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FEASIBILITY STUDY OF AN INTERNAL PAIR FORMATION SPECTROMETER FOR NEUTRON CAPTURE SPECTROSCOPY

W. MICHAELIS, D. LANGE*
Institut fur angewandte Kernphysik
and
G. WILHELMI
Datenverarbeitungszentrale,
Kernforschungszentrum Karlsruhe,
Karlsruhe, Federal Republic of Germany

Abstract

FEASIBILITY STUDY OF AN INTERNAL PAIR FORMATION SPECTROMETER FOR NEUTRON CAPTURE SPECTROSCOPY. The internal pair formation process may be utilized to investigate the multipolarity of high-energy transitions in neutron capture spectroscopy. For such measurements an instrument is needed that is based on the energy summing principle and that exhibits high resolution and sensitivity. The paper discusses the properties of a spectrometer which consists of a silicon detector telescope, two NaI(Tl) scintillation detectors and a superconducting magnet. The detection efficiency and the multipolarity discrimination power are calculated using the Born approximation. The sensitivity to background radiation is investigated. A system of suitable design is proposed. The spectrometer may also be used for measurements of internal conversion electrons.

1. Introduction

Investigations of the radiative neutron capture process have proved to be a useful means for the study of nuclear structure. Considerable aid in interpreting the transition diagram is provided by a knowledge of the primary transition multipolarities. Information about these high-energy transitions can be derived from measurements of the internal conversion and, in fact, magnetic electron spectrometers have been applied in neutron capture spectroscopy up to several MeV [1]. An alternative process which can be utilized is the internal pair formation process, i.e. the emission of a positron-electron pair instead of a gamma ray. As yet studies of this process have not attracted much effort, mainly because of the continuous character of the single radiation. The pair production coefficient is fairly high, being about 10^{-3} in an average case. While the internal conversion coefficient is small for low atomic numbers and decreases with increasing transition energy, the dependence of pair formation upon atomic number is only slight and the coefficient increases with increase of gamma-ray energy. A comparison of both processes is given in Table I. Over a wide range of energy and atomic number utilization of pair formation may be preferable to internal conversion.

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* Present address: Strahlenklinik, University of Heidelberg, Heidelberg, Federal Republic of Germany.

Table I

Comparison of internal pair formation and internal conversion Coefficient ($\times 10^4$)

	Z = 33; k = 5[a]			Z = 84; k = 5			Z = 84; k = 7		
	E1	E2	M1	E1	E2	M1	E1	E2	M1
Conversion[b]	0.32	0.53	0.52	4.72	10.8	15.4	2.8[c]	6.0[c]	6.7[c]
Pair formation[d]	9.6	5.3	4.1	8.8	4.5	3.5	13.9	8.0[c]	

[a] The energy, k, is given in units of mc^2
[b] From tables of Sliv and Band [2]
[c] Extrapolated values
[d] Interpolated values from exact calculations performed by Jaeger and Hulme [3]

Most of the presently available data on internal pair production have been accumulated by means of lens type or semicircular focussing magnetic spectrometers [2]. Other techniques are the detection by annihilation radiation and the use of two double coincidence Geiger or scintillation counter telescopes. The sources were mainly beta-instable isotopes or proton-induced reactions on light nuclei. From the standpoint of resolution and/or sensitivity the hitherto known spectroscopic methods are hardly applicable to the complex neutron capture process. In this field a high-resolution system is needed that makes use of the energy summing principle and that collects the positron-electron pairs with high efficiency. Two of us [4] therefore proposed a new instrument which is based on the resolution, sensitivity and coincidence capabilities of semiconductor detectors. It is the purpose of this study to investigate the properties of such a system in more detail.

2. General Considerations

The main components of the proposed spectrometer are two silicon detectors and two NaI(Tl) scintillation counters operated in a fourfold coincidence. The semiconductor diodes are stacked to form a dE/dE + E telescope. They determine the total kinetic energy of the positron-electron pair when both particles are emitted into the solid angle subtended by the detectors. The annihilation quanta arising from the positrons stopped in the E-detector are selected by the scintillation counters which are placed on an axis perpendicular to the axis of the telescope.

