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Relations between Relevant Parameters for Inspection Procedures

R. Avenhaus, W. Gmelin, D. Gupta, H. Winter

GESELLSCHAFT FUR KERNFORSCHUNG M.B.H.

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RELATIONS BETWEEN RELEVANT PARAMETERS FOR INSPECTION PROCEDURES[1)]

by

R.Avenhaus, W. Gmelin, D. Gupta, H. Winter

Gesellschaft für Kernforschung m.b.H., Karlsruhe

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Im Rahmen der Untersuchungen zur instrumentierten Spaltstoffflußkontrolle wird das Problem der Erstellung einer Mengenbilanz in einer kerntechnischen Anlage behandelt. Im Gegensatz zu früheren Arbeiten, in denen nur eine Inventurperiode betrachtet wurde, wird der Fall einer Folge von Inventurperioden betrachtet, bei denen eine Reihe von neuen Parametern eine Rolle spielen, wie zum Beispiel der Startwert des Inventars für eine neue Inventurperiode, die verschiedenen Möglichkeiten zur Definition der Entdeckungswahrscheinlichkeit, die Strategie des Betreibers sowie das Konzept der Entdeckungszeit. Im ersten Teil werden diese Fragen analytisch behandelt. Da die Möglichkeiten der analytischen Behandlung jedoch beschränkt sind, werden im zweiten Teil Simulationsrechnungen für eine realistische Aufarbeitungs- bzw. Fabrikationsanlage durchgeführt, an Hand derer der Einfluß der Wahl des Startwertes sowie der Betreiberstrategie auf die Entdeckungswahrscheinlichkeit bzw. Entdeckungszeit untersucht wird.

In the framework of the analysis of an instrumented safeguards system the problem of the establishment of a material balance in a nuclear facility is treated. Contrary to former papers in which only one inventory period was considered, the case of a sequence of inventory periods is considered. Here, a number of new parameters is important, for example the way of estimating the inventory at the beginning of a new inventory period, the different possibilities for the definition of the probability of detection, the strategy of the operator and the concept of the detection time. In the first part these questions are treated analytically. As the possibilities for the analytical treatment are limited, in the second part a reprocessing plant and a fabrication plant are simulated on the computer. With the help of these simulations the influence of the choice of the starting inventory and the strategy of the operator on the probability of detection and on the detection time are investigated.

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R. Avenhaus, W. Gmelin, D. Gupta, H. Winter

Institut für Angewandte Reaktorphysik
Kernforschungszentrum Karlsruhe
Germany

Introduction

One of the most important safeguards measures in any safeguards system is the establishment of a material balance. Only this measure permits fulfilment of the safeguards objective namely, detection in case of a diversion, in a quantifiable manner. The prevention of a diversion can also be achieved implicitely by varying the time of establishing a material balance.

In the case of a single inventory period in which a material balance has been completed, the possible statistical statements, mainly the probability of detection have already been investigated [1, 2, 3]. The optimisation problem becomes much more complex if the more realistic case of a sequence of inventory periods is taken up for investigation. In that case a number of new parameters have to be considered. One of them is the method of estimation of the start inventory for the subsequent material balance period. It may be based on the book inventory, on the measured inventory and finally, on a linear combination of the two according to the maximum likelihood method. The chosen method influences the probability of detection which has been shown to be one of the important parameters for optimisation of safeguards systems.

Another important parameter is the strategies of diversion which have to be assumed in determining the probability of detection.

A new aspect in considering the sequence of inventory periods is the detection time. It is important to note that for the proper design of an effective safeguards system not only the amount of diversion, but the time taken to detect

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a diversion after it has taken place has also to be taken into consideration.

In the present paper, a number of technical parameters have been chosen for analysis which would influence the probability of detection and the detection time for a sequence of inventory periods. They are, the method of estimation for start inventory, errors in throughput and inventory measurements, diversion strategies and the number of inventories in a year.

In the first part of this paper, the problems have been treated analytically. Analytical expressions for the probability of diversion for different estimation methods and strategies of diversion have been developed. It has been shown that the detection time which is a random variable can also be expressed analytically with certain restrictions.

As the possibilities of such analytical investigations are somewhat limited, the same type of analyses have been carried out in the second part by simulating these parameters in a digital computer. For this purpose a typical reprocessing and a fabrication plant have been considered. These simulations may be considered equivalent to measurement experience in actual plants as all the random events occuring in reality, have been incorporated in these simulations. The results of these simulations have been analysed with respect to the influence of the above mentioned technical parameters on the probability of detection and the detection time.

Part I

Analytical Investigation of the Probability of Detection and the Detection Time for a Sequence of Inventory Determinations

I.1 Sequence of Inventory Determinations

In a nuclear facility, physical inventory taking may take place at the time t_n and t_{n+1} for establishing a material balance. The difference between the input and output measurements during the time interval (t_n, t_{n+1}) together with the in-plant fissile material inventory at t_n gives the so called book inventory J_{n+1}. Because of the measurement errors, J_{n+1} is a random variable, the variance of it is given by $\sigma^2_{J_{n+1}}$. Let the physical inventory taken at the time t_{n+1} be I_{n+1}, and the variance of it be $\sigma^2_{I_{n+1}}$. A sequence of inventory determinations will be considered in which the input and output of the facility between two subsequent inventories is approximately the same. Therefore, the variances of the input and output measurements between two subsequent inventories are always the same: σ^2. Also, the physical inventory has been assumed to be approximately the same when the inventory is taken, so that $\sigma^2_{I_n} = \sigma^2_{I_{n+1}}$. If the values of J_{n+1} and I_{n+1} do not differ significantly, the problem arises as to the choice of the value for the starting inventory for the next time interval.

I.2 Estimation of the Starting Inventory

Three theoretical possibilities exist for the estimation of the starting inventory:

1. I_{n+1} will be taken as the estimator. This is the most natural choice because I gives the actual value of the inventory.

2. J_{n+1} will be taken as the estimator. This may be reasonable in case the physical inventory is rather inaccurate compared to the book inventory.

3. **A combination of I_{n+1} and J_{n+1} will be taken as the estimator. In this paper a linear combination of J and I denoted as S,** will be chosen in such a way that the variance for the linear combination will be smaller than the variances of the either. This procedure, based on the maximum likelihood method has also been suggested by K. Stewart of Battelle Northwest, Laboratories, USA.

The use of the first and the second case does not give any difficulty, whereas the application of the third case is associated with a number of analytical problems which require attention.

Let the linear combination S be given by (K = const)

$$S = K \cdot J + (1-K)I \qquad (1.1)$$

If it is assumed that J and I are independent (which is normally the case), the variance of S is given by

$$\sigma_S^2 = K^2\sigma_J^2 + (1-K)^2\sigma_I^2. \qquad (1.2)$$

σ_S^2 is minimum for

$$K = \frac{\sigma_I^2}{\sigma_I^2+\sigma_J^2} \qquad (1.3)$$

Substituting this value of K in eqn. (1.1) the value of S is given by

$$S = \frac{\frac{J}{\sigma_J^2} + \frac{I}{\sigma_I^2}}{\frac{1}{\sigma_J^2} + \frac{1}{\sigma_I^2}} \qquad (1.4)$$

The variance σ_S^2 of S is given by

$$\frac{1}{\sigma_S^2} = \frac{1}{\sigma_J^2} + \frac{1}{\sigma_I^2} \qquad (1.5)$$

Eqn. (1.5) shows that σ_S^2 is less than both σ_J^2 and σ_I^2. Furthermore it is seen that J and I are weighted according to their variances, i.e. if σ_J^2 is very small then $S \approx J$ and vice versa.

I.3 Propagation of the Variances with Time

1.3.1 Physical Inventory as **Estimator**

In this case

$$S_n^1 = I_n = I$$

Therefore, the variance of S_n^1 is

$$\sigma_{S_n}^2 = \sigma_I^2 \qquad (1.6)$$

1.3.2 Book Inventory as **Estimator**

Here,

$$S_n^2 = J_n = S_{n-1}^2 + \Delta J$$

where

$$\Delta J = J_n - J_{n-1}$$

Therefore the variance of S_n^2 is

$$\sigma_{S_n}^2 = \sigma_{S_{n-1}}^2 + \sigma_\Delta^2 \qquad (1.7)$$

1.3.3 Maximum Likelihood Estimation

Here,

$$S_n^2 = \frac{\frac{J_n}{\sigma_{J_n}^2} + \frac{I_n}{\sigma_I^2}}{\frac{1}{\sigma_{J_n}^2} + \frac{1}{\sigma_I^2}}$$

The inverse of the corresponding variance is

$$\frac{1}{\sigma_{S_n}^2} = \frac{1}{\sigma_{J_n}^2} + \frac{1}{\sigma_I^2} \qquad (1.8)$$

or

$$\frac{1}{\sigma_{S_n}^2} = \frac{1}{\sigma_I^2} + \frac{1}{\sigma_{S_{n-1}}^2 + \sigma_\Delta^2}$$

With the abbreviations

$$\sigma_I^2 = b; \; \sigma_\Delta^2 = c \qquad (1.9)$$

from (1.8) the following recursion formula is obtained

$$\frac{1}{\sigma_{S_n}^2} = \frac{1}{b} + \frac{1}{c+\sigma_{S_{n-1}}^2} \qquad (1.10)$$

The boundary condition is

$$\sigma_{S_n}^2 = \sigma_{S_o}^2 \text{ for } t = t_o \qquad (1.11)$$

<u>Properties of the recursion formula (1.10)</u>

a) The asymptotic value of $\sigma_{S_n}^2$, which is denoted by σ_S^2, is given by

$$\frac{1}{\sigma_S^2} = \frac{1}{b} + \frac{1}{c+\sigma_S^2} \qquad (1.12)$$

or

$$\sigma_S^2 = -\frac{c}{2} \underset{(-)}{+} \left(\frac{c^2}{4} + bc\right)^{\frac{1}{2}} \qquad (1.13)$$

(the second solution is meaningless as < 0)

b) The following statement can be made:

for $\sigma_{S_o}^2 > \sigma_S^2$, $\sigma_{S_n}^2$ **decreases continuously**

for $\sigma_{S_o}^2 = \sigma_S^2$, $\sigma_{S_n}^2$ is constant

for $\sigma_{S_o}^2 < \sigma_S^2$, $\sigma_{S_n}^2$ increases continuously

This statement the proof of which is given in Appendix I is illustrated in Fig. I-1 to clarify its meaning. If the starting value $\sigma_{S_o}^2$ forthe series of variances σ_S^2 be higher (lower) than the asymptotic value σ_S^2, then the series converges monotoneously towards σ_S^2 from above (below).

Differential equation

The recursion formula(1.10) can be expressed in the following form

$$\sigma^2_{S_n} = \frac{bc+b\sigma^2_{S_{n-1}}}{b+c+\sigma^2_{S_{n-1}}}$$

or in the form

$$\frac{\sigma^2_{S_n} - \sigma^2_{S_{n-1}}}{n-(n-1)} = \frac{bc-c\sigma^2_{S_{n-1}} - (\sigma^2_{S_{n-1}})^2}{b+c+\sigma^2_{S_{n-1}}} \qquad (1.14)$$

This difference equation can be approximated by a differential eqn.:

$$\sigma^2_{S_n} \rightarrow y;\; n \rightarrow x;\quad \frac{\sigma^2_{S_n} - \sigma^2_{S_{n-1}}}{n-(n-1)} \rightarrow \frac{dy}{dx} \qquad (1.15)$$

with this, the eqn. (1.14) can be expressed in the following form

$$-\frac{dy}{dx} = \frac{y^2+cy-bc}{y+b+c} \qquad (1.16)$$

The properties of the differential eq. (1.16) and its solution have been discussed in Appendix II.

In Fig. I-5, the results of the recursion formula (1.10) and the differential equation (1.16) have been shown. The following input data for the measurements in a hypothetical nuclear facility have been used:

Throughput [kg Pu/d]	2		
Process inventory [kg Pu]	20		
Rel.St.dev/throughput measurement [%]	(Min) 0.1	(Med) 0.5	(High) 2.5
Rel.St.dev/inventory measurement [%]	1	5	25

The resulting values of c (cumulative variance for the throughput measurement) and b (variance for inventory), for a total throughput of 500 kg Pu, as a function of the number of inventories, are shown below

No. of inventories	b			c		
	Low	Med.	High	Low	Med.	High
10	0.04	1	25.	$1.99\ 10^{-4}$	0.0049	0.125
5	0.04	1	25.	$3.98\ 10^{-4}$	0.0098	0.249
1	0.04	1	25.	$19.9\ 10^{-4}$	0.0490	1.246

The curves in Fig. I-5 show a number of interesting points:

a) There is a good agreement between the results obtained from recursion formula (1.10) and the differential eqn. (1.16).

b) Both the curves (one $y_o > y_1$ and the other $y_o < y_1$) show the trends predicted earlier (compare Figs. I-1, I-4).

c) A comparison of the two estimation methods namely the book inventory (1) and the maximum-likelihood method (2) has been made in the lower part of Fig. I-5. In the case of 1, the variance of the estimated value increases linearly according to the relation $y_o + r \cdot c$, whereas, that in the case of 2 decreases continuously, with an increasing number of inventories/ yr.

I.4 Probability of Detection

I.4.1 Definition of the Probability of Detection in the Case of a Sequence of Inventory Determinations

It has been pointed out earlier [1,2,3], that the probability of detection plays a key role as it may be used as a criterion for the effectiveness of any safeguards measure. In the case of several inventory determinations logically a number of different possibilities for the definition of the probability of detection may be worked out. (This problem did not exist in the case of only one inventory determination.)

First possibility: ${}^{1}p^{r}(E/m_o)$ is the probability to detect a diversion for the first time after the r-th inventory, if between the 0-th and the r-th inventory the amount m_o of fissile material will be diverted. The notation $p(E/m_o)$ for the probability of detection is chosen to indicate that it is the probability of detection under the condition that m_o will be diverted.

Second possibility: ${}^{2}p^{r}(E/m_o)$ is the probability to detect a diversion up to the r-th inventory at least **once**, if between the 0-th and the r-th inventory the amount m_o of fissile material will be diverted.

Third possibility: ${}^{3}p^{r}(E/m_o)$ is the probability to detect a diversion after the r-th inventory **irrespective of what has happened in former inventory determinations**, if between the 0-th and the r-th inventory the amount m_o of fissile material will be diverted.

Let p^r be the probability that a diversion is detected after the r-th inventory, then (for independent p^r)

$$ {}^{1}p^{r}(E/m_o) = (1-p^1)(1-p^2)\ \dots\ (1-p^{r-1})p^r \tag{1.17}$$

$$ {}^{2}p^{r}(E/m_o) = 1-(1-p^1)(1-p^2)\ \dots\ (1-p^r) = 1-\prod_{v=1}^{r}(1-p^r) \tag{1.18}$$

$$ {}^{3}p^{r}(E/m_o) = p^r \tag{1.19}$$

In this paper for the rest of the first part, only the third definition of the probability of detection will be used. It will be denoted simply by p^r.

