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Institut für Experimentelle Kernphysik

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Coupling losses and the measurement of Q-values of superconducting cavities

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For the investigation of the surface resistance of superconductors at high frequencies the Q-value of resonating cavities is usually measured. This is done by observing the decay time of the energy stored in the cavity. The decay time is inversely proportional to the losses in the cavity walls.

In order to fill the cavity and to observe the decay time, coupling elements have to be provided which, however, give rise to additional losses. These consist of two parts. First energy is lost through the coupling apperture by radiation. Second ohmic losses might occur in the coupling elements. Usually the quality factor Q_o of the cavity without coupling has been determined from the relation

$$Q_o = Q_L \, (1 + \beta) \qquad (1)$$

where Q_L is the loaded Q-value. The coupling coefficient β can be determined by a measurement of the reflected power at the cavity input by using the relation

$$\frac{P_r}{P_i} = \left(\frac{1-\beta}{1+\beta}\right)^2 \quad \text{or} \quad \frac{P_e}{P_i} = \frac{4\beta^2}{(1+\beta)^2} \qquad (2)$$

where P_i is the incident, P_r is the reflected power and P_e is the emitted power at t = t' (fig.1).

During our measurements we made the experience that a reliable determination of β is rather difficult[1] since it requires a perfect matching of the network which is used to couple power into the cavity and to observe the decay time. Therefore a method will be described here which permits to determine the Q-value of the unloaded cavity without measuring β. Furthermore it will be shown how the radiative and ohmic losses can be determined independently. It was found that the ohmic losses are not negligible, if coupling loops are used.

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[1] J. Halbritter, P. Kneisel, Ext.Bericht 3/67-3, 1967, Institut für Exp.Kernphysik, Karlsruhe

We start by observing that the losses are proportional to the inverse of the quality factor. By adding up the individual losses we obtain the relation

$$\frac{1}{Q_M} = \frac{1}{Q_o} + \frac{1}{Q_r} + \frac{1}{Q_K} = \frac{1}{Q_o'} + \frac{1}{Q_r} \qquad (3)$$

with $Q_M = \hat{\tau}\,\omega$ where $\hat{\tau}$ is the measured decay time. Q_o is the quality factor of the cavity which we want to determine. $(1/Q_r)$ corresponds to the radiation losses and $(1/Q_K)$ indicates the losses in the coupling system.

The radiation losses can be written as

$$\frac{1}{Q_r} = \frac{ß}{Q_o'} = G \quad . \qquad (4)$$

In accordance with equ. (1) we find that $Q_M = Q_L$, if the losses in the coupling system are negligible. It can be shown that $ß/Q_o' = G$ is a quantity which depends only on the geometrical dimensions of the coupling system.

The cavity can be connected to the generator and the measuring device either by a wave guide[2] or a coaxial cable.[1][3] We prefered the second possibility since it is most convenient as far as cryogenic and vacuum technique is concerned and it allows to change the coupling coefficient. A coupling loop is usually used to couple the cavity fields to the fields in the coaxial line. If the coupling loop does not protrude into the cavity but is withdrawn into the tube of the coaxial line a cut-off mode will build up in the coupling tube with an exponentially decreasing field. By changing the position of the coupling loop in this field the coupling coefficient ß can easily be changed over a large range of values. In this case one has

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[2] P.B. Wilson, Nucl.Instr. and Methods 20 (1963), 336-340

[3] Hahn et al, Measurement of Q of Superconducting TE_{011} Cavities by Frequency Sweep Technique (Paper presented at the 6th Internat.Conf. on High Energy Acc., Cambridge, Sept. 11-15, 1967)

$$G(z) = G_o \, e^{-2\alpha \, z} \qquad (5)$$

where z gives the position of the loop. The constant α in a first approximation does not depend on the frequency and is given by $\alpha = j'_{11}/a$ where a is the radius of the coupling tube$^+$. Of course equ. (5) will hold only, if the coupling loop does not distort the fields of the cut-off mode.

Ohmic losses in the coupling system will arise in the coupling loop and in the walls of the coupling tube. It seems reasonable to assume that the losses in the loop are proportional to the square of the electric field at the loop. Since this field is again determined by the cut-off mode one can write

$$\frac{1}{Q_{K_{loop}}} = K_L \, e^{-2\alpha z} \qquad (6)$$

The losses in the tube walls are very approximately given by

$$\frac{1}{Q_{K_{wall}}} = K_W \, (1 - e^{-2\alpha z}) \qquad (7)$$

since beyond the loop one has the coaxial line with practically negligible losses.

