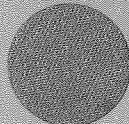


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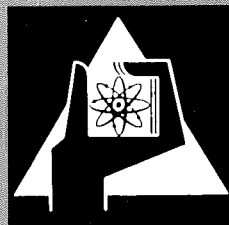
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Institut für Angewandte Reaktorphysik

CRASH: a Computer Programme for the Analysis of Creep and
Plasticity in Fuel Pin Sheaths

M. Guyette

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plasticity in fuel pin sheaths

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S u m m a r y

A computer programme, calculating the thermal and mechanical behaviour of fuel pin sheaths, has been written in the framework of the development of the sodium cooled fast breeder reactor SNR.

This programme, named CRASH (CReep Analysis in a fuel pin SHEath), allows to determine the evolution in the time, of the tri-axial stress and strain states, when creep or plasticity takes place in the material.

The basic assumption for this calculation is the axi-symmetry of the sheath. The loads, which can be taken into account by the programme, are: an outer pressure, an inner pressure, a temperature gradient and a fuel-clad radial and axial interaction. These loads can be any functions of the time.

The programme has been conceived in a sufficiently general way as to allow modifications, as soon as the knowledge of the material behaviour under irradiation would improve.

It is so that the programme can use any creep or plastic law and that the material properties can be any functions of the temperature.

On the other hand, in spite of this general character, the computing time is sufficiently small as to allow parametric studies. The possibilities of application of the programme are thus rather large: they cover the design of fuel pins as well as the analysis of experimental results of irradiation on sheath materials.

Z u s a m m e n f a s s u n g

Im Rahmen der Entwicklung des Schnellen Natriumgekühlten Brutreaktors SNR wurde ein Maschinenprogramm geschrieben, das das thermische und mechanische Verhalten der Brennelementhülle berechnet.

Das Programm mit dem Namen CRASH (Creep Analysis in a fuel pin Sheath) bestimmt die dreidimensionalen Spannungs- und Dehnungszustände in Abhängigkeit von der Zeit bei Kriech- oder Plastizitätsverhalten des Materials.

Die Grundvoraussetzung der Rechnung ist die Axialsymmetrie der Hülle. Die Belastungen der Hülle, die in dem Programm berücksichtigt werden können, sind: der Außendruck, der Innendruck, der Temperaturgradient und die axiale und radiale Wechselwirkung des Brennstoffs mit der Hülle. Diese Größen können beliebige Funktionen der Zeit sein.

Das Programm wurde in einer genügend allgemeinen Form verfaßt, um leicht Umschreibungen vornehmen zu können, die den Fortschritt im Wissen um das Festkörperverhalten unter Bestrahlung berücksichtigen. So kann das Programm beliebige Kriech- oder Plastizitätsgesetze verwenden und die Materialeigenschaften können ebenso dem neuesten Kenntnisstand entsprechend eingegeben werden.

Andererseits ist die Rechenzeit trotz des allgemeinen Charakters genügend klein, um Parameterstudien durchführen zu können. Die Möglichkeiten der Anwendung des CRASH-Programmes sind so recht groß: sie erfassen den Entwurf eines Brennstabes ebenso wie die Analyse experimenteller Ergebnisse der Bestrahlungen von Hüllmaterialien.

S o m m a i r e

Un programme de calcul du comportement mécanique et thermique des gaines des éléments combustibles a été écrit dans le cadre du développement du réacteur rapide, surgénérateur, refroidi au sodium SNR.

Ce programme, appelé CRASH (CReep Analysis in a fuel pin SHeath), permet de déterminer l'évolution dans le temps, des états tri-axiaux de contrainte et d'allongement des gaines, lorsque du fluage ou de la plasticité a lieu dans le matériau.

L'hypothèse de base pour ce calcul est que la gaine présente une symétrie de révolution. Les sollicitations, qui peuvent être prises en compte par le programme, sont: une pression extérieure, une pression intérieure, un gradient de température et une interaction radiale et axiale entre le combustible et la gaine. Ces sollicitations peuvent être des fonctions quelconques du temps.

Le programme a été conçu de façon très générale, de manière à pouvoir être amélioré à chaque progrès des connaissances sur le comportement mécanique des matériaux sous irradiation.

C'est ainsi que le programme peut traiter n'importe quelle loi de fluage ou de plasticité et que les propriétés du matériau considéré sont toutes des fonctions quelconques de la température.

D'autre part, malgré le caractère général du programme, le temps de calcul est suffisamment réduit que pour permettre des études paramétriques.

Les possibilités d'application du programme sont donc vastes: elles vont du design d'éléments combustibles à l'analyse de résultats expérimentaux d'irradiation sur des matériaux de gainage.

0. Introduction

A computer programme calculating the thermal and mechanical behaviour of fuel pin sheaths has been written in the framework of the development of the sodium cooled fast breeder reactor (SNR).

The report summarizes the method of calculation used and gives a brief description of the programme. In the first part the basic equations for the calculations of the stresses and strains will be given.

The second part shows how these equations are applied in the programme and describes briefly the auxiliary calculations.

The third part gives a description of the programme, its input and output data.

1. General method of calculation

1.1. Basic assumptions

One has assumed, throughout this report, that the fuel pin sheath could be considered as a perfect circular cylinder.

On the other hand, all the loads on the sheath (inner and outer pressure, axial force, temperature distribution) are considered as axisymmetric.

The sheath is considered as a long tube and the calculations are performed for cross sections far enough from the ends, in order to have negligible end effects. Furthermore, axial variations of the loads, as an axial temperature gradient or an axial variation of the fuel sheath contact pressure, are considered as negligible.

The material is assumed isotropic in what concerns all its properties.

The creep or plasticity occurs at constant volume.

All the calculations will be handled in cylindrical coordinates (r , θ , z).

1.2. Components of the stresses and strains

The analysis of stresses and strains in a solid body requires the determination of the various components of the stresses and the strains for all the points of the solid.

In a general problem in cylindrical coordinates, one must determine the 6 components of the stresses σ_r , σ_θ , σ_z , $\tau_{r\theta}$, τ_{rz} , $\tau_{\theta z}$ and the 6 components of the total strain ϵ_r , ϵ_θ , ϵ_z , $\gamma_{r\theta}$, γ_{rz} , $\gamma_{\theta z}$.

