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A Statistical Method for Evaluation of Hot Channel Factors in Reactor Design

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A b s t r a c t

Previous methods of hot channel analysis assumed all the cooling channels in a core to be independent. Actually we have to distinguish between two types of uncertainties, namely: local uncertainties, which may vary from point to point within a reactor, and global uncertainties which do not vary within a reactor, or at least within parts of a reactor. The global uncertainties introduce evidently a correlation among the temperatures of the coolant in the channels they affect in the same way.

This paper gives a quantitative assessment of the different effects of these two types of uncertainties and proposes a new method of evaluating the hot channel factor, taking into account this correlation.

The method was developed with reference to sodium cooled fast reactors, but is quite general and may readily be applied to other reactor types.

The paper shows moreover an example for the reactor Na-2 and a comparison with the previous methods. The actual temperature profile of the coolant in the reactor is taken into account.

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1. Introduction

In the thermal design of nuclear reactors, the operating temperature is limited by the requirement that the temperature of the core components does not exceed critical values. For instance,with reference to sodium cooled fast reactors, the coolant temperature at the cooling channel outlet should be sufficiently below the sodium boiling point, the canning temperature should not exceed the critical value, which, depending upon the material and applied stresses, provokes an unacceptable creep rate in the canning; and the fuel temperature should be lower than the melting point of the fuel.

In the reactor thermal design, it is necessary therefore to assume adequate safety margins, which take into account all the uncertainties affecting the parameters, upon which the core temperature depends. These uncertainties are of different nature, namely measurement errors,fabrication tolerances, inexact knowledge of physical properties of the core materials, calculation imprecision etc...

On the other hand, the assumption of large safety margins corresponds to a decrease in the operating temperature, with consequent reduction of the plant efficiency, and to an increase in the unitary cost of the delivered power.

Starting from these considerations,the initial deterministic concept of calculation of these safety coefficients (the so called hot channel and hot spot factors) was substituted by statistical analysis and several statistical methods were developed in the literature.

These methods assumed all the cooling channels in a core to be independent and treated all the uncertainties in the same manner. Actually we have to distinguish between two types of uncertainties, namely: local uncertainties, which may vary from point to point within a reactor, and global uncertainties which do not vary within a reactor, or at least within parts of a reactor. The global uncertainties introduce evidently a correlation among the temperatures of the coolant in the channels they affect in the same way. This paper gives a quantitative assessment of the different effects of these two types of uncertainties and proposes a new method of evaluating the hot channel factor, taking into account this correlation.

This method was developed with reference to sodium cooled fast reactors, but is quite general. It gives an overall hot channel factor for flat power reactors, and can be extended to the hot spot factors when the most critical zone only, in every fuel pin, is taken into account. A more complete hot spot analysis should be object of further work.

Moreover, this paper shows a numerical example for the reactor Na-2, with a comparison with the previous methods and an approach to take into account the actual temperature profile of the coolant in the reactor.

2. Coolant outlet temperature

The coolant temperature (ϑ_o) at a cooling channel outlet is given by the relation:

$$\vartheta_o = \vartheta_i + \Delta\vartheta_c \qquad (1)$$

with

$$\Delta\vartheta_c = \int_0^l \frac{L_s}{c_p\, Q}\, dz \qquad (2)$$

where:

ϑ_o = coolant temperature at the channel outlet

$\Delta\vartheta_c$ = coolant temperature span across the channel

l = length of the active zone of the channel

L_s = thermal power delivered in the channel per unit of length

c_p = specific heat of the coolant

Q = coolant mass flow rate through the channel

z = abscissa along the channel axis.

For sodium cooled reactors, the coolant channel arrangement is shown in fig.1. The uncertainty on the value of ϑ_o arises from the uncertainties affecting the individual parameters upon which ϑ_o depends. Namely, ϑ_i is known in the limits of a measurement error; l depends upon the length of the fuel pellet stack, which, due to the mechanical tolerances, can be different among the several fuel pins; the uncertainty on the value of L_s depends upon calculation imprecisions of the neutron flux distribution, deformations of the theoretical flux distribution due to the actual control rod position, fabrication tolerances of the pellet density and plutonium enrichement, possible eccentricity of the cladding and measurement errors of the actual total power of the reactor; c_p is assumed to be known exactly; the uncertainty on the value of Q depends upon the inexact calibration of the orifice at the inlet of the subassembly to which the channel belongs and upon the mechanical tolerances of the pitch and outer diameter of the pins which bound the channel.

3. Definition of hot channel factor.

Let ϑ_{crit} be the critical value, which should not be exceeded by the coolant temperature at the channel outlet, we define, as hot channel factor (F_{hc}), the safety factor, which takes into account all the design uncertainties, in such a way that:

$$\vartheta_o = \vartheta_{i-nom} + F_{hc} \Delta\vartheta_{c-nom} \leq \vartheta_{crit} \tag{3}$$

where ϑ_{i-nom} and $\Delta\vartheta_{c-nom}$ are the coolant inlet temperature and the coolant temperature span in the considered channel, when all the parameters, upon which they depend, assume their nominal values. This definition can be extended from the individual cooling channels to the whole cooling system of the core: the over-all hot channel factor will be indicated as F^t_{hc} and defined as:

$$\vartheta'_o = \vartheta_{i-nom} + F^t_{hc} \Delta\vartheta^{max}_{c-nom} \leq \vartheta_{crit} \tag{4}$$

where ϑ_{i-nom} is assumed constant along the core radius and $\Delta\vartheta_{c-nom}^{max}$ is the nominal maximum temperature span of the coolant across the core.

Provided that the most limiting parameter, in the thermal design of the core, is the coolant critical temperature, relation(4) permits to determine the maximum allowable coolant temperature span, given the inlet temperature and the power profile, if F_{hc}^{t} was previously determined.

4. Deterministic and statistical hot channel factor.

The first approach to evaluate the hot channel factor was the cumulative or deterministic method [1]. This method assumes that in the channel, which is in the most critical conditions, that is at maximum nominal temperature and power, all the uncertainties coincide in the most unfavourable way: that is, for instance, in this channel the maximum possible peak factor of the flux occurs together with the maximum possible density and enrichement of the fuel pellets, the minimum flow rate, etc..

Defining a subfactor for each individual parameter:

$$F_i = \frac{\text{maximum coolant temperature span due to the uncertainty"i"}}{\text{nominal coolant temperature span}}$$

F_{hc} is given in this case by:

$$F_{hc} = \prod_{i=1}^{n} F_i \qquad (5)$$

where n is the total number of uncertainties.

This factor is referred to a single channel, namely to the most critical one; but, for the way, by which it is calculated, it assures that no channel in the core exceeds the critical temperature. The same authors [1], however, noted that the probability of the occurence of all worst deviations at one location is extremely low, and let the problem open to a statistical analysis.

In statistical concept, the hot channel factor is not an absolute factor, but is a function of the safety assigned to the reactor design.
When the function

$$F^{t}_{hc} = F^{t}_{hc} \text{ (confidence level)}$$

is known, the designer can weigh the cost of the plant against the safety and consequently choose his safety coefficient: the advantage of the statistical method lies properly in the possibility to give a quantitative figure to the safety, avoiding to assume margins, which increase sensibly the costs, without increasing significatively the safety of the plant.

5. Literature survey.

Methods, to combine statistically the hot channel subfactors (5), were proposed by Rude and Nelson [2] and Parrette [3]. Successively Tingey [4] and Abernathy [5] introduced a simpler method derivated from the error propagation theory. The Enrico Fermi reactor was the first reactor to use hot spot factors calculated on a statistical basis [6].
These methods limited themselves to the calculation of a statistical factor for the most critical channel. A semistatistical method was proposed by Chelemer and Tong [7]: this method assumes that only some parameters, namely the fabrication tolerances, are actually of statistical nature. Many reactors, among which the Na-2 [8], assumed a similar concept.
The extension of the statistical analysis to the whole core was successively performed by Nelson and Minkler [9] and Judge and Bohl [10], [11].
Businaro and Pozzi [12] introduced the "spot" concept, which was applied to the Orgel design [13]: this method takes into account the probability of every pellet in a fuel pin to be "hot", against the previous methods which considered the most critical pellet only.

The extension of the spot method to the whole core was performed by Gueron and Fenech [14] [15].

The various methods under review assume that all the coolant channels in a core are independent: this paper points out the importance of the correlation introduced by the uncertainties which affect the whole core or whole groups of channels, and proposes a new method of analysis. All the parameters are assumed to be independent of time: principally, the hot spot analysis has the scope to determine the performance of the plant during the design stage; when the reactor is in operation, a new analysis should be performed starting from the data measured by the in core instrumentation, in order to verify the assumptions and eventually to remove some precautional limitations [16], [17].

6. Distribution of the uncertainties.

A statistical hot channel analysis requires, as a basis, the knowledge of the statistical distribution of the uncertainties.

A continuous statistical distribution is characterized by the probability frequency function $p(x)$: $p(\bar{x})\,dx$ is the probability that a random variable x assumes a value within $\bar{x}$ and $\bar{x} + dx$. The probability $P(\bar{x})$, that x will not exceed the value $\bar{x}$, is given by:

$$P(\bar{x}) = \int_{-\infty}^{\bar{x}} p(x)\,dx$$

The main parameters of a statistical distribution are the mean(m) and the standard deviation (σ), defined respectively by:

$$m = \int_{-\infty}^{+\infty} x p(x) dx \qquad \sigma = \int_{-\infty}^{+\infty} p(x)\ (x - m)^2 dx$$

The most common distribution is the normal distribution, for which:

$$p(x) = \frac{1}{\sqrt{2\pi}\ \sigma}\ e^{-\frac{1}{2}\frac{(x-m)^2}{\sigma^2}} \qquad (6)$$

In the following we will abbreviate this expression by the symbol (m, σ). The values of $p(x)$ and $P(x)$ for the normal distribution are available in mathematical tables [18]. Table n.1 gives some values of the probability that in a normal distribution a value x will be exceeded, as function of the parameter λ, ratio of the deviation from the mean to the standard deviation.

