matrix for an oscillating valve is given as (Wylie and Streeter 1993, Lee *et al.* 2005)

$$\begin{Bmatrix} q \\ h \end{Bmatrix}^{n+1} = \begin{bmatrix} 1 & 0 \\ -\dfrac{2\Delta H_{V0}}{Q_{V0}} & 1 \end{bmatrix} \begin{Bmatrix} q \\ h \end{Bmatrix}^{n} + \begin{Bmatrix} 0 \\ \dfrac{2\Delta H_{V0}\Delta\tau}{\tau_0} \end{Bmatrix} \qquad (10)$$

where ΔH_{V0} and Q_{V0} are the steady-state head loss across, and the flow through, the valve, the mean dimensionless valve opening size is τ_0 and the magnitude of the dimensionless perturbation in the valve opening is $\Delta\tau$. The assumption embedded within the derivation is that $\Delta\tau$ is much smaller than τ_0.

Lee *et al.* (2005) derived the governing equation for the head perturbation at the upstream face of the valve by combining the transfer matrices for all the components in a pipeline system with a leak. Analysis of this governing equation indicated that a leak could introduce a sinusoidal pattern in the resonant response. The period of this sinusoidal leak-induced pattern is related to the location of the leak, and the magnitude of the pattern is governed by the size of the leak. As a result, the FRD of a pipeline system can be used to locate the leak as well as estimate its size.

It is clear that approximation and linearisation are involved throughout the analysis of a leaking pipe by the transfer matrix method. The FRD derived from the transfer matrix method actually represents the linear frequency response of a pipeline system. Lee *et al.* (2002, 2003, 2005) studied the discrepancy between the frequency response of a pipeline derived from the linear transfer matrix method and that resulted from the nonlinear MOC. It is found that, when the magnitude of the sinusoidal driving function ($\Delta\tau$) was large and thus violated the linearisation approximations, the nonlinear effects could transfer energy between different frequency components and distort the FRD. The *linearisation error*, which was defined as the percentage reduction in the amplitude of the output (head oscillation) calculated by MOC from the linear output derived from the transfer matrix method at the input frequency, is proportional to the value of $\Delta\tau$ with an exponential pattern.

3 Selection of appropriate excitation signals to minimise the nonlinear response of a pipeline

This research demonstrates that the nonlinear behaviour of a pipeline system is related not only to the value of $\Delta\tau$, but also to the characteristics of the input signal. By using the appropriate excitation signal, part of the nonlinear response of a pipe system can be suppressed, thereby yielding frequency responses close to the linear dynamics.

In general, the output signal $y(n)$ of a sampled nonlinear system can be described by a Volterra series expansion (Godfrey 1993, Roinila *et al.* 2010), which is a summation of discrete convolutions of both the linear portion and the nonlinear components, and it can be written as

$$\begin{aligned} y(n) &= \sum_{k=0}^{M} s_1(k)u(n-k) \\ &+ \sum_{k_1=0}^{M}\sum_{k_2=0}^{M} s_2(k_1,k_2)u(n-k_1)u(n-k_2)+\cdots \\ &+ \sum_{k_1=0}^{M}\cdots\sum_{k_i=0}^{M} s_i(k_1,\cdots,k_i)u(n-k_1)\cdots u(n-k_i) \end{aligned} \tag{11}$$

where $u(k)$ is system input, M is the length of total data sequence of interest, s_1 is linear kernel, and $s_2,\cdots,s_i$ are nonlinear kernels. If the system is linear, then it can be described by the first convolution only, which is consistent with classic linear system theory.

Applying the correlation algorithm (Godfrey 1993) to Eq. (11), the cross-correlation function $\phi_{uy}(n)$ between the input and the output can be written as

$$\begin{aligned} \phi_{uy}(n) &= \sum_{k=0}^{M} s_1(k)\phi_{uu}(n-k) \\ &+ \sum_{k_1=0}^{M}\sum_{k_2=0}^{M} s_2(k_1,k_2)\phi_{uu}(n-k_1)\phi_{uu}(n-k_2)+\cdots \\ &+ \sum_{k_1=0}^{M}\cdots\sum_{k_i=0}^{M} s_i(k_1,\cdots,k_i)\phi_{uu}(n-k_1)\cdots\phi_{uu}(n-k_i) \end{aligned} \tag{12}$$

where $\phi_{uu}(n-k_1)\cdots\phi_{uu}(n-k_i)$ represents the i th order autocorrelation function of the input $u(n)$ and it can be described by the symbol $\phi_{uu}(n)_i$, which in turn may be written by

$$\phi_{uu}(n)_i = \sum_{k_1,\cdots,k_i}^{M} u(k)u(n-k_1)\cdots u(n-k_i)\,. \tag{13}$$

