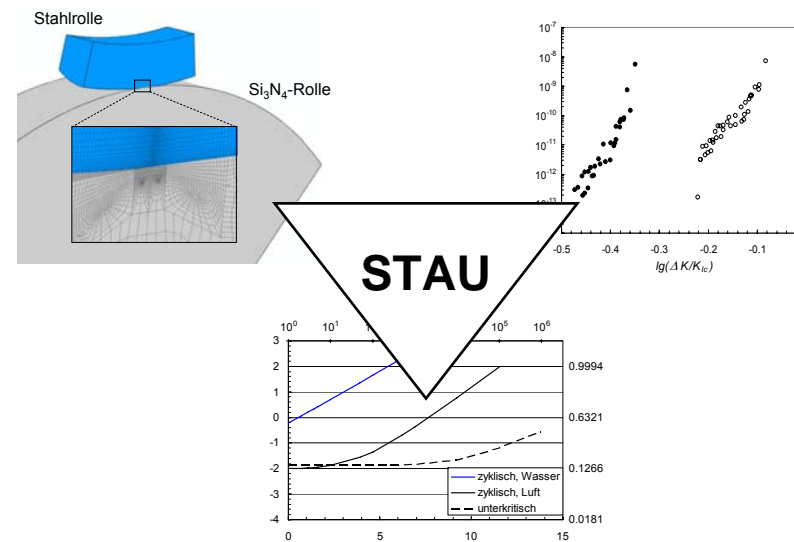


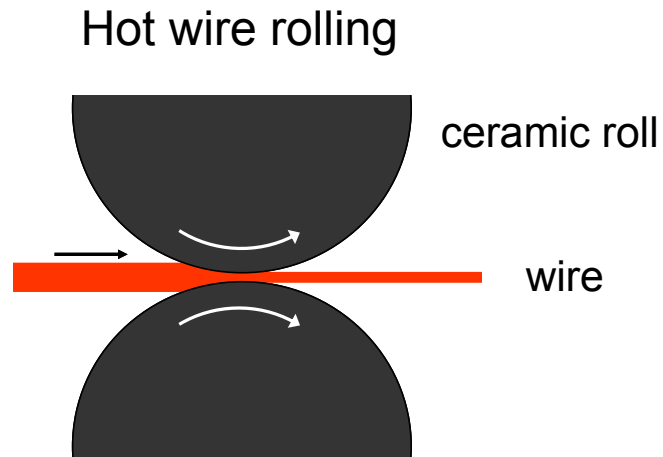
PROBABILISTIC ANALYSIS OF A ROLLING CONTACT FATIGUE TEST FOR SILICON NITRIDE

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Karlsruhe Institute of Technology (KIT), Institute for Applied Materials (IAM)



Motivation



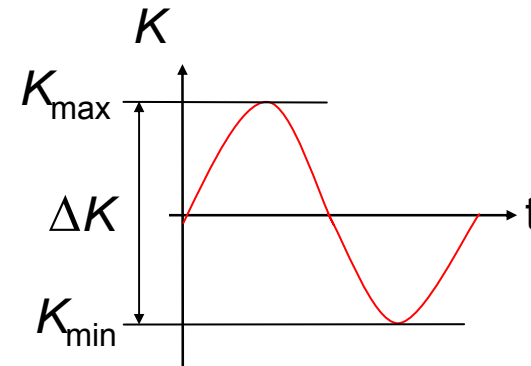
Fraunhofer Gesellschaft, IWM, Freiburg

- High strength ceramics (Si_3N_4) are used in rolling applications
- Time-dependent failure due to slow crack propagation
- Design of such components requires probabilistic methods
- Rolling contact fatigue test as model system for rolling applications

Theory: Crack propagation mechanisms

■ Crack loading

$$\Delta K = K_{\max} - K_{\min} \quad R = \frac{K_{\min}}{K_{\max}}$$



Subcritical crack propagation

- Stress corrosion/ chemical reaction
- Quasi-static effect

$$\frac{da}{dt} = A_S \cdot \left(\frac{K_I}{K_{Ic}} \right)^{n_S}$$

A_S, n_S : material properties

Cyclic crack propagation (fatigue)

- Depends on load sequence
- Degradation of strengthening effects (grain bridging)

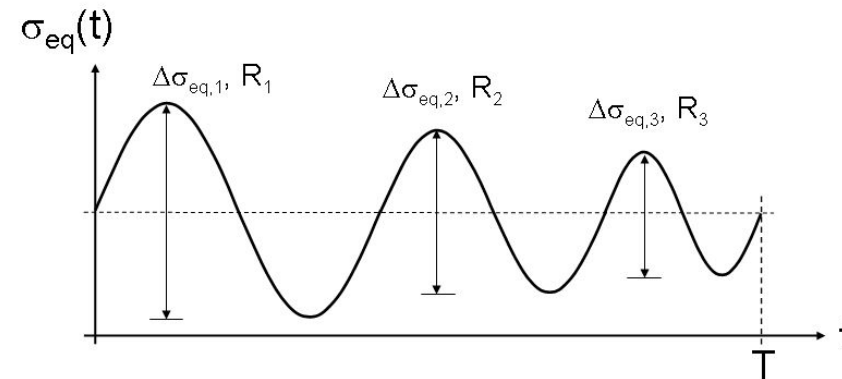
$$\frac{da}{dN} = \frac{A}{(1-R)^{n-p}} \cdot \left(\frac{\Delta K_I}{K_{Ic}} \right)^n$$