I.4.2 Calculation of the Probability of Detection for Different Strategies of Diversion

In the case of a single **inventory determination, the strategy of diversion** - for example single diversion of m_o kg Pu or n-times the diversion of $\frac{m_o}{n}$ kg Pu - has no influence on the probability of detection. For several inventories the probability of detection p^r depends on the strategy of diversion. **As an example two cases have been considered here:**

Case I: The same fraction of the amount m_o will be diverted uniformly during each inventory period, i.e. the amount $\frac{m_o}{k}$ will be diverted per inventory period so that after r inventories the total amount of diverted material will be $r \cdot \frac{m_o}{k}$.

Case II: The amount m_o will be diverted at one time between the r-1'th and the r'th inventory.

It is evident that the way, how the starting inventory for the next period is estimated after an inventory, has also a great influence on the probability of detection. All the three methods of estimation discussed in part I.2 have to be considered.

Case I It is assumed that the amount $\frac{m_o}{k}$ is diverted in each interval.

Measured inventory (I) as the **estimator:** In this case if no detection of a diversion takes place after an inventory taking the measured value of the inventory I will be taken as the **estimator,** with the assumption that no diversion has taken place. The following relation will then be valid for the difference of the expectation values EJ_r, EI_r:

$$EJ_r - EI_r = \frac{m_o}{k} \qquad (1.20)$$

The variance is given by

$$\sigma^2(J_r - I_r) = \sigma_I^2 + \sigma_\Delta^2 + \sigma_I^2 \qquad (1.21)$$

Let g be the threshold value for the difference $d = J_r - I_r$ beyond which the inspector declares a diversion. Then the corresponding probability of error α is given by

$$\alpha = 1-\phi\ (g) \qquad (1.22)$$

The probability of detection for the amount $r \cdot \frac{m_o}{k}$ assumed to be diverted is then

$$p_{11}^{r}\ (E/m_o) = \phi\left(\frac{\frac{m_o}{k}}{(\sigma_\Delta^2 + 2\sigma_I^2)^{\frac{1}{2}}} - g\right) \qquad (1.23)$$

Book inventory (J) as the **estimator:** The following relation is valid for the difference of the expectation values after the rth inventory

$$EJ_r - EI_r = r \cdot \frac{m_o}{k} \qquad (1.24)$$

Let σ_o^2 be the variance of the **estimator** at the beginning. The variance for $J_r - I_r$ is then given by:

$$\sigma^2(J_r - I_r) = \sigma_o^2 + r \cdot \sigma_\Delta^2 + \sigma_I^2 \qquad (1.25)$$

and the probability of detection

$$p_{12}^{r}\ (E/m_o) = \phi\left(\frac{r \cdot \frac{m_o}{k}}{(\sigma_o^2 + r\ \sigma_\Delta^2 + \sigma_I^2)^{\frac{1}{2}}} - g\right) \qquad (1.26)$$

Maximum-likelihood value: Let the **estimator** of the inventory at the beginning i.e. $t = t_o$ be S_o with the variance σ_o^2.

1st inventory: J_1, I_1; $EJ_1 - EI_1 = \frac{m_o}{k}$

No diversion is detected. The **estimator** S is then given by

$$S_1 = \frac{1}{N_1}\left(\frac{J_1}{\sigma_{J_1}^2} + \frac{I_1}{\sigma_I^2}\right);\ N_1 = \frac{1}{\sigma_{J_1}^2} + \frac{1}{\sigma_I^2} \qquad (1.27)$$

The expectation value of this S is then

$$ES_1 = \frac{1}{N_1}\left(\frac{EI_1 + \frac{m_o}{k}}{\sigma_{J_1}^2} + \frac{EI_1}{\sigma_I^2}\right) = EI_1 + \frac{m_o}{k}\ \frac{\sigma_I^2}{\sigma_I^2 + \sigma_{J_1}^2} = EI_1 + D_1 \qquad (1.28)$$

eqn. (1.28) shows that the true value of the inventory will be estimated a little too high by the amount D_1 (for $\sigma_I^2 \to 0$: $D_1 \to 0$)

2nd inventory: $J_2 = S_1 + \Delta J,\ I_2$

$$EJ_2 - EI_2 = \frac{m_o}{k} + D_1 \tag{1.29}$$

Again no diversion is detected. The **estimator** S_2 will then be given by

$$S_2 = \frac{1}{N_2}\left(\frac{J_2}{\sigma^2_{J_2}} + \frac{I_2}{\sigma^2_I}\right)$$

The expectation value of the S_2 is

$$ES_2 = EI_2 + D_2\ ;\ D_2 = \frac{\sigma^2_I}{\sigma^2_I + \sigma^2_{J_2}}\left(\frac{m_o}{k} + D_1\right) \tag{1.30}$$

nth inventory: $J_n = S_{n-1} + \Delta J,\ I_n$

$$EJ_n - EI_n = \frac{m_o}{k} + D_{n-1} \tag{1.31}$$

D_n is given by the following recursion formula:

$$D_n = \frac{\sigma^2_I}{\sigma^2_I + \sigma^2_{J_n}}\left(\frac{m_o}{k} + D_{n-1}\right) \tag{1.32}$$

where

$$\sigma^2_{J_n} = \sigma^2_{S_{n-1}} + \sigma^2_\Delta \tag{1.33}$$

$\sigma^2_{S_n}$ has been discussed in section two of this chapter.

The probability of detection for this case is given by

$$p^r_{13}(E/m_o) = \phi\left(\frac{\frac{m_o}{k} + D_{r-1}}{(\sigma^2_I + \sigma^2_{J_r})^{\frac{1}{2}}} - g\right) \tag{1.34}$$

Case II It is assumed that the amount m_o will be diverted between the r-1'th and the r'th inventory. The probabilities of detection for the three possibilities of **estimators** are given below.

Measured inventory (I) as the **estimator:** In this case also the same expression as in the first case is obtained excepting that the the total amount m_o has to be used.

$$p_{21}^{r} \ (E/m_o) = \phi\left(\frac{m_o}{(\sigma_\Delta^2+2\sigma_I^2)^{\frac{1}{2}}} - g\right) \qquad (1.35)$$

Book inventory (J) as the **estimator:** In this case the same expression is obtained as in the first case (1.32) excepting that the total amount of m_o instead of $r \frac{m_o}{k}$, has to be introduced

$$p_{22}^{r} \ (E/m_o) = \phi\left(\frac{m_o}{(\sigma_o^2+r\cdot\sigma_\Delta^2+\sigma_I^2)^{\frac{1}{2}}} - g\right) \qquad (1.36)$$

Maximum-likelihood value: In this case all the values (inventories I and **estimators** S) are the same as their expectation values up to the r-1th inventory. Only after the rth inventory the relation $EJ_r - EI_r = m_o$ is valid. With this, the probability of detection is given by

$$p_{23}^{r}(E/m_o) = \phi\left(\frac{m_o}{(\sigma_{J_r}^2+\sigma_I^2)^{\frac{1}{2}}} - g\right) \qquad (1.37)$$

I.4.3 Numerical Examples

Some typical numerical examples have been discussed in this section to investigate the different analytical expressions developed and discussed in the previous sections of this part.

I.4.3.1 Parametric Study for the Probability of Detection

Figs. I-6 and I-7 show how the probability of detection is influenced by the estimation methods used for the starting inventory (measured inventory 1; book inventory 2; maximum likelihood 3). It is to be noted that instead of the function $p^r(E/m_o)$ the argument $X_{\mu\nu}$ in the eqn.

$$p^r_{\mu\nu}(E/m_o) = \phi(X_{\mu\nu})$$

has been plotted against the number of inventories/yr. For the purpose of comparison this is adequate as p is the monotonous function of $X_{\mu\nu}$.

The following data have been used in generating these curves:

Throughput [kg Pu/yr]	279 kg		
Process inventory [kg Pu]	20 kg		
Rel.st.dev./throughput measurement [%]	0.068;	0.345;	1.72
Rel.st.dev./inventory measurement [%]	1	5	10

The calculated values of b and c are given below

No. of inventories	b			c		
	Low	Med.	High	Low	Med.	High
12	0.04	1.	4.	$0.0495 \cdot 10^{-4}$	0.000128	0.00317
7	0.04	1.	4.	$0.0848 \cdot 10^{-4}$	0.000219	0.00543
1	0.04	1.	4.	$0.5940 \cdot 10^{-4}$	0.001536	0.03876

In both the figures three sets of combinations of b and c have been considered. Fig. I-6 corresponds to case I ($\frac{m_o}{k}$ amount assumed to be diverted uniformly during each inventory period) and Fig. I-7 to case II (m_o assumed to be diverted once between the r-lth and the rth inventory). In both the cases m_o was assumed to be = 5 kg/yr.

The following points are worth noting from the two curves:

a) The estimation method 1 (measured inventory) is found to be the worst of the three methods. The main reason is the fact that with this method only a diversion which has happened during an inventory period can be detected. A diversion which could have happened in the previous inventory period is not considered. Besides this, the variance associated with the measured inventory is considerably higher than that for the throughput measurements. This fact reduces the probability of detection of a given amount for this method.

b) For continuous diversion, case I (Fig. I-6), the methods 2 and 3 appear to be comparably good for the set of conditions considered here. For high accuracy in inventory measurement (b_{min}) the 3 becomes less effective than 2 with increasing number of inventories. Because of the high accuracy of inventory measurement, the estimated value gets a higher weightage from the inventory measurement as a result of which a part of the diverted amount in the previous inventory periods is ignored.

c) For case II (Fig. I-7), the estimation methods 2 and 3 appear equivalent in their quality with respect to the probability of detection.

I.4.3.2 Accumulation of the Amounts of Diverted Material in Different Inventory Periods

Fig. I-8 shows how the summation of the diverted amounts in the previous inventory periods are taken into consideration by the three estimation methods 1, 2 and 3. The same campaign data as in the case of Figs. I-6 and I-7 have been used in this case. The upper curve is for 12 inventories/yr. The lower curve is for 7 inventories/yr.

It may be noted that in 2 (book inventory) the sum of the actual amounts diverted ($r \cdot \frac{m_o}{k}$) is taken into consideration, whereas for 3 (maximum likelihood) the amount diverted tends to an asymptotic value which is lower than the sum of the amounts diverted. This fact is described by the recursion formula 1.32. D_n is the fraction of the amount $\frac{m_o}{k}$ (which has been diverted in the $(n-1)^{th}$ interval) taken into consideration for the nth **inventory determination.**

In this particular case, the amount D_n reaches an asymptotic value of around 1.28 kg for a total diverted amount of 5 kg for the maximum likelihood method. In case 1, only the amount diverted during a single inventory period is considered and it is reduced to zero after each inventory period.

I.5 Mean Time of Detection

I.5.1 General Formula

As mentioned in the beginning, it is important to know the mean time of detection of a diversion as a function of the important parameters (variances, error probability etc.). This mean detection time again depends on the strategy of the operator; in this section only the strategy of continuous diversion (case I, section I.4.2) will be studied.

The "mean time of detection" can be calculated in the following way:

Let the random variable T_d (detection time) be the number of inventories which is necessary to detect a diversion for the first time. p^r is the probability that after the r-th inventory a diversion is detected. Then (for independent p^r)

$$p(T_d=1) = p^1$$
$$p(T_d=2) = (1-p^1)p^2$$
$$p(T_d=r) = (1-p^1)(1-p^2)\ \ldots\ (1-p^{r-1})p^r$$

The mean detection time ET_d is defined as the expectation value of the random variable T_d:

$$ET_d = \sum_{r=1} r \cdot p(T_d=r)$$

or

$$ET_d = \sum_{r=1} r(1-p^1)(1-p^2)\ \ldots\ (1-p^{r-1})p^r \qquad (1.38)$$

This general formula can only be treated analytically in special cases.

I.5.2 Special Case

In the case that the physical inventory is taken as the estimation value for the starting inventory, the probability p^r (in the case of continuous and constant diversion) is independent of r: $p^r=p$. Therefore from (1.38)

$$ET_d = \sum_{r=1} r(1-p)^{r-1} \cdot p = \frac{1}{p} \qquad (1.39)$$

In a similar way, the variance of the random variable T_d can be calculated. One obtains

$$\mathrm{var}T_d = \frac{1-p}{p^2} \qquad (1.40)$$

According to (1.23), the mean time of detection ET_d is given by

$$ET_d = \frac{1}{\phi\left(\dfrac{\frac{m_o}{k}}{(\sigma_\Delta^2+2\sigma_I^2)^{\frac{1}{2}}} - g\right)} \qquad (1.41)$$

From (1.41) one has for $m_o = 0$, that is in case of **no** diversion:

$$ET_d = \frac{1}{\phi(-g)} = \frac{1}{1-\phi(g)}$$

or with (1.22)

$$ET_d = \frac{1}{\alpha} \qquad (1.42)$$

This means, that also in the case of no diversion on an average a "detection" will take place after $\frac{1}{\alpha}$ inventories. It should be pointed out that in such cases a wrong decision can be avoided by chosing suitable action levels.

In Fig. I-9 the mean time of detection and the probability of diversion have been shown as a function of m_o/k. As an example the case c_{med}/b_{min} with 12 inventories of section I.4.3 has been selected. This figure shows that in the case of no diversion (probability of error $\alpha = 2{,}5$ %) on an average after 40 inventories a "detection" will take place. With increasing m_o/k the mean detection time tends to 1 very quickly. A diversion of 500 g Pu between two inventories will be detected on an average after 2 inventory determinations in this particular case.

As mentioned in I.5.1, an analytical expression for the mean time of detection can be obtained only for very restricted cases.

The relationship between the mean time of detection, the probability of detection, the measuring accuracy and the error probability, is very important for effectivity considerations. In the cases where analytical solutions cannot be given another approach consists in simulating a campaign and investigating this problem in the framework of a parameter study. This is carried out in the second part of the present paper.

Part II

Digital Simulation of Measuring Processes in Typical Nuclear Facilities and Analysis of Simulation Results

In this part complete measurement experiments in two typical nuclear facilities have been simulated with a digital computer. The purpose of such simulations is firstly to investigate and analyse the interdependence of different parameters which would influence the quality of a safeguards system and secondly to determine the numerical values of such parameters (e.g. of the detection time) in more complex cases which can only be determined by analytical methods under certain, rather simplifying assumptions.

II.1 Assumptions, definitions and methods used

II.1.1 Assumptions and definitions

Although most of the terms which are used frequently in this part have been defined in part I, the most important of these terms are summarized below for ready reference.