Inserting (5), (6) and (7) into equ. (3) one obtains

$$\frac{1}{\omega \tau} = \frac{1}{Q_M} = \frac{1}{Q_o} + K_W + (G_o + K_L - K_W) \, e^{-2\alpha z} \qquad (8)$$

This relation is the basis for a determination of Q_o without knowing ß. To this end one has to measure the decay time τ for various positions of the coupling loop. If one plots the measured value $1/\tau$ (or more acurately $1/\omega\tau$) as a function of z one expects a curve that can be fitted by a constant term $1/Q_o + K_W$ and an exponential function with the slope 2α. Unfortunately only the sum $1/Q_o + K_W$ can be determined in this way where K_W corresponds to the total losses in the tube

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$^+$and $j'_{11} = 1{,}841$ the first zero of J'_1

wall. However, by covering not only the cavity but also the first part of the coupling tube with the superconducting material, K_W can be made small compared to $1/Q_o$. This is not true for K_L since the loop is not in contact with the cooling bath and hence cannot be made superconducting without great difficulties. However, this does not impaire the measurement of Q_o.

In order to verify the dependence of (8) measurements were performed at various temperatures, frequencies and different types of coupling loops. Some examples are shown in Fig. 2 and 3. One sees that the measured curves can indeed be decomposed into a constant term and an exponential function whose slope agrees reasonably well with the theoretical expectation. If the coupling loop is made too broad the field distortion becomes appreciable and a more complicated dependence on z is found.

Besides Q_o one can also determine the coupling losses $G_o + K_L - K_W$. They are given by the value of the exponential at $z = 0$, i.e. if the coupling loop is positioned right at the cavity wall. A separation of the radiation losses G_o and the ohmic losses $K_L - K_W$ is possible, if the coupling coefficient ß is measured directly according to equ. (2). With ß and Q_o known, G_o can be calculated from (4). If the coupling tube wall is superconducting one has $K_W \ll K_L$ and one obtains

$$\frac{K_L}{G_o} = \frac{1}{\beta}\left(1 - \frac{Q_M}{Q_o}(1 + \beta)\right) \qquad (9)$$

For our experimental set-up we find at f = 2,46 GHz that the losses in the coupling loop are comparable with the radiation losses. K_L becomes smaller at 2^{o}K since the temperature of the loop changes whereas G_o is independent of temperature.

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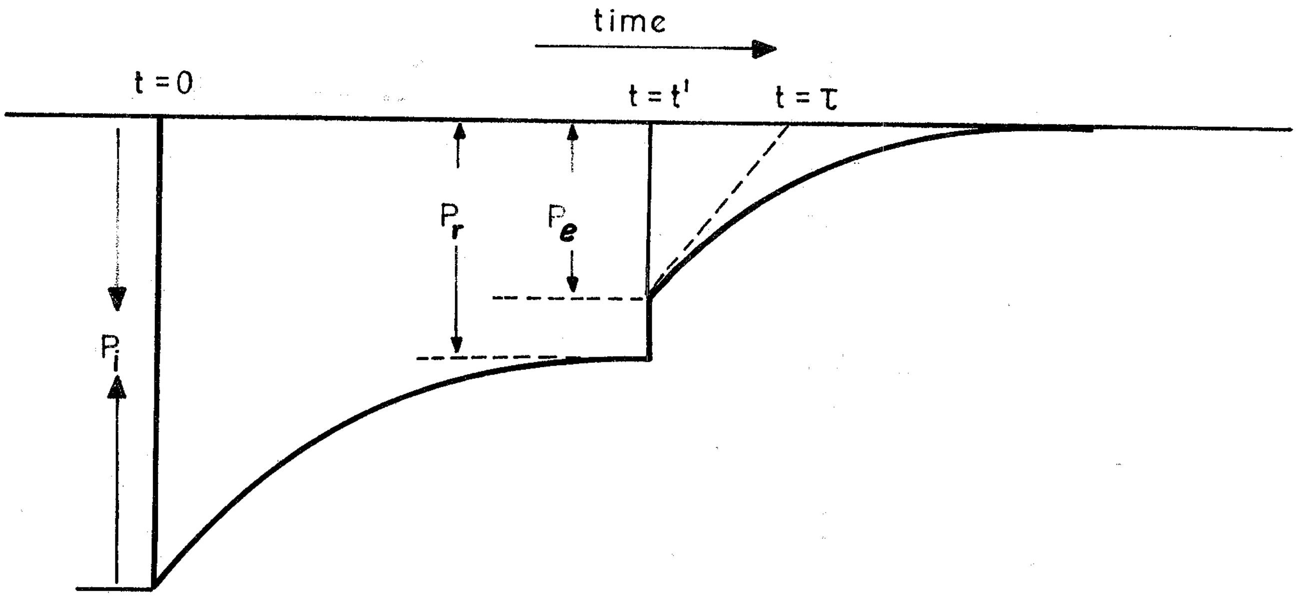