A, n, p : material properties

Theory: Failure probability

Weibull Theory

- Integration over component surface/ volume and flaw orientation
- Equivalent stress σ_{eq} : local failure criterion



Subcritical crack propagation

$$P_f = 1 - \exp \left[- \frac{1}{A_0} \int_A \frac{1}{2\pi} \int_{\alpha} \left(\max_{t \in [0, T]} \left\{ \left(\frac{\sigma_{eq}(t)}{\sigma_0} \right)^{n_s - 2} + \frac{\sigma_0^2}{B} \int_0^t \left(\frac{\sigma_{eq}(t')}{\sigma_0} \right)^{n_s} dt' \right\} \right)^{\frac{m}{n_s - 2}} d\alpha dA \right]$$

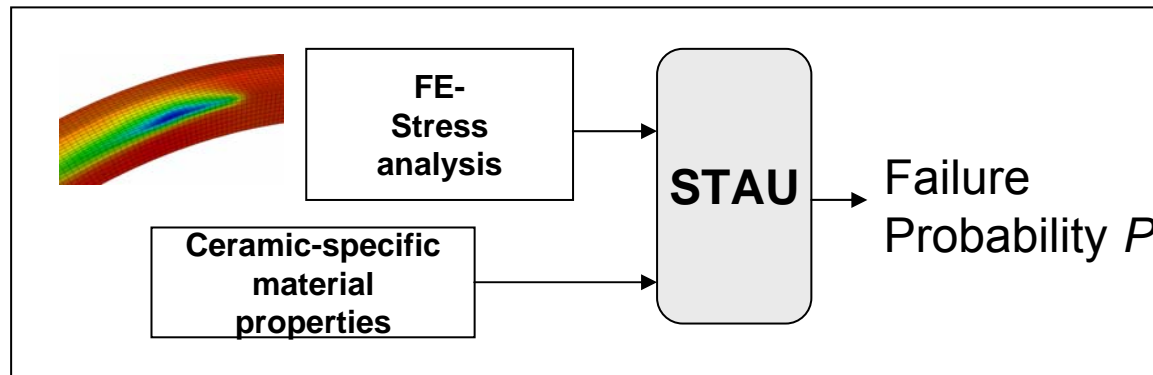
Time/Cycle dependent load history

Cyclic crack propagation

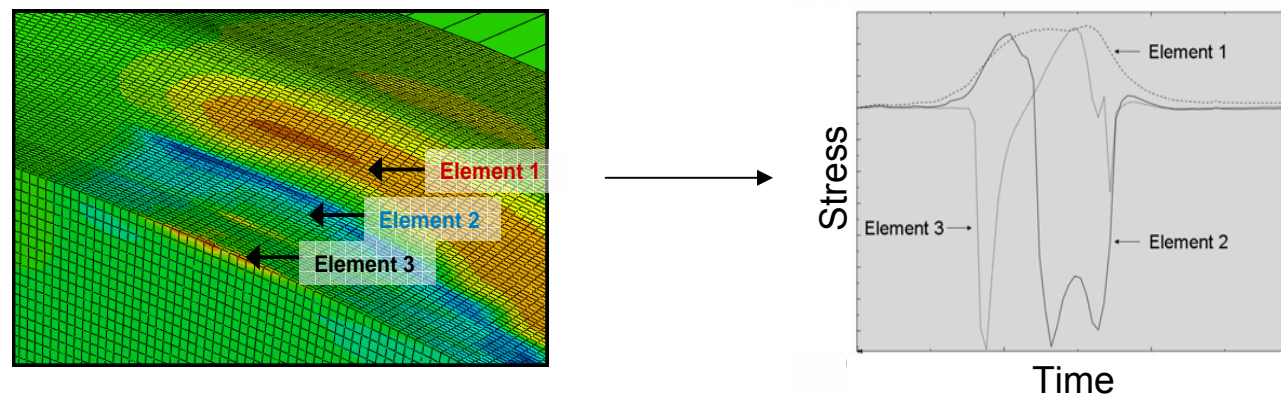
$$P_f = 1 - \exp \left[- \frac{1}{A_0} \int_A \frac{1}{2\pi} \int_{\alpha} \left(\max_{\eta \in [N_1, N_k]} \left\{ \left(\frac{\sigma_{eq, \max}(\eta)}{\sigma_0} \right)^{n - 2} + \frac{\sigma_0^2}{B} \int_{N_1}^{\eta} \left(\frac{\sigma_{eq, \max}(N')}{\sigma_0} \right)^n (1 - R(N'))^p dN' \right\} \right)^{\frac{m}{n - 2}} d\alpha dA \right]$$

Theory: Numerical evaluation

STAU¹: Finite-Element Postprocessor for reliability assessment of ceramics



Reliability analysis for rolls → Complex load history must be considered



¹ H. Riesch-Oppermann, M. Härtelt, O. Kraft, *Int. J. Mat. Res.* 99 (2008)