The dE/dx-detector should be very thin (100 - 200 μm). The electrons then suffer only a small energy loss and reach the E-counter with high counting probability. Another important feature is that more than 99.84 % of the gamma rays emitted into the solid angle of the telescope pass through a 150 μm detector without undergoing any interaction, if the photon energy is higher than 2 MeV. For electrons with energies $\geq$ 500 keV the specific energy loss is nearly independent of energy and amounts to 0.35 - 0.40 keV per μm silicon. Thus the internal pair formation events which involve the simultaneous passage of two particles through the counter can be pulse-height selected.

The solid angle defined by the telescope should be as large as possible for two reasons. The first requirement is simply the postulate of an adequate detection efficiency. The second reason is the necessity to minimize the influence of the positron-electron angular correlation on the response characteristics. When utilizing the total coefficient or any other quantity correlated with it one has to take into account that the differential cross section for production of pairs with a small angular separation is almost independent of multipole order, whereas the differential cross section for production of pairs with a large angular separation depends critically upon multiple order. In a neutron beam experiment the solid angle is limited not only by the obtainable detector size, but also by shielding problems and the maximum total counting rate in the detectors. A very powerful and favourable method for arriving at a large solid angle is provided by the use of a superconducting magnet, if the axis of the solenoid coincides with the axis of the telescope. With this system the silicon counters can be remote enough from the target that the total counting rate is significantly attenuated while at the same time the magnetic field insures the collection of positron-electron pairs emitted into a large solid angle. Both particles describe helical paths about the magnetic field lines the sense of rotation being of opposite sign. In the case of a homogenous field of sufficient strength 50 % of all pairs leaving the target will reach the telescope, if the detectors are only a little wider than the target. Thus with regard to intensity considerations the external target geometry, i.e. the extraction of a neutron beam from the reactor, implies no disadvantage compared to internal target geometry while at the same time the energy summing principle can be applied. A loss by a factor of 2 can easily be overcompensated by using a neutron beam with large radiating area. Because of these interesting features it is worthwhile to study the response characteristics of the proposed system in more detail.

3. Response Characteristics

When considering the detection efficiency and multipolarity discrimination power we have to deal with the problem to calculate the probability that for a given energy and multipolarity both the electron and the positron are emitted into a given solid angle Ω. It is convenient to take the Born approximation as a basis of such calculations. This approximation assumes that $Z\alpha/(v/c) \ll 1$, where v is the velocity of either the electron or the positron and $\alpha = 1/137$. In addition, we shall neglect the influence of the atomic number on the formation process. These simplifications are justified for the following reasons. Exact calculations [3] on pair production have been made using the exact Coulomb wave function solutions of the Dirac equation. The numerical computation is very tedious and only a few values of atomic number and transition energy were treated. However, the results show that the change in the total coefficient is small between $Z = 0$ and $Z = 84$ (less than 15 %) and that for sufficiently high energies agreement is found with the Born approximation to within 15 % [+)]. Unsatisfactory results are obtained from the Born approximation, if the energy distribution of positrons or electrons is considered, but in our case we shall integrate over the energy division and concentrate on the angular distribution in which even in the exact treatment the atomic number does not enter.

+) An exhaustive compilation of the literature is given in Ref. [5].

We start with the differential internal pair formation coefficient for production of pairs with a definite energy division and a definite angular separation ϑ, as obtained by Rose [6, 7] from the Born approximation:

$$F(k,W_+,\vartheta) = \frac{2\alpha}{\pi(l+1)} \frac{p_+p_-}{q} \frac{(q/k)^{2l-1}}{(k^2-q^2)^2}$$

$$\times \Big\{ (2l+1)(W_+W_- + 1 - 1/3\, p_+p_- \cos\vartheta)$$

$$+ l\Big[(q^2/k^2) - 2\Big](W_+W_- - 1 + p_+p_- \cos\vartheta)$$

$$+ 1/3\,(l-1)\, p_+p_- \Big[(3/q^2)(p_- + p_+\cos\vartheta)(p_+ + p_-\cos\vartheta)$$

$$- \cos\vartheta\Big]\Big\}$$

for electric multipole transitions of order 2^l and

$$F(k,W_+,\vartheta) = \frac{2\alpha}{\pi} \frac{p_+p_-}{q} \frac{(q/k)^{2l+1}}{(k^2-q^2)^2}$$

$$\times \Big\{ 1 + W_+W_- - \frac{p_+p_-}{q^2}(p_- + p_+\cos\vartheta)(p_+ + p_-\cos\vartheta)\Big\}$$

for magnetic multipole transitions.
Here the energy, k, is given in units of mc^2 and

$$W_+ + W_- = k \quad ; \quad p_+ = \sqrt{W_+^2 - 1}\ ;$$
$$p_- = \sqrt{W_-^2 - 1} \quad ; \quad q^2 = p_+^2 + p_-^2 + 2p_+p_- \cos\vartheta$$

W_+ and ϑ cover the intervals $1 \leq W_+ \leq k - 1$ and $0 \leq \vartheta \leq \pi$, respectively.