Table n.1

confidence level $P(x - m \leq \lambda\sigma)$	$P(x-m \geq \lambda\sigma)$	$\lambda = \frac{x-m}{\sigma}$
84.13 (%)	15.87 (%)	1
93.32	6.68	1.5
97.72	2.28	2
99.38	0.62	2.5
99.87	0.13	3
99.9968	$3.2 \cdot 10^{-3}$	4
99.9999999	10^{-7}	6

When the uncertainties are known only as tolerance limits (-H, +H) about the mean m, and the type of distribution is not known, it is common practice to assume that all the values within these limits are equally probable, that is the distribution is rectangular: the frequency function consists of a rectangle on the range (m-H, m+H), the standard deviation is $\sigma = H/\sqrt{3}$
In hot channel analysis, the knowledge of the mean and standard deviation only, and not of the type of distribution, can be sufficient in most practical cases, provided that the number of the uncertainties is large (see item 7). When these parameters are not given as data, they can be estimated through measurements as follows:

$$m = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - m)^2$$

where n is the number of measurements and x_i is the value assumed by x at the "i"th measurement. The confidence level of this "estimation" depends upon the number n of measurements [18].

7. Error propagation in hot channel analysis.

A general method, affected by a first order approximation, for calculating the mean and the standard deviation of the coolant temperature derives from the statistical error propagation theory [4,5]. We can assume that, in eq.(1), ϑ_i has actually its nominal value, and consider the uncertainty on ϑ_i as a further uncertainty in $\Delta\vartheta_c$. Let us write now $\Delta\vartheta_c$, as an explicit function of the several parameters x_i, upon which it depends:

$$\Delta\vartheta_c = \Delta\vartheta_c\,(x_1, x_2, \ldots\ldots\ldots x_i, x_n) \qquad (7)$$

then, we can develop in Taylor's series, about the nominal value:

$$\Delta\vartheta_{c\text{-}nom} = \Delta\vartheta_c(x_{1nom}, x_{2\,nom}, \ldots\ldots\, x_{n\ nom})$$

If we neglect the terms of superior order, we obtain:

$$\Delta\vartheta_c = \Delta\vartheta_{c\text{-}nom} + \left(\frac{\partial\Delta\vartheta_c}{\partial x_1}\right)_{x_1\,nom} dx_1 \ \ldots\ldots + \left(\frac{\partial\Delta\vartheta_c}{\partial x_n}\right)_{x_n\ nom} dx_n \qquad (8)$$

Now dx_i is a statistical variable with mean 0 and standard deviation σ_i. From the statistical theory [18], the linear function:

$$a_1x_1 + a_2x_2 \ldots\ldots\ldots\ldots + a_nx_n \qquad (9)$$

where a_i's are constant and x_i's n statistical independent variables, with mean m_i and standard deviation σ_i,

has the mean: $m = a_1m_1 + a_2m_2 + \ldots\ldots\ldots\ldots + a_nm_n$ (10)

and the standard deviation: $\sigma^2 = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \ldots\ldots + a_n^2\sigma_n^2$. (11)

In our case we get then for $\Delta\vartheta_c$:

$$m = \Delta\vartheta_{c\ nom} \qquad \sigma^2 = \sum_i \left(\frac{\partial \Delta\vartheta_c}{\partial x_i}\right)^2_{x_{i\ nom}} \sigma_i^2 \qquad (12)$$

These expressions were derived without any hypothesis on the distribution type of x_i. If all x_i's have a normal distribution, then also $\Delta\vartheta_c$ is normally distributed [18].
Generally, however, the Central Limit Theorem [18] states that the linear function (9) tends to a normal distribution, with m and σ given by (10) and (11) respectively, when $n \rightarrow \infty$, whichever is the distribution of x_i. When a correlation exists among the x_i's, the expression (12) becomes:

$$\sigma^2 = \sum \left(\frac{\partial \Delta\vartheta_c}{\partial x_i}\right)^2_{x_{inom}} \sigma_i^2 + \sum 2 \left(\frac{\partial \Delta\vartheta_c}{\partial x_i}\right)_{x_{inom}} \left(\frac{\partial \Delta\vartheta_c}{\partial x_j}\right)_{x_{jnom}} \rho_{ij}\sigma_i\sigma_j$$

where ρ_{ij} is the correlation coefficient between the variables i and j.
In this case the Central Limit Theorem, is still valid over very general conditions [18].
In the following it will be assumed that $\Delta\vartheta_c$ is normally distributed with m and σ given by (12).

8. Hot channel factor for the individual cooling channels.

In order to evaluate the hot channel factor F_{hc}, it is more practical to calculate separately the effects of each parameter an $\Delta\vartheta_c$, instead of expliciting the equation (7). Evaluating the contribution of the uncertainty on the parameter "i" to the total uncertainty on $\Delta\vartheta_c$ as:

$$\sigma^i_{\Delta\vartheta_c} \simeq \left(\frac{\partial \Delta\vartheta_c}{\partial x_i}\right)_{x_{i\,nom}} \frac{\sigma_i}{\Delta\vartheta_{c-nom}} \simeq \frac{\Delta\vartheta(x_1, \dots x_i+\sigma_i, x_n) - \Delta\vartheta_{c-nom}}{\Delta\vartheta_{c-nom}} \qquad (13)$$

we obtain by (12) the total standard deviation of $\Delta\vartheta_c$, referred to the nominal temperature span:

$$\sigma^2_{\Delta\vartheta_c} = \sum_{i=1}^{n} \left(\sigma^i_{\Delta\vartheta_c}\right)^2 \qquad (14)$$

The distribution being normal, F_{hc} is then given by:

$$F_{hc} = 1 + \lambda \, \sigma_{\Delta\vartheta_c} \qquad (15)$$

where λ , as function of the confidence level, is given in table n.1.

9. Total hot channel factor in the case of independent cooling channels.

The assumption of the previously derived factor (15), referred to the most critical channel in the core [4, 5, 6] is a very optimistic one: actually a core can have many channels equally limiting, moreover the other channels, which are at a lower nominal temperature, give always a not negligible contribution to the total hot channel probability of the core.

Let P^i_{hc} be the probability of the channel "i" to be "hot", $1-P^i_{hc}$ is then the probability that the critical temperature will not be exceeded in the channel "i". If the total number of channel is N and the channels are assumed to be independent, the probability, that no channel in the core is hot, results in $\prod_{i=1}^{N}(1-P^i_{hc})$. The total hot channel probability (probability that in the core at least one channel is "hot") is given then by:

$$P^t_{hc} = 1 - \prod_{i=1}^{N} (1 - P^i_{hc}) \qquad (16)$$

In "flat" power reactors, all channels have the same $P^i_{hc} = P_{hc}$, in this case we get:

$$P^t_{hc} = 1 - (1 - P_{hc})^N \qquad (17)$$

At ref [10] a practical method is proposed in order to evaluate the total hot channel factor in these hypotheses; this method, reported here for sake of completeness, can be summarized as follows: let us call $P_N(\xi)$ the probability that at least one element out of N samples drawn from the normal distribution (0,1) exceed a certain deviation ξ :

$$P_N(\xi) = 1-(1 - \int_{\xi}^{+\infty} \frac{1}{\sqrt{\pi}} \; e^{-\frac{1}{2}x^2} dx)^N \qquad (18)$$

Fig n.2 shows the $P_N(\xi)$ distribution for several values of N.

Let us assign, to the total hot channel factor, the confidence level C_λ which corresponds to the deviation $\lambda\sigma$ in a normal distribution: drawing, in fig.2, a straight line, parallel to the abscissa, through the point at $\lambda\sigma$ in the normal distribution, we obtain the coefficient (ξ), by which the standard deviation $\sigma_{\Delta\vartheta_c}$ must be multiplied, as the abscissa of the crossing point of this line with the actual $P_N(\xi)$ distribution: the total hot channel factor, for "flat" power reactor results in:

$$F_{hc}^{tf} = 1 + \xi\,\sigma_{\Delta\vartheta_c} \tag{19}$$

10.A useful approximation of the $P_N(\xi)$ distributions.

From fig.n.2, the $P_N(\xi)$ distributions result to be not normal. For the further use of these distributions, as well as for a more practical evaluation of the hot channel factor at item 9, it is useful to approximate $P_N(\xi)$, for each N, by a more pessimistic normal distribution - that is, at given confidence level, allowing larger deviations than the actual $P_N(\xi)$ - in the practical range of interest. This is a conservative approach with respect to reactor design.

Taking in fig.2, for each N, the point at 50% probability as mean of the distribution to evaluate, it is easy to calculate the standard deviation of the normal distribution, which pessimizes the actual one up to a desired confidence level.

Fig.n.3 shows - as function of N - the mean (h_N^m) and the standard deviation (h_N^σ) of the normal distributions which pessimize the $P_N(\xi)$'s up to 3σ conf.level (99,8%) and 6σ conf.level (10^{-9} probability of occurence) respectively.

Fig.n.3 being referred to the standardized normal distribution (0,1), h_N^m and h_N^σ indicate actually a ratio of deviations to standard deviation.

The total hot channel factor F_{hc}^{tf} for "flat" power reactor, with N independent cooling channels, can be evaluated as:

$$F_{hc}^{tf} = 1+h_N^m\,\sigma_{\Delta\vartheta_c}+h_N^{\sigma}\,\lambda\,\sigma_{\Delta\vartheta_c} \tag{20}$$

where λ, as function of the confidence level, is given in table n 1. It is clear however that λ must not exceed actually the value which was used as the basis of approximation at fig.3, otherwise the expression (10) gives no longer a conservative value. The pessimization, involved in the performed approximation, is practically negligible. For instance if we want to calculate F_{hc}^{tf} in the range $2\sigma \div 3\sigma$ of confidence level, we shall use h_N^{σ} corresponding to the 3σ approximation; the factors given by (20) and (19) will obviously coincide at 3σ conf.level, and will differ of 1%, as a maximum, at 2σ conf.level.

11. Assessment of the different effects of the local and global uncertainties.

With the previous procedure, the outlet temperature of the coolant of each channel was considered to be statistically independent from the other ones, and all the uncertainties were treated in the same way.
Actually, only some uncertainties vary from channel to channel within the reactor in a statistically independent manner, while others affect whole groups of channels or all the channels in a correlated way.
For instance, the power measurement error affects the whole core temperature profile, while the fuel density affects the individual channels in an independent manner ; the uncertainty on the orifice calibration affects equally the temperature of all the channels in a subassembly, while the pin pitch affects the temperature of the individual channels in the subassembly. It is easy to find a great number of similar examples.
Moreover from the examination of the production tolerances of the core components, it results that the tolerances of components belonging to the same production batch are usually smaller than the tolerances among components of different batches. It is possible to consider these mechanical tolerances as a result of two statistical

distribution: the first one is the distribution of the average value of the considered parameter among different production batches, the second one is the distribution of the considered parameter about the mean within every batch. As a consequence, in a generation of reactors with the same design, a certain parameter can vary among the individual reactors in an independent way, but among the individual channels in each reactor in a correlated way.

