

Supercurrent in a mesoscopic proximity wire

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Recent experiments on the proximity induced supercurrent in mesoscopic normal wires revealed a surprising temperature dependence. They suggest clean-limit behavior although the wires are strongly disordered. We demonstrate that this unexpected scaling is actually contained in the conventional description of diffusive superconductors and find excellent agreement with the experimental results. In addition we propose a SQUID-like proximity structure for further experimental investigations of the effects in question.

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1. Introduction

A normal metal in direct contact with a superconductor acquires superconducting properties.^{1,2} Although this proximity effect has been discussed for a long time, it has recently attracted new attention because of the dramatic progress in nanotechnology which allows the fabrication and study of metallic structures in the mesoscopic regime. Due to the proximity effect a supercurrent can flow through a normal metal between two superconductors.³⁻⁵ Recently, this current has been detected by Courtois et al.⁶ in a thin normal wire in the diffusive regime with superconducting strips deposited on its top (see Fig. 1).

The supercurrent is determined by an overlap of Cooper-pair wave functions penetrating into the normal wire from the superconducting strips. Since in the diffusive limit the effective penetration length is of order^{1,2} $\xi_N = \sqrt{D/2\pi T}$ one should expect that the supercurrent in the system is proportional to^{4,5} $I_c(T) \propto e^{-\sqrt{T/T_0}}$. In contrast, the experimental data of Ref.⁶ showed a much better fit to the dependence $I_c(T) \propto e^{-T/T_1}$, which is typical for ballistic systems.³ Furthermore, the fitting parameter T_1 deviated

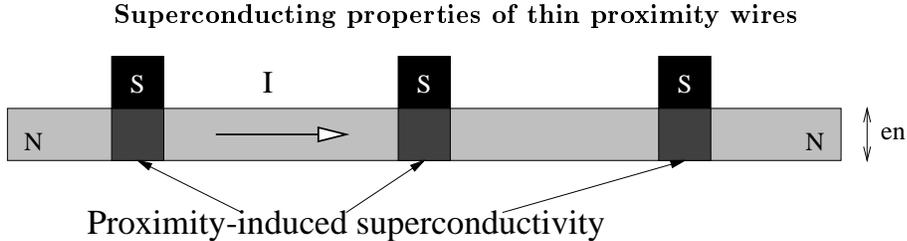


Fig. 1. Experimental geometry

from standard clean-limit results by more than one order of magnitude. This has raised the question, whether the usual criterion distinguishing between ballistic and diffusive limits is correct when applied to proximity induced superconductivity in the normal metal, or whether the above phenomenon has to be attributed to quantum effects not contained in the quasiclassical theory of superconductivity.

Below we will demonstrate that neither of these conjectures is true. We employ a quasiclassical calculation of the supercurrent in the proximity structures of Fig. 1 and show the corresponding results agree well with the experimental data.⁶ Furthermore, we will also propose a new, equivalent experiment, where superconducting material is deposited onto a normal ring. In this 'proximity SQUID' the magnetic flux through the ring is the equivalent of the phase difference across the normal wire.

2. The Model and the Formalism

Since the critical current of the structure of Fig.1 is determined by the longest SNS-cell which serves as a "bottleneck" it is sufficient to study a single cell. We denote the distance between these two adjacent superconducting strips as d and consider the case where d is much larger than the strips' thickness.

We will use the standard formalism of quasiclassical Green's functions⁷ in the diffusive limit⁸ For a thin normal wire with a thickness $\ll \xi_N$ (this condition is well justified in the experiment⁶) the Green functions in the wire directly below superconducting strips are equal to those of a superconductor for all relevant energies.⁹ In what follows we further assume that the superconductors have bulk properties, neglecting any suppression of the superconducting gap Δ in the strips in the vicinity of SN boundaries.

3. Solution and Results

Details of the formalism and the calculation will be published in a future publication, so we will only present the key results here.