The figure 1 shows a scheme of a small element of volume, in cylindrical coordinates, with the various components of the stresses on each face. The arrows indicate the positive directions of the various stress components. The convention of sign is the same as in [1]:

- the normal components are positive if they produce tension and negative if they produce compression
- the positive direction of the shear components is the same as the positive direction of the coordinates axes if a tensile normal stress has the positive direction on the corresponding axis.

1.3. Symmetry considerations

With the assumptions of symmetry made hereabove (para 1.1.), some of the stress and strain components disappear:

- i) In axisymmetric problems the stress components $\tau_{r\theta}$, $\tau_{\theta z}$ and the total strain components $\gamma_{r\theta}$, $\gamma_{\theta z}$ must be equal to zero, to satisfy the condition of symmetry. Moreover, all the other components are independent of the coordinate θ [1].
- ii) In long tubes, where axial variations of the loads are assumed negligible, all the cross sections remain plane and perpendicular to the axis of the cylinder after deformation. This can be easily shown: consider two adjacent axial slices in the sheath (fig. 2a). These two elements have identical deformations. Furthermore both elements are symmetrical with respect to their mid-plane and, as the loads are symmetrical with respect to this plane, the deformations are also symmetrical with respect to the mid-plane. If the cross sections don't remain plane, the two elements can no more be applied on each other after deformation (fig. 2b). The continuity of the material could no more comply with. The cross sections must thus remain plane after deformation (fig. 2c).

From the condition of plane axial deformation, it results that γ_{rz} is equal to 0. This fact imposes in turn that τ_{rz} is equal to 0.

As the loads variation is axially negligible, the stresses and strains don't vary with z or, in other words, all their components are independent of z . Only one section in the whole sheath is thus considered for the calculation.

However, if one wants to know the influence of the temperature level in the sheath or of the power per unit length, calculations can be done for several cross sections in the sheath. Nevertheless, these calculations assume that the axial temperature gradient or the axial variation in power distribution have no influence on the stress and strain distribution in the considered cross-section.

With the assumptions of symmetry made, only three components of the stresses (σ_r , σ_θ and σ_z) and three components of the total strains (ϵ_r , ϵ_θ and ϵ_z) are different of zero. This means, in other words, that the principal directions are the coordinate axes directions.

Moreover, all the components of the stresses and strains are functions of the radial coordinate only.

1.4. Basic equations for the determination of the stresses and strains

In the theory of elasticity [1], use is made of three types of equations to determine the stress and strain states of elastic materials: the equilibrium equations, the compatibility equations and the relations between stresses and strains. As will be shown later, in the two first types of equations, no assumption is made about the behaviour (elastic, creeping or plastic) of the material. These two first types of equations can thus also be used for creep or plastic analysis. The third type of equations describes the behaviour of the material. This will be first investigated.

1.4.1. Relations between stresses and strains

In creep or plasticity problems, the total strains are usually divided in three parts:

- the elastic strains (noted by the index el)
- the thermal strains (noted by the index th)
- the creep or plastic strains (noted by the index c)

The elastic strains are well known [1]. They are expressed in terms of the stresses by the classical Hooke's laws:

$$\epsilon_{r \text{ el}} = \frac{1}{E} [\sigma_r - \mu(\sigma_\theta + \sigma_z)] \quad (1.1a)$$

$$\epsilon_{\theta \text{ el}} = \frac{1}{E} [\sigma_\theta - \mu(\sigma_r + \sigma_z)] \quad (1.1b)$$

$$\epsilon_{z \text{ el}} = \frac{1}{E} [\sigma_z - \mu(\sigma_r + \sigma_\theta)] \quad (1.1c)$$

The thermal strains are the same in the three directions. This comes from the assumption of isotropy of the material (para 1.1). One has:

$$\epsilon_{r \text{ th}} = \epsilon_{\theta \text{ th}} = \epsilon_{z \text{ th}} = \alpha T \quad (1.2)$$

In this expression, the value α represents the thermal expansion coefficient. If the thermal expansion is not linear with the temperature, α is a function of the temperature.

The value T represents in fact the temperature difference between the state for which the stresses and strains are calculated and a reference state, where the stresses are assumed equal to zero and for which the dimensions are given. The value T is generally a function of the coordinates. In this particular problem, it is a function of r only.

The relations between the creep or plastic strains and the stresses are empirical equations. These empirical equations are generally obtained from experimental results for uniaxial stress states. The empirical creep or plastic laws are generally of the form [2], [3]:

$$\epsilon_{\text{eq c}} = f(\sigma_{\text{eq}}, T, t, \dots) \quad (1.3)$$

These empirical laws are very often intricate and non linear. This fact brings up many difficulties in the mathematical solution of the equations.

The relation (1.3) has, of course, not the same expression for plasticity and creep. Its expression can also differ from one material to the other.

However, as will be shown later, an iterative solution of the equations is possible with the assumption that the permanent strains are known functions of the coordinates. The discussion of the form of the equation (1.3) will be made in more details later in para 2.6. and 2.7.

As the permanent strains obtained with equation (1.3) are written for a uniaxial state, the question arises then to find an equivalence between a uniaxial stress or strain state and a triaxial one. This question is discussed with some details in [4].

In this report use will be made of the Mises or of the Tresca theories:

- the Mises theory states that two states of stress or strain are equivalent when the deformation energy is equal for the two states. Knowing the state of stress or strain at a point in a tridimensionnal state, one can define an equivalent stress or strain in a monoaxial state, by equating the equality of the deformation energy for the two states. One gets finally:

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2} \quad (1.4)$$

and, in the same way, for the strains:

$$\epsilon_{eq} = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_r - \epsilon_\theta)^2 + (\epsilon_\theta - \epsilon_z)^2 + (\epsilon_r - \epsilon_z)^2} \quad (1.5)$$

- The Tresca theory states that two states of stress or strain are equivalent when the maximum shear stress is equal for the two states. This leads finally to the following definition of the equivalent stress:

$$\sigma_{eq} = \sigma_{max} - \sigma_{min} \quad (1.6)$$

where σ_{\max} and σ_{\min} are respectively the largest and the smallest values of σ_r , σ_θ and σ_z .