Let us assume for instance, that the uncertainty of the coolant temperature depends only upon the power measurement error and the pin pitch tolerances; the actual hot channel probability to evaluate is, in this case, the probability that the sum of the power error contribution, drawn once, and of the pitch contribution, drawn N times, exceeds the critical value. It is intuitively clear that the previous methods, which consider the statistical sum of the two contributions drawn an equal number of times, is more conservative than the actual one. In order to assess quantitatively the difference between the two methods, only two statistical distribution will be considered, $f(x)$ and $g(y)$.

Let us call $P_a(\xi)$ the actual probability that the sum $x + y$ exceeds a certain value ξ, when x is once sampled and y a number N of times. $P_a(\xi)$ is given by the following integral:

$$P_a(\xi) = \int_{-\infty}^{+\infty} f(x) \left[1 - \left(\int_{-\infty}^{\xi - x} g(y)\, d\, y \right)^N \right] dx \tag{21}$$

In fact:

$f(x)dx$ is the probability that x falls within x and $x + dx$

$1 - \left(\int_{\infty}^{\xi - x} g(y)dy \right)^N$ is the probability that $y \geq \xi - x$ at least in one out of N samples

x and y being independent, the product of the previous terms gives the probability that when x falls within x and $x + dx$, y is so large that $x + y \geq \xi$ at least in one out of N samples. Since x can

assume all the values between $-\infty$ and $+\infty$, we get (21).
In the case that f(x) and g(y) are the normal distributions $(0, \sigma_g)$ and $(0, \sigma_l)$ respectively, by (6) the expression (21) becomes:

$$P_a(\xi) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma_g}\, e^{-\frac{1}{2}\frac{x^2}{\sigma_g^2}} \left[1 - \left(\int_{-\infty}^{\xi - x} \frac{1}{\sqrt{2\pi}\,\sigma_l}\, e^{-\frac{1}{2}\frac{y^2}{\sigma_l^2}}\, dy\right)^N\right] dx \qquad (22)$$

This relation has been solved an a digital computer for different values of the parameters N, ξ, σ_g and σ_l: namely for different values of the ratio $R = \sigma_g/\sigma_l$, with $\sigma_g^2 + \sigma_l^2 = 1$. The resulting distributions are represented and compared with the $P_N(\xi)$'s in fig.3 and 4, for N = 100 and 10 000 respectively. It appears that the results of the two methods are actually very different, and a not negligible reduction of the total hot channel factor can be expected, especially when N and R are large, by considering the exact distributions (22).
It appears moreover that the more limiting uncertainties are the local ones: these, in fact, are sampled many times, and have, therefore, a greater probability to exceed large deviations, than the global uncertainties, which are sampled once or few times only.

12. Proposed method for "flat" power reactors.

Since the evaluation of the hot channel probability, starting from the $P_a(\xi)$ distributions, would involve the resolution of a large number of integrals of the type (21), a practical method will be proposed, using the approximation introduced at item 10.
With reference to sodium cooled reactors, the core is assumed to consist of a number N_s of subassembly, each subassembly having a number N_c of channels. We assume, for the moment, a "flat" power distribution. In order to calculate F_{hc}^{tf} in these hypotheses, the uncertainties must be divided into three groups, namely:

1. uncertainties affecting the individual N_c channels in a subassembly
2. uncertainties affecting individually the whole subassemblies

3. uncertainties affecting the whole core temperature.

The standard deviations $\sigma^{ij}_{\Delta\vartheta_c}$ of the uncertainty i belonging to the same group "j"(j=1,2,3 respectively) can be statistically added as usually:

$$\sigma_j = \sqrt{\Sigma\,(\sigma^{ij}_{\Delta\vartheta_c})^2}$$

Let us now introduce a practical graphical convention: a block ~~(fig.6) indicates a normal distribution (m, σ); the series of two~~ or more blocks indicates the sum of the corresponding distributions; and the parallel of N blocks indicates that the corresponding normal distributions are drawn N times.

We can substitute, then, a block $\left(\sum_{i=1}^{n} m_i, \sqrt{\sum_{i=1}^{n} \sigma_i^2}\right)$ to the series of n blocks (m_i, σ_i), and, according to item 10, a block$(m+h^m_N\sigma, h^\sigma_N\sigma)$ to the parallel of N identical blocks (m, σ), (fig.6).

With these conventions, the actual representation of the uncertainties in the reactor is shown in fig.7. As shown in fig.7, executing the operations indicated by the block diagram, we are able to substitute a simple normal distribution (M_R, σ_R) to the complex actual distribution.*

The total hot channel factor for a "flat" power distribution results now in:

$$F^{tf}_{hc} = 1 + M_R + \lambda\sigma_R \tag{23}$$

where λ depends upon the desired confidence level as in table n.1. It is useful to remember, that this confidence level expresses the probability that no channels in the core exceed the factor (23): its complement, the probability that at least one out the $N=N_s \cdot N_c$ channels exceed the factor (23).

* Evaluating numerically the exact sum of a $P_N(\xi)$ distribution and a normal distribution, and comparing with the sum of the corresponding approximate distribution and the normal one, it has been proved that the pessimization of the introduced approximation is still held.

13. Numerical example for the reactor Na-2

The principal data for the reactor Na-2 are given in table n.2

Table n.2

Parameter	Nominal value	standard deviation
Core power	$L = 730\ Mw_{th}$	$\sigma'_L = 3.5\%$
Sodium inlet temperat.	$\vartheta_i = 380^\circ C$	$\sigma_{\vartheta_i} = 1^\circ C$
Maximum temperature span in the most limiting channels	$\Delta\vartheta^{max}_{c-nom} = 208^\circ C$	
Number of subassemblies	$N_s = 150$	
Number of cooling channels in a subassemb.	$N_c = 336$	
Number of pellet in a pin	$N_p = 95 \pm 1$	
Length of the active zone of a fuel pin	$l = 950$ mm	$\sigma_l = 3$ mm
Fuel pellet density	δ = 80% of the theoretical value	$\sigma'_\delta = 2.2\%$
Pu - enrichment	a = 21.14% (I zone) a = 31.14% (II zone)	$\sigma'_a = 1.7\%$
Pin outer diameter	$d_o = 6$ mm	$\sigma_{do-M} = 0.008$ mm (among diff. batches) $\sigma_{do-B} = 0.003$ mm (in the same batch)
Canning eccentricity	$e = 0$	$\sigma_e = 0.006$ mm
Pin pitch	$p = 7.9$ mm	$\sigma_p = 0.13$ mm
Number of pin spacer grids	$N_g = 10$	
Flow rate calibration in a subassembly	F = varies along the core radius according to the power profile	$\sigma'_F = 2.2\%$
Neutron flux uncertainty 1. due to calculation 2. due to the control rod position		 $\sigma'_\phi = 2\%$ $\sigma'_{\phi_R} = 3.9\%$

The partial contributions to the coolant outlet temperature are reported in table n 3.

Table n 3

Channel $\sigma^{i1}_{\Delta\vartheta_c}$	Subassembly $\sigma^{i2}_{\Delta\vartheta_c}$	Core $\sigma^{i3}_{\Delta\vartheta_c}$
Pellet density $\sigma^{\delta}_{\Delta\vartheta_c} = 0.002$	Orifice calibration $\sigma^{F}_{\Delta\vartheta_c} = 0.023$	Power $\sigma^{L}_{\Delta\vartheta_c} = 0.035$
Pellet enrichment $\sigma^{a}_{\Delta\vartheta_c} = 0.001$	Neutron flux (control rod position) $\sigma^{\phi_R}_{\Delta\vartheta_c} = 0.039$	Inlet coolant temp. $\sigma^{\vartheta_i}_{\Delta\vartheta_c} = 0.006$
Pin* outer diameter in the same batch $\sigma^{do-B}_{\Delta\vartheta_c} = 0.001$		Pin* outer diameter (in different batches) $\sigma^{do-M}_{\Delta\vartheta_c} = 0.008$
Pin pitch $\sigma^{p}_{\Delta\vartheta_c} = 0.012$		Neutron flux (calculation imprecision) $\sigma^{\phi}_{\Delta\vartheta_c} = 0.020$
Canning eccentricity $\sigma^{e}_{\Delta\vartheta_c} = 0$		
Active length $\sigma^{l}_{\Delta\vartheta_c} = 0.002$		
$\sigma_j = \sqrt{\Sigma(\sigma^{ij}_{\Delta\vartheta})^2}$ $\sigma_1 = 0.012$	$\sigma_2 = 0.045$	$\sigma_3 = 0.043$

* The reactor is assumed to be constructed with pins deriving from the same batch. No difficulty exists to take into account the actual number of batches.

The $\sigma^{ij}_{\Delta\vartheta_c}$'s were calculated according to(13) from the well known relations among $\Delta\vartheta_c$ and the several parameter "i". For the channel uncertainties, the procedure indicated at [12] and [14] was adopted. Namely, it was assumed that the density, the enrichment and the diameter are constant along the pellet length, and they vary statistically among the several pellets in a channel: the corresponding $\sigma^{i}_{\Delta\vartheta_c}$'s, derived by (13), were then divided by the square root of the total number of pellets in a channel, namely by $\sqrt{3N_p}$, in order to evaluate their average contributions to $\Delta\vartheta_c$*
The pitch was assumed to be constant between two pin spacer grids, and vary statistically along the channel axis; the pitch contribution given by (13) was then divided by $\sqrt{3Ng}$. For the active length of a channel, the number of pins (namely 3) was taken into account.
Table n 4. summarizes then the calculation of F^{tf}_{hc} according to fig.7. F^{tf}_{hc} (conf.level 97,7%) = $1+M_R+2\sigma_R = 1.26$

* The mean of n statistical values, assumed by a variable with the normal distribution (m, σ), has the normal distribution $(m, \sigma/\sqrt{n})$. [18]

Table n. 4

Channel	Subassembly	Core
$\sigma_1 = 0.012$	$\sigma_2 = 0.045$	$\sigma_3 = 0.043$
$N_c = 336$ $h^{m}_{336} = 2.9$ * $h^{\sigma}_{336} = 0.53$ *(3σ approximation)	$N_s = 150$ $h^{m}_{150} = 2.63$ * $h^{\sigma}_{150} = 0.55$	
$M_c = h^{m}_{336}\,\sigma_1 = 0.035$ $\sigma_c = h^{\sigma}_{336}\,\sigma_1 = 0.006$	$M'_s = M_c = 0.035$ $\sigma'_s = \sqrt{\sigma_2^2 + \sigma_c^2} = 0.045$	
	$M_s = M'_s + h^{m}_{150}\,\sigma'_s = 0.15$ $\sigma_s = h^{\sigma}_{150}\,\sigma'_s = 0.025$	$M_R = M_s = 0.15$ $\sigma_R = \sqrt{\sigma_3^2 + \sigma_5^2} = 0.05$
F^{tf}_{hc} (97,7% conf.lev.) $= 1 + M_R + 2\,\sigma_R$		1.26

14. Comparison among the different methods.

Table n.5 presents a comparison among the different methods. For the deterministic method each subfactor is given at the same confidence level as the statistical ones (97,7 %), in order to allow a comparison.
The channel uncertainties were assumed to be constant along the axis of the channel for the deterministic method and the first simple statistical approach, which considered only one channel: in the first case, because this is the assumption of the deterministic concept; in the second, because the more complete analysis, which allows smaller channel subfactors, was developed simultaneously to the concept of the plurality of the statistical occurences in the several channels and pellets.