It is known that, for a linear system, the FRF can be estimated by the ratio of the Fourier transform of the cross-correlation function between the input and the output [$\phi_{uy}(n)$] to the Fourier transform of the autocorrelation function of the input [$\phi_{uu}(n)$]. This algorithm was used for the estimation of the FRF of a real pipeline system by Lee *et al.* (2006). When the pipeline system has nonlinear dynamics, as described by Eqs. (11) and (12), the FRF estimated using the preceding algorithm will have errors when it is compared with the theoretical linear FRF.

It has been verified by earlier research (Godfrey 1993, Roinila *et al.* 2010) that all the even kernel components shown in Eq. (12) can be cancelled out if the input signal $u(n)$ is persistent, periodic and antisymmetric, i.e. it can be written by

$$u(n) = -u(n + S/2)\,. \tag{14}$$

where S is the length of one period of the input signal. Since the contributions of the lower-order kernels on the output are usually dominating, once the nonlinear effect caused by the second-order kernel is removed, the estimation of the linear part of the system dynamics can be obtained more accurately.

Among persistent, periodic and antisymmetric signals, the inverse repeat sequence (IRS) is suitable for extracting the linear portion of the FRF of a system and it has been successfully applied to system identification of electrical devices (Godfrey 1993, Roinila *et al.* 2010). One period of an IRS can be obtained by doubling a period of a maximum-length binary sequence (MLBS) and toggling every other digit of the doubled sequence. The MLBS is one special type of pseudo-random binary sequences (PRBS). PRBS are two-level, predetermined and periodic signals. A detailed definition of PRBS is given by Godfrey (1993).

The MLBS can be generated by an *n*-bit shift register with exclusive or (XOR) feedback from the last stage and one or more of the other stages to the first stage. The stages of feedback must be chosen in such a way that a maximum periodical length of $2^n - 1$ is achieved. The process for generating the MLBS using shift registers is illustrated in Fig. 1.

Lee *et al.* (2008) developed a customised solenoid side-discharge valve to generate MLBS-based excitation signals. The valve was electronically controlled to produce pulses that follow a MLBS-based pattern; that is, only when the output of the shift register is 1, is a sharp pulse generated by abruptly opening and then closing the side-discharge valve. The movement of the valve was measured and the variation of the dimensionless valve opening throughout the test was used as the input signal.

This research proposes that the IRS is better than the MLBS in extracting the linear frequency response of a pipeline for the purpose of leak detection. The IRS inherits all the desirable properties for system identification from the MLBS, such as wide bandwidth, persistent and periodic. In addition, the IRS is antisymmetric, thereby enabling undesired effect of nonlinear dynamics of a pipeline to be suppressed.

4 Comparison of the MLBS and the IRS in the accuracy of linear FRD extraction

Numerical simulations are conducted to verify the fact that the IRS is superior to the MLBS in suppressing the nonlinear dynamics of a pipeline in the process of FRD extraction. Preliminary numerical studies have been performed by the authors on a leaking pipe in a reservoir-pipeline-valve (RPV) system (Gong *et al.* 2011). It was demonstrated that when the dimensionless amplitude of the valve oscillation ($\Delta\tau$) is 0.2 and thus exceeding the linearisation approximation, the FRD calculated from the nonlinear MOC model with IRS excitation was closer to the theoretical FRD derived from the linear transfer matrix method. This research conducts further investigation by comparing the accuracy of the frequency response extracted by the MLBS and the IRS under various values of $\Delta\tau$. Single frequency sine waves are also used in the study to provide reference information. Since a sine wave is also persistent, periodic and antisymmetric, its ability in suppressing the nonlinear dynamics of a pipeline should be similar to that of an IRS.

The pipeline system used in this study is a RPV system. The pipeline is intact and has a uniform internal diameter (D) of 40 mm and a total length (L) of 100 m. The head of the reservoir is 30 m. The wave speed (a) is 1000 m/s uniformly and the Darcy-Weisbach friction factor (f) is 0.02. The oscillating in-line valve has an initial opening of 6 mm in diameter and a discharge coefficient of 0.9. As a result, the steady-state opening area of the valve is 2.83×10^{-5} m^2, and the steady-state flow rate through the valve (Q_{V0}) is 6.1×10^{-4} m^3/s (velocity is 0.485 m/s, Reynold number is 17,017). The IRS-based excitation is generated by oscillating the dimensionless valve opening between two positions $\tau_0+\Delta\tau$ and $\tau_0-\Delta\tau$, where τ_0 is set to be unity in this study.