For ϵ_{eq} one has:

$$\epsilon_{eq} = \frac{2}{3} (\epsilon_{\max} - \epsilon_{\min}) \quad (1.7)$$

One has further assumed that the permanent strains verify the Mises relations [3], [5], [6]:

$$\frac{\epsilon_{rc} - \epsilon_{\theta c}}{\sigma_r - \sigma_\theta} = \frac{\epsilon_{rc} - \epsilon_{zc}}{\sigma_r - \sigma_z} = \frac{\epsilon_{\theta c} - \epsilon_{zc}}{\sigma_\theta - \sigma_z} \quad (1.8)$$

and the condition of constant volume [3], [5], [6]:

$$\epsilon_{rc} + \epsilon_{\theta c} + \epsilon_{zc} = 0 \quad (1.9)$$

The combination of the equation (1.8) and (1.9) with, either the relations (1.4) and (1.5), or (1.6) and (1.7), leads to the following relations between the creep strains and the stresses:

$$\epsilon_{rc} = \frac{\epsilon_{eq} c}{\sigma_{eq}} \left[\sigma_r - 1/2(\sigma_\theta + \sigma_z) \right] \quad (1.10a)$$

$$\epsilon_{\theta c} = \frac{\epsilon_{eq} c}{\sigma_{eq}} \left[\sigma_\theta - 1/2(\sigma_r + \sigma_z) \right] \quad (1.10b)$$

$$\epsilon_{zc} = \frac{\epsilon_{eq} c}{\sigma_{eq}} \left[\sigma_z - 1/2(\sigma_r + \sigma_\theta) \right] \quad (1.10c)$$

The relations (1.10) are known as the Soderberg equations [7]. In conclusion of this paragraph, one can give the expression of the relations between the total strains and the stresses:

$$\epsilon_r = \frac{1}{E} [\sigma_r - \mu(\sigma_\theta + \sigma_z)] + \alpha T + \epsilon_{rc} \quad (1.11a)$$

$$\epsilon_\theta = \frac{1}{E} [\sigma_\theta - \mu(\sigma_r + \sigma_z)] + \alpha T + \epsilon_{\theta c} \quad (1.11b)$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \mu(\sigma_r + \sigma_\theta)] + \alpha T + \epsilon_{zc} \quad (1.11c)$$

In these equations, the values of the creep or plastic strains can be replaced by their values given in (1.10).

1.4.2. Equilibrium equations

These equations express that each element of the solid body is in equilibrium under the various forces acting on it.

In tri-axial problems, there are generally three equilibrium equations, one for each of the three coordinate directions. For this particular problem, the projection of the forces gives a non trivial equation in the radial direction only [1]. On the figure 3, one has represented the stresses on the 4 lateral faces of an element of volume. The resulting forces on the various faces are worth:

- face 1 $(\sigma_r + d\sigma_r)(r + dr)d\theta dz$
- face 2 $-\sigma_r d\theta dz$
- face 3 or 4 $\sigma_\theta dr dz$

Summing the projection of these forces in the radial direction, one gets:

$$(\sigma_r + d\sigma_r)(r + dr)d\theta dz - \sigma_r r d\theta dz - 2\sigma_\theta dr dz \sin \frac{d\theta}{2} = 0 \quad (1.12)$$

This leads finally to:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1.13)$$

1.4.3. Compatibility equations

These equations express that the solid remains continuous after deformation. The deformation of an element of volume are not arbitrary. Due to the presence of neighbouring elements, some restrictions exist to the deformations, which are expressed by relations between the components of the total strains.

With the symmetry considerations made in para 1.3, one can deduce very easily the compatibility equations. If u is the radial displacement, the value of the radial total strain is then:

$$\epsilon_r = \frac{du}{dr} \quad (1.14)$$

Consider now a circumference, having a radius r before the deformation of the solid. After deformation, its radius is $(r + u)$. The circumferential change in length is thus:

$$\Delta L = 2\pi(r + u) - 2\pi r = 2\pi u \quad (1.15)$$

and the expression of the tangential total strain:

$$\epsilon_\theta = \frac{\Delta L}{L} = \frac{2\pi u}{2\pi r} = \frac{u}{r} \quad (1.16)$$

Finally, from the symmetry considerations, the axial total strain is independent of r , θ and z . One gets thus:

$$\epsilon_z = C_3 = \text{constant} \quad (1.17)$$

1.5. Solution of the equations for known permanent strains

In this paragraph, a solution of the system composed of the equilibrium equation (1.13), the compatibility equations (1.14), (1.16) and (1.17) and the relations between stresses and total strains (1.11) will be given by assuming that the permanent strains ϵ_{rc} , $\epsilon_{\theta c}$ and ϵ_{zc} are known functions of the radial coordinate [3].

The values of the stresses, as functions of the total strains and of the permanent strains, are first determined from the equations (1.11).

One gets:

$$\sigma_r = \frac{E}{1-2\mu} \left[\frac{\mu \epsilon_t}{1+\mu} - \alpha T \right] + \frac{E}{1+\mu} (\epsilon_r - \epsilon_{rc}) \quad (1.18a)$$

$$\sigma_\theta = \frac{E}{1-2\mu} \left[\frac{\mu \epsilon_t}{1+\mu} - \alpha T \right] + \frac{E}{1+\mu} (\epsilon_\theta - \epsilon_{\theta c}) \quad (1.18b)$$

$$\sigma_z = \frac{E}{1-2\mu} \left[\frac{\mu \epsilon_t}{1+\mu} - \alpha T \right] + \frac{E}{1+\mu} (\epsilon_z - \epsilon_{zc}) \quad (1.18c)$$

with:

$$\epsilon_t = \epsilon_r + \epsilon_\theta + \epsilon_z \quad (1.19)$$

One replaces then the stresses in the equilibrium equation (1.13) by their values as a function of the strains, given by the equations (1.18).

One gets:

$$\begin{aligned} \frac{\mu}{1-2\mu} \frac{d\epsilon_t}{dr} - \frac{1+\mu}{1-2\mu} \frac{d(\alpha T)}{dr} + \frac{d\epsilon_r}{dr} - \frac{d\epsilon_{rc}}{dr} \\ + \frac{\epsilon_r - \epsilon_\theta}{r} - \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} = 0 \end{aligned} \quad (1.20)$$

One uses then the compatibility equations (1.14), (1.16), (1.17) to replace the values of ϵ_r , ϵ_θ and ϵ_z in the equation (1.20).