Fig.1 Response Signal of the Cavity at Resonance

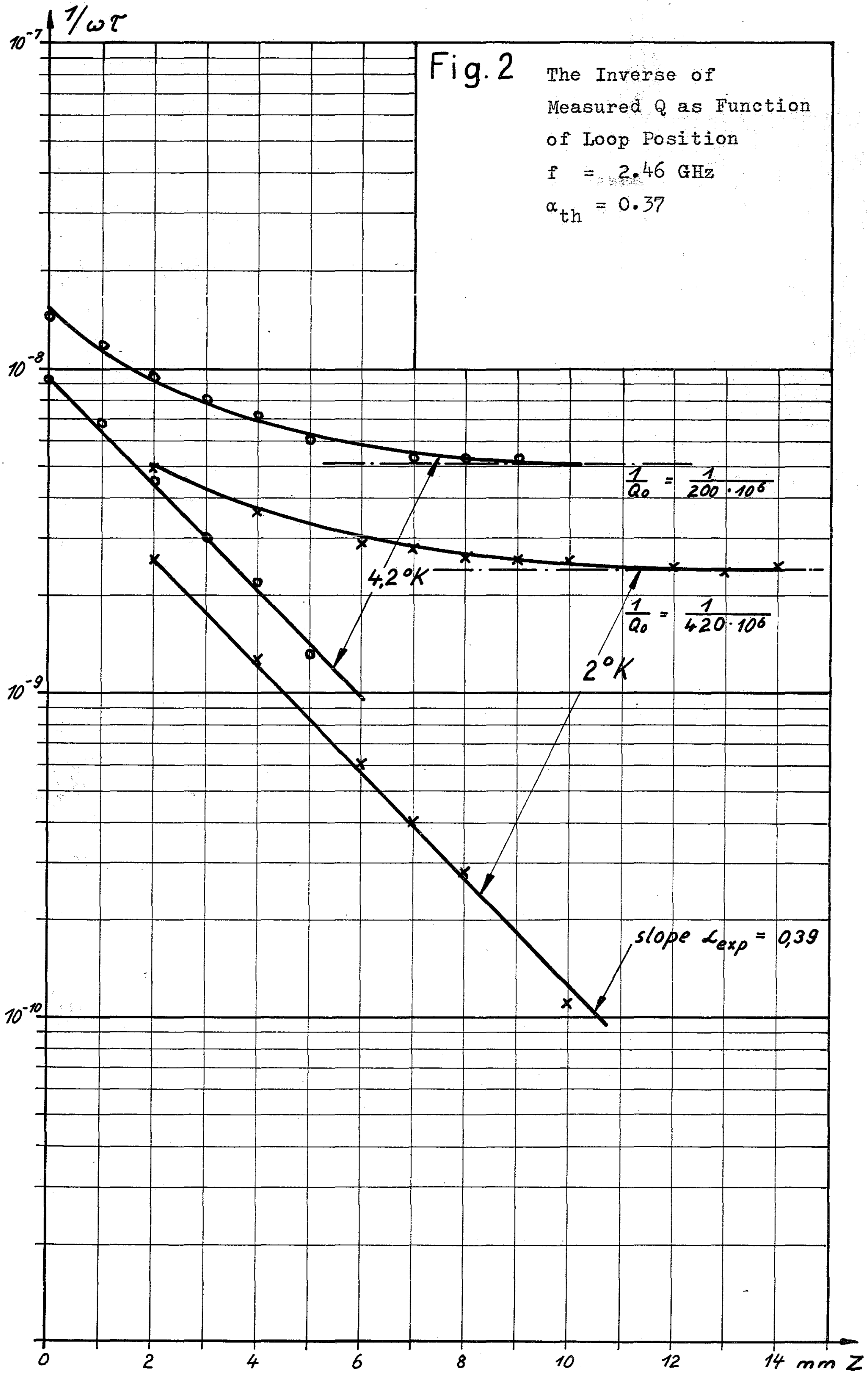

Fig. 2 The Inverse of Measured Q as Function of Loop Position

$f = 2.46$ GHz