MOC simulations are conducted by oscillating the valve in patterns of MLBS, IRS and sine wave respectively, with various values of $\Delta\tau$. The MLBS used in this study is generated from a 10-stage shift register with a clock frequency of 100 Hz. As a result, the MLBS has a period of 10.23 s, an effective bandwidth of 44.3 Hz (the frequency where the power spectrum line drops to half of the maximum) and a resolution in its spectrum of 0.098 Hz (the interval between two power spectrum lines). The IRS used in this study is obtained by doubling one period of the MLBS and toggling each other digit of the double sequence. Accordingly, the period of the IRS is 20.46 s, but the effective bandwidth and frequency-domain resolution are the same as those of the MLBS. Considering that the fundamental frequency of the RPV model is 2.5 Hz [$a/(4L)$], the MLBS and the IRS are capable of determining the frequency response of the preceding pipeline system up to its 17th harmonics. The sine wave used in the numerical simulation has a frequency same as the fundamental frequency of the pipe (2.5 Hz), so that the first resonant response can be extracted. Each MOC simulation has a time step of 1 ms and a modelling duration of 2000 s. As a result, 195 periods of the MLBS or 97 periods of the IRS are used in one MOC modelling.

The values of the percentage *linearisation error* [as defined in Lee et al. (2005) and explained in Section 2 in this paper] are calculated for the first resonant response extracted from the preceding MLBS, IRS and sine signals with various values of $\Delta\tau$. The results are given in Fig. 2. Since the FRD extracted from the MLBS and the IRS are sampled discrete sequences, linear interpolation is used to estimate the frequency response at the specific resonant frequency 2.5 Hz.

The results shown in Fig. 2 demonstrate that the linearisation error introduced by the MLBS, as expected, is in general higher than the error resulted from the IRS and the sine signal. The values of the linearisation error introduced by the IRS and the sine signal are comparable and proportional to the value of $\Delta\tau/\tau_0$ in an exponential pattern. The comparability is consistent with the inference that the IRS and the sine signal have similar ability in suppressing the nonlinear dynamics because both of them are antisymmetic; while the exponential pattern is consistent with the results presented in Lee et al. (2005). When the value of $\Delta\tau/\tau_0$ is greater than 0.4, it seems that the MLBS behaves better, although significant error is observed for all these three cases. However, it should be noted that the results shown in Fig. 2 are for the first resonance only and the interpolation process is involved for the results from the MLBS and the IRS. The FRD (the first three resonances only) extracted from the MLBS and the IRS when the value of $\Delta\tau/\tau_0$ equals 0.4 are shown in Fig. 3. It can be seen that the FRD extracted from the MLBS has greater discrepancy from the FRD obtained from the linear transfer matrix method. The discrepancy is attributable to the effect of nonlinearity in the nonlinear MOC simulation.

5 Conclusions

This research has analysed the effect that transient excitation has on the nonlinear behaviour of a pipeline. A detailed analysis of the nonlinearity of a pipeline and its effect on the FRD-based leak detection techniques has been conducted. Because all the FRD-based leak detection techniques are based on linear theory, the FRD generated for leak detection is desired to only represent the linear response of the pipeline.

This research demonstrates that wide bandwidth, persistent, periodic and antisymmetric signals are suitable for the extraction of the linear FRD. The antisymmetric property enables even kernels of the nonlinear dynamics of a system to be cancelled out; therefore, the linear part of the system dynamics can be estimated more accurately. The inverse-repeat sequence (IRS), in particular, is found to be an appropriate excitation signal. Numerical studies conducted in this research demonstrate that the IRS is better than the maximum-length binary sequence (MLBS) that has been used in earlier research for FRD extraction.