From table n.5, the conservatism of the deterministic method and the optimism of the first statistical approach become evident. Moreover, in this case, the proposed method allows, at equal confidence level (97,7%), a total hot channel factor (1.26) ~ 5% lower than the factor evaluated assuming all channels to be independent (1.31). (The factor 1.31 would occur at a confidence level of 99.9 %, according to the proposed method).

Uncertainties	subfactors $F_i' = 1+2\,\sigma^{i'}_{\Delta\vartheta_c}$ *	Deterministic method	Statistical method - one channel only	Subfactors $F_i = 1+2\sigma^{i}_{\Delta\vartheta_c}$	$N_s \cdot N_c = 50{,}400$ independent channels	proposed method
Pin pitch	1.14	$F_{hc} = \prod F_i'$	$F_{hc} = 1 + \lambda \sqrt{\Sigma(F_i' - 1)^2}$	1.024	$F^{tf}_{hc} = 1 + \frac{\xi}{2} \sqrt{\Sigma(F_i - 1)^2}$	$F^{tf}_{hc} = 1 + M_R + \lambda\sigma_R$
Pin outer diameter (in one batch)	1.04			1.002		
Pellet density	1.04			1.002		
Pellet enrichment	1.04			1.002		
Channel active lenght	1.01			1.004		
Orifice calibration	1.046			$F_i = F_i'$		
Flux distribution -control rod effects-	1.078					
Flux distribution -calculation-	1.040					
Core power	1.070					
Coolant inlet temp.	1.012					
Pin outer diameter (from batch to batch)	1.016					
Hot channel factors	F_{hc}	1.68	1.18 Conf.lev. (97,7%)	F^{tf}_{hc}	1.31 Conf.lev. (97.7%)	1.26 Conf.lev. (97.7%)

* The "channel"subfactors, in the first two methods, were not averaged over the number n of their independent occurences in a channel (see item 13 and 14)

15. Hot channel factor for the actual temperature distribution.

The assumption that all the channels have the same nominal temperature span is conservative, and some reduction of the hot channel factor can be expected, when the actual temperature distribution is taken into account.

In the case of the reactor Na-2, this reduction is not expected to be very large: in fact, the calibration of the flow rate has been designed in such a way, that the maximum coolant temperature span is constant in each subassembly, namely it is 208°C [8]; the minimum temperature span is, however, a function of the distance of the subassembly from the core center. Therefore the temperature profile concerns more the channels within a subassembly, rather than the several subassemblies. Due to the actual ratio of the uncertainties affecting the individual channels to those affecting the subassemblies, the minimum value, which can be expected for the hot channel factor taking into account the actual temperature profile, is 1.23 (at 97.7% conf.level), which corresponds to the limit case where the channel contributions are neglected (see table n.4). The exact consideration of the temperature profile cannot, therefore, reduce the total hot channel factor more than $\sim 3\%$. The expected number of "hot" channels is, however, very different: in fact, in the case that the whole coolant temperature in a subassembly exceeds a large deviation, the total number of channels would be "hot" at a "flat" temperature profile, while at the actual temperature profile, the number of "hot"channels would be much smaller. The difference does not appear to be important in the total hot channel probability, because we have calculated the probability that <u>at least one channel</u> is "hot", and each subassembly has a certain number of channels at the same maximum temperature span. It is clear that for different importance of the channels uncertainties in respect to the subassembly ones, a larger reduction might be expected.

In order to cover the most general cases, it is, therefore, worth while to show how the actual temperature profile can be taken into account.

Let us define as "form" factor the factor $f_f = 1+h$, where

$$h = \frac{\Delta\vartheta_{c-nom} - \Delta\vartheta^{av}_{c-nom}}{\Delta\vartheta^{av}_{c-nom}}$$

, with $\Delta\vartheta^{av}_{c-nom}$ the nominal average temperature span in a subassembly; h is defined over the interval $(-H_1, +H_2)$, where

$$H_2 = \frac{\Delta\vartheta^{max}_{c-nom} - \Delta\vartheta^{av}_{c-nom}}{\Delta\vartheta^{av}_{c-nom}} ,$$

and

$$H_1 = \frac{\Delta\vartheta^{av}_{c-nom} - \Delta\vartheta^{min}_{c-nom}}{\Delta\vartheta^{av}_{c-nom}} .$$

Let us evaluate the probability that for no one out of Nc channels in a subassembly, the product $f_f = 1 + h$, and $f_{hc} = 1 + x$, where x has the normal distribution $(0, \sigma_1)$, exceeds a certain value. Assuming in accordance to par 7;

$$(1 + h) \cdot (1 + x) \simeq 1 + h + x , \tag{24}$$

we must evaluate the probability that the sum h + x, does not exceed a certain deviation z for any channel in a subassembly.

If $N_c(h)\,\Delta h$ is the number of channels which have a nominal deviation from $\Delta\vartheta^{av}_{c-nom}$, within h and $h + \Delta h$, the probability that no one out of $N_c(h)\,\Delta h$ channels exceeds a total deviation z is:

$$\left[1 - P(x \geq z-h)\right]^{N_c(h)\,\Delta h}$$

where

$$P(x \geq z-h) = \int_{z-h}^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma_1} e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}} dx$$

And the total probability $P_p(z)$, that no channel in a subassembly exceeds a deviation z is:

$$P_p(z) = \prod_{i=1}^{n} \left[1 - P(x \geq z-h)\right]^{N_c(h)\Delta h} \tag{25}$$

where n is the number of the intervals Δh, into which the range $(-H_1, + H_2)$ has been divided.

By (25)

$$\log P_p(z) = \sum_{i=1}^{n} N_c(h) \log \left[1 - P(x \geq z-h) \right] \Delta h$$

and for $\Delta h \to 0$

$$\log P_p(z) = \int_{-H_1}^{+H_2} N_c(h) \log \left[1 - P(x \geq z-h) \right] dh \qquad (26)$$

In our case the temperature profile in each subassembly can be assumed to be approximatively linear [8], therefore $H_1 = H_2 = H$; in this case $N_c(h)$ has a rectangular distribution in the range $(-H, +H)$, of height $\frac{N_c}{2H}$. By (26) we have:

$$\log P_p(z) = \frac{N_c}{2\,H} \int_{-H}^{+H} \log \left[1 - P(x \geq z-h) \right] dh \qquad (27)$$

By (27) the $P_p(z)$ distribution can be evaluated. In order to give a practical method to take into account the temperature profile, a different approach will be followed.

According to [11]*, let us define as Neq, the equivalent number of channels, which would give the same probability $P_p(z)$, if all channels have the maximum nominal deviation +H:

$$P_p(z) = \left[1 - P(x \geq z - H) \right]^{Neq}$$

* In ref.11, the author evaluates the "equivalent" number of equally limiting channels in the case of a cosine distribution of the power, for N independent cooling channels. In his paper, however, the equivalence is evaluated at the same expected number of hot channels rather than at the same hot channel probability; moreover no attempt is performed to take into account that the number of channels having a certain nominal temperature is itself a function of the core radius. Applicating a procedure similar to that given above, with the remark that in this case the linear approximation (24) cannot be held, we should obtain:

$$\frac{N_{eq}}{N} = \frac{\int_0^{\pi/2} \sin 2\alpha \, \log\left(1 - \int_r^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma \cos\alpha} e^{-\frac{1}{2}\frac{(y-\cos\alpha)^2}{(\sigma\cos\alpha)^2}} dy\right) d\alpha}{\log\left(1 - \int_r^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(y-1)^2}{\sigma^2}} dy\right)}$$

where $r = \dfrac{\Delta\vartheta_c \text{ (allowable)}}{\Delta\vartheta_{c\text{-nom}}^{max}}$

and

$$\log P_p(z) = N_{eq} \log \left[1 - P(x \geq z-H)\right] \tag{28}$$

By (27) and (28), we get:

$$\frac{N_{eq}}{N_c} = \alpha = \frac{1}{2H} \frac{\int_{-H}^{+H} \log\left[1-P(x \geq z-h)\right] dh}{\log\left[1-P(x \geq z-H)\right]} \tag{29}$$

By (29), we get the important result that α, ratio of N_{eq} to N_c, does not depend upon N_c, it depends however upon z, σ_1, and H. Fig.8, shows α as function of H, for different values of σ_1, when z = H, that is when the equivalence is evaluated at the probability to exceed the maximum nominal temperature span in the subassembly. For larger values of z, α decreases very rapidly (Fig.9).

Fig.8 gives therefore the more conservative values of α and is assumed as basis of the further calculations. The two limit cases in the reactor Na-2 are the subassemblies at the core center, for which H = 0.015 and at the core boundary for which H = 0.24; in the first case we obtain α= 23%, in the second α= 2%; that is N_{eq} = 78 and 7 respectively. Assuming all the subassemblies equal to the central ones, F^t_{hc} would result 1.25 (at 97,7% conf.level); while, assuming all the subassemblies equal to the peripherical ones, F^t_{hc} would result 1.24, according to the procedure indicated at item 13. In this case, it is evidently not necessary to evaluate more exactly the hot channel factor F^t_{hc} , which is well defined by these limit values.

In a more general case, however, a core can be divided in N_z zones, each one with a number N_s^j of identical subassemblies, with N_{eq}^j channels. The proposed procedure allows to evaluate then an equivalent normal distribution (M_z^j, σ_z^j) for each zone, as indicated in fig.10. M_z^j, σ_z^j being not equal in each zone, it is not possible to apply the method derived in item 12 and illustrated by fig.6.