II.1.1.1 Basis for inspector's statement

The inspector makes his statement with regard to a diversion on the basis of two sets of measured values i.e. the book and the physical inventory.

a) He calculates the book inventory $\hat{J}$ with the corresponding variance σ_J^2 on the basis of throughput measurements in the time interval (t_1, t_2).

b) He measures the physical inventory $\hat{I}$ at t_2 with the corresponding variance σ_I^2 and adds to that the inventory $\hat{S}$ estimated to be present in the plant at t_1 with the corresponding variance σ_S^2.

The estimated value $\hat{d}$ (MUF) is then given by

$$\hat{d} = \hat{J} - \hat{I} + \hat{S} \qquad (2.1)$$

with the corresponding variance

$$\sigma_d^2 = \sigma_J^2 + \sigma_I^2 + \sigma_S^2 \qquad (2.2)$$

As well known, MUF may consist of different components as diversion, biased measurements, mal operation etc. However it is assumed in this paper, that the only components of MUF are diversion and fluctuations on account of the random nature of the measurements. Furthermore it is assumed, that the measuring errors are normally distributed. Then the inspector makes the following alternative statements:

Alternative results	Statement
$\hat{d} \geq \sigma_d \cdot g_{1-\alpha}$	Material has been diverted
$\hat{d} < \sigma_d \cdot g_{1-\alpha}$	No diversion has taken place

where $g_{1-\alpha}$ is connected to the probability of the type I error α by:

$$1-\alpha = \phi\,(g_{1-\alpha}) \qquad \text{(see 1.22 in part I)}$$

For the digital simulation an $\alpha = 0.025$ with the corresponding value of $g_{1-\alpha} = 1.96$ has been chosen.

II.1.1.2 Probabilities of detection

Several different possibilities of the definition of a probability of detection have been investigated in the first part. For the digital simulation the values according to the three different definitions of the probability of detection have been calculated.

i. The probability to detect a diversion for the first time at the r'st inventory taking, which (for mutual independent p^i) is given by (see 1.17)

$$^1p^r = p^r \prod_{i=1}^{r-1} (1-p^i) = (1-\beta_r) \prod_{i=1}^{r-1} \beta_i \qquad (2.4)$$

where

β_r - the probability of an type II error.

ii. The probability to detect a diversion at the end of a single inventory period r independent of the preceding inventory periods (see 1.19)

$$^3p^r = (1-\beta_r) = \phi\left(\frac{m_{or}}{\sigma_{d_r}} - g_{1-\alpha}\right) = p^r \qquad (2.5)$$

m_{or} - the amount assumed to be diverted during the r'th inventory period.

iii. The probability to detect at least once a diversion in r inventory periods (for mutual independent p^i):

$$^2p^r = 1 - \prod_{i=1}^{r} (1-p^i) = 1 - \prod_{i=1}^{r} \beta_i \qquad (2.6)$$

(see 1.18)

This probability of detection, referred to as cumulative probability of detection, is highly dependent on the strategy of diversion.

It can easily be shown, that the following inequalities hold:

$$^1p^r \leq {^3p^r} \leq {^2p^r} \qquad \text{for all } r$$

In table II-3, β_r has been presented, for the three estimation methods for the standing inventory (see I.2 and II.1.3).

The strategy of a continuous diversion (see I.4.2) has been chosen for the calculation of β_r as well as for the subsequent digital simulation.

II.1.1.3 Detection time (T_d)

The probabilities of detection as defined in part I and in I.1.1.2 of part II can be calculated a priori for an amount assumed to be diverted. However, a probability of detection is only one of several parameters which describe the capability of a system to detect diversions. A further parameter is the detection time T_d, which is a random variable too (see part I). Up till now the relationship between the detection time and the probabilities of diversion had not been investigated.

In part I the calculation of T_d has been demonstrated for a simple case where the starting values are set to be the results of the inventory measurements and also the relationship between the probability of detection and T_d has been investigated.

For other cases of the starting values the evaluation of T_d by analytical means leads to mathematical difficulties and therefore T_d had to be simulated.

II.1.2 Methods for the generation of data

Fig. II-1 shows the scheme for the generation of random measurements by digital simulation. It may be noted that three different random events have been built into the simulation scheme. These events have been assumed to occur either sequentually or in parallel.

a) The throughput and the inventory amounts for the two plants were found to vary within a certain range as shown in Tables II-1 and II-2. The true values for the individual data were generated with the help of a pseudo random generator assuming that all the values within the given ranges have equal probabilities of occurance. This was done to take into account the natural variations on the throughput and the inventory amounts.

b) The measurement values for the throughput obtained by the inspection system were generated randomly from a normal distribution which was constructed around the true values (obtained in a) with the corresponding given variances. This procedure was considered adequate to simulate the randomness of measurement in reality by the inspection authority.

c) In the case of inventory measurements,the true values were first generated from which the amount assumed to be diverted was subtracted. A normal distribution was constructed around the resulting value with the corresponding variance (which is a function of the amount and the accuracy of inventory measurement) and the measurement values obtained by the inspection authority were simulated randomly from the normal distribution.

The detection time T_d was calculated to be that time interval $(0, t_i)$ in the course of a year, in which a diversion as defined in (I.42) was detected for the first time during the ith inventory. This part of the simulation was repeated several times to get an idea on the magnitude of the oscillation in the values of T_d. **An estimate for the mean detection time E T_d with the associated standard deviation could then be estimated from this repetitions.**

II.1.3 Choice of estimators

All the three variations for choosing the **estimators** for the initial inventory, as discussed in chapter I namely,

a) accepting the book value J
b) accepting the measured inventory I
c) maximum-likelihood estimation

were built in the simulation process. In section II.3.1 the results from these three variations have been discussed.

II.2 Data on the nuclear facilities considered

A fabrication plant for plutonium containing fuel elements and a reprocessing plant for irradiated fuel elements from a light water type reactor, have been used as the basis for simulation. The relevant data are given in Tables II-1 and II-2. The tables are self explanatory.

II.3 Analysis of the results of simulation

Only a small part of the simulation results has been selected here for analysing the following questions:

3.1 Which method of estimation for the starting inventory should be selected.

3.2 What are the parameters which influence the propagation of variance σ_d with time.

3.3 How is the detection time and the probability of detection influenced by the different parameters considered.

3.4 Importance of the different parameters investigated.

II.3.1 Choice for the method of estimation

The detection time T_d and the cumulative probability of detection P_T have been taken as the two criteria according to which the quality of the three estimation methods can be tested. In Table II-3, these two parameters have been listed for the three methods for different throughput measurement errors, number of inventories/yr and the amount diverted (assumed to be diverted uniformly throughout the year, case I, **part I.**) Similar to the conclusion drawn in section 4, **part I,** the results in this table show that the method 2 (inventory estimation) is the worst of the three and that the method 1 (book inventory)and 3 (maximum likelihood method) are equivalent, for the range of throughput and inventory accuracies, amount diverted and the number of inventories/yr,considered in this paper.

For this reason, all the subsequent figures are based on the method 1. Other results of the simulation indicate , that the probability of detection is mainly influenced by the accuracy of the inventory measurement for the range considered, the probability of detection decreases linearly with increasing variance in the inventory measurement. The variance in a throughput measurement has very little influence on the probability of detection for the range of accuracies and the mode of diversion considered in this paper.

II.3.2 Propagation of σ_d in time

The propagations of the variance σ_d in the course of a year, for the reprocessing and the fabrication plant discussed in section 2, are shown in Figs. II-2 and II-4 respectively. The three sets of curves in each of these figures are for three different values of throughput measurement errors indicated on the margin of the curves. The upper most set gives very low values, the middle set gives values which are attainable at present and the lowest set gives values which are very high. The parameter in each set is the accuracy of the inventory measurement. All the relevant data used to generate these curves are given in Tables II-1 and II-2. Each of these curves are constructed from experimental values for 1, 6 and 12 inventories/yr. These values are shown as an example by a cross, a circle and a square respectively, in the upper most set of curves in Fig. II-2. The rest of the curves is not fitted with these markings.

Points of interest in Figs. II-2, II-4.

a) The propagation of variance σ_d is independent of the number of inventories carried out in a year, for the ranges of measurement errors for the throughput and the inventory, considered in the simulation. The slope of almost all the curves is zero, because of b).

b) The absolute value of σ_d is determined mainly by the measurement error of the inventory, excepting for the lowest curves in the two figures. In these cases the extremely large errors in the throughput measurements give equivalent values of variances as those obtained for the inventory with 1 % or 0.5 % accuracies. Only in these two cases σ_d^2 increases linearly with time.

Figs. II-3 and II-5 show the development of σ_d as a function of the overall throughput measurement variance σ_T with the accuracy of inventory measurement as the parameter. They illustrate once more the overwhelming influence of the accuracy of the inventory measurement on the total variance σ_d:

a) For 10 % or 5 % accuracy an inventory measurement for the reprocessing plant and 5 % accuracy for the fabrication plant, the throughput accuracies can vary by a factor of 25 without significantly influencing the σ_d. Since σ_d determines the probability of detection, this means that the probability of detection is also influenced mainly by the variance of the inventory measurement in this range.

b) The picture is however changed for an inventory measurement accuracy of 1 % or 0.5 %. With these accuracies the influence of the errors of the throughput measurements on σ_d cannot be neglected.

II.3.3 <u>Detection time</u>

In Figs. II-6 and II-7, the detection time T_d has been shown as a function of a number of inventories/yr for the reprocessing and the fabrication plant. Again, the accuracy of the inventory measurement has been used as the parameter. The three sets of curves in each figure correspond to the three levels of accuracies for throughput measurement indicated in the margin. A diversion of 5 kg has been assumed spread equally over all the inventory periods.

The following trends can be seen in Figs. II-6 and II-7.

a) With the highest accuracy of inventory measurement (1 % or 0.5 %), the detection time decreases with increasing inventories/yr up to a certain number of inventories/yr, after which a further increase does not bring any improvement in the detection time.

b) With decreasing accuracy in inventory measurement, the detection time tends to go through a minimum. This may be partly because of the fact that with the present scheme of diversion, the amount assumed to be diverted per inventory period gets reduced with increasing number of inventories/yr. Since σ_d, which is a direct measure of the detection probability, is mainly determined by the σ_I (which remains independent of the number of inventories), and therefore is also independent of the number of inventories, the probability of detection reduces with increasing number of inventories, and therefore the detection time increases.

c) The lower limit of the detection time is given by the number of inventories/yr. For example, the detection time cannot be less than 0.5 yr if two inventories (which are assumed to be equidistant) are carried out per year.

d) Within the range of the accuracies for the inventory measurement considered here, the detection time appears to be a linear function of the accuracy.

II.3.4 Importance of the parameters

As mentioned at the beginning three main parameters have been varied in the present simulation, namely, the accuracy of the inventory taking, the accuracy of the throughput measurement, and the number of inventory takings/yr. For the set of conditions considered, the accuracy of the inventory taking appears to be the most important parameter, as it influences and determines both the detection time and the detection probability in a very significant manner. The accuracies of the throughput measurement which are available at present (the medium level shown in Figs. II-2, 4, 5, 6) appear to be adequate, unless inventory measurement accuracy is reduced below 1 % or the inventory amount is reduced considerably. The number of inventories/yr is also strongly influenced by the accuracy of the inventory taking. For example for 5 % accuracy in a reprocessing plant, or 1 % accuracy in a fabrication plant, a larger number of inventories/yr than 2, would not give any additional advantage with regard to the detection time.

Conclusion

A large volume of data has been generated in the present paper both from the analytical part and from the part on digital simulation. These data permit a number of generalized conclusions. They are however, valid under the conditions specified in this paper. These conclusions are to be viewed in relation to the objectives formulated in the introduction of this paper.

1. Among the three estimation methods for the starting inventory to be used for the subsequent material balance, the book inventory and the maximum likelihood estimate appear to be equivalent both with respect to the probability of detection and the detection time. The method based on measured inventory is worse than the other two.

2. The propagation of the total variance for the establishment of material balance, as a function of time and its absolute value, is almost a unique function of the variance in the inventory measurement. Only when the variances of the throughput measurements attain comparable values (the variances already attainable at present are lower than such values), as those obtained by inventory measurement, does the throughput variance influence the total variance.

3. The detection time is almost a linear function of the stand.deviation of the inventory measurement. Depending on the absolute value of the inventory variance, and the amount assumed to be diverted (spread uniformly over all the inventory period), the detection time as a function of the number of inventory/yr may go through a minimum. This is however, mainly because of the mode of diversion assumed for this study.

4. The cumulative probability of detection falls monotonously with increasing variance of the inventory measurement for the range of amounts assumed to be diverted, considered in this study. The variance of the throughput measurements has negligible influence on the probability of detection.

5. The dominating parameter, which influences and determines the two criteria for evaluation (probability of detection and the detection time), is the variance of the inventory measurement. An improvement of the variance by reducing either the standard deviation for inventory measurement or the absolute amount of the inventory in a plant, will lead to an improvement of the probability of detection and the detection time. The standard

deviations for the throughput measurements which are attainable today, appear adequate, as long as the variance of inventory measurement is not improved.

Acknowledgement

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Appendix I

Properties of the recursion formula 1.10

In this appendix, the following statement has been proven. Let $\sigma^2_{S_n}$ be defined by the recursion formula 1.10, and let σ^2_S be defined by (1.13):

$$\sigma^2_S = -\frac{c}{2} + \left(\frac{c^2}{4} + bc\right)^{\frac{1}{2}}$$

Then

$\sigma^2_{S_n}$ decreases continuously for $\sigma^2_{S_o} > \sigma^2_S$

$\sigma^2_{S_n}$ is constant for $\sigma^2_{S_o} = \sigma^2_S$

$\sigma^2_{S_n}$ increases continuously for $\sigma^2_{S_o} < \sigma^2_S$

Prove by induction:

a) To be shown $\sigma^2_{S_1} < \sigma^2_{S_o}$ for $\sigma^2_{S_o} > \sigma^2_S$ or

$$\frac{1}{\sigma^2_{S_1}} > \frac{1}{\sigma^2_{S_o}} \quad \text{for} \quad \sigma^2_{S_o} > \sigma^2_S$$

Now the following relation is valid

$$\frac{1}{\sigma^2_{S_1}} = \frac{1}{b} + \frac{1}{c+\sigma^2_{S_o}} > \frac{1}{\sigma^2_{S_o}}$$

as the sign of equality is given for $\sigma^2_{S_o} = \sigma^2_S$ and the sign > is valid for $\sigma^2_{S_o} > \sigma^2_S$.

b) Conclusion from n-1 to n

The condition $\sigma^2_{S_{n-1}} > \sigma^2_{S_n}$ is valid.