A characteristic energy scale is provided by the Thouless energy $E_d =$

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D/d^2 , where D is the diffusion constant. For high temperatures, $T \gg E_d$ (equivalent to the geometrical condition $d \gg \xi_N$), the mutual influence between the superconductors can be neglected. In this approximation, we get the current-phase relation $I = I_c \sin(\varphi)$ with critical current

$$I_c = \frac{64\pi}{3 + 2\sqrt{2}} \frac{T}{eR_N} \frac{d}{\xi_N} \exp\left(-\frac{d}{\xi_N}\right) \propto T^q \exp\left(-\sqrt{\frac{T}{T_0}}\right) \quad (1)$$

with $2\pi T_0 = E_d$, $q = 3/2$ and R_N is the normal state resistance of the normal part. For comparison, the expression following from the Ginzburg-Landau analysis² has the same T -dependence but with $q = 1/2$. On the other hand, in the clean limit one expects a temperature dependence $I_c \propto e^{-T/T_1}$.

At this stage we note a remarkable mathematical artefact caused by the exponent $q = 3/2$. The logarithmical derivative of (1)

$$\frac{dI_c(T)}{I_c(T)dT} = \left(\frac{3}{2T} - \frac{1}{2\sqrt{TT_0}} \right)$$

has a minimum at $T = 36T_0$ and varies very slowly at higher temperatures, so $\log I_c$ is almost linear in T . As a good approximation for the slope in a logplot, we can take the logarithmical derivative in the minimum and get $I_c \propto e^{-T/T^*}$ where $T^* = 24T_0$. This implies that within a considerable temperature interval a “quasi-clean” scaling is found also in dirty SNS systems, which explains the behavior found in the experiments of Courtois et al.⁶

In the low temperature limit $T = 0$ another simple estimate for the critical current can be derived.

$$I_c = \frac{\Delta}{R_N e} \arctan\left(\frac{E_d}{2\Delta}\right) \xrightarrow{E_d \ll \Delta} \frac{D}{2R_N e d^2}. \quad (2)$$

This demonstrates the importance of the Thouless energy as a relevant energy scale in SNS-junctions at $E_d \ll \Delta$. The Thouless energy also determines the proximity induced effective gap in the quasiparticle spectrum of the N-metal (see Ref.¹⁰ and further references therein). We further note that (2) resembles the well-known Ambegaokar-Baratoff formula for the critical current of a Josephson tunnel junction at $T = 0$ if we substitute D/d^2 by the gap $\sim \Delta$. Thus our results emphasize that the Thouless energy in a diffusive proximity coupled normal wire plays the same role as the gap Δ in a “strong” superconductor and does not only determine the density of states, but also the critical current.

A solution of the full problem as obtained numerically is shown in fig 2. The result matches quantitatively to the experiments, if d is chosen slightly larger than the average cell length of the experiment. This reflects the fact, that the critical current of the whole chain is determined by the

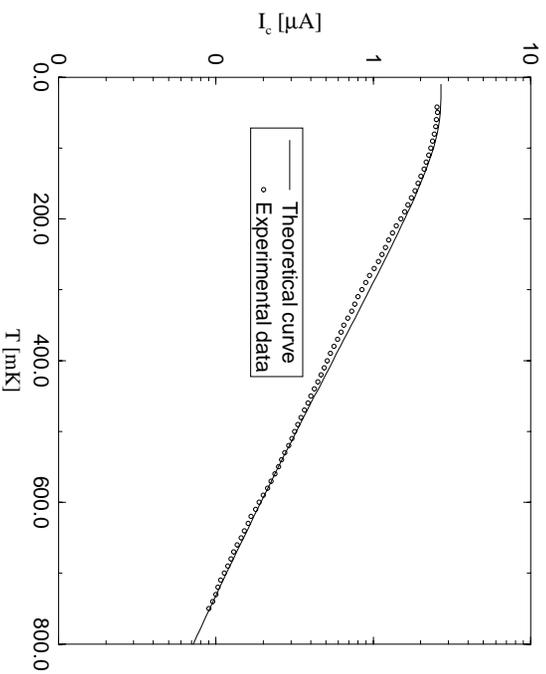


Fig. 2. Critical current for the experimental parameters [6], experimental data are supplied by H.Courtois

longest SNS-cell with the lowest I_c . The adjustment lies within the range of experimental accuracy. In general the current-phase relation $I(\varphi)$ may deviate from sinusoidality.^{4,5} We investigated also this property, but found in the SNS geometry nearly no deviation down to very low temperatures (see Fig. 3).