Results: Material parameters Si₃N₄-SL200

Strength

$$\sigma_{4PB} = 1044 \text{ MPa}$$

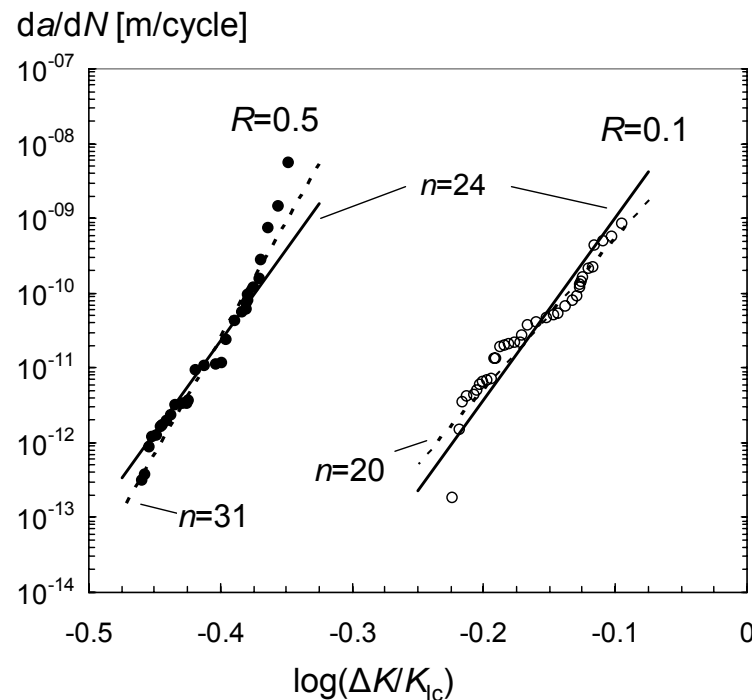
$$m = 11.5$$

Subcritical crack growth

$$n = 42, A_S = 10^{-6} \text{ m/s}$$

Lube and Dusza, *J.Eur.Ceram.Soc.*, 27(2-3),2007

Cyclic fatigue parameters (air)



Härtelt et al., *J Am Ceram Soc*, 2011, in press

Crack growth exponent n depends on load ratio R ($n=20,31$)

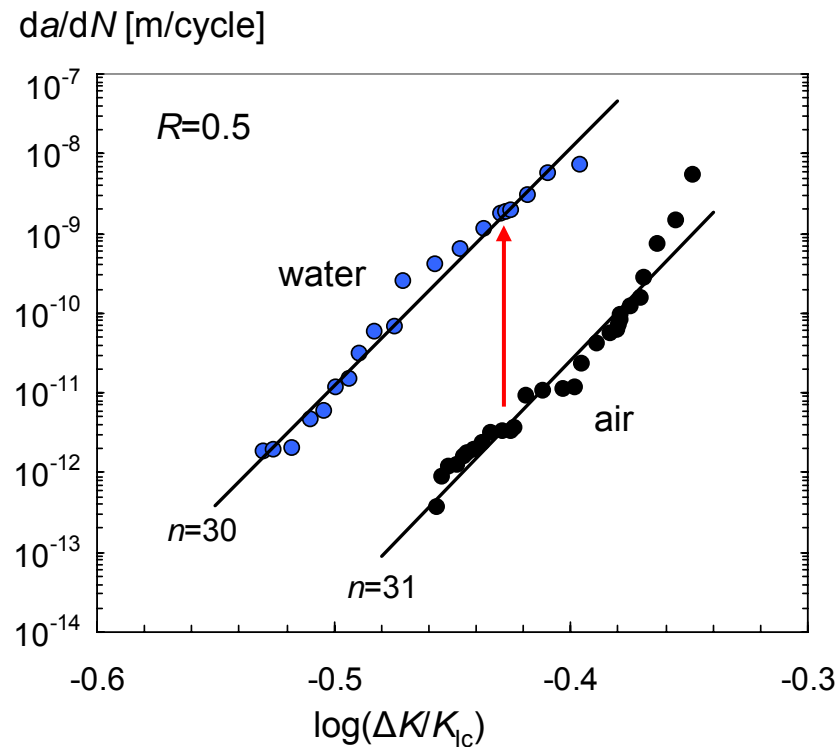
Curves must be represented by common exponent n :

$$\frac{da}{dN} = \frac{A}{(1-R)^{n-p}} \cdot \left(\frac{\Delta K_I}{K_{Ic}} \right)^n$$

$$n=24, p=2.2, A=3 \cdot 10^{-8} \text{ m/cycle}$$

Results: Material parameters Si_3N_4

■ Cyclic fatigue parameters (water)

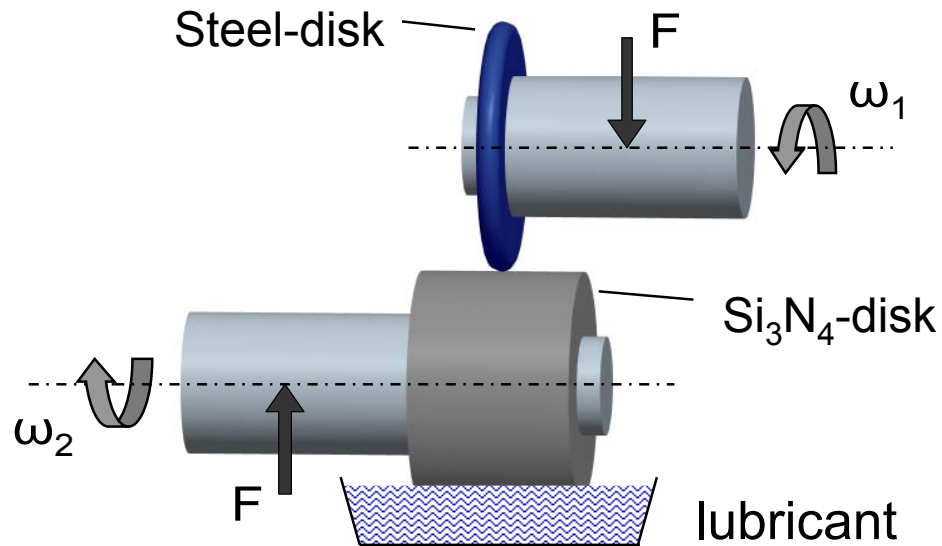


- Water enhances fatigue effect!
- Measurements for $R=0.5$: $n=29.9$
- Parameters are evaluated assuming $p=2.2$ (air):