Noting that:

$$\epsilon_t = \epsilon_r + \epsilon_\theta + \epsilon_z = \frac{du}{dr} + \frac{u}{r} + C_3 \quad (1.21)$$

one has:

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = \frac{1-2\mu}{1-\mu} \left[\frac{d\epsilon_{rc}}{dr} + \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} \right] + \frac{1+\mu}{1-\mu} \frac{d(\alpha T)}{dr} \quad (1.22)$$

The equation (1.22) allows to find the expression of the radial displacement u . The integration is performed as follows:

The equation (1.22) can still be written:

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d(ur)}{dr} \right] = \frac{1-2\mu}{1-\mu} \left[\frac{d\epsilon_{rc}}{dr} + \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} \right] + \frac{1+\mu}{1-\mu} \frac{d(\alpha T)}{dr} \quad (1.23)$$

A first integration between the limits a and r gives, after grouping all the constants into C_1 :

$$\frac{d(ur)}{dr} = \frac{1-2\mu}{1-\mu} \left[r\epsilon_{rc} + r \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right] + \frac{1+\mu}{1-\mu} \alpha T r + C_1 r \quad (1.24)$$

A second integration leads to:

$$ur = \frac{1-2\mu}{1-\mu} \left[\int_a^r r\epsilon_{rc} dr + \int_a^r r dr \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right] + \frac{1+\mu}{1-\mu} \int_a^r \alpha T r dr + C_1 r^2 + C_2 \quad (1.25)$$

Making use of the formula of integration by parts and of the Leibnitz formula, the term with double integral sign becomes:

$$\int_a^r r dr \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr = \frac{r^2}{2} \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr - \frac{1}{2} \int_a^r (\epsilon_{rc} - \epsilon_{\theta c}) r dr \quad (1.26)$$

One gets finally for the expression of the radial displacement:

$$u = \frac{1 - 2\mu}{2(1 - \mu)} \left[\frac{1}{r} \int_a^r r (\epsilon_{rc} + \epsilon_{\theta c}) dr + r \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right] + \frac{1 + \mu}{1 - \mu} \frac{1}{r} \int_a^r \alpha T r dr + C_1 r + \frac{C_2}{r} \quad (1.27)$$

Replacing now in equations (1.16) and (1.14) the value of u by the expression (1.27) one gets for the tangential and radial total strains:

$$\epsilon_{\theta} = \frac{1 + \mu}{1 - \mu} \frac{1}{r^2} \int_a^r \alpha T r dr + \frac{1 - 2\mu}{2(1 - \mu)} \left[\frac{1}{r^2} \int_a^r r (\epsilon_{rc} + \epsilon_{\theta c}) dr + \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right] + C_1 + \frac{C_2}{r^2} \quad (1.28)$$

$$\epsilon_r = -\epsilon_{\theta} + \frac{1 + \mu}{1 - \mu} \alpha T + \frac{1 - 2\mu}{1 - \mu} \left[\epsilon_{rc} + \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right] + 2C_1 \quad (1.29)$$

The equations (1.28), (1.29) and (1.17) allow to determine the total strains and, with the help of equations (1.18), the stresses in the sheath, at the condition that the permanent strains are known functions of the radius.

The constants C_1 , C_2 and C_3 , which appears in these equations, are integration constants. These last ones must be determined by the help of boundary conditions. This is the object of the two next paragraphs.

In the first one (para. 1.6), an analytical expression of the integration

constants will be given in the case where there is no contact between the fuel and the sheath.

In the second one (para. 1.7), the case of contact between fuel and sheath will be treated.

1.6. Determination of the integration constants. Boundary conditions in the absence of contact

The boundary conditions can be expressed, in this case, as follows:

- the radial stress at the inner face of the sheath must be equal to the inner gas pressure
- the radial stress at the outer face of the sheath is equal to the outer pressure (coolant pressure)
- the integral of the axial stress on a cross section of the sheath is equal to the axial force acting on the tube

To equate the two first conditions, one needs the expression of the radial stress. This last one is obtained by replacing, in equation (1.18a), the values of ϵ_r , ϵ_θ and ϵ_z by their expressions (1.29), (1.28) and (1.17). One gets finally:

$$\begin{aligned} \sigma_r = E \left[- \frac{1}{1-\mu} \frac{1}{r^2} \int_a^r \alpha T r dr + \frac{1}{2(1-\mu^2)} \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr \right. \\ \left. - \frac{1-2\mu}{2(1-\mu^2)} \int_a^r (\epsilon_{rc} + \epsilon_{\theta c}) r dr \right. \\ \left. + \frac{C_1 + \mu C_3}{(1+\mu)(1-2\mu)} - \frac{C_2}{(1+\mu)r^2} \right] \end{aligned} \quad (1.30)$$

One equates now that:

$$(\sigma_r)_{r=a} = -p_a \quad (1.31)$$

$$(\sigma_r)_{r=b} = -p_b \quad (1.32)$$

The minus sign, appearing in the second member of these two equations, is due to the fact that a positive pressure corresponds to a compressive stress.

Putting:

$$\frac{1}{b^2} \int_a^b \alpha T r dr = L \quad (1.33)$$

$$\frac{1}{b^2} \int_a^b (\epsilon_{rc} + \epsilon_{\theta c}) r dr = M \quad (1.34)$$

$$\int_a^b \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr = N \quad (1.35)$$

the conditions (1.31) and (1.32) become finally:

$$\frac{C_1}{1-2\mu} - \frac{C_2}{2} + \frac{\mu}{1-2\mu} C_3 = - (1+\mu) \frac{p_a}{E} \quad (1.36)$$

$$\begin{aligned} \frac{C_1}{1-2\mu} - \frac{C_2}{b^2} + \frac{\mu}{1-2\mu} C_3 &= \frac{1+\mu}{1-\mu} L + \frac{1-2\mu}{2(1-\mu)} M \\ &- \frac{1}{2(1-\mu)} N - (1+\mu) \frac{p_b}{E} \end{aligned} \quad (1.37)$$