4. The Proximity SQUID

The proximity induced supercurrent should lead to an interesting flux-periodic behavior of the “proximity loop” structure shown in Fig. 4. A normal ring is contacted over a range of length d_S by superconducting material. If the ring is narrow it can be mapped onto the linear system discussed above by absorbing the vector potential in the gauge invariant phase $\varphi(r) = \varphi_0(r) + 2e \int_0^r dr' A(r')$. I.e. the anomalous Green’s function F carries the phase factor $\exp[i\varphi_0(r)]$, but $\alpha(r)$ and $\varphi(r)$ satisfy the Usadel equation with appropriate boundary conditions and the substitution $d = L - d_S$. Since the Green’s functions must be single-valued at every point of the ring, the “real” phase φ_0 can change only by multiples of 2π when circling around the

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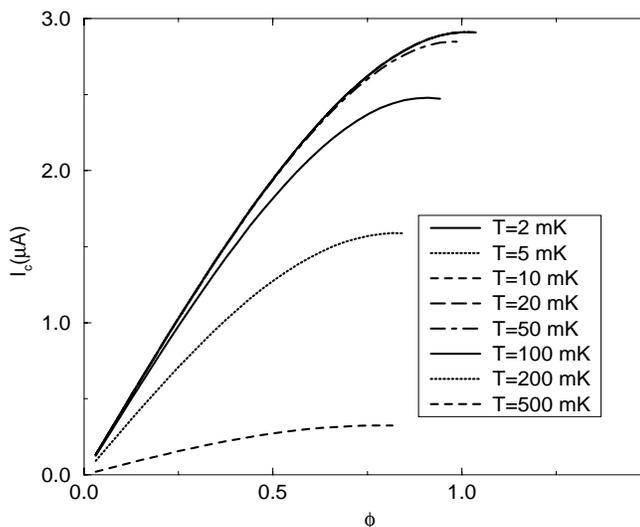


Fig. 3. Current phase relation, parameters as in the experiment, see Fig. 2 ring. For this reason, at $x = 0$ the gauge-invariant phase drops by

$$\phi = \varphi(0+) - \varphi(0-) = 2e \oint dr A(r) = \frac{2e\Phi}{\hbar},$$

where Φ is the magnetic flux through the ring.

Taking into account the self-inductance, this system is completely equivalent to a standard SQUID formed by a superconducting loop interrupted by a weak link. The configuration considered here is complementary: a normal loop is interrupted by a narrow superconducting strip, but due to the proximity effect it shows the same properties as the standard SQUID.

At relatively high temperatures the supercurrent in the normal proximity loop is exponentially small. However, as the temperature is lowered below the corresponding Thouless energy the supercurrent becomes large (cf. (2)) and can be easily detected experimentally. Also the current-phase relation of a diffusive SNS structure can be easily studied.

5. Conclusions

Making use of a standard quasiclassical formalism of the superconductivity theory we evaluated the supercurrent in mesoscopic proximity wires. Our results match quantitatively with the experimental data⁶ thus demonstrating that the quasiclassical theory of superconductivity is sufficient for

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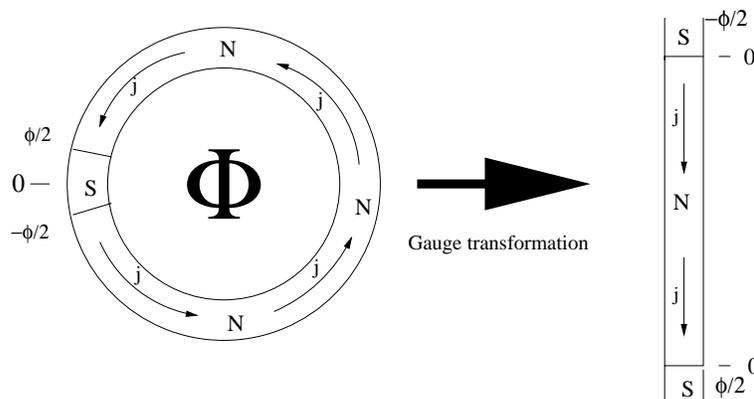


Fig. 4. Proximity SQUID

the description of these systems. The deviations of the current-phase relation from sinusoidality are found to be small even at very low temperatures. Finally we argue that a normal-metal loop interrupted by a narrow superconducting strip has the same properties as a standard SQUID and can be used for further experimental investigations of the proximity effect in mesoscopic systems.

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