KERNFORSCHUNGSZENTRUM

KARLSRUHE

November 1968

KFK 882
EUR 4161 e

Institut für Neutronenphysik und Reaktortechnik

# Comment on the Calculation of Neutron Lifetime and Material Worth

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# Comment on the Calculation of Neutron Lifetime and Material Worth *

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*Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung m.b.H., Karlsruhe.

## A b s t r a c t

Bilinear averaging has been widely accepted as the adequate procedure for establishing few group perturbation cross sections. The most important differences compared to the usual flux-averaging procedure are generally obtained for the material worths especially of predominantly scattering materials. In addition to these results the present work gives a strong indication that the calculation of the neutron lifetime is also markedly influenced by the averaging scheme used, leading to an underprediction of the calculated value, a fact which is most times observed in comparison with experimental results.

In the recently published literature (1), (2), (3), it has been pointed out that one has to be careful in collapsing group constants for perturbation calculations with a reduced number of groups. Bilinear averaging has been accepted as the adequate procedure for establishing few group perturbation cross-sections. The differences in the results compared to the results obtained with the normal flux averaging procedure are most pronounced for the material worths of light elements, which predominantly scatter neutrons. In the present work some numerical results are given for the assembly SNEAK-3A-2 which is described by Schröder (4).

Besides the reaction rates and the material worths, the most important quantity considered is the neutron lifetime, which turns out to be influenced considerably by the collapsing procedure.

The formulae used in this work for different collapsing schemes (i.e. with normal flux-, adjoint flux- , and bilinear weighting) are the same as that of Pitterle (1) whose treatment of the scattering matrix seems to be most reasonable compared to that of other authors. The reference case for the following comparison has been calculated with a 26-group set (5) established in the Institut für Neutronenphysik und Reaktortechnik at the Kernforschungszentrum Karlsruhe. For all cases one-dimensional diffusion calculations in radial direction have been performed. This assures that the weighting functions can be assumed to be exact and all differences observed are caused by the group collapsing and not by any spatial effects.

As a first step a group collapsing to five groups has been performed which contain the following somewhat arbitrarily chosen groups of the 26-group set with the Russian ABN group structure (6).

| New group | I | II | III | IV | V |
|---|---|---|---|---|---|
| Groups of the 26-group set | 1-5 | 6-11 | 12-18 | 19-24 | 25-26 |

In table 1 the most interesting integral results of the 5 group diffusion calculations are shown normalized to the corresponding 26-group results.

Table 1 essentially confirms the conclusions of others (see e.g. the statements in the work of Pitterle (1)). Therefore, they will not be repeated here.

In addition to these cases we have performed also perturbation calculations and have determined the neutron lifetime which can be calculated by a perturbation formalism too taking 1/v as perturbation cross-section. The corresponding results are given in table 2.

| weighting spectrum for the few-group constants | normalized criticality factor $k_{eff}$ | normalized conversion ratio of the core | normalized breeding ratio of the assembly | normalized power fraction of the core | normalized value of $\sigma_c/\sigma_f$ of fissile material in the core | normalized fraction of fissions in fertile material in the core |
|---|---|---|---|---|---|---|
| $\phi$ | 0.99944 | 0.99958 | 1.00001 | 1.00006 | 0.99896 | 1.00051 |
| $\phi^+$ | 1.00265 | 0.82101 | 0.88898 | 0.93208 | 1.20771 | 1.69308 |
| $\phi^+\phi$ | 1.00245 | 0.99584 | 0.97470 | 1.00105 | 0.99630 | 0.99804 |

Table 1: Results of 5 group calculations normalized to 26 group results

| case | weighting function of the few-group cross-sections for calculating | | | number of few-groups | neutron life-time $\ell$ | $\beta/\ell$ $\beta$=effective fraction of delayed neutrons | central reactivity worth of | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\phi$ | $\phi^+$ | $\delta\Sigma$ | | | | Al | C | Fe | Mo | Ni | $U^{235}$ | $U^{238}$ |
| a | $\phi$ | $\phi$ | $\phi$ | 5 | 0.94457 | 1.06655 | 0.72827 | 0.62102 | 0.59319 | 0.96927 | 0.89767 | 1.05405 | 0.95045 |
| b | $\phi^+$ | $\phi^+$ | $\phi^+$ | 5 | 1.31657 | 0.82514 | -1.11220 | 0.18263 | 3.77791 | 1.40423 | 3.75433 | 0.75670 | -0.08161 |
| c | $\phi^+\phi$ | $\phi^+\phi$ | $\phi^+\phi$ | 5 | 0.99473 | 0.988986 | 0.97618 | 0.97907 | 0.99242 | 0.98418 | 0.98575 | 0.98523 | 0.98657 |
| d | $\phi$ | $\phi^+$ | $\phi^+\phi$ | 5 | 0.99597 | 0.99270 | 0.98666 | 0.98774 | 0.99982 | 0.99232 | 0.99583 | 0.99459 | 0.99350 |
| e | $\phi$ | $\phi$ | $\phi^+\phi$ | 5 | 0.98625 | 1.02145 | 1.03345 | 1.01493 | 0.95948 | 1.00119 | 0.98440 | 1.01614 | 1.00155 |
| f | $\phi$ | $\phi$ | $\phi$ | 11 | 0.97277 | 1.02587 | 0.94936 | 0.99867 | 0.98934 | 0.99916 | 0.93495 | 1.00676 | 1.01218 |
| g | $\phi^+\phi$ | $\phi^+\phi$ | $\phi^+\phi$ | 11 | 0.99928 | 0.99898 | 0.99315 | 0.99297 | 0.99372 | 0.99533 | 0.99549 | 0.99600 | 0.99583 |