In the third boundary condition, use is made of the axial stress. This last one is obtained with the help of equation (1.18c), in the same way as for σ_r hereabove. One has:

$$\begin{aligned} \sigma_z = E \left[- \frac{1}{1-\mu} \alpha T + \frac{\mu}{1-\mu^2} \epsilon_{rc} + \frac{\epsilon_{rc} + \epsilon_{\theta c}}{1+\mu} \right. \\ \left. + \frac{\mu}{1-\mu^2} \int_a^r \frac{\epsilon_{rc} - \epsilon_{\theta c}}{r} dr + \frac{2\mu C_1 + (1-\mu)C_3}{(1-2\mu)(1+\mu)} \right] \end{aligned} \quad (1.38)$$

On the other hand, the boundary condition on σ_z can be written:

$$2\pi \int_a^b \sigma_z r dr = F_z \quad (1.39)$$

The axial force on the sheath can be expressed in function of the outer and inner pressure. One gets:

$$F_z = \pi(p_a a^2 - p_b b^2) \quad (1.40)$$

The boundary condition (1.39) becomes finally:

$$\begin{aligned} \frac{2\mu}{1-2\mu} C_1 + \frac{1-\mu}{1-2\mu} C_3 = \frac{b^2}{b^2-a^2} \left[2 \frac{1+\mu}{1-\mu} L - \frac{2-\mu}{1-\mu} M \right. \\ \left. - \frac{\mu}{1-\mu} N \right] - \frac{1+\mu}{E} \frac{b^2 p_b - a^2 p_a}{b^2 - a^2} \end{aligned} \quad (1.41)$$

The three equations (1.36), (1.37) and (1.41) constitute a system of equations in the three unknowns C_1 , C_2 and C_3 . The solution of this system is:

$$\begin{aligned} C_1 = \frac{b^2}{b^2-a^2} \left[\frac{1-3\mu}{1-\mu} L + \frac{1}{2(1-\mu)} M - \frac{1-2\mu}{2(1-\mu)} N \right] \\ - \frac{1-2\mu}{E} \frac{p_b b^2 - p_a a^2}{b^2 - a^2} \end{aligned} \quad (1.42)$$

$$\begin{aligned} C_2 = \frac{a^2 b^2}{b^2-a^2} \left[\frac{1+\mu}{1-\mu} L + \frac{1-2\mu}{2(1-\mu)} M - \frac{1}{2(1-\mu)} N \right. \\ \left. - (1+\mu) \frac{p_b - p_a}{E} \right] \end{aligned} \quad (1.43)$$

$$C_3 = \frac{2b^2}{b^2 - a^2} (L - M) - \frac{1 - 2\mu}{E} \frac{p_b b^2 - p_a a^2}{b^2 - a^2} \quad (1.44)$$

These three constants, brought in the equations (1.28) and (1.29), allow to determine the total strain distribution, satisfying the boundary conditions and compatible with the permanent strain distribution.

1.7. Determination of the integration constants. Boundary conditions in the case of contact between fuel and sheath

The solution of this problem is only possible, if some assumptions are made about the behaviour of the fuel itself, when a contact occurs with the sheath.

It has been assumed that the actual outer fuel radius, when the contact is established, is equal to the deformed inner sheath radius and is worth:

$$a + u_a = r_f - p_c / \alpha_r \quad (1.45)$$

In this equation, $a + u_a$ represents the deformed inner sheath radius, assumed equal to the outer fuel radius after contact, r_f is an hypothetical outer radius, that the fuel would have in absence of contact, p_c is the contact pressure and α_r is a radial fuel strength coefficient.

The values of r_f and α_r are assumed to be known functions depending of the time only.

Furthermore, it has been assumed that when the contact is established, the friction between the fuel and the sheath is such that the fuel and sheath strains in the axial direction are equal [8]. The fuel-sheath axial interaction can then be equated as follows:

$$\epsilon_z = \epsilon_{zf} - F_{zf} / \alpha_z \quad (1.46)$$

In this equation, ϵ_z is the sheath axial strain, ϵ_{zf} is an hypothetical fuel axial strain, that would exist in absence of axial force on the

fuel, the force F_{zf} is the axial force introduced in the sheath by the pressure of the fuel (positive if tensile) and α_z is an axial fuel strength coefficient. The values ϵ_{zf} and α_z are assumed to be known functions of the time. This last condition (1.46) must only be satisfied when ϵ_{zf} is larger as ϵ_z . Indeed the use of the boundary condition (1.46) with ϵ_z greater as ϵ_{zf} would mean that a tensile force could be generated in the fuel column. This is, of course, not possible, due to the division in pellets and due to the properties of the fuel column itself. In this last case, the boundary condition in the axial direction is determined as in para 1.6. (equations 1.39 and 1.40).

The swelling and plastic behaviour of the fuel can be taken into account by a proper choice of the four functions of the time r_f , α_r , ϵ_{zf} and α_z .

The boundary condition on the inner face of the sheath can then be written:

$$(\sigma_r)_{r=a} = -p_a - p_c \quad (1.47)$$

Taking the value of p_c given by the equation (1.45), one gets:

$$(\sigma_r)_{r=a} = -p_a - \alpha_r(r_f - a - u_a) \quad (1.48)$$

The equations (1.30) and (1.27) allow to determine respectively the values of $(\sigma_r)_{r=a}$ and u_a .

Putting these values into (1.48), one gets finally the following equation:

$$\left[\frac{1}{1-2\mu} - \frac{(1+\mu)\alpha_r}{E} r \right] C_1 - \left[\frac{1}{a^2} + \frac{(1+\mu)\alpha_r}{Ea} r \right] C_2 + \frac{\mu}{1-2\mu} C_3 = - \frac{1+\mu}{E} \left[p_a + \alpha_r(r_f - a) \right] \quad (1.49)$$

On the other hand, the boundary condition in the axial direction is expressed by:

$$2\pi \int_a^b \sigma_z r dr = \pi(a^2 p_a - b^2 p_b) + F_{zf} \quad (1.50)$$

Using the value of F_{zf} given in (1.46), the equation (1.50) becomes:

$$2\pi \int_a^b \sigma_z r dr = \pi(a^2 p_a - b^2 p_b) + \alpha_z (\epsilon_{zf} - \epsilon_z) \quad (1.51)$$

This leads finally to the expression:

$$\begin{aligned} \frac{2\mu}{1-2\mu} C_1 + \left[\frac{1-\mu}{1-2\mu} + \frac{1+\mu}{E} \frac{\alpha_z}{\pi(b^2 - a^2)} \right] C_3 = \\ \frac{b^2}{b^2 - a^2} \left[2 \frac{1+\mu}{1-\mu} L - \frac{2-\mu}{1-\mu} M - \frac{\mu}{1-\mu} N \right] \\ - \frac{1+\mu}{E} \left[\frac{p_b b^2 - p_a a^2}{b^2 - a^2} - \frac{\alpha_z \epsilon_{zf}}{\pi(b^2 - a^2)} \right] \end{aligned} \quad (1.52)$$

The boundary condition on the outer face of the sheath can be expressed in the same way as in the para. 1.6. The equation (1.37) remains still valid in this case.