The probability for emission of an internal pair with angular separation ϑ irrespective of the energy division is readily obtained from

$$G(k,\vartheta) = \int_1^{k-1} F(k,W_+,\vartheta)\, dW_+$$

and the total coefficient is given by

$$\alpha(k) = \int_0^{\pi} G(k,\vartheta) \sin\vartheta\, d\vartheta$$

Let us assume that the given solid angle Ω is defined by the aperture angle 2ϕ. If φ denotes the angle between detector axis and direction of a particle emitted into Ω, then the probability for emission of both particles into the telescope referred to gamma emission into 4π follows from

$$H(k,\Omega) = \int_{\vartheta=0}^{\mathrm{Min}[\pi,2\phi]} \int_{\varphi=0}^{\phi} \sin\varphi\, d\varphi\, g^*(\vartheta,\phi,\varphi)\, G(k,\vartheta) \sin\vartheta\, d\vartheta$$

where $g^*(\vartheta,\phi,\varphi)$ represents a suitable weighting function. Since G is independent of φ, the integration over φ can be included into the weighting function and we get

$$H(k,\Omega) = \int_{0}^{\mathrm{Min}[\pi,2\phi]} g(\vartheta,\phi)\; G(k,\vartheta) \sin\vartheta\, d\vartheta$$

Thus the problem reduces to calculate $g(\vartheta,\phi)$. It is convenient to determine also the function $f(\vartheta,\phi)$ which refers to the case that at least one particle is emitted into Ω.

If $0 < \phi \le \frac{\pi}{2}$, we obtain using the abbreviation

$$x(\vartheta,\phi,\varphi) = \mathrm{Arc}\cos\left(\frac{\cos\phi - \cos\varphi\, \cos\vartheta}{\sin\varphi\, \sin\vartheta}\right)$$

$$g(\vartheta,\phi) = \begin{cases} \frac{1}{2}\left[1-\cos(\phi-\vartheta)\right] + \frac{1}{2\pi}\int_{\phi-\vartheta}^{\phi} x(\vartheta,\phi,\varphi)\sin\varphi\, d\varphi; & 0 \le \vartheta \le \phi \\ \frac{1}{2\pi}\int_{\vartheta-\phi}^{\phi} x(\vartheta,\phi,\varphi)\sin\varphi\, d\varphi; & \phi \le \vartheta \le 2\phi \\ 0; & 2\phi \le \vartheta \le \pi \end{cases}$$

The function $f(\vartheta,\phi)$ is calculated to be

a) $0 < \phi \le \frac{\pi}{2}$; $0 \le \vartheta \le 2\phi$:

$$f(\vartheta,\phi) = \begin{cases} \frac{1}{2}\left[1-\cos\phi\right] + \frac{1}{2\pi}\int_{\phi}^{\phi+\vartheta} x(\vartheta,\phi,\varphi)\sin\varphi\, d\varphi; & \vartheta+\phi \le \pi \\ \frac{1}{2}\left[\cos(\phi+\vartheta)-\cos\phi\right] + \frac{1}{2\pi}\int_{\phi}^{2\pi-(\phi+\vartheta)} x(\vartheta,\phi,\varphi)\sin\varphi\, d\varphi; & \vartheta+\phi \ge \pi \end{cases}$$