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Notation

A = cross-sectional area of a pipeline (m^2)

A_L = opening area of a leak orifice (m^2)

a = wave speed (m/s)

C_{Ld} = coefficient of discharge for a leak orifice (-)

D = internal pipe diameter (m)

f = Darcy-Weisbach friction factor (-)

g = gravitational acceleration (m/s^2)

H = head (m)

H_0 = steady-state head (m)

H_{L0} = head across a leak (m)

h = complex head perturbation (m)

j = imaginary unit, $\sqrt{-1}$ (-)

L = length of pipe (m)

M = length of total data sequence of interest (-)

n = number of stages of a shift register, or a positive integer (-)

Q = flow (m^3/s)

Q_0 = steady-state flow (m^3/s)

Q_{L0}, Q_{V0} = discharge through a leak and a valve (m^3/s)

q = complex discharge perturbation (m^3/s)

R = linearised resistance term, $fQ_0 / (gDA^2)$ (s/m)

S = length of one period of a signal (-)

s_1 = linear kernel of a system (-)

$s_2, \cdots, s_i$ = nonlinear kernel of a system (-)

t = time (s)

u = input signal (-)

x = distance (m)

y = output signal (-)

Greek symbols

ΔH_{V0} = steady-state head loss across a valve (m)

ϕ_{uu} = autocorrelation function of the input (-)

ϕ_{uy} = cross-correlation function between the input and the output (-)

ω = angular frequency (rad/s)

τ_0 = mean dimensionless valve aperture size (-)

$\Delta\tau$ = magnitude of the dimensionless valve aperture perturbation (-)

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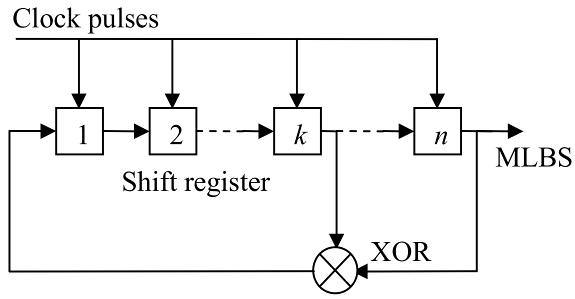

Figure 1 An *n*-Stage shift register with XOR feedback for MLBS generation.

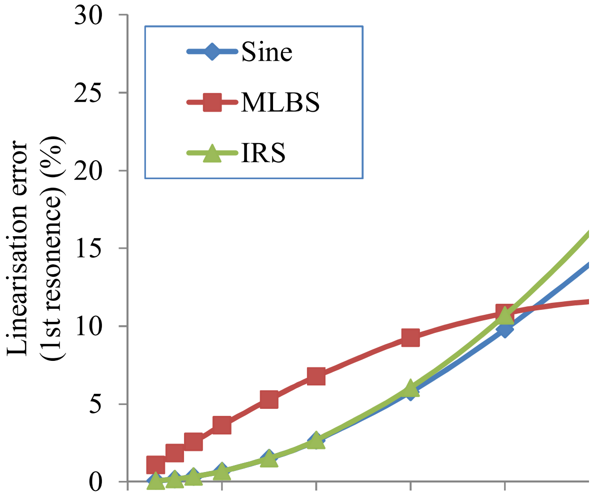

Figure 2 Percentage error at the first resonance (frequency response at 2.5 Hz) as a function of the magnitude of valve perturbation.

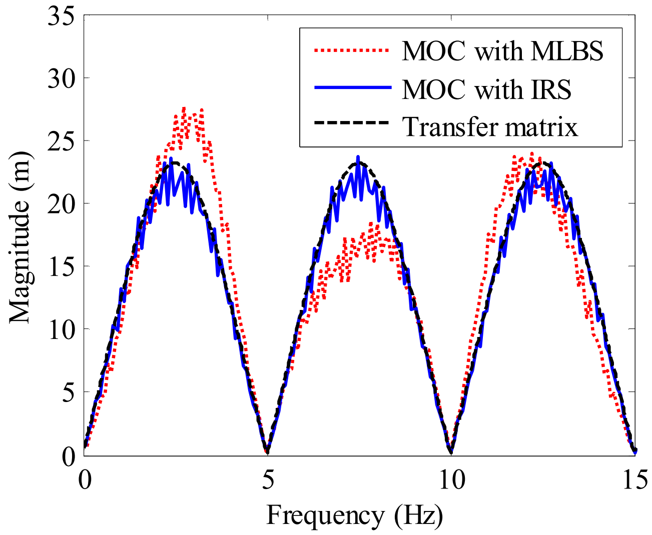

Figure 3 Comparison between the FRD from MOC using the MLBS and the IRS excitation with $\Delta\tau / \tau_0 = 0.4$ and the linear FRD from the transfer matrix method.