$$\frac{da}{dN} = \frac{A}{(1-R)^{n-p}} \cdot \left(\frac{\Delta K_I}{K_{Ic}} \right)^n$$

$$n=29.9, p=2.2, A=5 \cdot 10^{-5} \text{ m/cycle}$$

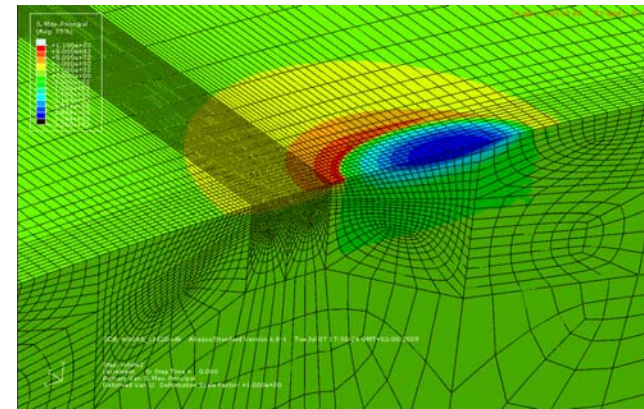
Results: Rolling contact fatigue (RCF) test



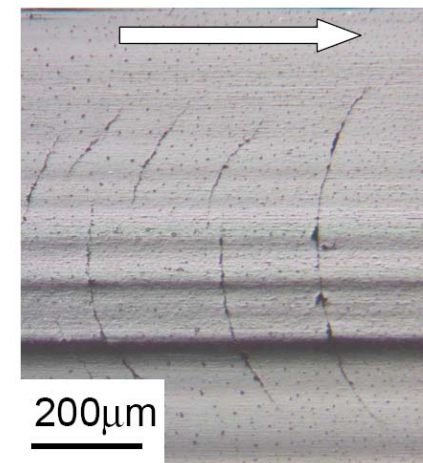
- lubricant: friction coefficient $\mu=0.085$
- $F=1700\text{N}$
- relative slip: $\sim 22\%$
- max. principal stress: $\sim 1100\text{ MPa}$