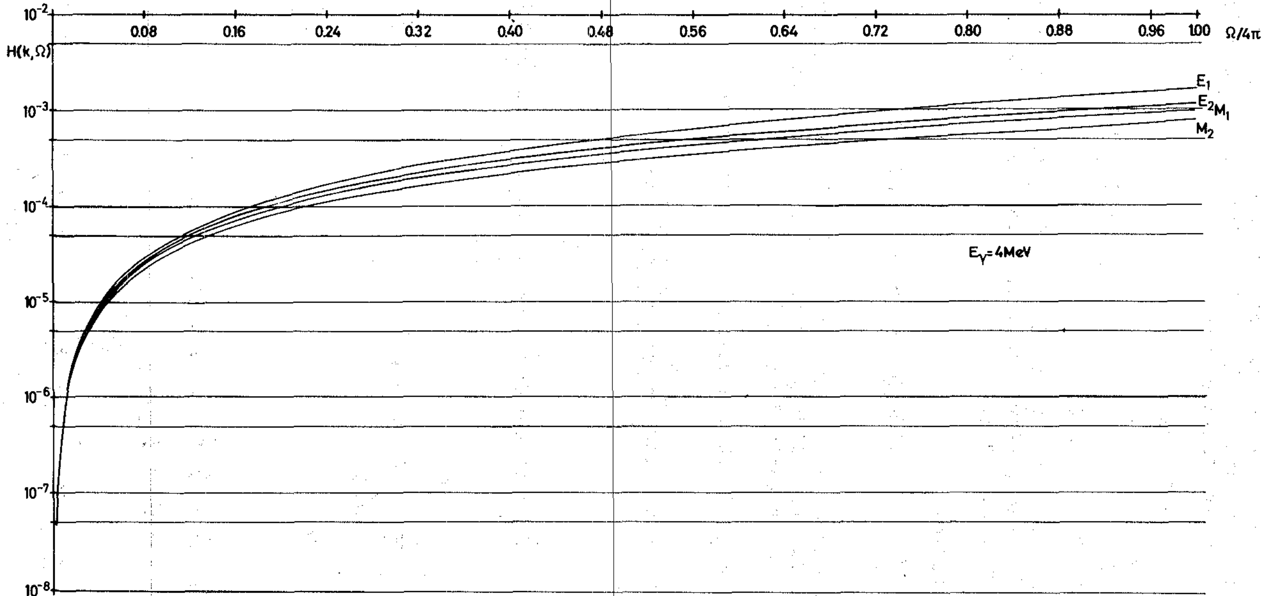


FIG. 1. Response characteristics. Logarithmic plot for E_γ = 4 MeV.

b) $0 < \phi \leq \frac{\pi}{2};\ 2\phi \leq \vartheta \leq \pi$:

$$f(\vartheta,\phi) = \begin{cases} \frac{1}{2}\left[1-\cos\phi\right] + \frac{1}{2\pi}\int\limits_{\vartheta-\phi}^{\vartheta+\phi} x(\vartheta,\phi,\varphi)\ \sin\varphi d\varphi; & \vartheta+\phi \leq \pi \\ \frac{1}{2}\left[\cos(\phi+\vartheta)-\cos\phi\right] + \frac{1}{2\pi}\int\limits_{\vartheta-\phi}^{2\pi-(\phi+\vartheta)} x(\vartheta,\phi,\varphi)\ \sin\varphi d\varphi; & \vartheta+\phi \geq \pi \end{cases}$$

In the case $\frac{\pi}{2} \leq \phi \leq \pi$ the corresponding formulae are readily obtained from the relationships

$$g(\vartheta,\phi) = 1 - f(\vartheta,\pi-\phi)$$
$$f(\vartheta,\phi) = 1 - g(\vartheta,\pi-\phi)$$

With the above expressions numerical computations have been performed for the multipolarities E1, M1, E2, M2 and the transition energies 4, 5 and 6 MeV. In Fig. 1 the ratio of the number of internal pairs emitted into Ω to the number of gamma rays emitted into 4π is plotted as a function of Ω for E_γ = 4 MeV. The results show that the detection efficiency is strongly dependent on solid angle. Increasing Ω from 1 % to 50 % yields nearly three orders of magnitude in the counting rate. In addition, the multipolarity discrimination power is improving with increasing solid angle. Since without magnet Ω is limited to at best 10 % of 4π, the collecting action of a magnetic field provides a considerable improvement in the performance of the system. The relative differences in the detection efficiency for the most important multipoles are sufficiently large to allow an experimental determination of the multipole character. They are about 50 % for 4 MeV E1 and M1 radiation observed in 2π geometry. The corresponding values at 5 and 6 MeV are 40 % and 30 %, respectively. The ratios for $\Omega/4\pi$ = 1 in Fig. 1 are identical with the total pair formation coefficient.

In Fig. 2 the function

$$\frac{H\ (k,\Omega)}{H\ (k,\Omega_o)} \cdot \frac{\Omega_o}{\Omega}$$

with Ω_o = 0.25 % is plotted. This expression is a slowly varying function of Ω and allows a linear scale.