It is necessary therefore to evaluate numerically the final distribution:

$$P_{hc}(z) = \int_{-\infty}^{+\infty} p(x) \left\{ 1 - \prod_{j=1}^{N_Z} \left[1 - P^{j}_{(y \geq z-x)} \right] \right\} dx$$

where

$P_{hc}(z)$ = is the probability that at least one channel exceeds the deviation z

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_3} e^{-\frac{1}{2}\frac{x^2}{\sigma_3^2}}$$

$$P^{j}_{(y \geq z-x)} = \frac{1}{\sqrt{2\pi}\,\sigma_z^j} \int_{z-x}^{+\infty} e^{-\frac{1}{2}\frac{(y - M_z^j)^2}{(\sigma_z^j)^2}} dy$$

Then $F^t_{hc} = 1+z$, where z depends upon the conf.level as in the $P_{hc}(z)$ distribution.

16. Acknowledgments

The author wishes to thank Mr.E.G.Schlechtendahl for his interest and valuable suggestions on this subject.

17. Symbol list

f_f = form factor, which takes into account the temperature profile in a subassembly ($f_f = 1 + h$).

f_{hc} = uncertainty factor, which takes into account the statistical uncertainties of the channels in a subassembly [$f_{hc}=1+x$, where x has the normal distribution $(0, \sigma_1)$].

F_{hc} = hot channel factor for the individual channels.

F_{hc}^{t} = total hot channel factor for the whole core.

F_{hc}^{tf} = F_{hc}^{t}, when a "flat" power distribution in the core is assumed.

F_i = hot channel subfactor referred to the uncertainty "i".

h = nominal deviation from the mean of the temperature at a cooling channel outlet, referred to the nominal temperature.

H = limit value in a rectangular distribution, in particular maximum value of h.

h_N^m, h_N^σ = coefficients by which the standard deviation has to be multiplied, in order to obtain the mean and the standard deviation, respectively, of the normal distribution approximating the $P_N(\xi)$ one.

m = mean of a statistical distribution, generally referred to normal distributions.

(m, σ) = normal distribution with mean m, and standard deviation σ

M_c, M_s, M_R, M_z^j = means of the equivalent normal distribution of the uncertainties of the channels, subassembly, core and zone "j" respectively

N = total number of channels in the core

N_c = total number of channels in a subassembly

$N_c(h)$ = number of channels in a subassembly having a nominal temperature deviation within h and h + dh

N_{eq} = equivalent number of channels at the maximum deviation H, which would give the same probability to exceed a deviation z, than the $N_c(h)$ distribution.

N^j_{eq} = N_{eq} referred to the zone "j".

N_s = total number of subassembly in a core.

N^j_s = number of subassembly in the zone "j"

N_z = number of zone, in which the core can be divided, having N^j_s equal subassemblies.

$p(x)$ = probability frequency distribution, generally referred to normal distributions.

$P_{(x \geq K)}$ = probability to exceed a deviation K, generally referred to normal distributions.

$P_a(\xi)$ = actual probability that the sum of two normal distribution exceeds a value ξ, when the first is once sampled and the second N times.

P^i_{hc} = probability of the channel "i" to be "hot"

P^t_{hc} = total hot channel probability of the core

$P_N(\xi)$ = probability that at least one out of N samples, drawn from a normal distribution, exceeds a value ξ

$P_p(z)$ = probability that the sum of the deviation h due to the temperature profile in a subassembly and the channel uncertainties $(0, \sigma_1)$ exceeds a value z

α = ratio of the equivalent number of channels(N_{eq})to N_c

$\Delta\vartheta_c$ = coolant temperature span.

$\Delta\vartheta_{c-nom}$ = nominal coolant temperature span, function of the core radious

$\Delta\vartheta^{max,av,min}_{c-nom}$ = respectively the maximum, average and minimum $\Delta\vartheta_{c-nom}$ in a subassembly.

ϑ_{crit} = critical value which must not be exceed by the coolant temperature

ϑ_o = coolant outlet temperature

ϑ_i = coolant inlet temperature

λ = ratio of a deviation from the mean, to the standard deviation in a normal distribution.

ξ = coefficient corresponding to λ, in a $P_N(\xi)$ distribution.

σ = standard deviation in a statistical distribution, generally referred to normal distributions.

σ_c, σ_s, σ_R, σ_z^j = standard deviation of the equivalent normal distribution corresponding to the uncertainties of the channel, subassembly, core and zone "j" respectively

$\sigma_{\Delta\vartheta_c}$ = total standard deviation of $\Delta\vartheta_c$, for the individual channels referred to $\Delta\vartheta_{c-nom}$

$\sigma^i_{\Delta\vartheta_c}$ = contribution of the uncertainties "i" to $\sigma_{\Delta\vartheta_c}$, referred to $\Delta\vartheta_{c-nom}$

$\sigma^{ij}_{\Delta\vartheta_c}$ = contribution of the uncertainties "i" to σ_j

σ_j = (j=1,2,3) contribution of the uncertainties which affect the individual channels, the whole subassemblies, and the whole core, respectively, to the total $\sigma_{\Delta\vartheta_c}$, referred to $\Delta\vartheta_{c-nom}$.

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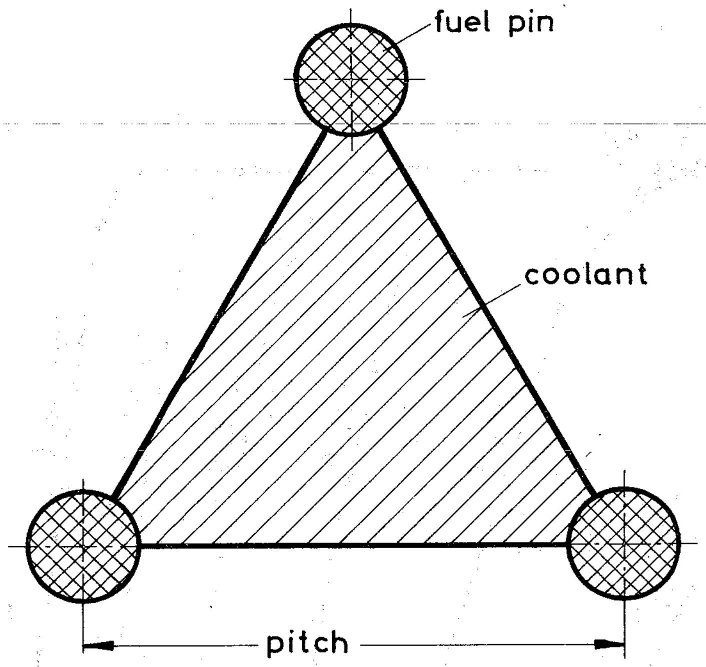


Fig. 1.Coolant Channel Arrangement

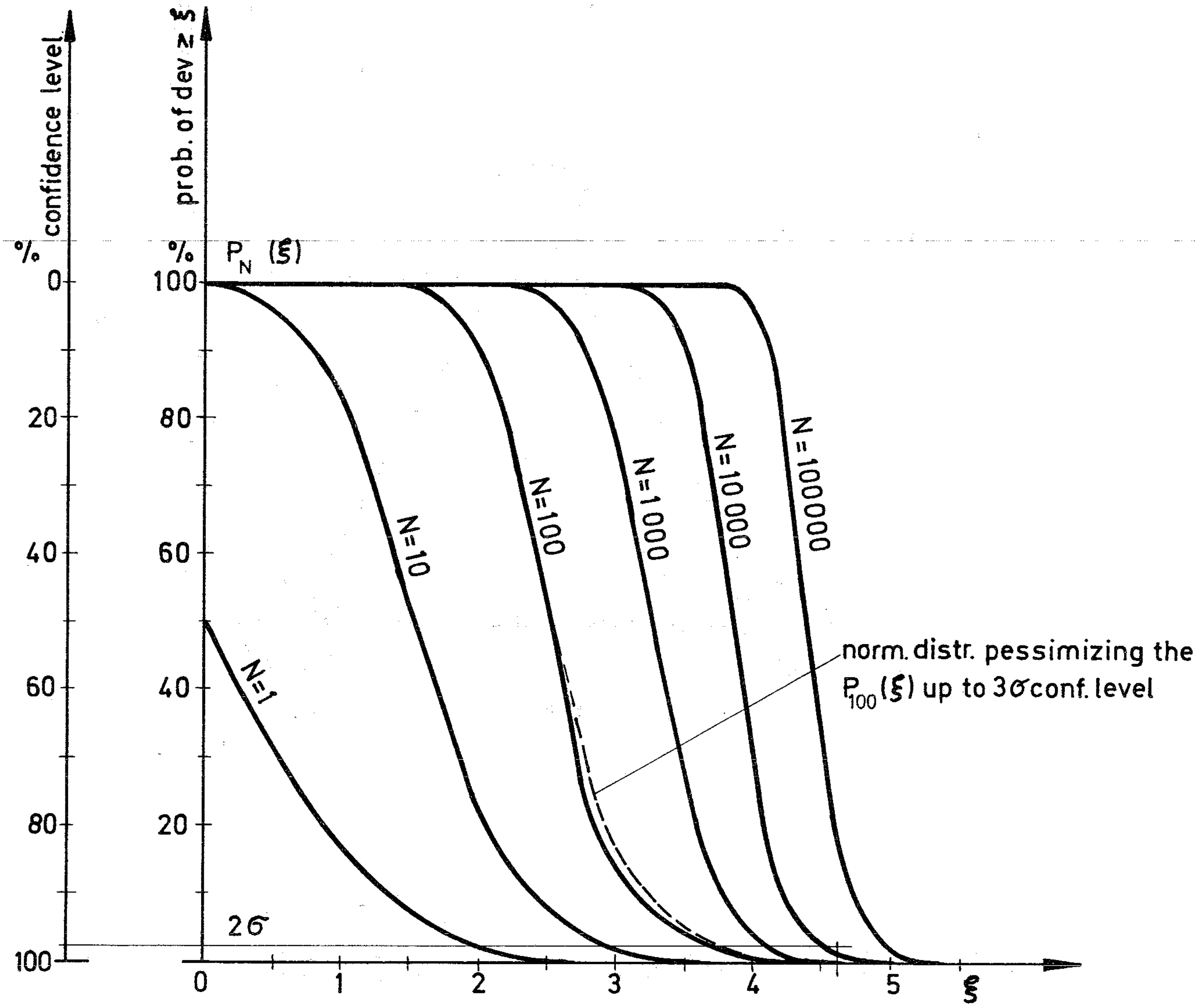


Fig.2. $P_N(\xi)$ Distributions

$P_N(\xi)$ = probability that at least one out of N samples drawn from the normal distribution (0,1) exceeds a deviation ξ.