Table 2: Results of the few-group perturbation calculations normalized to the 26-group results

The column "weighting function of the few-group cross-sections for calculating" $\phi$, $\phi^+$, $\delta\Sigma$ respectively, means that e.g. in case d the flux is determined by a 5-group diffusion calculation using flux-weighted group cross-sections, the adjoint flux by a 5-group diffusion calculation using adjoint flux-weighted group cross-sections and the perturbation cross-sections by a group-collapsing method using normal flux and adjoint flux weighting (bilinear weighting).

The results of table 2 support the following indications:

2a) The best results for the perturbation calculations are obtained, as one could expect, for case d of table 2, i.e. using flux and adjoint flux weighted constants for the few group flux and adjoint flux calculations, respectively, and bilinear weighting for the few group perturbation cross-sections.

2b) One gets somewhat less accurate but still quite satisfactory results if one uses the bilinear weighted group constants for all calculations, i.e. determining few group flux, adjoint flux and perturbation cross-sections, as done in case c of table 2. This is mainly caused by an improvement of the few group adjoint flux by this procedure compared to the usual flux weighting of case a which yields considerable errors in the adjoint flux energy distribution, especially in the importance differences $(\phi^+_i - \phi^+_j)$ in the low-energy region. These errors are responsible for the discrepancies in the material worth of scattering materials. Although the method used in case c gives reasonable results for perturbation calculations one should be aware of the fact that the reaction rates of table 1 are more in error with this method than with normal flux weighting.

2c) Only for those materials for which capture and/or fission events are the dominant ones, the material reactivity worth can be determined with reasonable accuracy by few group calculations using the usual flux weighted group constants.

In addition to the 5-group calculations we have performed two 11-group calculations as indicated in the lower part of table 2, taking the following distribution of the 26 groups within the 11 condensed groups as often used by P. Engelmann (7).

| new group | I | II | III | IV | V | VI | VII | VIII | IX | X | XI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| groups of the 26 group set | 1-2 | 3-4 | 5-6 | 7-8 | 9 | 10 | 11-12 | 13-14 | 15-16 | 17-19 | 20-26 |

3a) It can be seen that increasing the number of few groups the discrepancies between case a) and f) of table 2 are remarkably reduced, but it seems that there exists a systematic tendency for the few-group calculations to underpredict the neutron lifetime.

3b) Case c and especially case g of table 2 show that taking the bilinear weighted cross-sections for few group flux and adjoint flux calculations as well as for the determinations of the perturbation group constants one can obtain reasonably accurate perturbation results with minimal additional effort, i.e. with only small changes in the group collapsing program and only one diffusion calculation.

## Conclusions

Almost all presently available group sets with a number of groups of about 50 or less have been established by the usual flux weighting method. Besides the results concerning the adequate group collapsing procedure this work gives an indication that for use in perturbation calculations the normal flux weighting may lead to systematic deviations in the results obtained. At least, this may be one of the reasons why the values of the neutron lifetime and of the reactivity worths of scattering materials and perhaps also of $B^{10}$ calculated with these group sets do not agree satisfactory with the measured values. Table 2 shows that the usual method yields a systematic underprediction of the calculated neutron lifetime, a fact observed many times in comparing theoretical with experimental values. Therefore, it seems necessary to establish three different kinds of group sets: one for the calculation of the normal flux and reaction rates, one for the calculation of the adjoint flux, and a third one for the calculation of perturbation cross-sections. In establishing these different sets one has to consider carefully the question of resonance self-shielding which will be different for the normal and the adjoint flux, e.g. in a pure or predominantly fission resonance.

It would be very interesting to compare the perturbation results obtained with the laborious method outlined above with the corresponding ones obtained by the usual method for establishing group sets. Unless one does not know the magnitude of the error introduced by using the approximate calculational scheme as is presently done, one has to be very careful in the interpretation of differences in neutron lifetime or material worths between theory and experiment and between different theoretical results which are produced by different group sets.

It must be kept in mind, however, that the most times unadequate treatment of the averaged group constants for the calculation of the adjoint flux and for perturbation calculations is, of course, only one possible reason for a disagreement between the theoretical

and experimental values of the quantities mentioned above, others being e.g. the influence of sample size, heterogeneity effects, and mutual interaction between the sample and the surrounding zone.

Acknowledgement

I would like to thank Dr. H. Küsters for encouraging discussions during this study.

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