The calculations described here are only approximately valid, if a lower limit is set on the energy of either the positron or the electron by the pulse-height selection in the dE/dx-counter. For sufficiently high transition energies the fraction of pairs which is excluded in this way is small. Thus we can expect that more sophisticated calculations will not considerably alter the results. Studies which allow for the pulse-height selection will be performed in the near future. These claculations will also include a proper correction to the shape of the positron energy spectrum. Such a correction is suggested from the exact solutions of the Dirac equation.

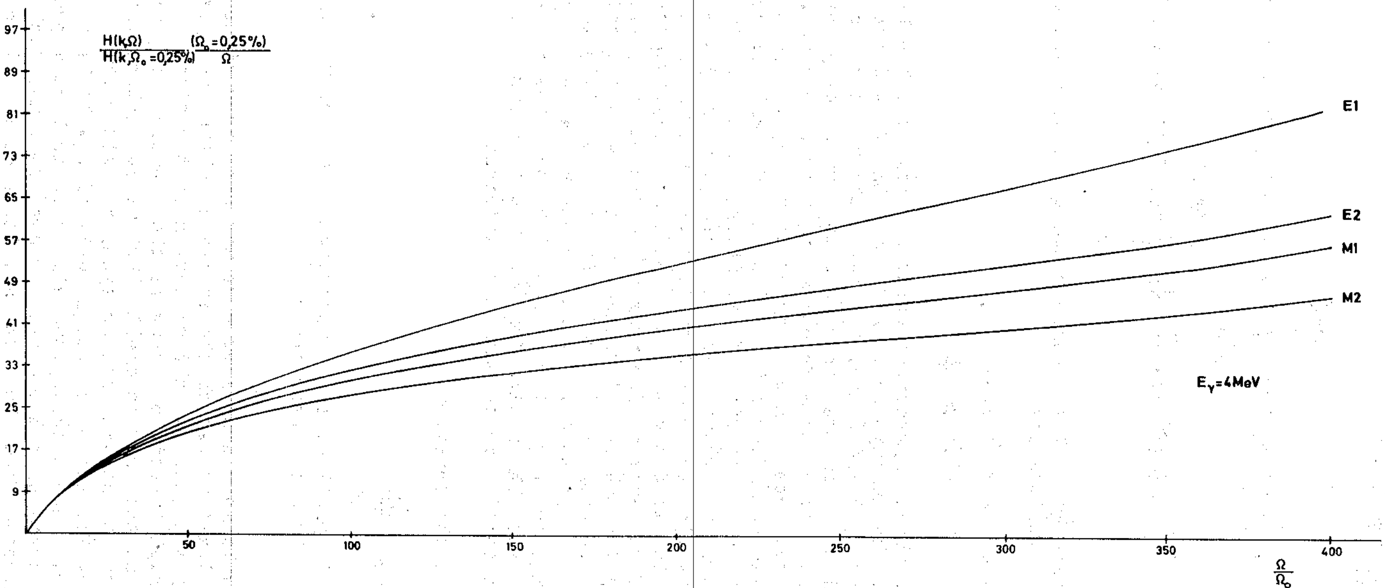


FIG. 2. Response characteristics. Linear plot in a relative scale. E_γ = 4 MeV.