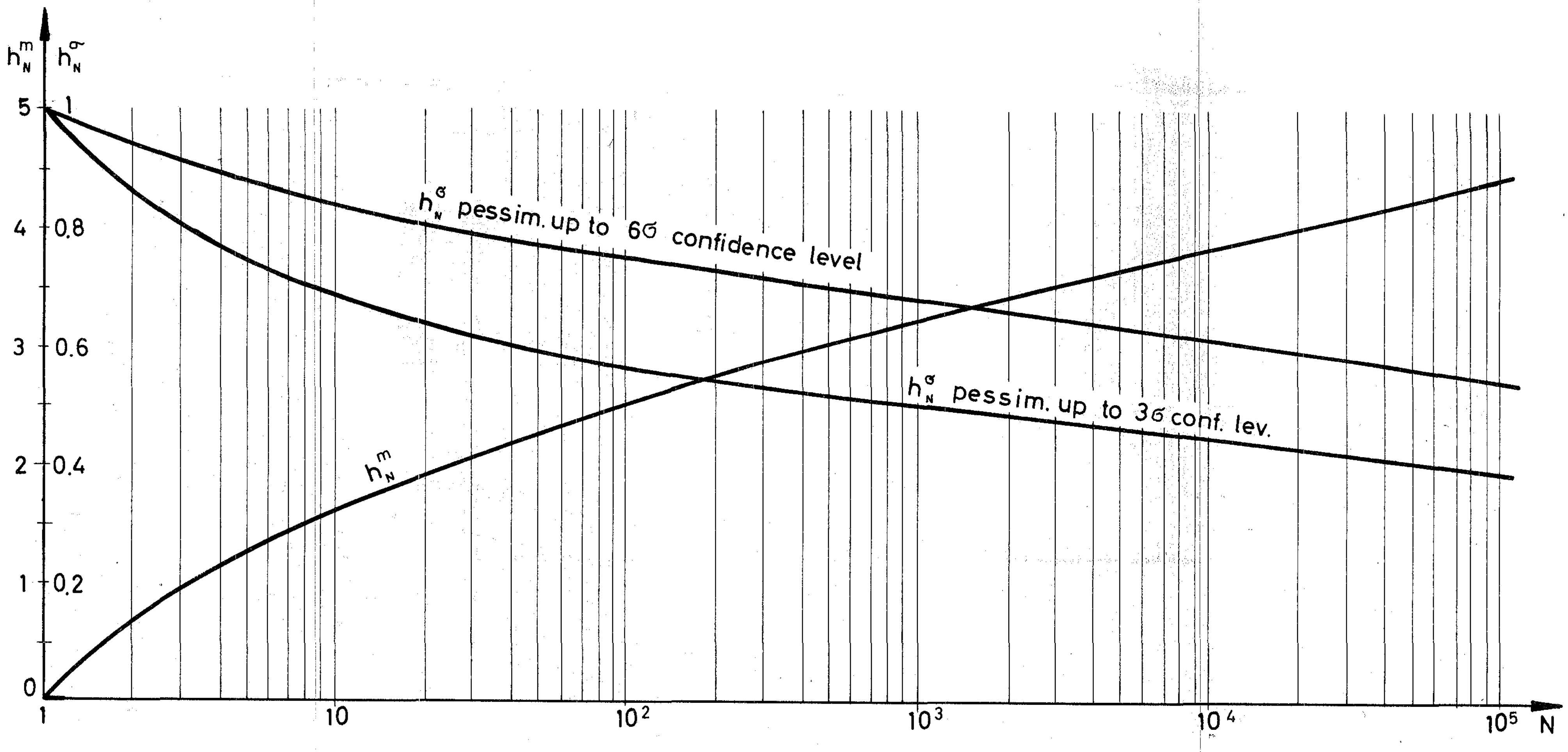


Fig. 3

Mean (h_N^m) and Standard Deviation (h_N^σ) of the Normal Distribution Pessimizing the $P_N(\xi)$ One (fig.2), Versus Number of Samples (N)

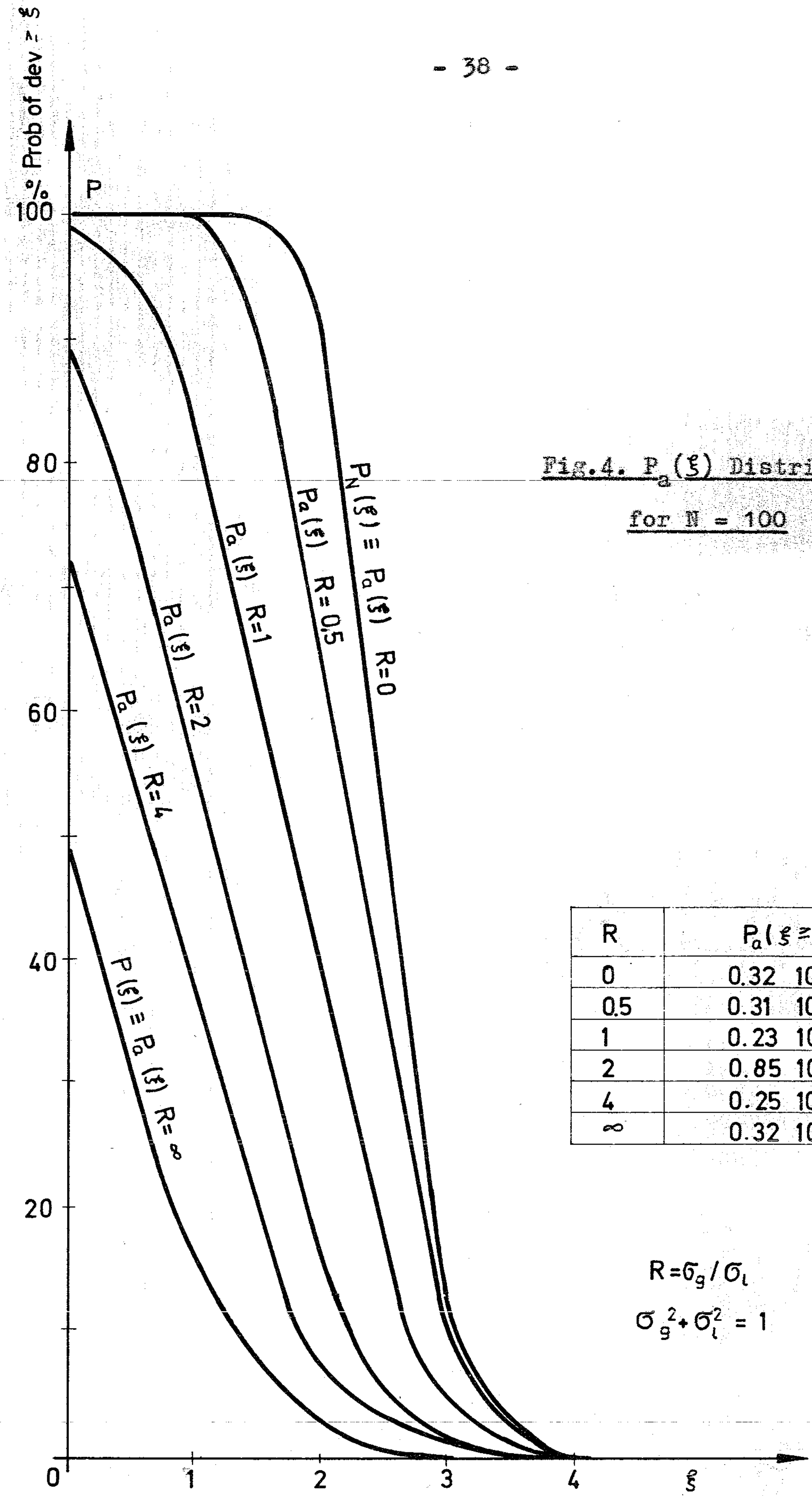


Fig.4. $P_a(\xi)$ Distributions for N = 100

R	$P_a(\xi \geq 4)$
0	$0.32 \; 10^{-2}$
0.5	$0.31 \; 10^{-2}$
1	$0.23 \; 10^{-2}$
2	$0.85 \; 10^{-3}$
4	$0.25 \; 10^{-3}$
∞	$0.32 \; 10^{-4}$

$P_a(\xi)$ = probability that the sum of the normal distributions $(0, \sigma_g)$ and $(0, \sigma_1)$ exceeds a deviation ξ, when the first is drawn once and the second N times

$P_N(\xi)$ = the same probability when both are drawn N times

$P \; (\xi)$ = normal distribution (0,1)