This means

$$\frac{1}{c+\sigma^2_{S_{n-1}}} < \frac{1}{c+\sigma^2_{S_n}}$$

or

$$\frac{1}{b} + \frac{1}{c+\sigma^2_{S_{n-1}}} < \frac{1}{b} + \frac{1}{c+\sigma^2_{S_n}}$$

or

$$\frac{1}{\sigma^2_{S_n}} < \frac{1}{\sigma^2_{S_{n+1}}}$$

Thus the given statement is proven.

Appendix II

Properties of the differential equation 1.16

In order to discuss the differential eq. (1.16)

$$-\frac{dy}{dx} = \frac{y^2+cy-bc}{y+b+c} \qquad \text{(II.1)}$$

it is transformed to

$$-\frac{dy}{dx} = \frac{(y-y_1)(y-y_2)}{y+b+c} \qquad \text{(II.2)}$$

where

$$y_1 = -\frac{c}{2} + \left(\frac{c^2}{4} + bc\right)^{\frac{1}{2}} \qquad \text{(II.3)}$$

$$y_2 = -\frac{c}{2} - \left(\frac{c^2}{4} + bc\right)^{\frac{1}{2}}$$

It is to be noted that these are the asymptotic values of σ_S^2 corresponding to eqn. (1.13) for the recursion formula (1.10).

The boundary condition is

$$y = y_o \text{ for } x = 0 \qquad \text{(II.4)}$$

The eqn. (II.2) can be reduced by partial fraction expansion to the form

$$-\frac{dx}{dy} = \frac{A}{y-y_1} + \frac{B}{y-y_2} \qquad \text{(II.5)}$$

where

$$A = \frac{1}{2} + \frac{\frac{1}{2}+t}{(1+4t)^{\frac{1}{2}}} \; ; \; B = \frac{1}{2} - \frac{\frac{1}{2}+t}{(1+4t)^{\frac{1}{2}}} \; ; \; t = \frac{b}{c} \qquad \text{(II.6)}$$

The general course of eqn. (II.5) is shown graphically in Fig. I-2.

Integration of eqn. (II.5) gives logarithmic singularities at y_1, y_2. The qualitative trend is shown in Fig. I-3.

Mirroring the Fig. I-3 at the y=x coordinate, the function $y(x)$ which is sought is obtained. This function is represented in Fig. I-4. The part of the function which is of interest, is given in the upper right hand quadrant in the x-y plan (shaded area). Here the same type of trend is recognizable as in the case of the recursion formula (Fig. I-1). For a starting value of $y_o' > y_1$, the curve falls monotoneously to the asymptotic value y_1 and vice versa. Besides this, one obtains the unique solution $y=y_1$ starting with the value $y_o=y_1$, as seen from eqn. (II.2).

By integration of eq. (II.5) with the boundary condition (II.4), the solution is obtained in the following implicit form:

$$y_o > y_1 \;:\quad A \ln \frac{y-y_1}{y_o-y_1} + B \ln \frac{y-y_2}{y_o-y_2} = -x$$

$$y_o < y_1 \;:\quad A \ln \frac{y_1-y}{y_1-y_o} + B \ln \frac{y-y_2}{y_o-y_2} = -x \qquad \text{(II.7)}$$

The slope of the curve at x = 0 determines the rate at which the curve approaches the asymptotic value. An analysis of the eqn. (II.1) shows that the larger the difference between the starting and the asymptotic value (y_o-y_1) the faster is the rate at which the curve approaches the asymptotic value.

List of Symbols

b	Variance of the inventory measurement
c	Variance of the flow measurement
D	Expectation value of difference between estimation value and true inventory for the maximum likelihood estimate in case of continuous diversion
Ea	Expectation value of the random variable a
g	Fractile for normal distribution
I	Physical inventory
J	Book inventory
m_o	Amount of fissile material to be diverted
${}^{i}p^{r}(E/m_o)$	Probability of detection
S	Estimation value of the physical inventory
t	Time
T_d	Detection time
$y(x)$	Differential equation for the variance of the estimation value
y_o	Boundary condition for y(x)
$y_{1,2}$	Asymptotic values of y(x)
α	Error probability
σ_a^2	Variance of the random variable a
$\phi(x)$	**Distribution function of the standard normal distribution**

Table I-1 : Formulae for probability of detection for different cases

Case I : Equal amount of $\frac{m_o}{k}$ diverted uniformly during each inventory period

Case II : The whole amount m_o diverted between the r-1th and the rth inventory

Estimation method[1)]	Case I	Case II
1. Book inventory J will be taken for the **estimator**	$p^r_{11}(E/m_o) = \phi\left(\frac{r\cdot\frac{m_o}{k}}{(\sigma_o^2+r\sigma_\Delta^2+\sigma_I^2)^{\frac{1}{2}}} - g\right)$	$p^r_{21}(E/m_o) = \phi\left(\frac{m_o}{(\sigma_o^1+r\sigma_\Delta^2+\sigma_I^2)^{\frac{1}{2}}} - g\right)$
Eqn.no. part	3.4 I	3.16 I
2. Measured inventory I will be taken for the **estimator**	$p^r_{12}(E/m_o) = \phi\left(\frac{\frac{m_o}{k}}{(\sigma_\Delta^2+2\sigma_I^2)^{\frac{1}{2}}} - g\right)$	$p^r_{22}(E/m_o) = \phi\left(\frac{m_o}{(\sigma_\Delta^2+2\sigma_I^2)^{\frac{1}{2}}} - g\right)$
Eqn.no. part	3.7 I	3.17 I
3. Maximum-likelihood estimate	$p^r_{13}(E/m_o) = \phi\left(\frac{\frac{m_o}{k}+D_{r-1}}{(\sigma_I^2+\sigma_{J_r}^2)^{\frac{1}{2}}} - g\right)$	$p^r_{23}(E/m_o) = \phi\left(\frac{m_o}{(\sigma_{J_r}^2+\sigma_I^2)^{\frac{1}{2}}} - g\right)$
Eqn.no. part	3.15 I	3.18 I

1) for the clarification of the symbols see text

Table II-1: Data for the Simulation of a Typical Reprocessing Plant

1. Data for Throughput Measurements

Stream	Input	Output	High active waste 1	High active waste 2
Average amount of Pu/batch [kg/batch]	1.4	0.698	$0.84\ 10^{-3}$	$0.7\ 10^{-3}$
No.of batches per year	200	400	100	66
Average process inventory	20 kg Pu			
Amount of solution A_1 or ceramic [kg]	1960	0.8	2800	1400
Variation of A_1 [kg or l]	$\pm$ 200	$\pm$ 0.04	$\pm$ 500	$\pm$ 120
Rel. accuracy of measurement of A_1 [%]	0.06-1.5	0.02-0.5	1-25	4-100
Pu-concentration A_2 [kg/l] or [-]	$0.1\ 10^{-2}$	0.873	$0.3\ 10^{-6}$	$0.5\ 10^{-6}$
Variation of A_2	$\pm\ 0.16\ 10^{-3}$	$\pm$ 0.005	$\pm\ 0.1\ 10^{-6}$	$\pm\ 0.3\ 10^{-6}$
Rel. accuracy of measurement of A_2 [%]	0.12-3.0	0.05-0.75	4-100	10-250
Density [kg/l]	1.4	-	-	-
Variation of density	$\pm$ 0.1	-	-	-
Rel. accuracy of measurement of density [%]	0.02-0.5	-	-	-

2. Data for Inventory Measurements

Rel.accuracies in % : 1,5,10

No.of inventories/yr: 1,2,6,12

3. Amounts assumed to be diverted : 1,5,10 kg Pu/yr

Table II-2 : Data for the Simulation of a Typical Fabrication Plant

1. Data for Throughput Measurements

Stream	Input	Output	Waste barrels	Waste bottles
Average amount of Pu/batch [kg/batch]	4.54	4.28	0.01	$0.5 \cdot 10^{-2}$
No.of batches per year	400	400	800	200
Average process inventory	80 kg Pu			
Amount ceramic or Pu [kg] B_1	5.2	4.28	0.01	$0.5 \cdot 10^{-2}$
Variation of B_1	$\pm$ 0.1	$\pm$ 0.1	$\pm 0.5\ 10^{-2}$	$\pm 0.2 \cdot 10^{-2}$
Rel.accuracy of measurement [%]	0.02-0.5	0.08-2.0	2-50	2-50
Concentration of Pu [-] B_2	0.823	-	-	-
Variation of B_2	$\pm 0.3 \cdot 10^{-2}$	-	-	-
Rel. Accuracy of measurement [%]	0.04-1	-	-	-

2. Data for Inventory Measurements

Rel.Accuracies % : 0.5, 1.0, 5.0

No. of inventories/yr: 1,2,6,12

3. Amounts assumed to be diverted :1,5,10 kg Pu/yr

Table II-3: Comparison Between the Different Estimation Methods for the Reprocessing Plant

Accuracy of inventory measurement : 5 %

No. of inventories/yr			2		6		12	
amounts diverted /‾kg/yr_7			1	5	1	5	1	5
P_T /‾%_7	F_{L_1}	Method 1	14.1	0.92	19.9	56.9	30.8	53.2
		Method 2	12.7	0.86	18.4	44.2	29.5	45.2
		Method 3	14.1	0.92	19.9	56.9	30.8	53.2
	F_{L_2}	Method 1	14.1	0.92	19.9	57.	30.8	53.
		Method 2	12.7	0.86	18.4	44.	29.5	45.
		Method 3	14.1	0.92	19.9	57.	30.8	53.
	F_{L_3}	Method 1	12.4	0.83	18.3	51.	30.4	50.
		Method 2	11.7	0.79	18.3	43.	29.5	44.
		Method 3	12.5	0.84	19.4	52.	30.5	51.
T_{dm} /‾yr_7	F_{L_1}	Method 1	0.95	0.6	0.82	0.72	0.65	0.81
		Method 2	0.95	0.6	0.87	0.87	0.83	0.93
		Method 3	0.95	0.6	0.82	0.72	0.65	0.81
	F_{L_2}	Method 1	0.9	0.65	0.95	0.62	1.0	0.85
		Method 2	0.9	0.65	0.93	0.83	1.0	1.0
		Method 3	0.9	0.7	0.95	0.62	1.0	0.85
	F_{L_3}	Method 1	1.0	0.6	0.93	0.73	0.81	0.77
		Method 2	1.0	0.6	1.0	0.83	1.0	0.94
		Method 3	1.0	0.55	1.0	0.65	0.74	0.95

Accuracy of Throughput Measurements F_2 /‾%_7

Level of accuracy for throughput measurements	Input	Output	High active waste 1	High active waste 2
F_{L_1}	0.136	0.03	4.1	10.8
F_{L_2}	0.68	0.15	20.6	53.9
F_{L_3}	3.4	0.75	103.1	269.3

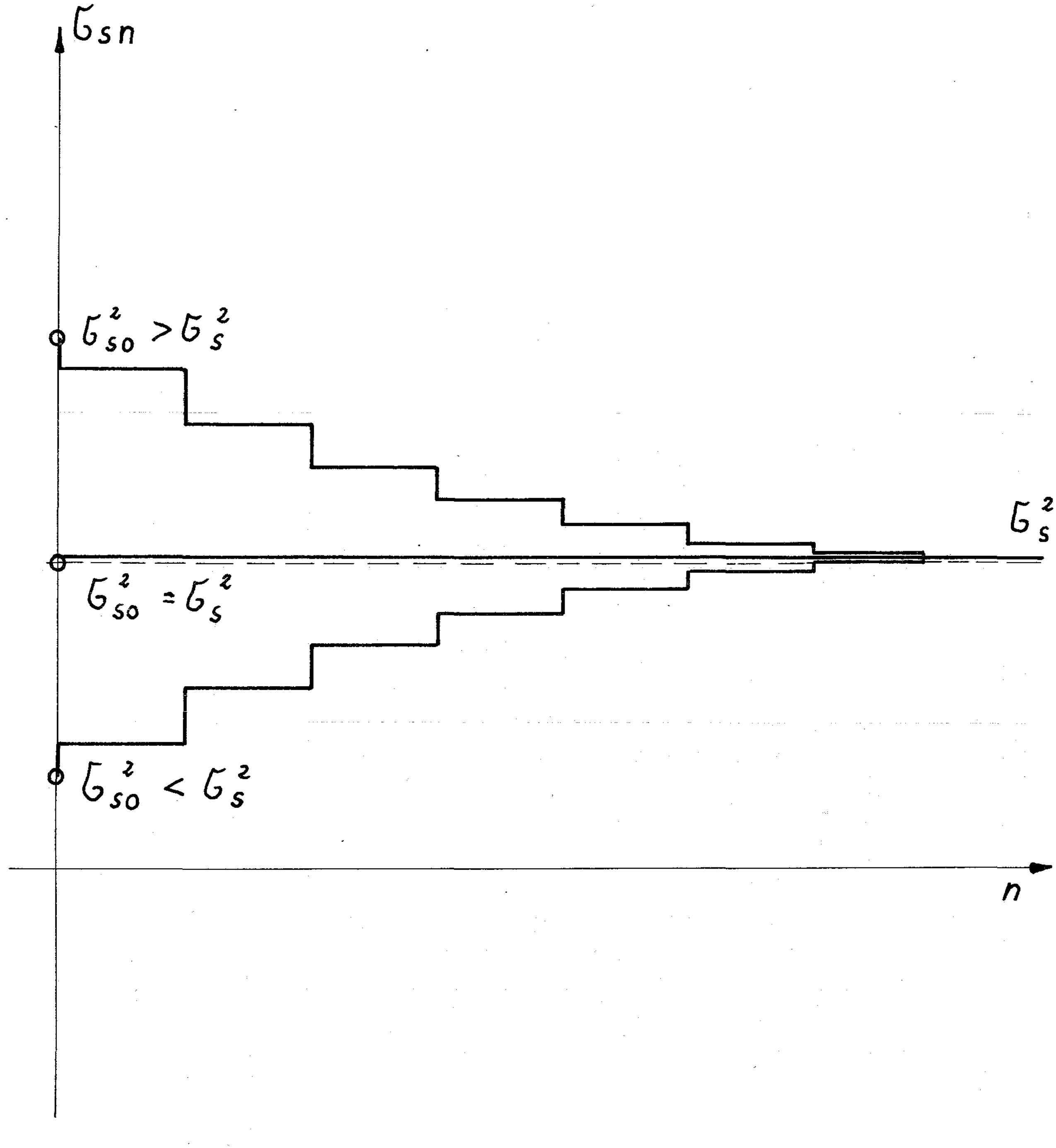


Fig. I-1 Time development of the σ_{sn}^2 for $\sigma_{so}^2 \gtreqless \sigma_s^2$