4. Discrimination Against Interfering Radiation

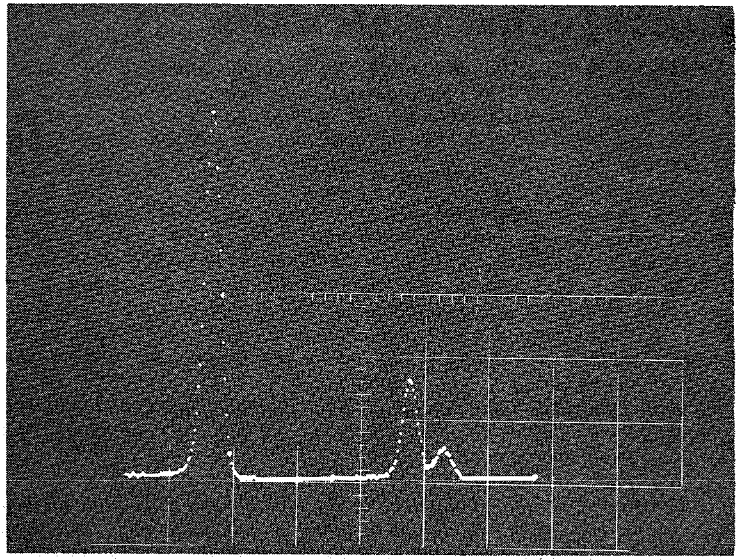
Since the values for H (k,Ω) range between 10^{-4} and 10^{-3}, it must be insured that the coincidence counting rate due to gamma rays emitted from the target is suppressed with high effectiveness. This problem was investigated experimentally. A telescope consisting of a 170 μm dE/dx-detector and a 2 mm E-counter was irradiated with conversion electrons from a ^{207}Bi source (without magnet). No distortion of the electron spectrum occurred when a gamma-ray source of ^{60}Co was added which had a source strength more than three orders of magnitude higher than that of the electron source. This is demonstrated in Fig. 3 where the spectra taken without and with Co source are shown. By using a superconducting magnet and thus a remote position of the telescope the sensitivity to gamma radiation can be further attenuated by more than an order of magnitude while the charged particle intensity is increased. Finally, the requirement of a fourfold coincidence relationship considerably reduces the gamma induced background. Thus we can conclude that interference from gamma radiation presents no severe problem.

Another source of interfering radiation is the internal conversion process. Such an event alone is not capable to trigger the coincidence circuit since there is no source of positrons. The same is true for events occurring in cascade. Another obvious possibility is the emission of a low-energy internal conversion electron into the dE/dx-counter with simultaneous production of a positron in the E-detector via external pair formation. The probability for such an event is very small because of the unfavourable geometry for gamma rays, the low atomic number of silicon and the pulse-height selection in the dE/dx-counter. Only in those cases where the energy of intense conversion electrons is such that the energy loss in the thin detector equals that of two high-energy electrons interference may occur with measurable intensity. However, the internal pair lines remain undisturbed. The background has either a continuous character or satellite peaks appear. Moreover, the possibility for this interference can easily be controlled.

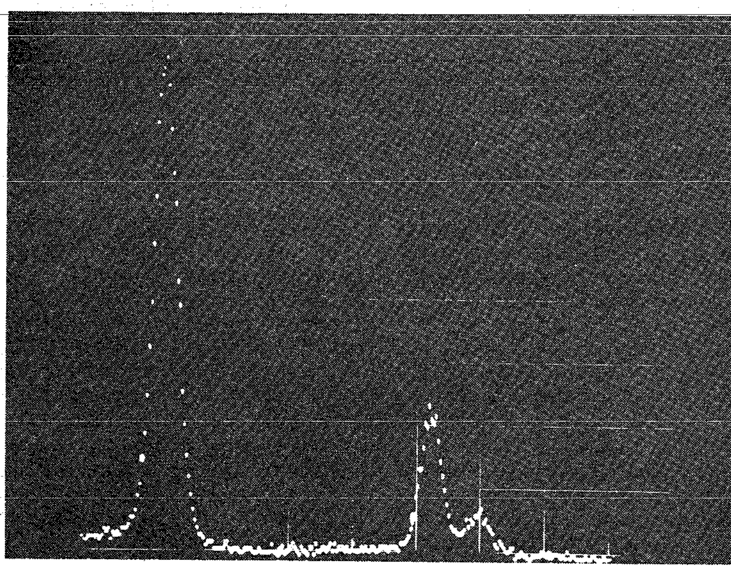
Summing with low-energy conversion electrons can also occur in the case of internal pair events, if there are highly converted coincident transitions and if a large solid angle is used. This interference results in a reduction of the pair lines and thus may falsify the measurement. A direct way to eliminate such summing effects is a reduction of the solid angle. This may be accomplished by placing the target in a diverging magnetic field. Then the efficiency is reduced by the magnetic mirror effect. As the electrons or positrons move towards the detector, the radius of the helical orbits will decrease due to the increase in magnetic field strength. Since the angular momentum of the particles remains constant, the velocity component perpendicular to the direction of the field will increase. As a result particles emitted from the target at an angle Θ relative to the field vector will be totally reflected, if Θ exceeds the angle Θ_o given by

$$\sin^2\Theta_o = \frac{B}{B_{max}}$$

Here B is the field strength at the target position and B_{max} is the maximum magnetic field strength on the path to the telescope. By varying B the collection efficiency and thus the magnitude of the



a)



b)

FIG. 3. K, L, M conversion electron spectrum from ^{207}Bi: (a) without ^{60}Co source; (b) with ^{60}Co source. See text.

summing effects can be controlled. A proper method to vary the field at the target position without changing the maximum field strength is discussed in section 6.