$\alpha_{th} = 0.37$

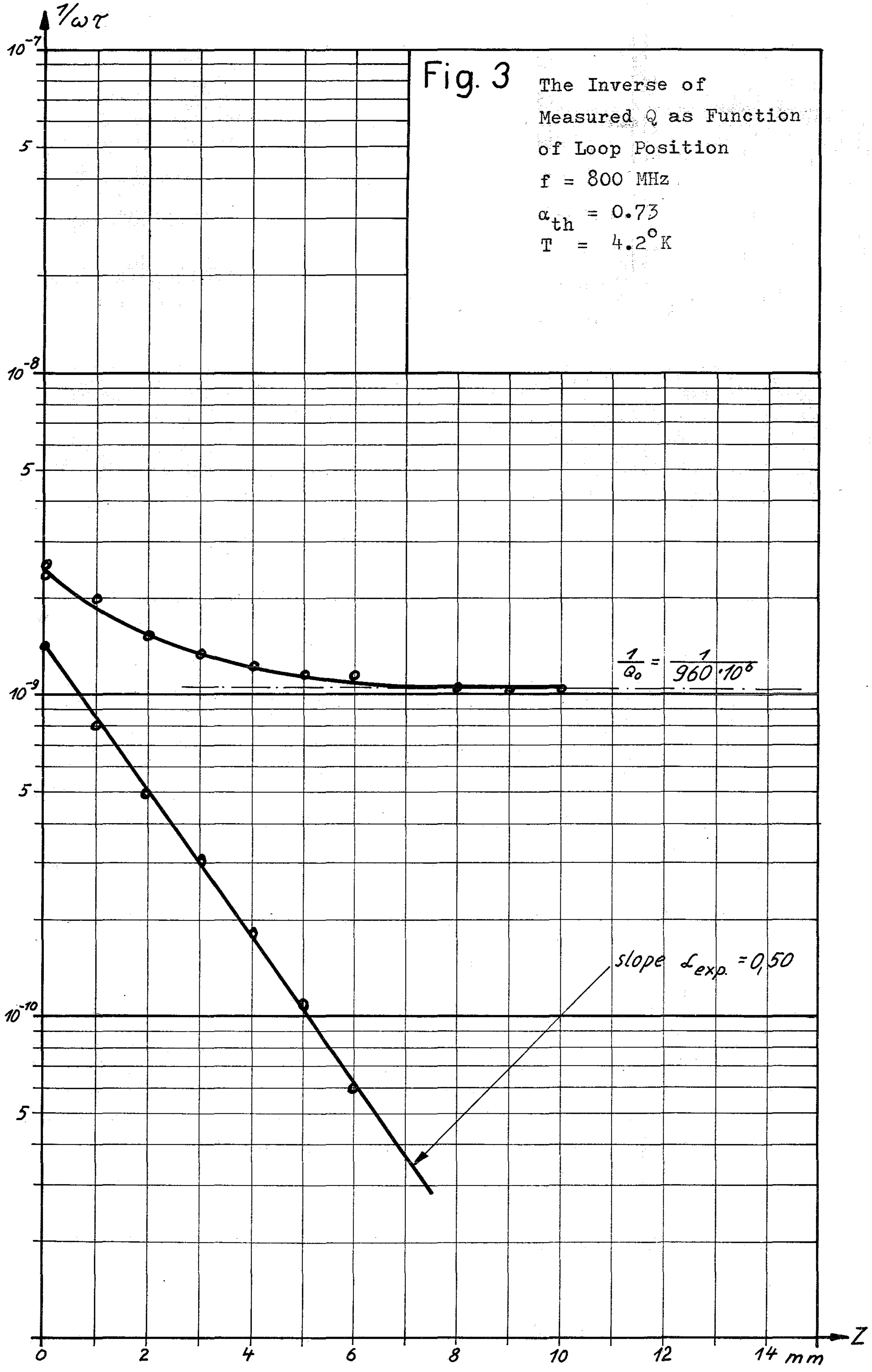

Fig. 3

The Inverse of
Measured Q as Function
of Loop Position
$f = 800$ MHz
$\alpha_{th} = 0.73$
$T = 4.2^\circ K$