RCF tests: Iyas Khader, Fraunhofer Institute IWM, Freiburg

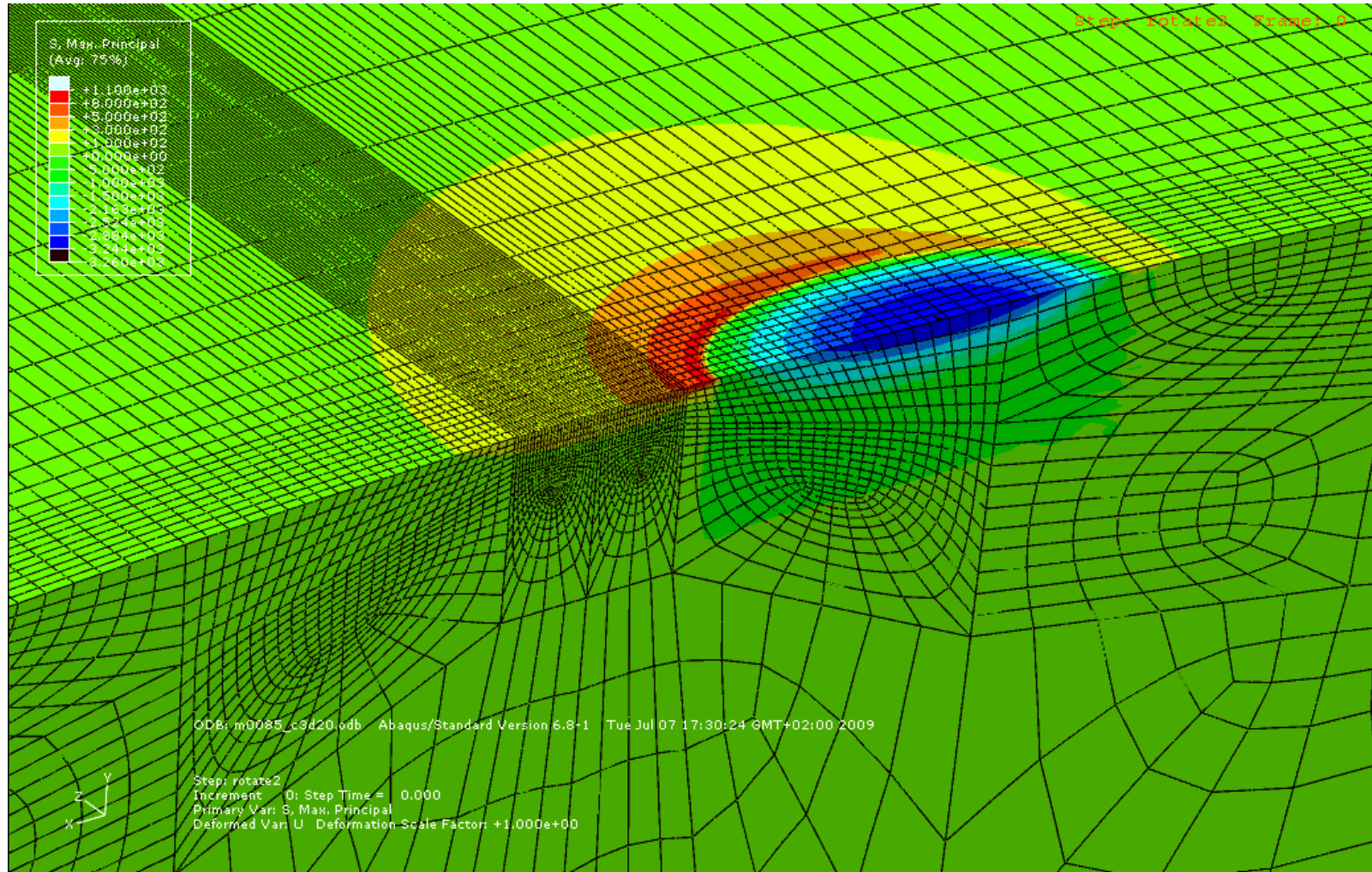
Stress distribution



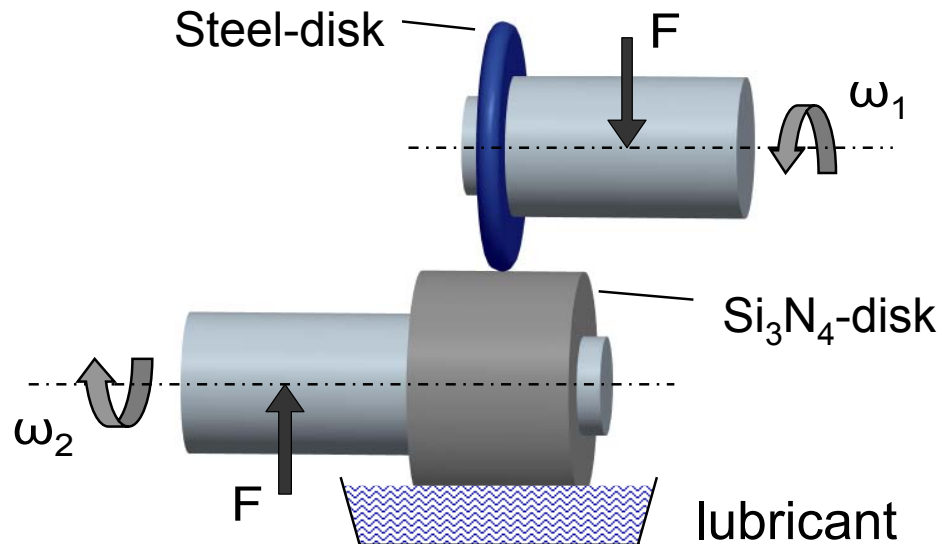
Damage after 10^5 rotations



Results: Rolling contact fatigue (RCF) test



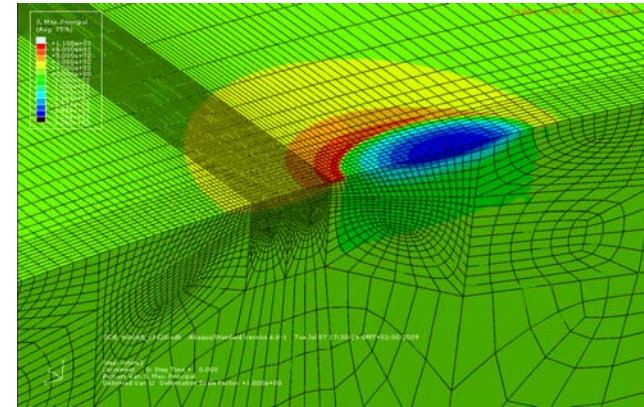
Results: Rolling contact fatigue (RCF) test



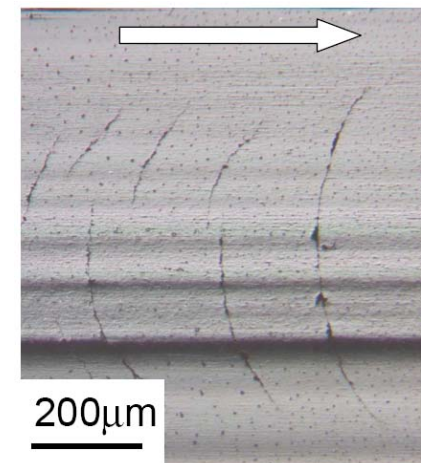
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Stress distribution