Another procedure for eliminating the influence of summing with conversion electrons might be to correct the pair lines for the intensity distributed over the sum peaks. This intensity may be obtained from a (simultaneous) measurement where an energy loss greater than that of two fast electrons is admitted in the dE/dx-counter.

5. Timing

Due to the remote position of the telescope and the collecting action of the magnetic field part of the electrons or positrons have to travel a long flight path before they reach the detector system. Therefore we have to study the time behaviour for unfavourable emission angles and energy divisions. In a homogeneous magnetic field the time of flight is given by

$$t = \frac{d}{c \cos \Theta \sqrt{1 - \left(\frac{m_o c^2}{m_o c^2 + E_{kin}}\right)^2}}$$

where d is the target - detector distance. Let us consider a total kinetic energy of 7 MeV. If one particle is emitted with 6.5 MeV at $\Theta = 0^o$ and the other particle with 0.5 MeV at $\Theta = 80^o$, then the difference in time of flight for d = 20 cm is calculated to be 3.8 nsec. Thus no problems arise both for the time and the energy resolution.

6. Design

A schematic lay-out of a proper spectrometer design is shown in Fig. 4. A beam of thermal neutrons enters an evacuated target chamber and impinges upon a thin target located on the symmetry axis of a superconducting solenoid system. The inside diameter of the magnet is 5 cm. The central field is approximately 50 kOe. Thus 9 MeV electrons describe helical paths with at most 0.64 cm radius and can reach a 3 cm dia. detector without striking the wall. The telescope is placed 19 cm above the target and is mounted on the end of a cold finger connected to a liquid-nitrogen cryostat. A liquid-helium cryostat shielded by liquid nitrogen houses the superconducting magnet. Target and detector system can be removed without disassembling the helium cryostat.

The main problem in designing a superconducting magnet with constant field strength over the whole distance between target and telescope arises from the requirement that the solenoid be divided for the passage of the neutron beam and the emergence of the annihilation quanta. A proper solution is indicated in Fig. 4. A suitable geometric arrangement of several coils insures a practically homogeneous magnetic field. The local dependence of the field strength as calculated for the spectrometer axis with a current