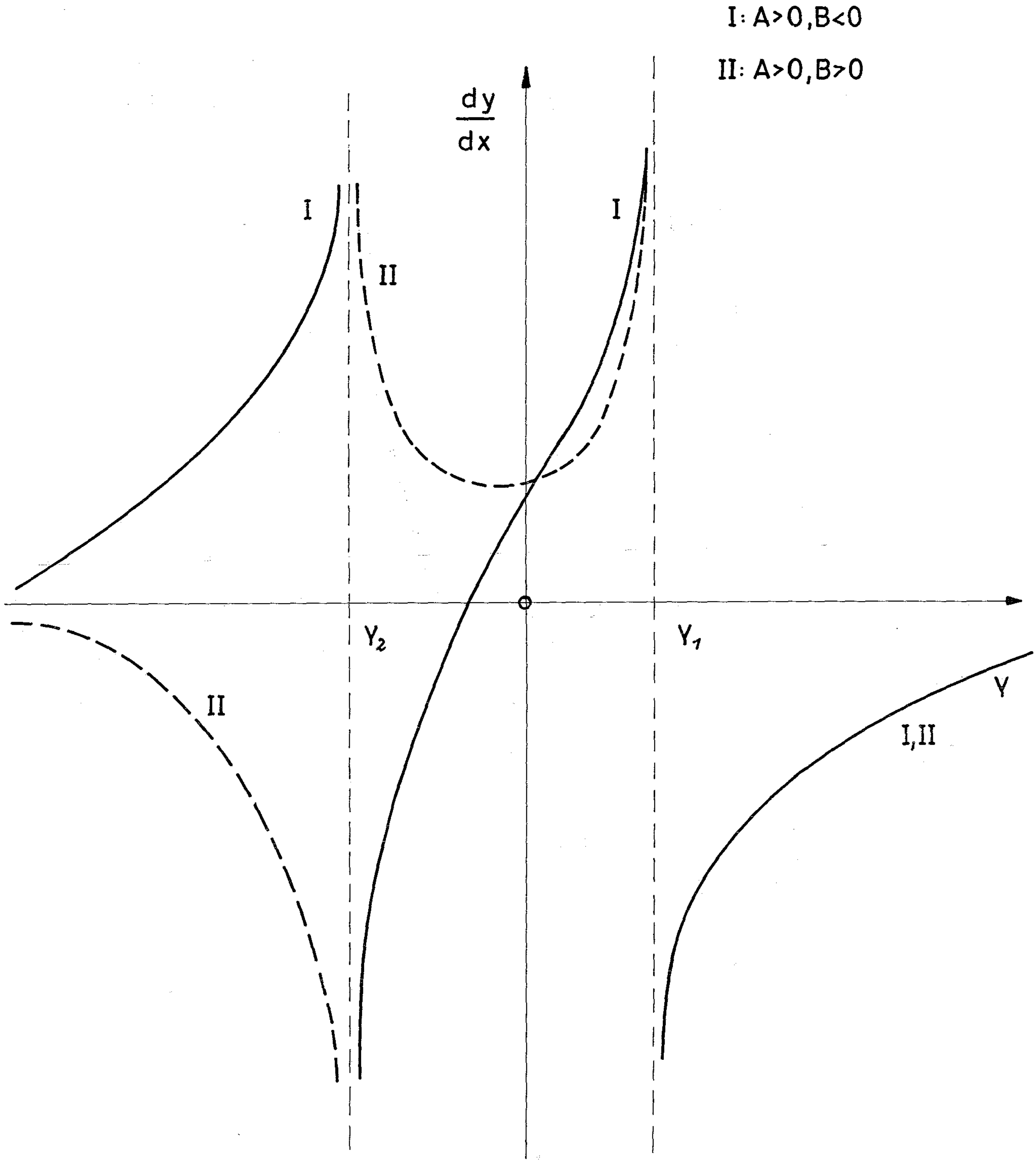


Fig. I-2: Graphical representation of the differential equation (2.13)

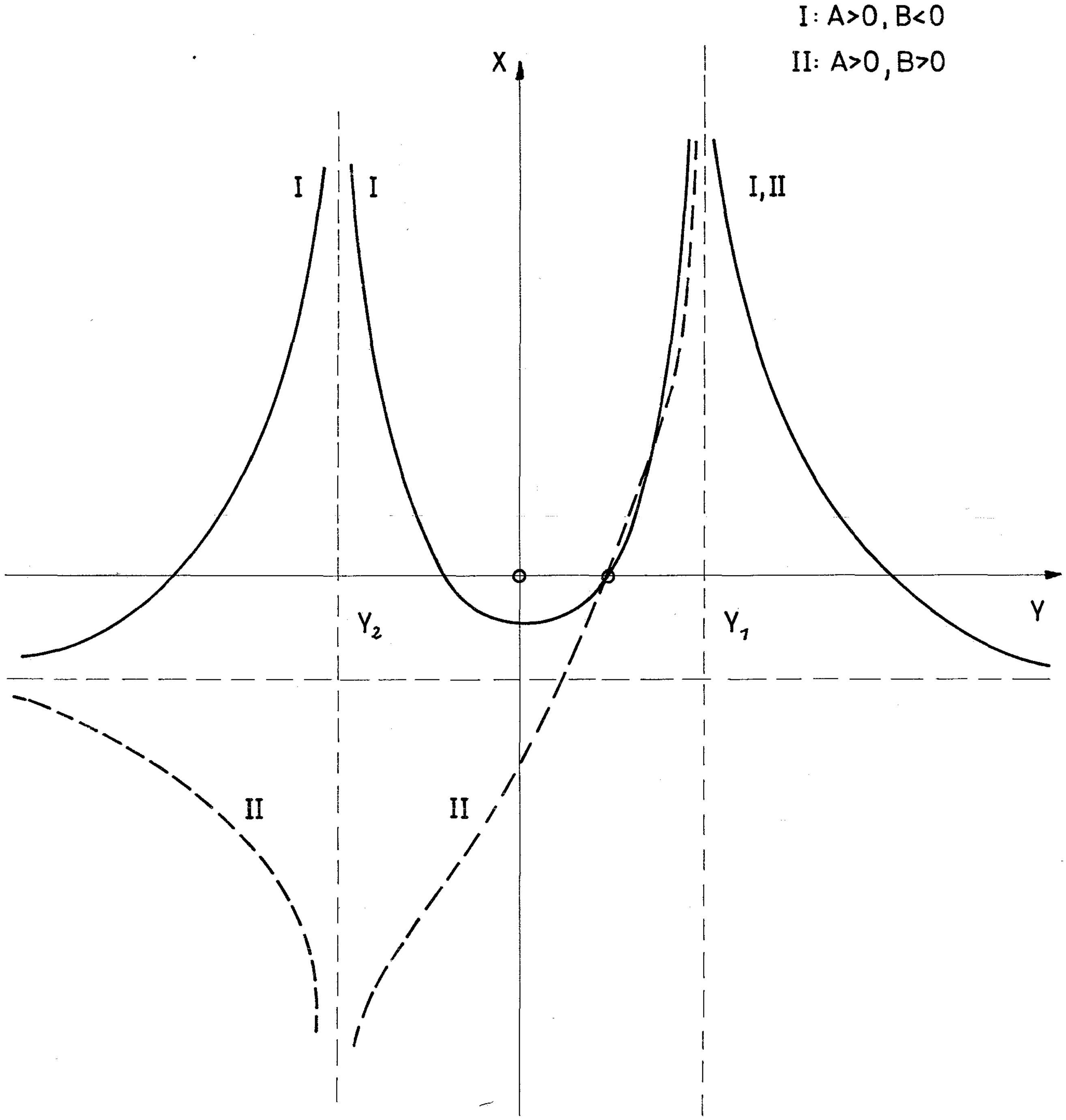


Fig. I-3: Graphical representation of the solution of the differential equation (2.13) x as a function of y

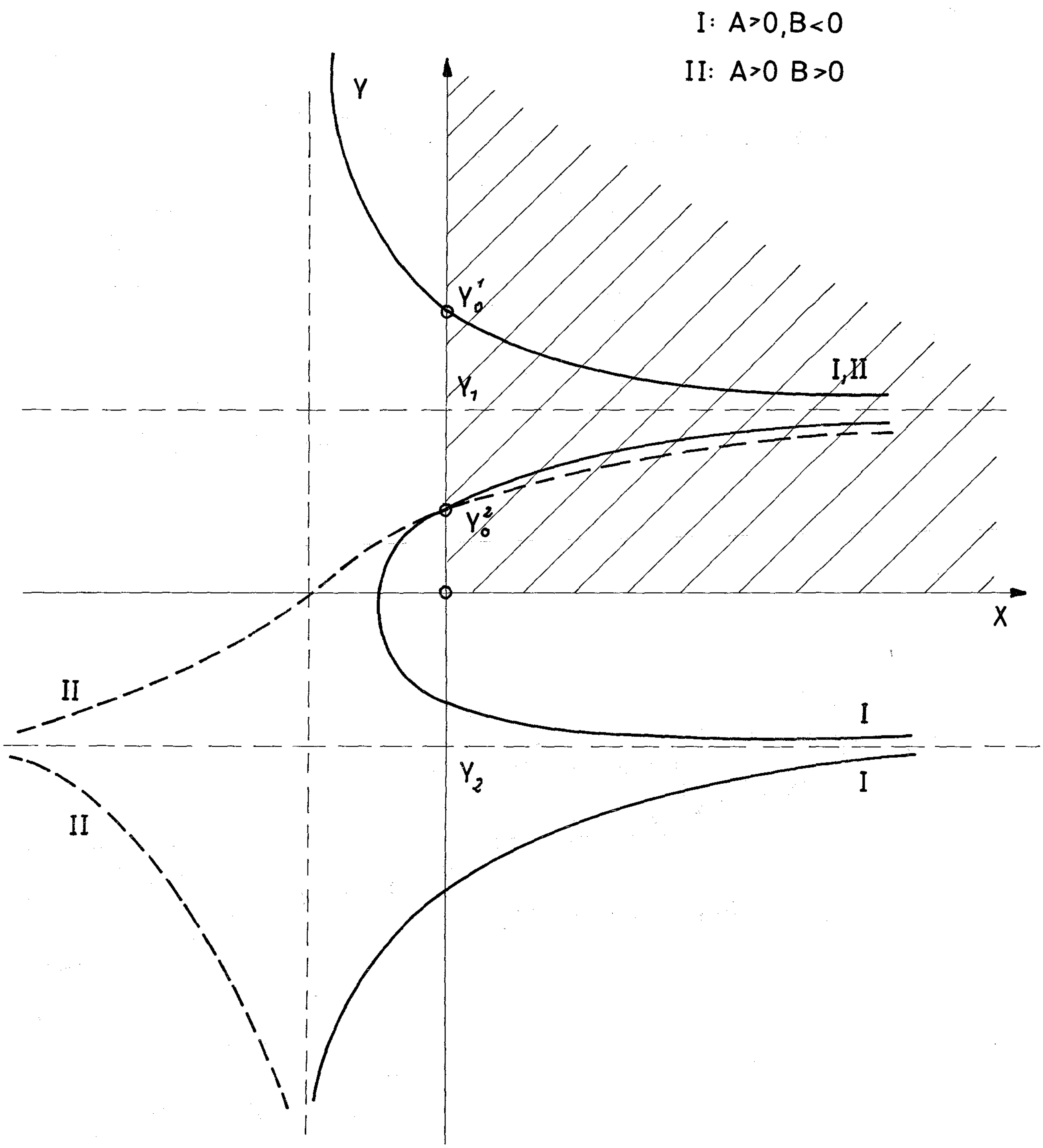


Fig. I-4: Graphical representation of the solution of the differential equation (2.13) y as a function of x.

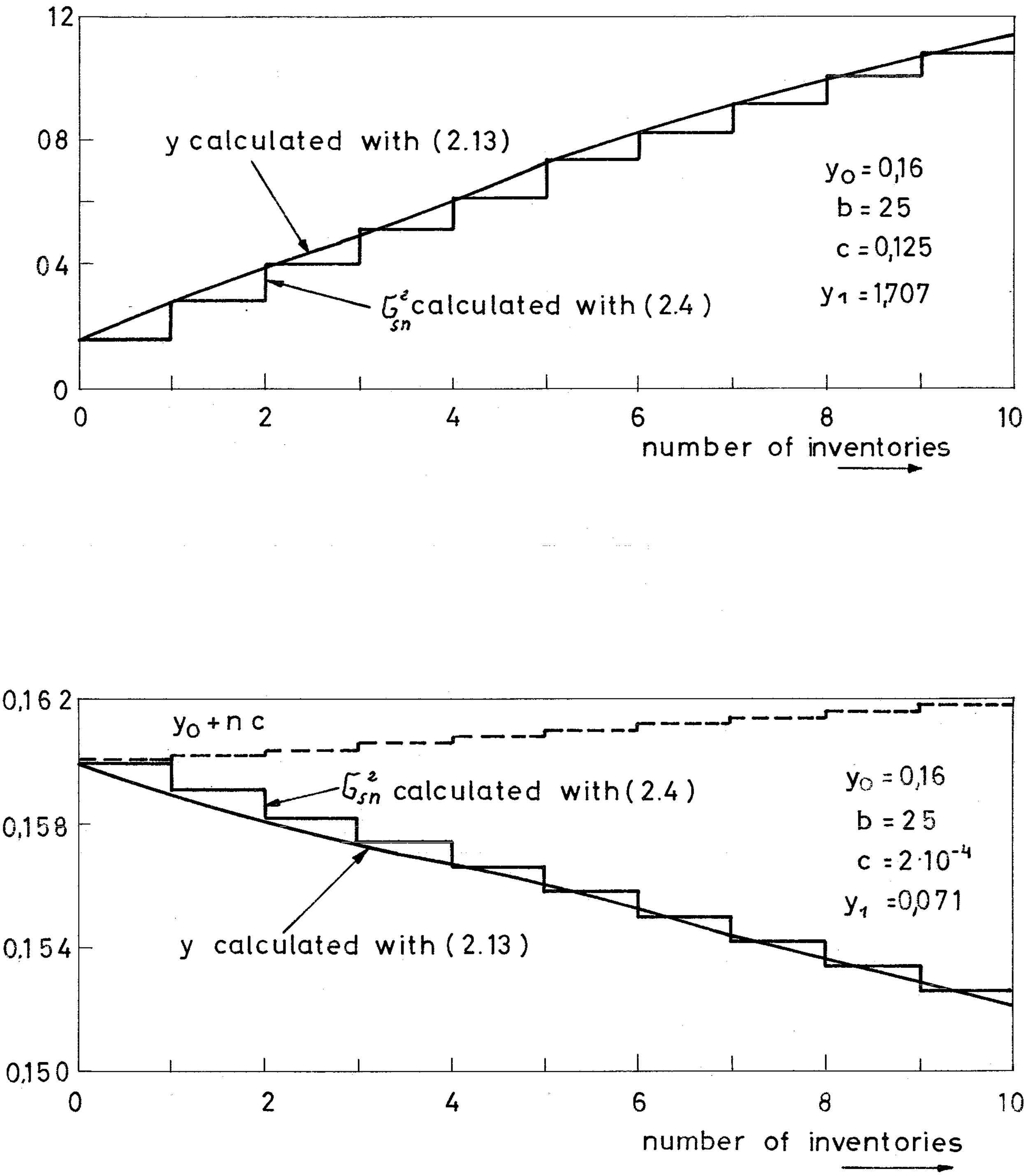


Fig. I-5: Variance of the estimator according to maximum likelihood method as a function of the number of inventories.