The equations (1.49), (1.37) and (1.52) constitute a system of 3 equations, allowing to determine the three unknowns C_1 , C_2 and C_3 . These equations are intricate and they will not be analytically solved here. In the programme, they are directly numerically solved.

1.8. Principle of the solution

In para 1.5, the equations allowing to calculate the stress state in a sheath for a known distribution of permanent strains, have been written. The integration constants appearing in the equations were determined in para 1.6 and 1.7.

Actually, the problem remains still unsolved since the permanent strains are not known.

However a solution is possible by an iterative method [3]. For each time where the states of stress and strain are to be calculated the following calculation steps can be performed:

1. Take a first guess of the distribution of permanent strains in the sheath for the time to be calculated
2. From this distribution of permanent strains and for the temperature distribution and the inner and outer pressure corresponding to the calculation time, determine the value of the integration constants with the formulas of para 1.6 or 1.7
3. Calculate the distribution of the total strains in the sheath with the equations (1.28) and (1.29)
4. Determine the state of stress in the sheath at the time of calculation from the values of the total strains with the help of equations (1.18)

This stress state satisfies automatically the boundary conditions and is compatible with the permanent strains state postulated at the begin of the process

5. With the help of a creep or plastic law, calculate the increments of permanent strains between the previous calculation time and the present one
6. Compare the calculated permanent strains obtained in step 5 with the assumed values of step 1.

If the differences are smaller as a given criterion, begin the calculation for the next time. If agreement is no reached, go to the next step

7. Determine a new estimate of the creep or plastic strains and begin the process again at step 2

The successive calculation steps summarized hereabove will be reviewed in more details in the part 2 hereafter.

2. Practical application of the method

2.1. Generalities

The states of stress and strain will be determined for various points in the thickness of the sheath, located in the same cross section, and at various times.

At time 0 and in cold state, one considers $n + 1$ points of calculation, equally spaced in the thickness of the sheath, the first one being located at the inner side of the sheath, the latest one at the outer side. If corrosion occurs during the life of the sheath, its thickness is reduced, but only the cold radius of the $n+1$ th point varies with time, the cold radii of the other points remaining constant. If the spacing between the two last points becomes 0, the number of points is reduced by 1.

The states of stress and strain will be determined for various values of the time called "the calculation times". In order to determine the boundary conditions for each of these calculation times, one needs to know the values of the inner and outer pressure p_a and p_b and, if a solid contact with the fuel is taken into account, the values of r_f , α_r , ϵ_{zf} and α_z , as explained in para 1.7.

Moreover, one needs some data to calculate, for each calculation time, the temperature distribution. As will be shown later (para 2.3), for each time of calculation, the temperature distribution can be determined from the value of the linear power q' and the coolant or average sheath temperature T_{cs} .

One has thus to determine, for each time of calculation, the values of the 8 or 4 variables functions of the time, respectively if a fuel contact is considered or not.

2.2. Determination of the variables functions of the time

The 8 variables functions of the time constitute a part of the input data of the programme. As explained hereafter three methods to give these input data are possible.

2.2.1. The 8 variables are parabolic functions of the time

With this method, one needs only to give, as input data, the 3 coefficients of the parabolas for each variable. Some coefficients can, of course, be equal to 0 in order to have a constant value or a linear increase or decrease with the time.

The intervals between the successive calculation times are equal with this method.

This method provides an easy input for the simple problems.

2.2.2. The 8 variables are given by points

For each time of calculation, the value of the time is given, followed by the values of the variables. One assumes that the functions are linear between two successive points of calculation. The length of the time intervals is no more a constant.

This method allows to follow, with a good accuracy, any functions of the time.

It needs however a large number of input data (Up to 9 values for each point of calculation).

2.2.3. Cycling of the time dependent variables is performed

If the time dependent variables are cycled, these cycles can, of course, be represented with the method of para 2.2.2. However, to save time in preparing the input data, a special subroutine allows to calculate the successive cycles, when the first one only is described by the method of para 2.2.2. A number of cycles is also specified and the programme proceeds until this number is reached.

One distinguishes two types of variables:

- the variables q' , T_{cs} , p_b , α_r and α_z are reproduced, at each point of any cycle, equal to the value they have at the corresponding point of the first cycle.
- the variables p_a , r_f and ϵ_{zf} are increased or decreased during each part of any cycle, of the same value they increased or decreased during the corresponding part of the first cycle.

This allows to simulate the increase of inner gas pressure due to the fission gas release and the radial and axial swelling of the fuel.

2.3. Calculation of the temperature distribution

The temperature distribution in the sheath is determined, for each time of calculation, from the linear power q' and the coolant or sheath average temperature T_{cs} .

It is assumed that a part of the power is generated in the sheath itself, mainly by γ -heating. The fraction of the heat generated in the sheath F_h is a datum of the problem. This datum is assumed to remain constant with the time. The heat sources in the sheath are thus worth:

$$q = \frac{F_h q'}{\pi(b^2 - a^2)} \quad (2.1)$$

One assumes first that the coolant temperature is given. Then:

$$T_{co} = T_{cs} \quad (2.2)$$

The wall outer temperature is calculated with the help of a heat transfer coefficient h . The value of this coefficient is assumed independent of the time and of the temperature.

$$T_{n+1} = T_{co} + \frac{q'}{2\pi bh} \quad (2.3)$$

The temperature distribution in the sheath is determined with the help of the conduction theory by assuming that the heat flows in radial direction only. This theory [9] states that the temperature distribution, for each ring between two points of calculation, is given by:

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = - \frac{q}{k} \quad (2.4)$$

where k is an average heat conductivity in the considered ring.