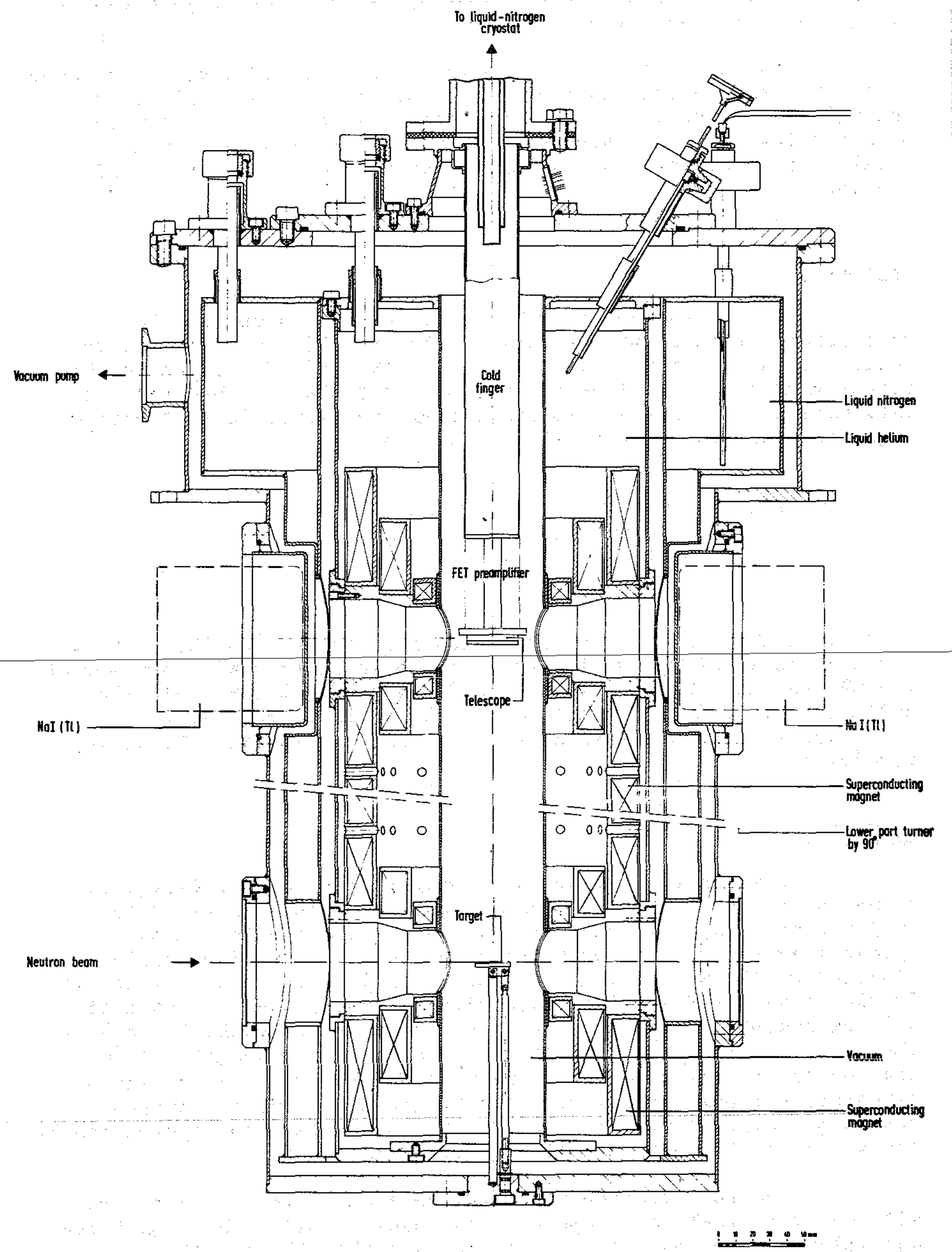


FIG. 4. Design of the spectrometer.

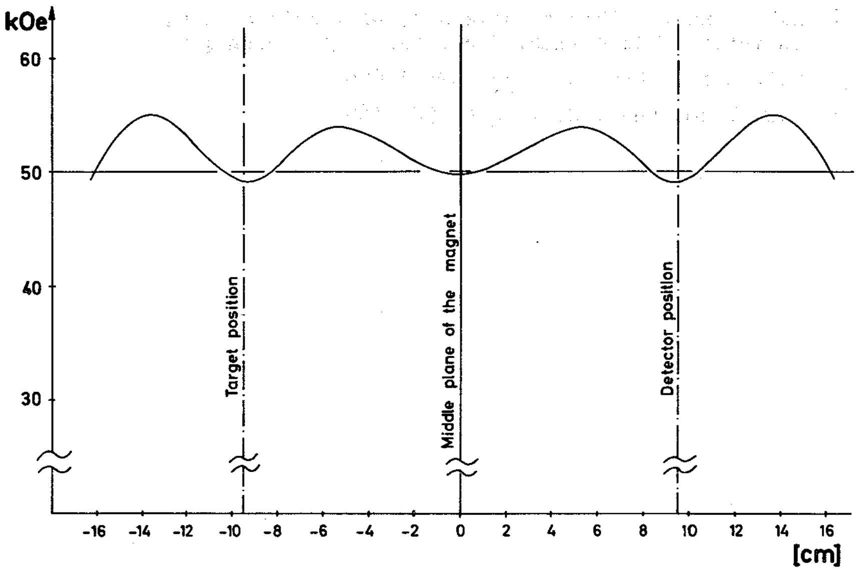


FIG. 5. Magnetic field strength on the spectrometer axis.

of 55 A is shown in +) Fig. 5. By changing the current through the auxiliary coils placed beyond the target position a diverging field can be produced at the target and the effective solid angle can be controlled by means of the magnetic mirror effect. In this way a very versatile instrument is obtainable.

The E-counter should be as thick as possible. Fully depleted detectors produced by ion implantation are very well suited for stacking a thick counter. The energy resolution of the telescope is essentially determined by the dE/dx-counter because of the large capacitance of this detector. With present techniques an optimum resolution of 0.07 % at 10 MeV can be attained.

It is worthwhile to remark that the principle discussed here may also be successfully applied for measurements of the internal conversion electron spectrum from neutron capture.

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