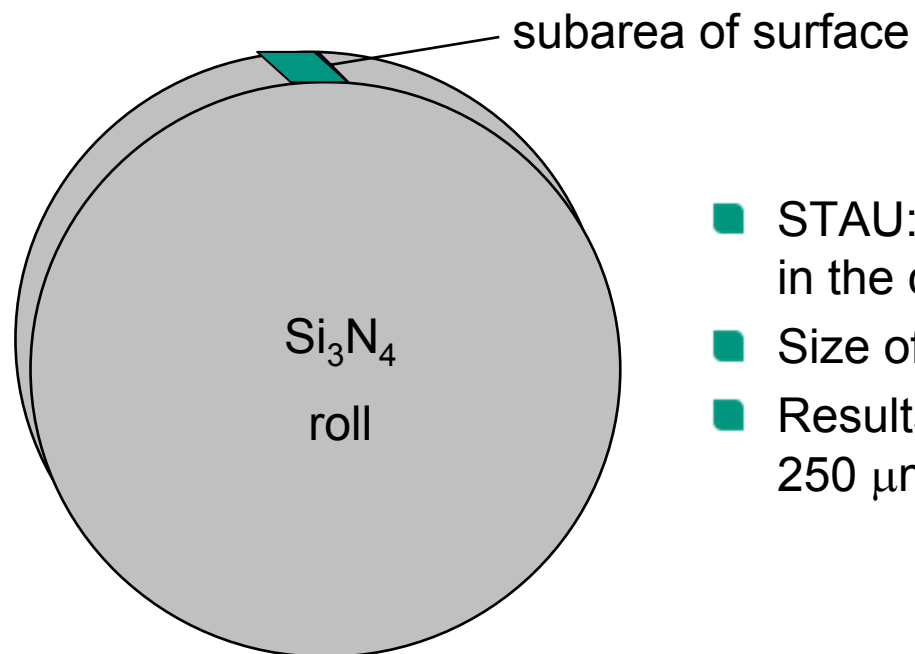
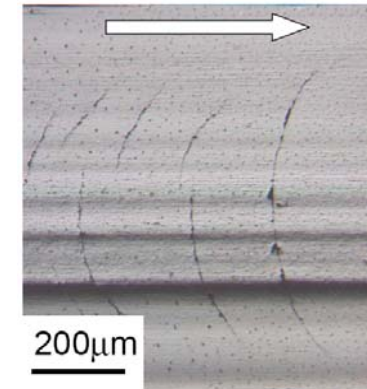


Damage after 10^5 rotations



Results: STAU analysis

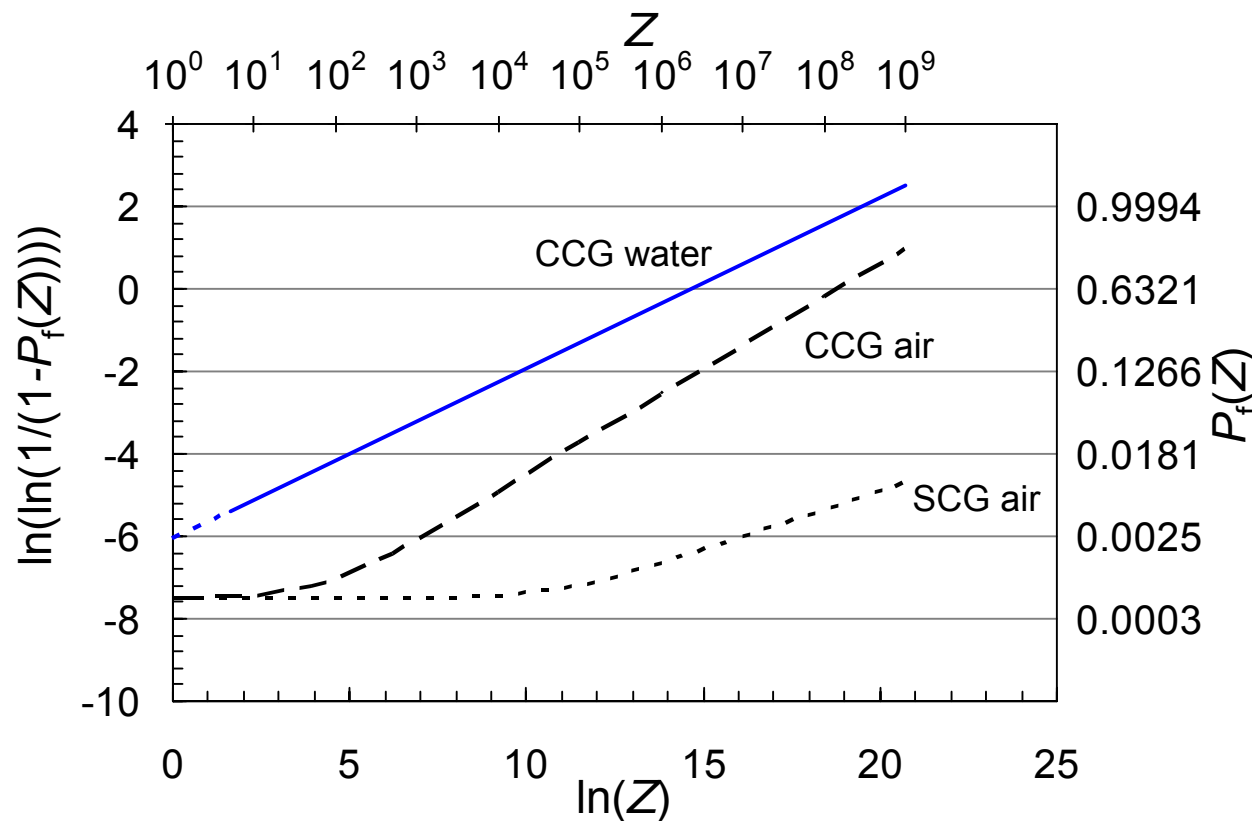
- Contact damage: initiation of macroscopic flaws
- Predicting of a certain flaw density on the surface