After integration, this equation leads to the following temperature distribution:

$$T = - \frac{q}{4k} r^2 + A \ln r + B \quad (2.5)$$

where A and B are integration constants, to be determined by the boundary conditions.

For the outer ring, the outer temperature and outer heat flux are known. One can thus determine the integration constants for the outer ring. This allows in turn, to calculate the temperature and the heat flux at the inner side of the ring, values which are equal to those at the outer side of the next ring.

A recurrence method allows then to calculate the temperature distribution in the successive rings. The knowledge of the outer temperature leads to:

$$T_{i+1} = - \frac{q}{4k} r_{i+1}^2 + A \ln r_{i+1} + B \quad (2.6)$$

The heat flux at the outer surface of the ring is known. Its value φ_{i+1} , determined by a heat balance, is expressed by:

$$\varphi_{i+1} = \frac{q' - \pi(b^2 - r_{i+1}^2)q}{2\pi r_{i+1}} \quad (2.7)$$

On the other hand, it is equal to:

$$\varphi_{i+1} = -k \left[\frac{dT}{dr} \right]_{r_{i+1}} = \frac{qr_{i+1}}{2} - k \frac{A}{r_{i+1}} \quad (2.8)$$

After determination of the integration constants with the help of the equations (2.6), (2.7) and (2.8), one gets finally the following expression for the temperature distribution:

$$T = T_{i+1} + \frac{q}{4k}(r_{i+1}^2 - r^2) + \frac{1}{k} \left[\frac{q'}{2\pi} - \frac{q}{2} b^2 \right] \ln \frac{r_{i+1}}{r} \quad (2.9)$$

This equation allows us to determine the temperature T_i , by making $r = r_i$:

$$T_i = T_{i+1} + \frac{q}{4k}(r_{i+1}^2 - r_i^2) + \frac{1}{k} \left[\frac{q'}{2\pi} - \frac{q}{2} b^2 \right] \ln \frac{r_{i+1}}{r_i} \quad (2.10)$$

Moreover, the average temperature of the ring can be determined by the equation:

$$T_{av i} = \frac{1}{\pi(r_{i+1}^2 - r_i^2)} \int_{r_i}^{r_{i+1}} 2\pi Tr dr \quad (2.11)$$

One gets finally:

$$T_{av i} = T_{i+1} + \frac{q}{8k} (r_{i+1}^2 - r_i^2) + \frac{1}{k} \left[\frac{q'}{2\pi} - \frac{q}{2} b^2 \right] \left[\frac{1}{2} - \frac{r_i^2}{r_a^2 - r_i^2} \ln \frac{r_a}{r_i} \right] \quad (2.12)$$

As said hereabove, the value of k in the preceding equations is the average heat conductivity in the considered ring. As the conductivity of the material is a function of the temperature, its value must be determined by an iterative process:

One calculates first the value of $T_{av i}$ with a value of k corresponding to T_{i+1} . Using now the value of k corresponding to the calculated $T_{av i}$, one determines a new value of $T_{av i}$. This process is repeated until the difference between two successive values of $T_{av i}$ is less as a convergence criterion. With the last value of k , one calculates then the inner temperature of the ring with the equation (2.10). This process can be repeated for all the rings starting from the outermost one.

The average temperature of the sheath is determined as follows:

$$T_{av} = \frac{\sum_{i=1}^n [T_{av i} (r_{i+1}^2 - r_i^2)]}{(b^2 - a^2)} \quad (2.13)$$

If the average sheath temperature is given, one proceeds by iterations. The following steps are successively made:

1. The outer sheath temperature is made equal to the given average temperature:

$$T_{n+1} = T_{cs} \quad (2.14)$$

2. A temperature distribution is calculated and an average temperature is determined with the method described hereabove
3. An error on the average temperature is calculated:

$$E = T_{cs} - T_{av} \quad (2.15)$$

If this error is greater as a convergence criterion, a new value of T_{n+1} , called T_{n+1}^* is calculated:

$$T_{n+1}^* = T_{n+1} + E \quad (2.16)$$

The process is repeated from step 2 until the convergence is reached.

The knowledge of the temperature allows to calculate the various properties of the material at each calculation point: expansion coefficient, creep or plasticity parameters. The Young's modulus and the Poisson's ratio, however, are the same for all the points of calculation. Their value is determined for the sheath average temperature.

2.4. First guess of the permanent strains at the end of a time interval

The values of the permanent strains at the beginning of a time interval are known. They are equal to the values at the end of the previous interval.

This postulates in fact that these permanent strains are known at the beginning of the first interval, it is to say at time 0. In what concerns the creep strains they are identical to 0 for time 0.

More difficult is, of course, the problem for the plastic strains because these last ones are not time dependent. This problem has been solved by the use of an hypothetical origin of the time (equal to -1 hr), at which all the loads on the sheath (inner and contact pressure, outer pressure, temperature gradient) are equal to zero. For this time, the stresses in the sheath are equal to zero and, consequently, also the plastic strains. The loads are then increased during the time interval between -1 hr and 0 hr up to their value at time 0.

This expedient allows to have a first time interval at the beginning of which the permanent strains are known. For the first iteration of a time interval, the permanent strains increments are determined as follows:

$$\Delta \epsilon_{jci} = \Delta \epsilon_{jci}^* \frac{\Delta t}{\Delta t^*} \quad \text{for } i=1, n+1 \quad (2.17)$$

where j is r or θ or z , $\Delta \epsilon_{jci}^*$ is the increment at the convergence for

the previous time interval, Δt the present time interval length and Δt^* the previous time interval length.

For the other iterations, corrections to the $\Delta \epsilon_{jci}$ are brought using the method of para 2.8.

The values of the assumed permanent strains at the end of a time interval are thus worth:

$$\epsilon_{jci} = \epsilon_{jci}^* + \Delta \epsilon_{jci} \quad (2.18)$$

2.5. Calculation of the states of stress and total strain for a given state of permanent strain

This part of the calculations is performed in three steps summarized at points 2, 3 and 4 of para 1.8. These steps will be described in more details hereunder:

2.5.1. Determination of the integration constants

The determination of the integration constants is made with the help of the expressions (1.42), (1.43) and (1.44) of para 1.6., in the case without fuel contact, or by solving the equations (1.49), (1.37) and (1.52), if a fuel contact is considered.