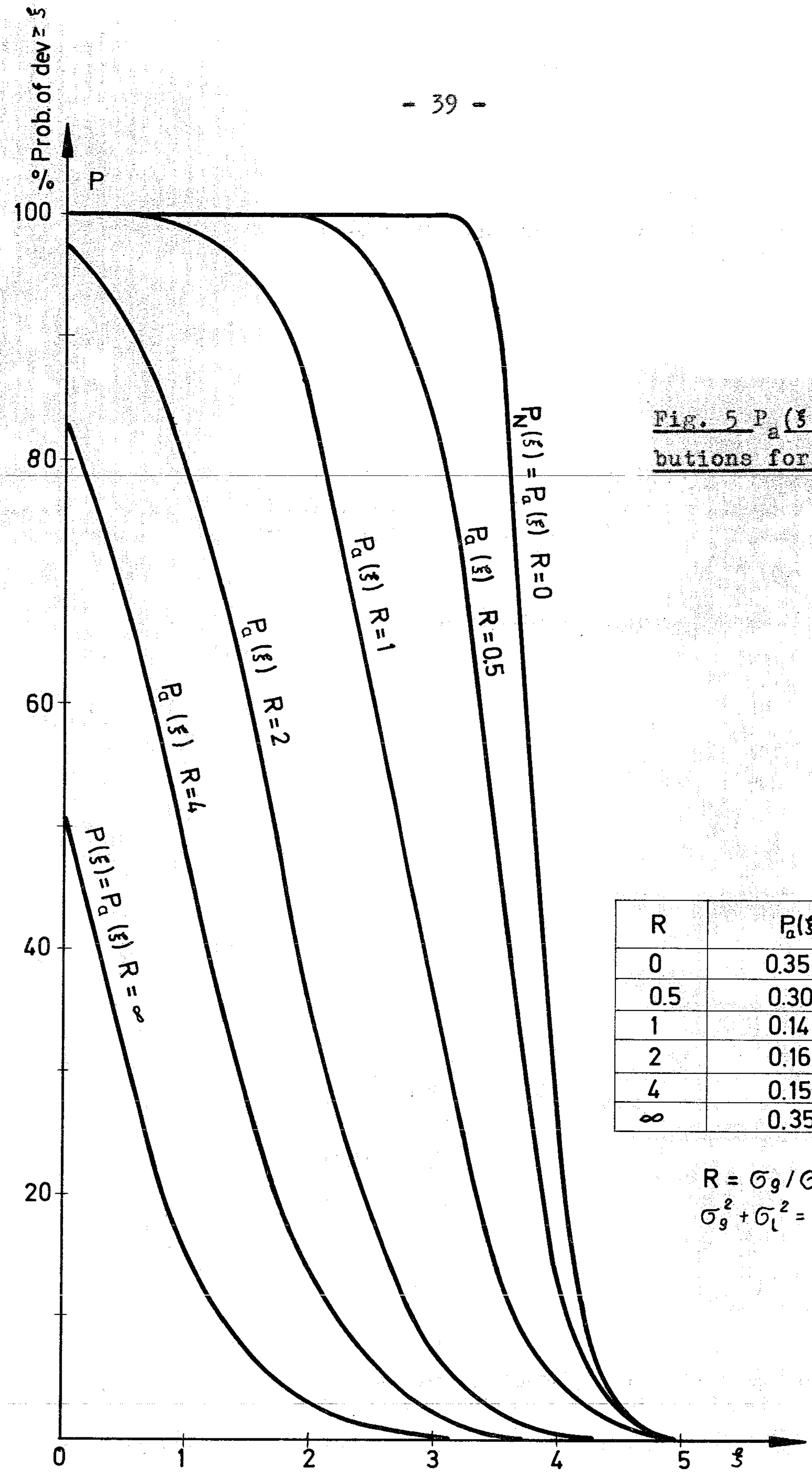


Fig. 5 $P_a(\xi)$ Distributions for N = 10 000

R	$P_a(\xi \geq 5)$
0	$0.35 \; 10^{-2}$
0.5	$0.30 \; 10^{-2}$
1	$0.14 \; 10^{-2}$
2	$0.16 \; 10^{-3}$
4	$0.15 \; 10^{-4}$
∞	$0.35 \; 10^{-6}$

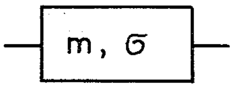
$R = \sigma_g / \sigma_l$

$\sigma_g^2 + \sigma_l^2 = 1$

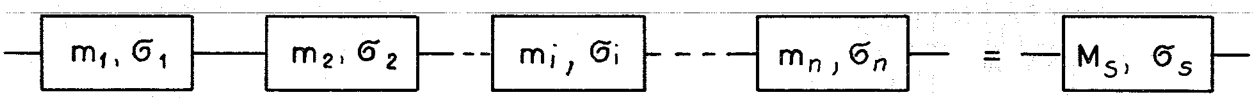
$P_a(\xi)$ = probability that the sum of the normal distributions $(0, \sigma_g)$ and $(0, \sigma_1)$ exceeds a deviation ξ, when the first is drawn once and the second N times.

$P_N(\xi)$ = the same probability when both are drawn N times

$P(\xi)$ = normal distribution (0,1)



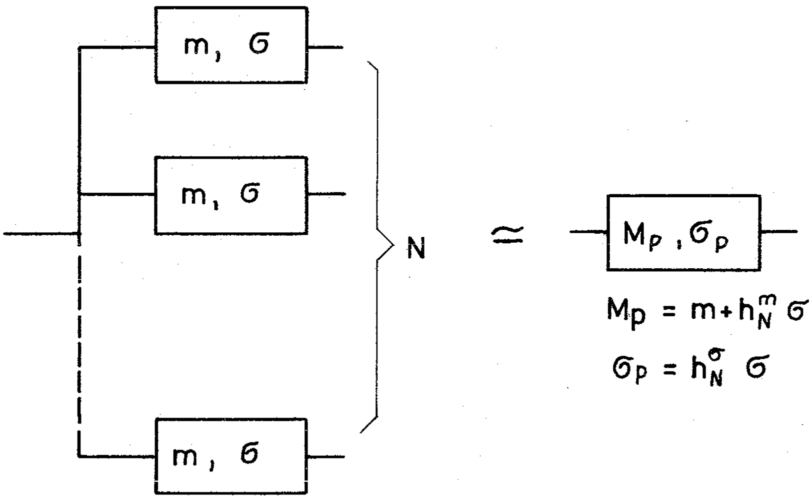
6a) normal distribution (m, σ)



$$M_S = \sum_{i=1}^{n} m_i$$

$$\sigma_S^2 = \sum_{i=1}^{n} \sigma_i^2$$

6b) sum of n normal distributions



6c) normal distribution drawn N times

Fig.6. Operations upon Normal Distributions Represented as Block Diagrams

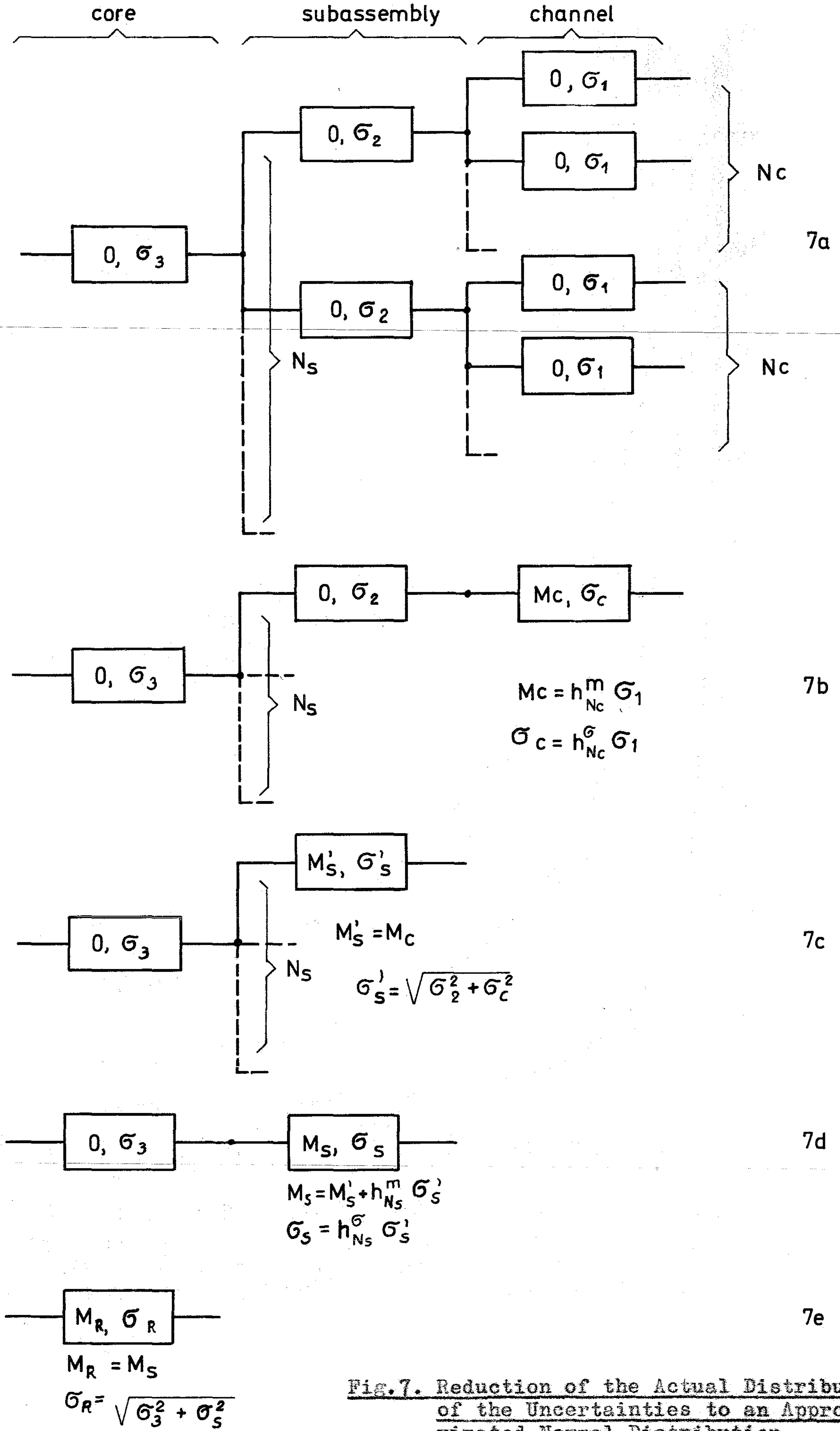


Fig.7. Reduction of the Actual Distribution of the Uncertainties to an Approximated Normal Distribution.