- STAU: probability of the initiation of one flaw in the considered subarea
- Size of subarea ↔ crack density
- Results refer to crack density of 1 crack per 250 μm along the circumference

Results: Failure probability

- Probability to initiate one macroscopic crack every 250 μm



- Weibull function

$$P_f = 1 - \exp \left[- \left(\frac{Z}{N_0} \right)^m \right]$$

m – slope of the curve

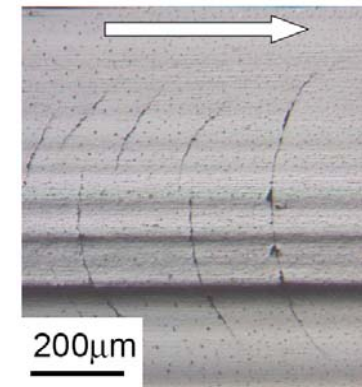
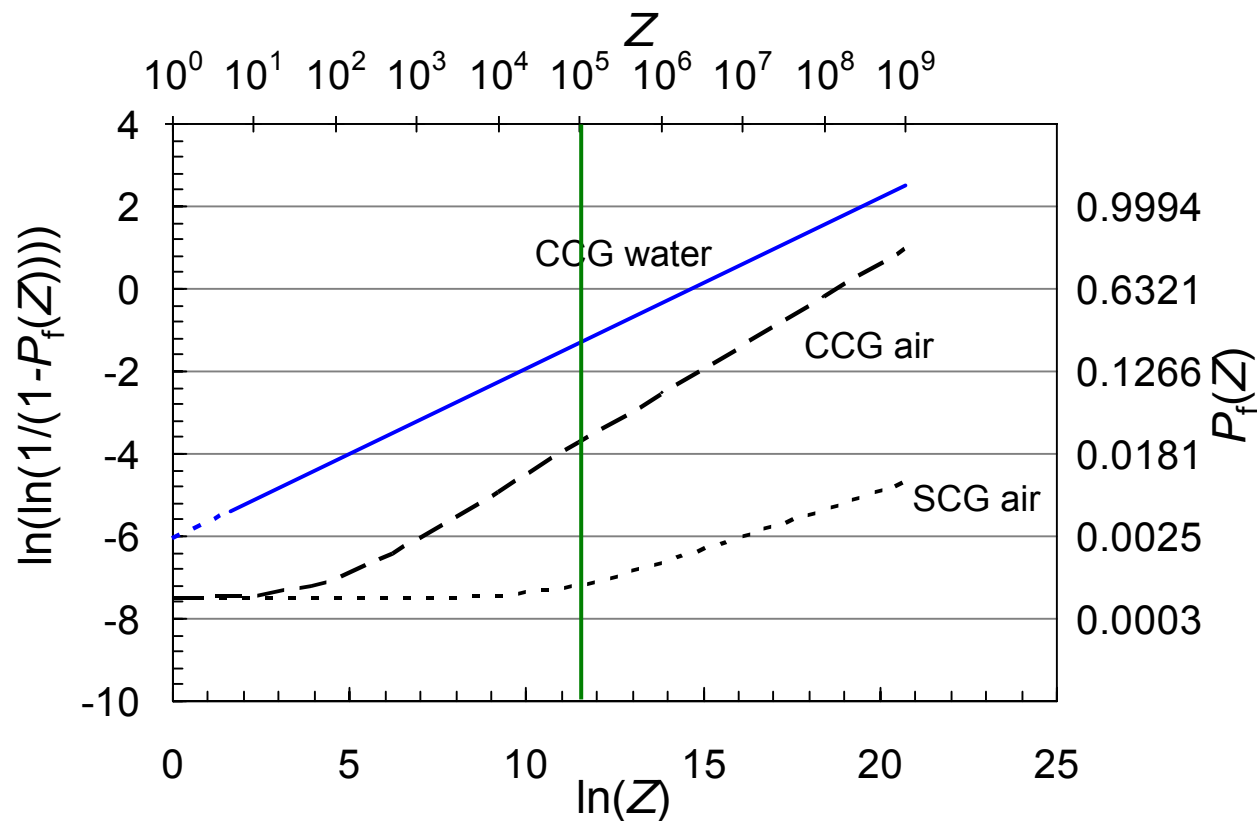
N_0 - characteristic lifetime
(63%-quantile)

Z – no. of rotations

- Highest failure probability obtained for fatigue parameters in water.