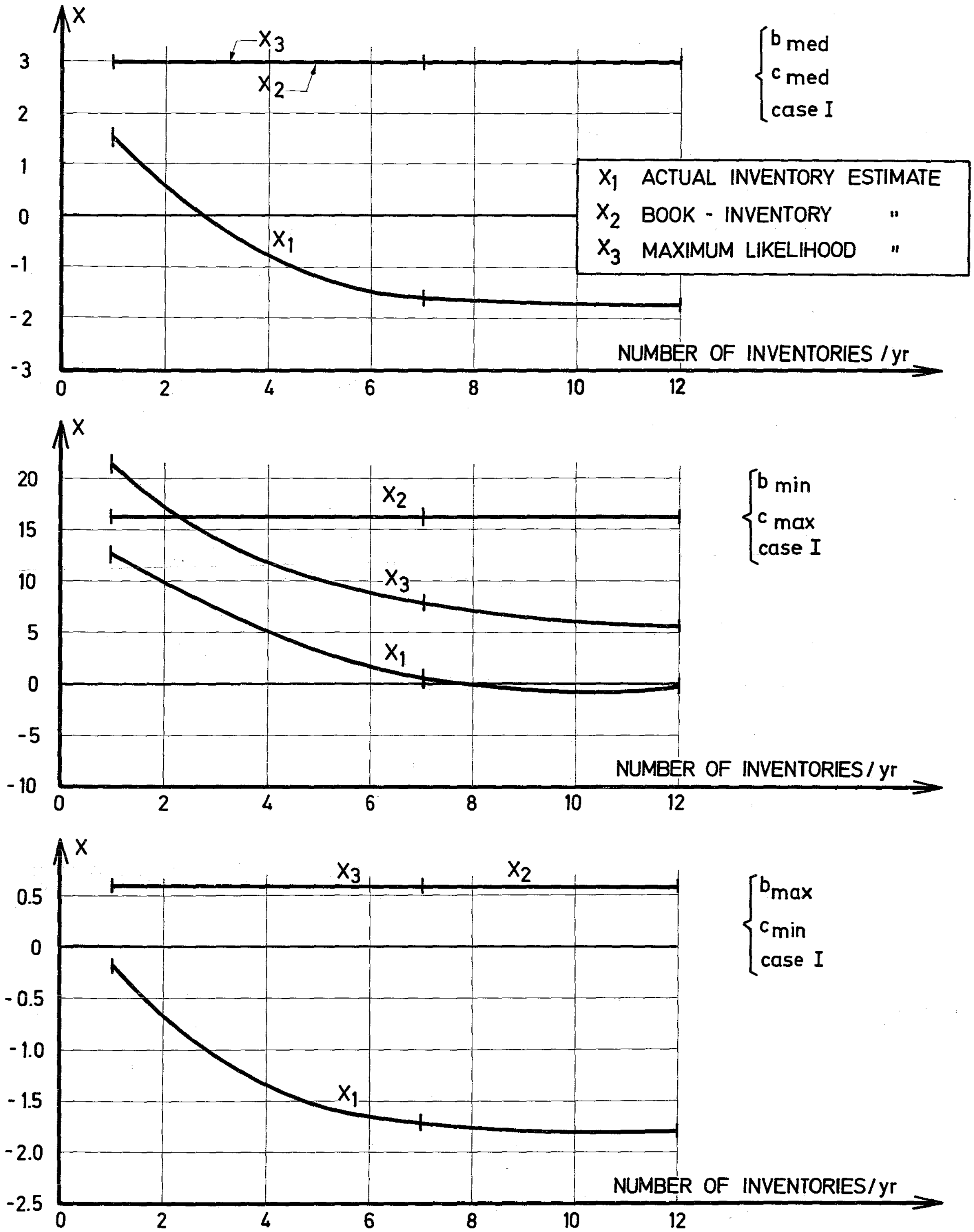


FIG. I-6 ARGUMENT X OF THE PROBABILITY OF DETECTION AS A FUNCTION OF THE NUMBER OF INVENTORIES PER YEAR, FOR EQUAL AMOUNTS DIVERTED IN EACH INVENTORY PERIOD

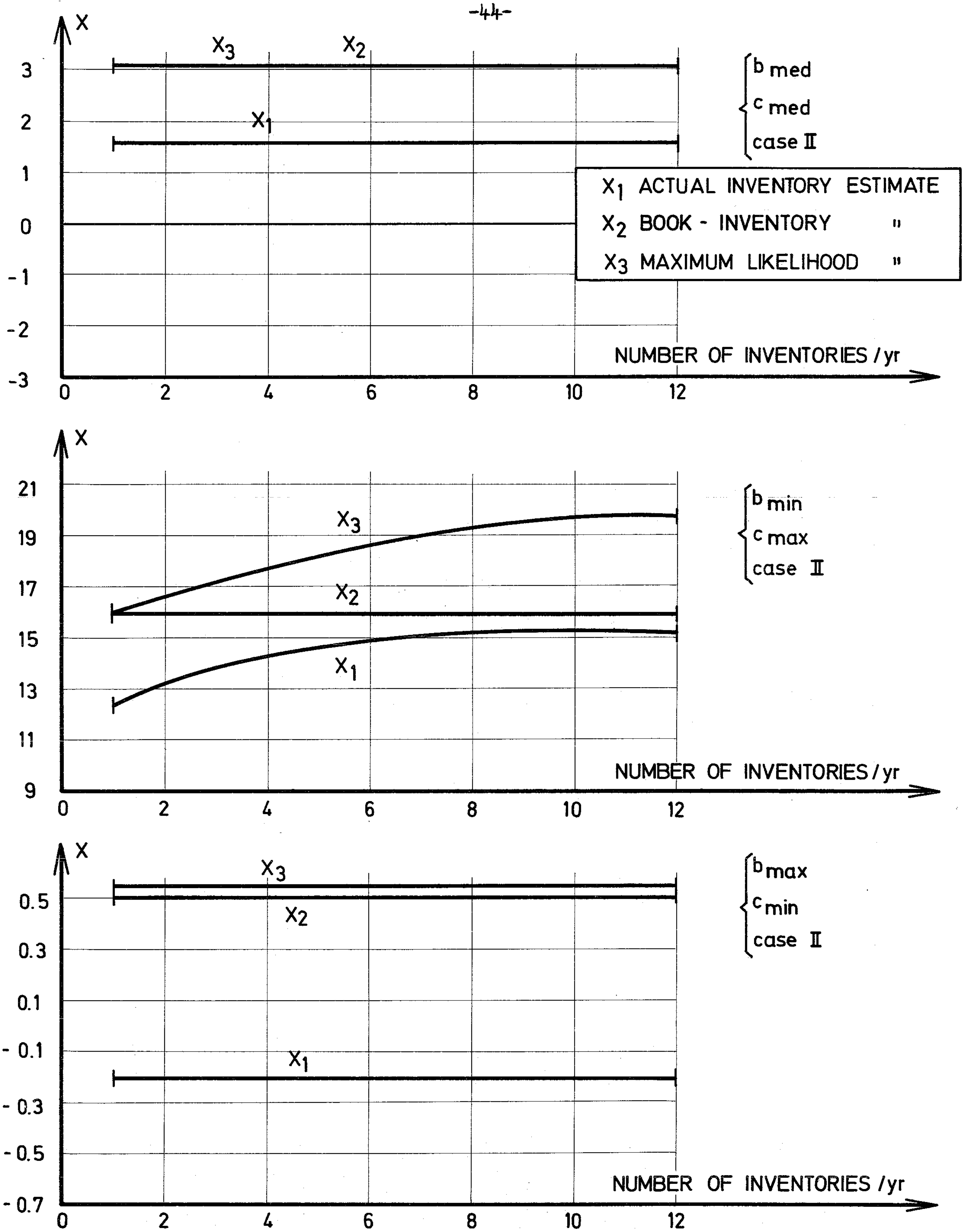


FIG. I-7 ARGUMENT X OF THE PROBABILITY OF DETECTION AS A FUNCTION OF NUMBER OF INVENTORIES PER YEAR, FOR A SINGLE CASE OF DIVERSION

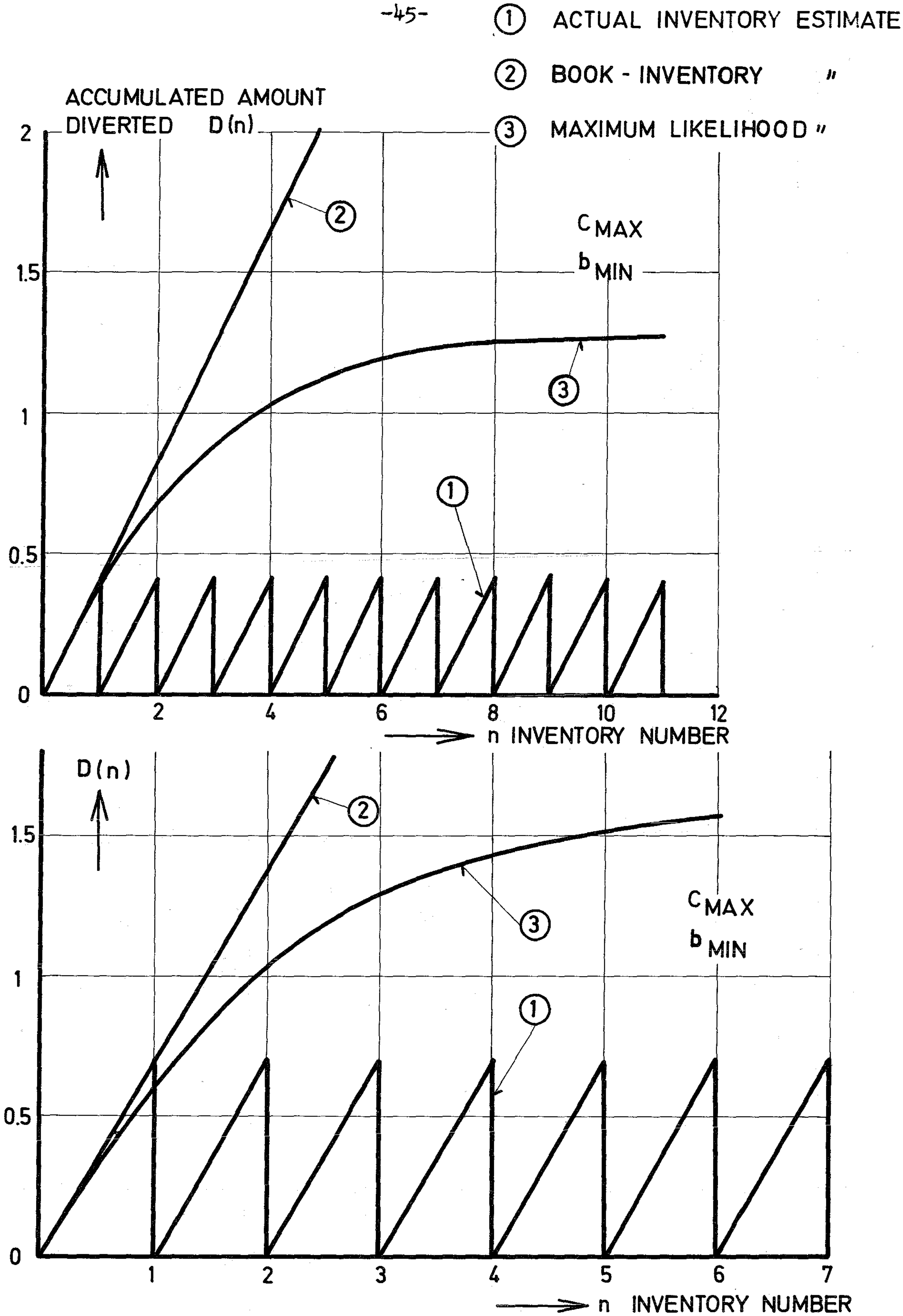


FIG.I-8 ACCUMULATED AMOUNTS OF DIVERSION AS CONSIDERED BY THE THREE ESTIMATION METHODS

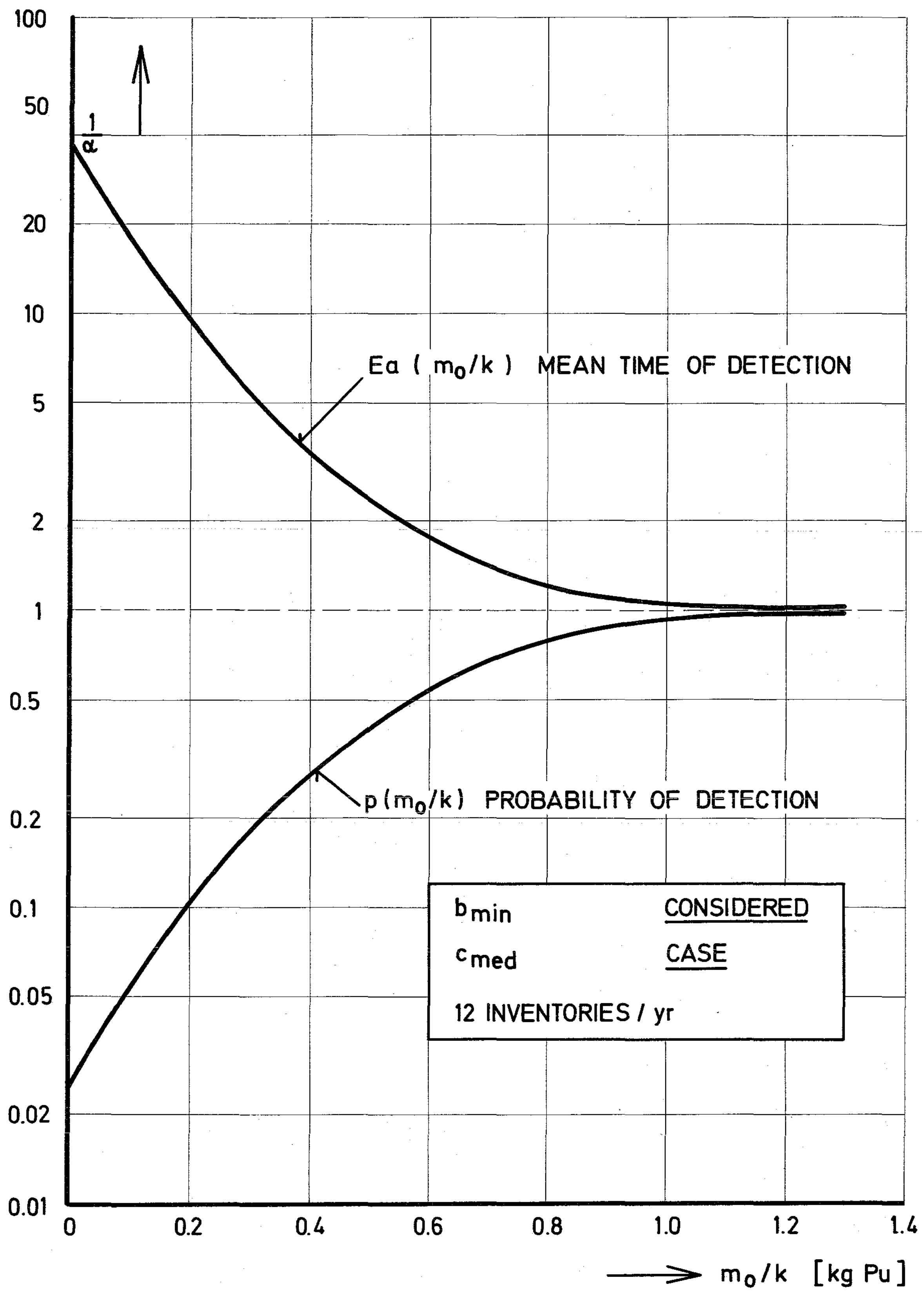


FIG. I - 9 MEAN TIME OF DETECTION AND PROBABILITY OF DETECTION AS A FUNCTION OF THE AMOUNT OF Pu, DIVERTED BETWEEN TWO INVENTORIES