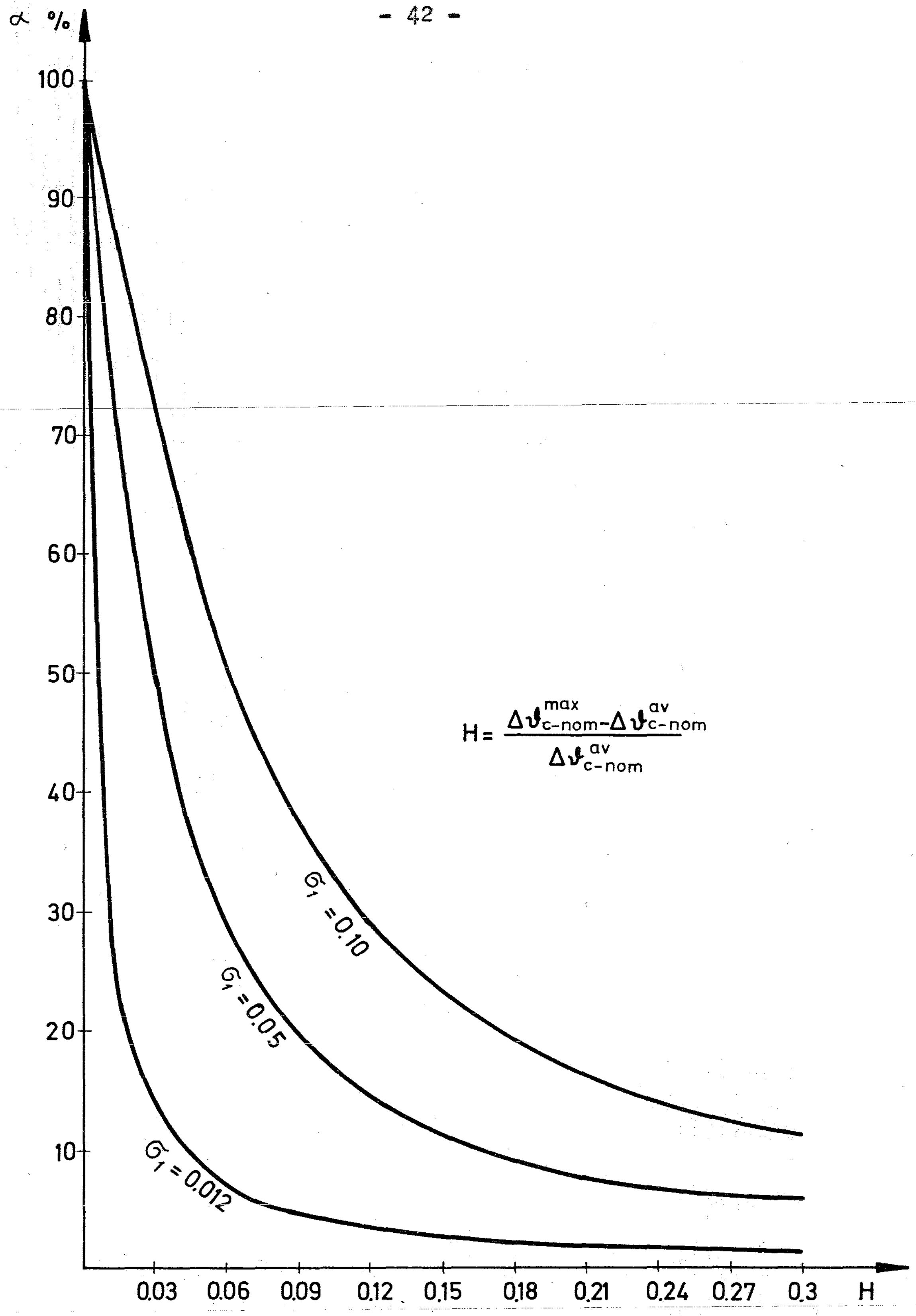


Fig. 8. Equivalent Fraction of Channels as Function of H

α = fraction of channels which, assumed at the same temperature $\Delta\vartheta^{max}_{c-nom}$, have the same probability to exceed this temperature, as the actual number of channels with the rectangular distribution (-H, +H).

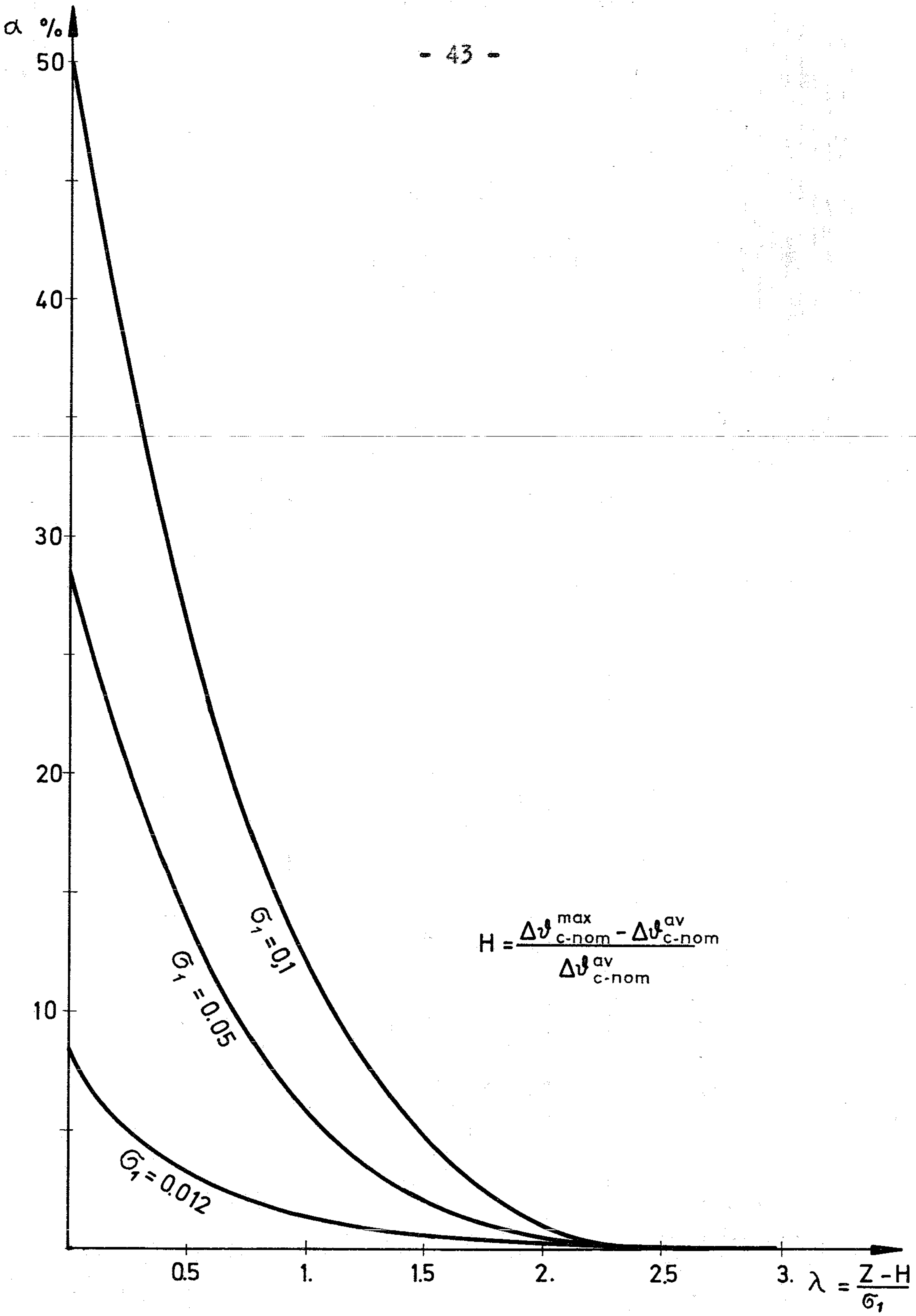


Fig. 9. Equivalent Fraction of Channels as Function of λ. $H=0.06$

α = fraction of channels which, assumed at the same temperature $\Delta\vartheta^{max}_{c-nom}$, have the same probability to exceed a deviation $z=H+\lambda\,\sigma_1$, as the actual number of channels with the rectangular distribution $(-H, +H)$.

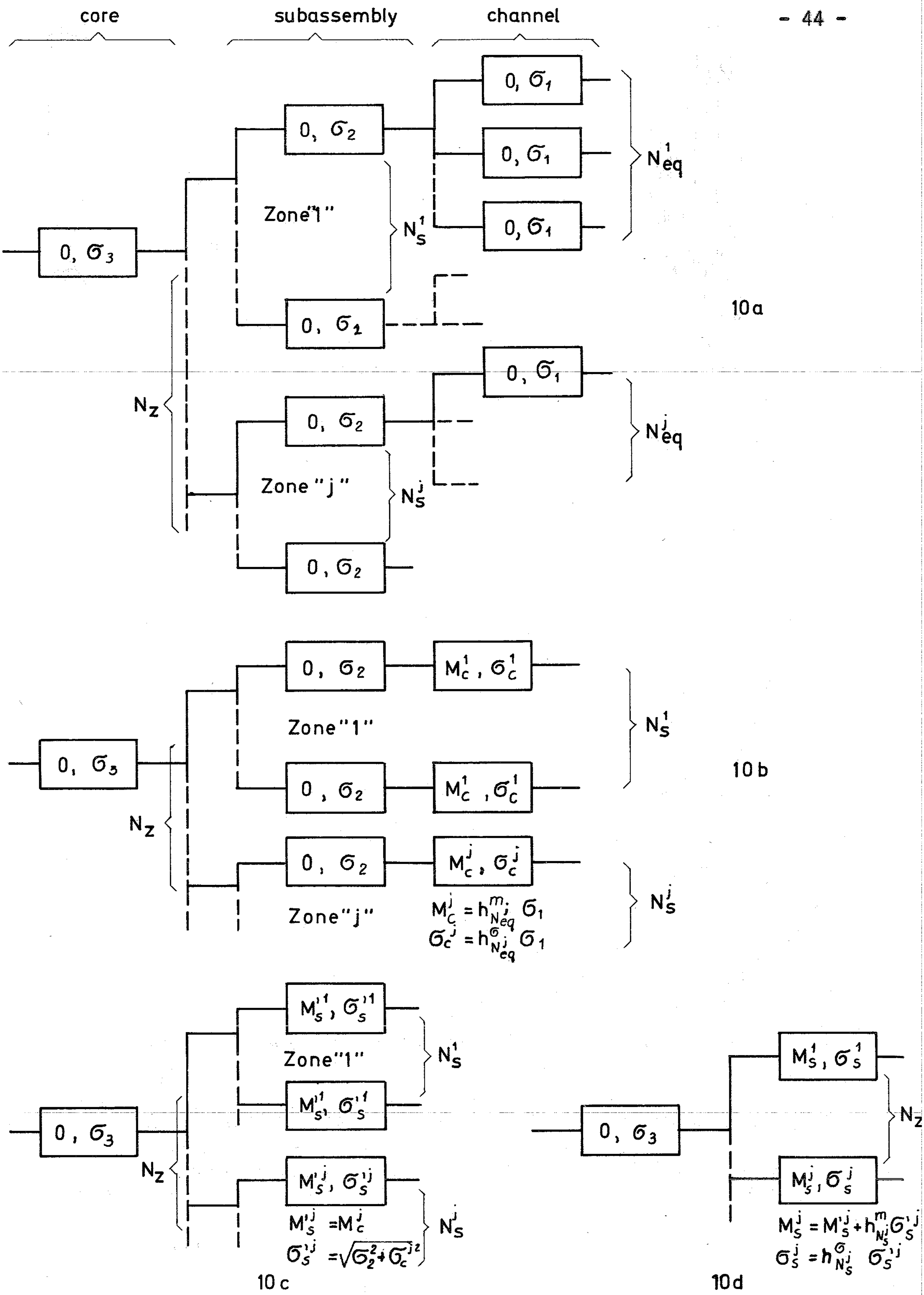


Fig. 10. Equivalent Distribution of the Uncertainties for the Actual Temperature Profile.