Results: Failure probability

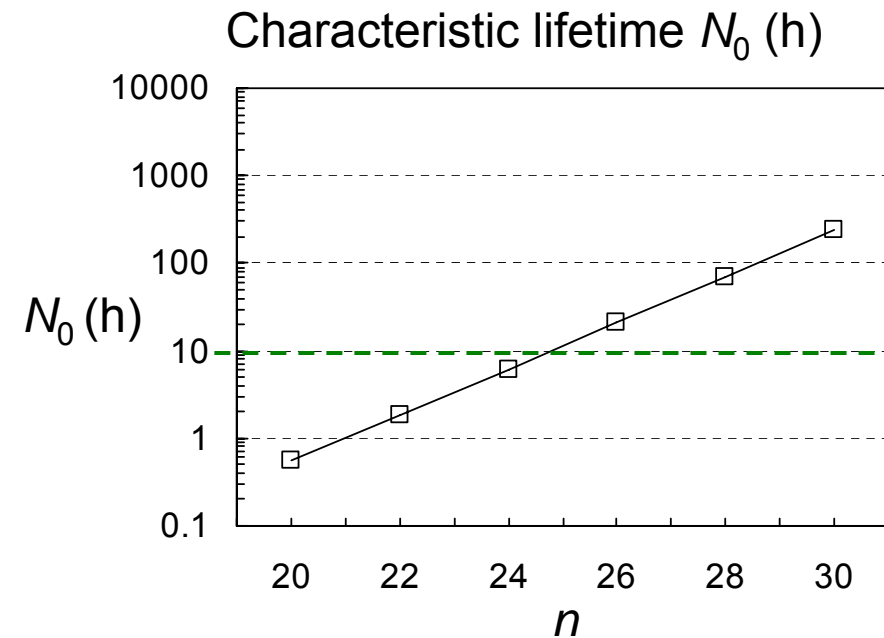
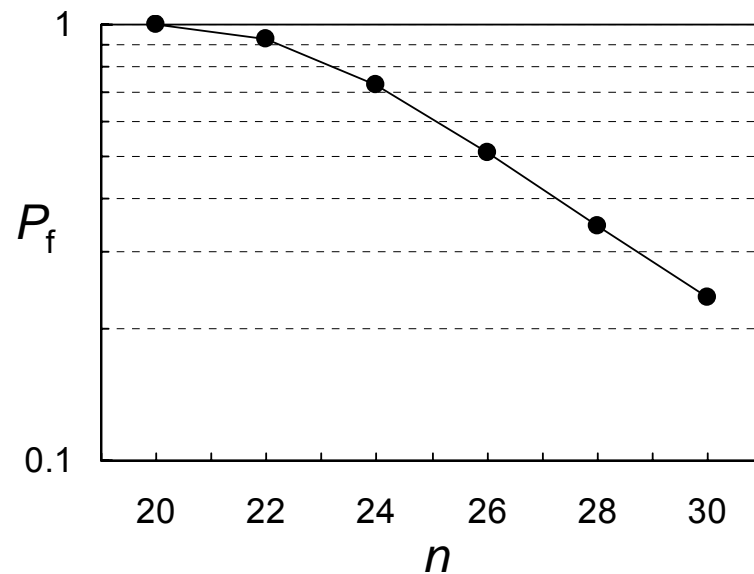
- Relation with experimental crack density after 10 h



- Initiation probability low for parameters in (air)

Results: Parametric study

■ Influence of crack growth exponent n

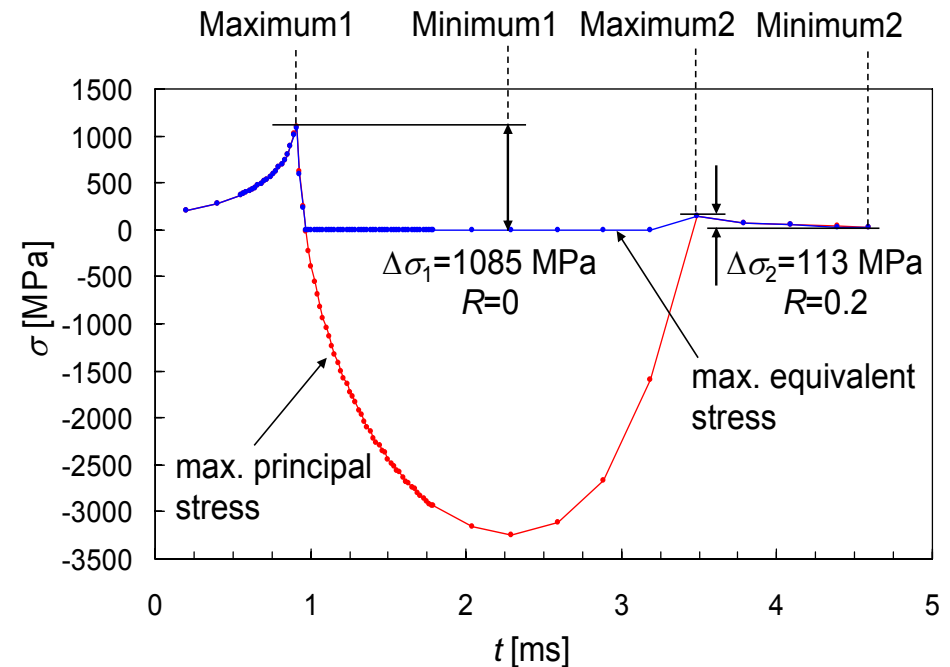


- Crack growth exponent has a strong influence on the results:
 For $n < 25$, crack initiation probability is $> 70\%$;
 Characteristic lifetimes are below 10 h

Discussion

- Agreement with experimental results for lower n -values if water parameters are used

- Stress history: $R=0$ dominates
- ➔ Using a lower n value ($n \approx 20$) in the case of water.



- Limitations of the analysis scheme:
 - Stress gradients in the range of natural flaws
 - Interaction of macroscopic cracks
 - Wear

Summary

- STAU as general tool for reliability assessment under complex loading
- prediction of evolving crack patterns on roll surface is possible
- sensitivity to crack growth parameters is challenging

Acknowledgements

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