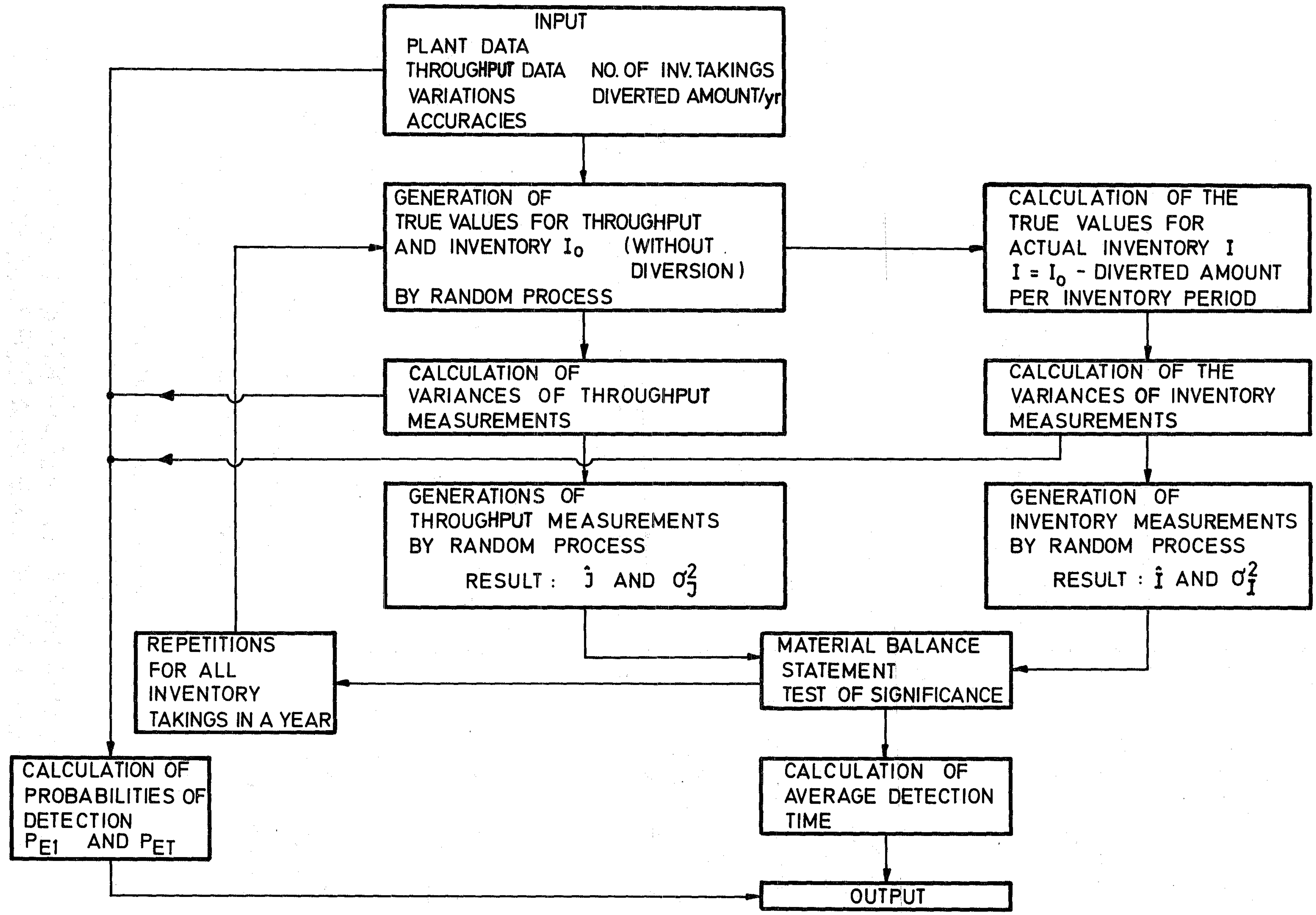


FIG. II-1 SCHEME OF THE GENERATION OF THE MEASUREMENTS BY DIGITAL SIMULATION

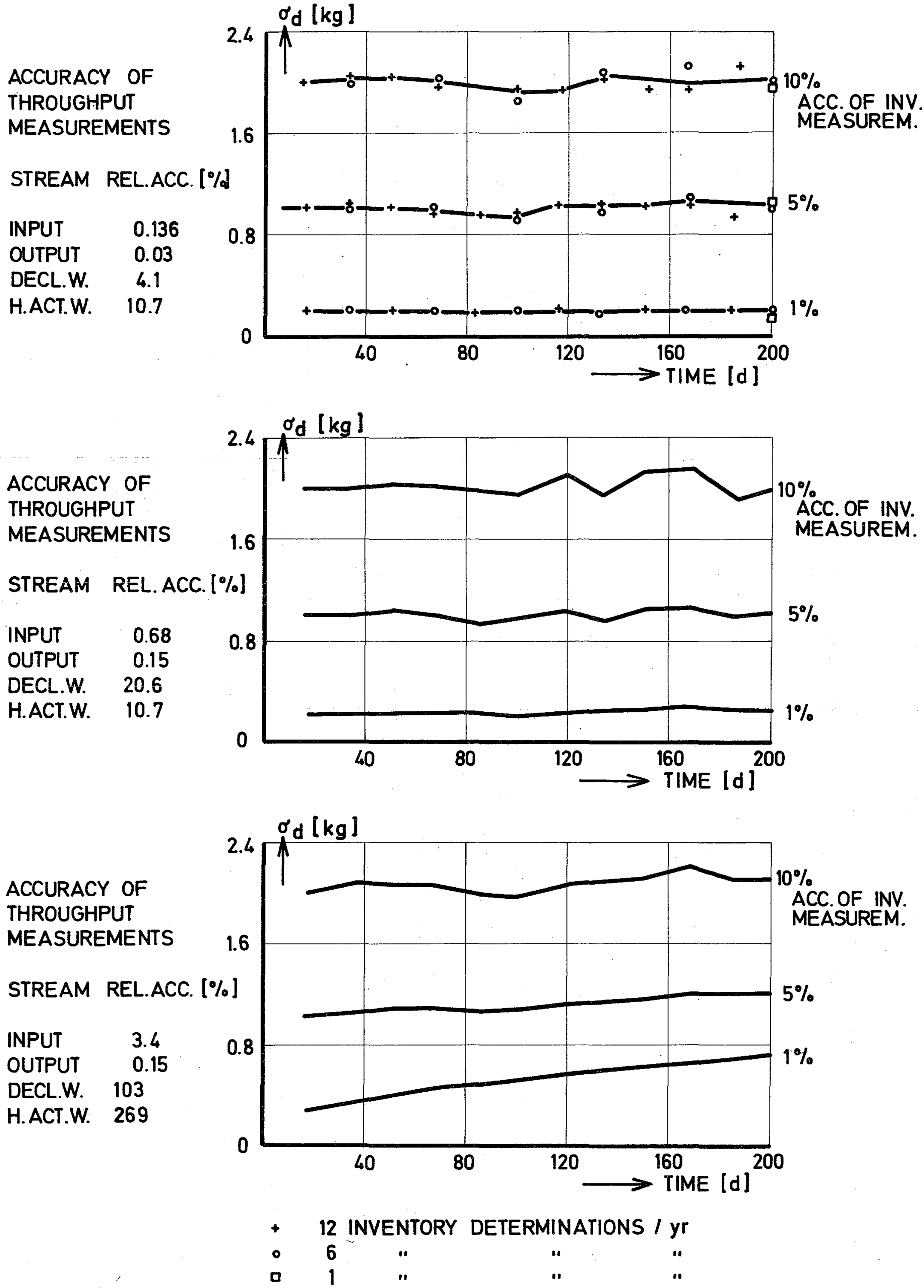


FIG. II-2 PROPAGATION OF MEASUREMENT ERROR AS A FUNCTION OF TIME IN A REPROCESSING PLANT

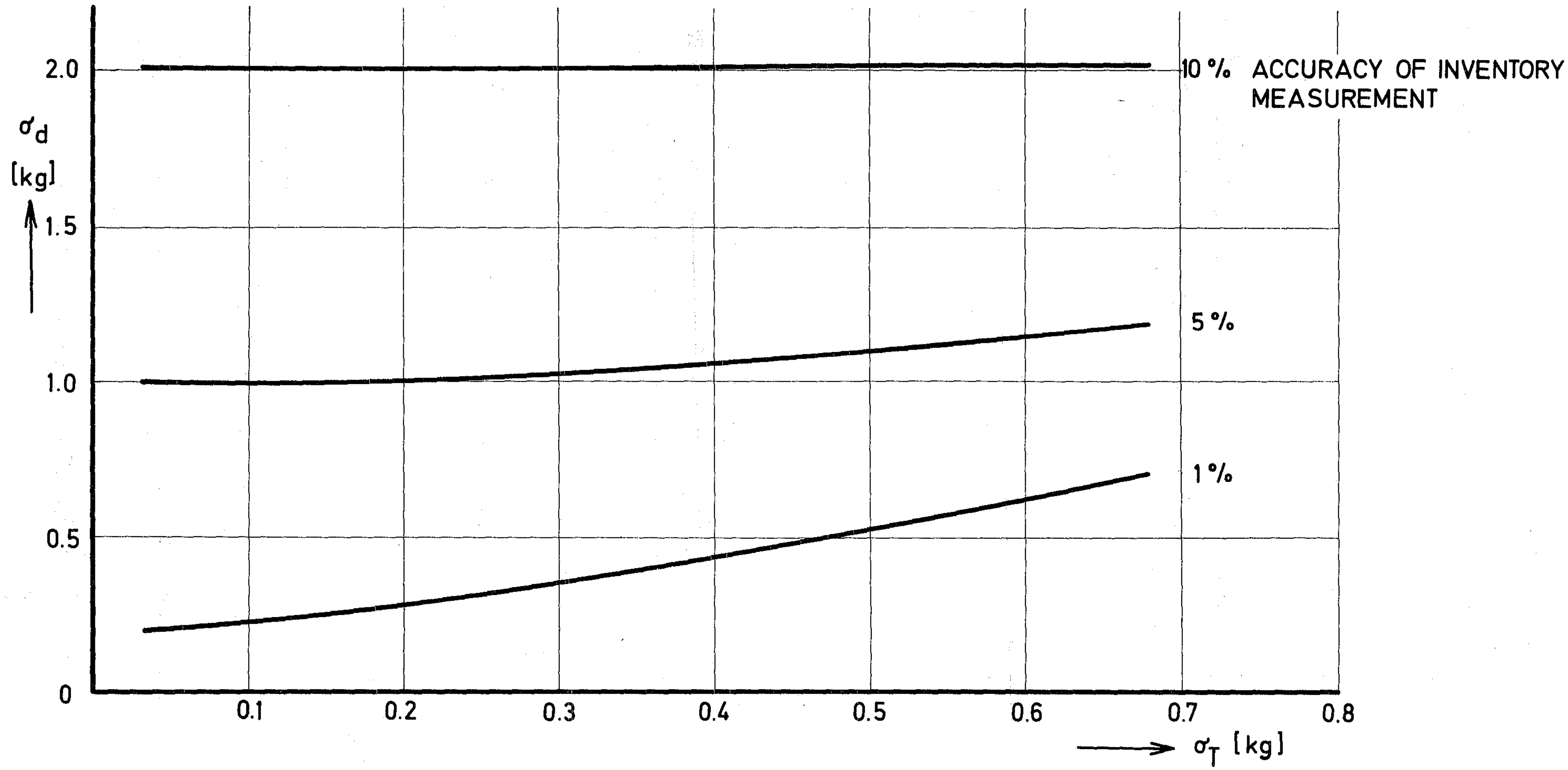


FIG.II-3 RELATIONS BETWEEN THE STANDARD DEVIATION OF THE MATERIAL BALANCE σ_d AND THE THROUGHPUT MEASUREMENT σ_T FOR A TYPICAL REPROCESSING PLANT

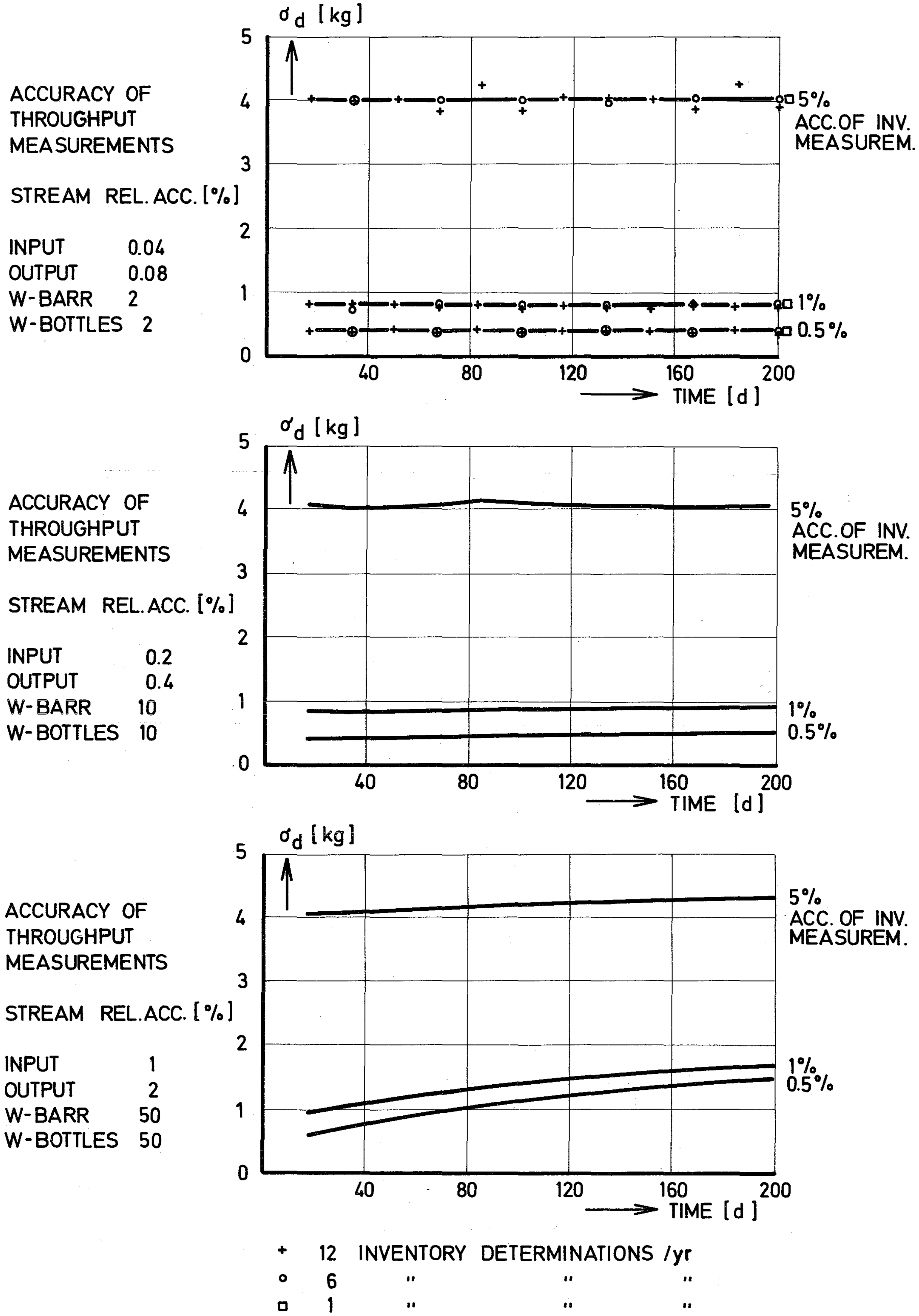


FIG. II-4 PROPAGATION OF MEASUREMENT ERROR AS A FUNCTION OF TIME IN A FABRICATION PLANT

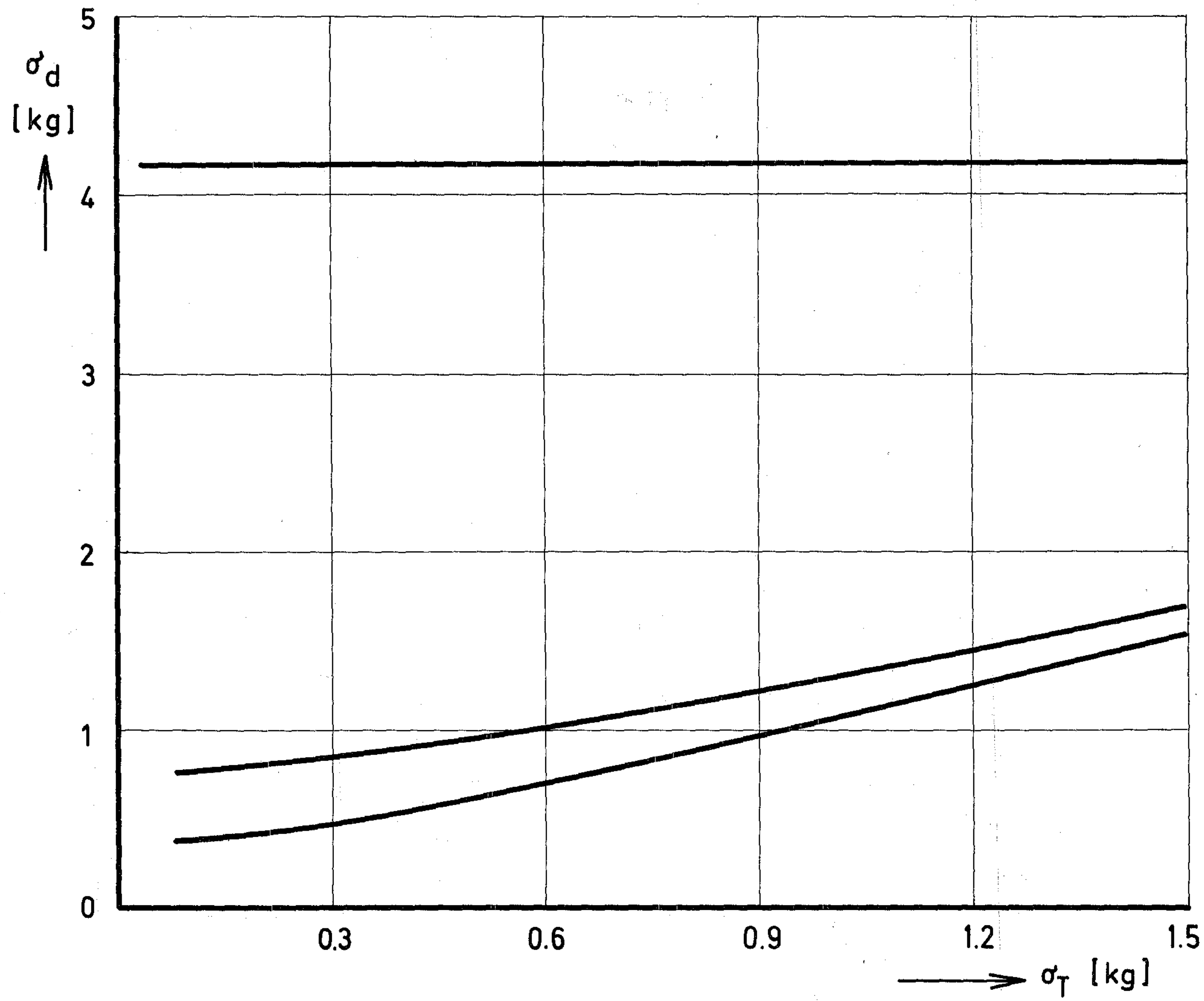
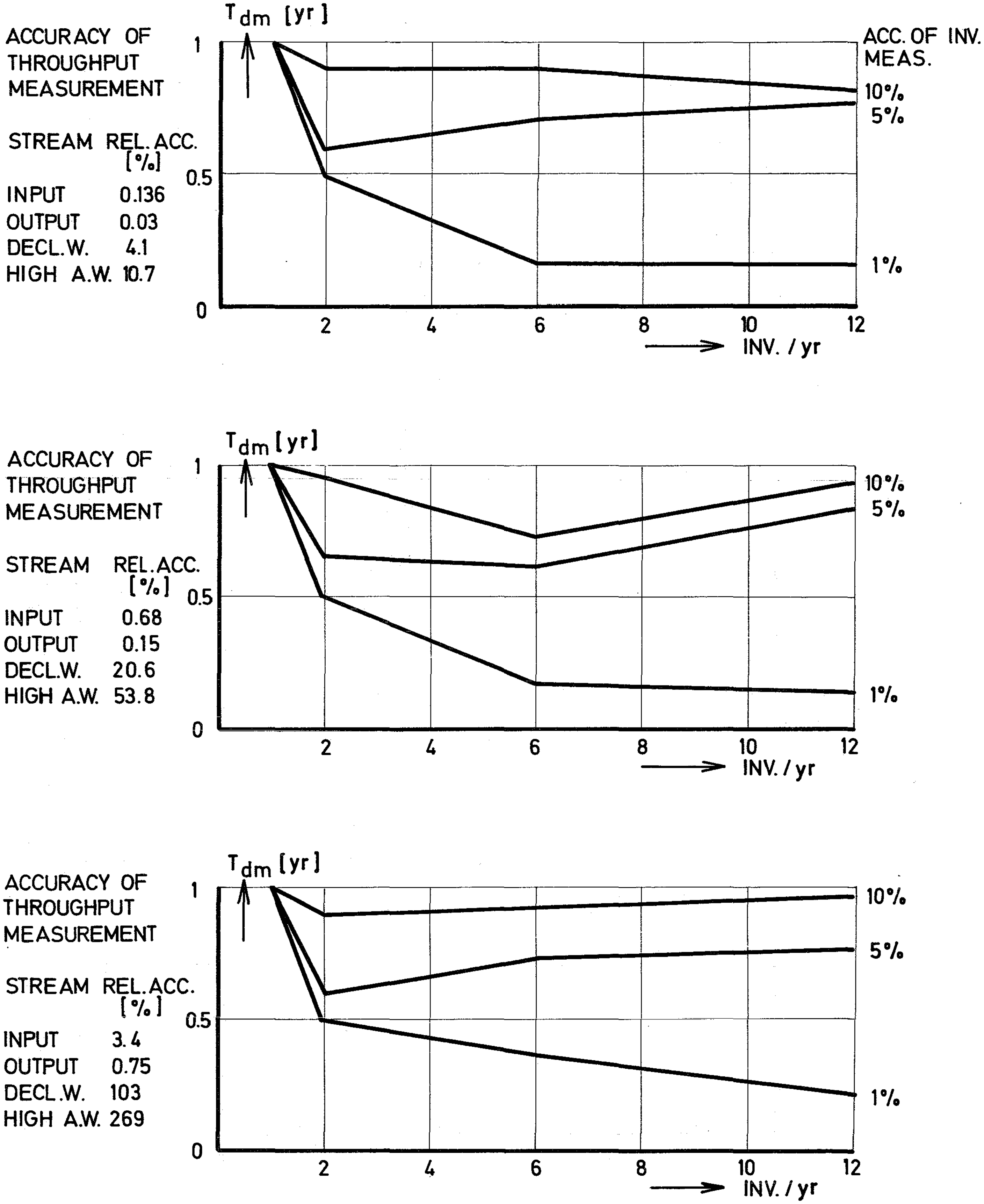
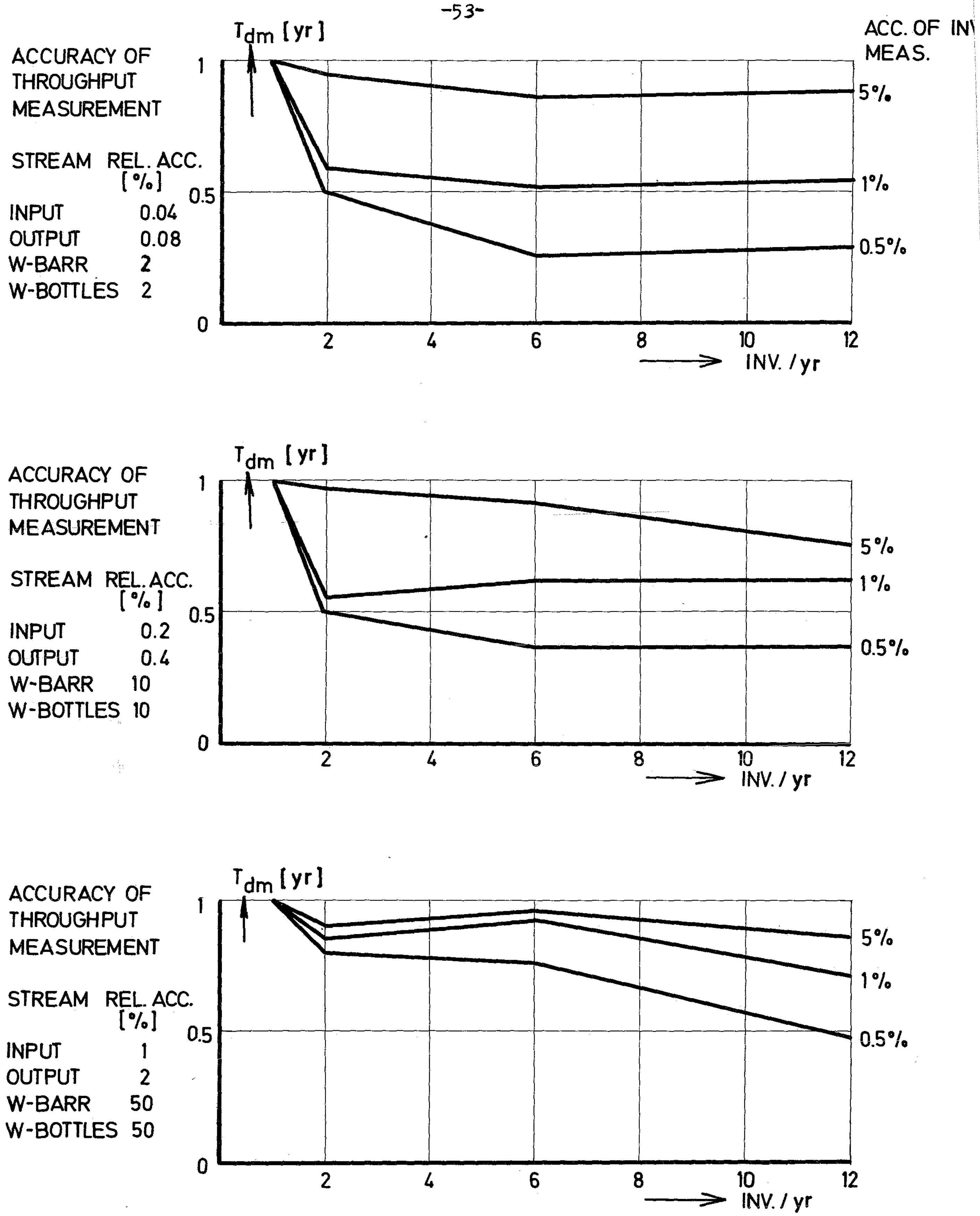


FIG. II - 5 DEPENDENCE BETWEEN THE STAND. DEVIATION OF THE MATERIAL BALANCE σ_d AND THE THROUGHPUT MEASUREMENT σ_T FOR A TYPICAL REFABRICATION PLANT



AMOUNT DIVERTED : 5 Kg Pu / yr

FIG. II-6 DETECTION TIME T_{dm} AS A FUNCTION OF THE NUMBER OF INVENTORY TAKINGS FOR A TYPICAL REPROCESSING PLANT



AMOUNT DIVERTED: 5Kg Pu/yr

FIG. II-7 DETECTION TIME T_{dm} AS A FUNCTION OF THE NUMBER OF INVENTORY TAKINGS FOR A TYPICAL FABRICATION PLANT