In these equations, appear the values of the integrals L, M and N given by the expressions (1.33), (1.34) and (1.35). These integrals are numerically calculated with the well known trapezoidal rule. The expression of these integrals become than:

$$L = \frac{1}{b^2} \sum_{i=1}^n \left[1/2 (\alpha(T_i) T_i r_i + \alpha(T_{i+1}) T_{i+1} r_{i+1}) (r_{i+1} - r_i) \right] \quad (2.19)$$

$$M = \frac{1}{b^2} \sum_{i=1}^n \left[1/2 ((\epsilon_{rc i} + \epsilon_{\theta c i}) r_i + (\epsilon_{rc i+1} + \epsilon_{\theta c i+1}) r_{i+1}) (r_{i+1} - r_i) \right] \quad (2.20)$$

$$N = \sum_{i=1}^n \left[\frac{1}{2} \left(\frac{\epsilon_{rc\ i} - \epsilon_{\theta c\ i}}{r_i} + \frac{\epsilon_{rc\ i+1} - \epsilon_{\theta c\ i+1}}{r_{i+1}} \right) (r_{i+1} - r_i) \right] \quad (2.21)$$

In the case of contact between fuel and sheath, the three equations (1.49), (1.37) and (1.52) are numerically solved, i.e. all their coefficients are numerically determined and brought into the subroutine SIMQ [10], which allows to solve a system of linear equations.

2.5.2. Calculation of the total strains

The knowledge of the integration constants allows then to calculate the total strains from the equations (1.28) and (1.29). In these equations, the value of r is successively made equal to the various r_i , in order to assess the total strains at the various points of calculation. The integrals appearing in these equations are determined with the trapezoidal rule in the same manner as for the integration constants hereabove.

2.5.3. Calculation of the stresses

When the total strains are known, one can determine the stresses for each point of calculation with the help of the equations (1.18). For the calculation of the permanent strain increments, one needs to know the equivalent stress at each calculation point. When creep calculations are performed, use is made of the values of the average equivalent stress on the time interval. For plastic calculations, the values at the end of the time interval are used. The equivalent stress values are determined with either the equation (1.4) or (1.6) at the beginning and at the end of the time interval. Average values on the time are then determined with the formula:

$$\bar{\sigma}_{eq\ i} = 1/2(\sigma_{eq\ i}^* + \sigma_{eq\ i}) \quad (2.22)$$

2.6. Determination of the creep strain increments corresponding to a stress state

The creep phenomenon is generally described in the literature [2], [5], [6] in terms of creep velocity, i.e. the equivalent creep rate is usually expressed as a function of the equivalent stress, the temperature and the time. Other parameters, as the equivalent creep strain or the neutron dose, can also be taken into account. The equivalent creep strain increments will thus be expressed as a product of the average equivalent creep rate during the time interval, by the time interval:

$$\Delta \epsilon_{eq\ i} = \bar{\epsilon}_{eq} \Delta t \quad (2.23)$$

Among the numerous creep laws available, two laws were chosen, the parameters of which can be easily found for many materials in the literature about creep. They are:

2.6.1. First creep law (Norton's law)

The expression of this creep law writes:

$$\dot{\epsilon}_{eq} = K \sigma_{eq}^n \quad (2.24)$$

In this equation, K and n are functions of the temperature. These functions are given in the programme in form of tables for various values of the temperature. The values for each point of calculation are determined by interpolation in this table.

The value of the temperature used for the interpolation is the arithmetic mean temperature during the time interval:

$$\bar{T}_i = \frac{T_i^* + T_i}{2} \quad (2.25)$$

As the curve of K as a function of the temperature is not too far from a straight line in a diagramme T-log K, the interpolations are performed on the values of log K.

2.6.2. Second creep law

This creep law is expressed by:

$$\dot{\epsilon}_{eq} = K \sigma_{eq}^n t^m \quad (2.26)$$

where K , n and m are functions of the temperature. Their values are interpolated from a table of properties in the same way as for the first law.

In this creep law, the equivalent creep velocity is an explicit function of the temperature, the stress and the time only. The various parameters appearing in the equation are deduced from experimental results performed at constant equivalent stress and constant temperature. In the equation so obtained, the equivalent creep rate can, however, be an implicit function of the creep strain.

This fact is of great importance, when use is made of this creep law with an equivalent stress which varies with the time.

The equivalent creep strain curve as a function of the time for each point of calculation will indeed strongly depend of this choice:

Consider curves of the equivalent creep strain in function of the time for various values of the equivalent stress (figure 4a and b). Assume that during the time interval 0 to t_1 the material is loaded with an average equivalent stress σ_1 . At the time t_1 , the representative point in the diagramme will be $A_t = A_s$; assume further that, during the time interval t_1 to t_2 , the material is loaded with a mean equivalent stress σ_2 and, between t_2 and t_3 , with a mean equivalent stress σ_3 . The figure 4a is relative to increasing values of the stress with time, the figure 4b to decreasing stresses. The problem arises now to determine the equivalent creep velocity between t_1 and t_2 . Two main theories are in presence [3]:

- the time hardening rule states that the equivalent creep velocity is a function of time only. The creep velocity for the points A_t and B_t will thus be respectively equal to the creep velocity at points A'_t and B'_t (fig. 4a and b).

The curve of the equivalent creep strain as a function of the time will thus be: $O A_t B_t C_t$

- the strain hardening rule states that the equivalent creep velocity is a function of the equivalent strain only. The creep velocities at points A_s and B_s are thus respectively equal to the creep velocities at points A'_s and B'_s . The curve of the equivalent creep strain as a function of the time is thus $O A_s B_s C_s$

The figures 4a and 4b show clearly that the strain-hardening rule leads to larger creep strains as the time-hardening rule, when the stress increases with time. The opposite conclusion is valuable when the stress decreases with time.

In the programme, one can perform calculations with any one of the two rules.

If the time hardening rule is used, the equivalent creep velocity is determined for a value of the time equal to the mid-interval time:

$$\bar{\epsilon} = K\sigma_{eq}^n \left[\frac{t^* + t}{2} \right]^m \quad (2.27)$$

When use is made of the strain-hardening rule, one calculates first the value of the time corresponding to the point A'_s on the figure 4. This time is worth:

$$t = \left[\frac{(m+1)\epsilon_1}{K\sigma_2^n} \right]^{1/(m+1)} \quad (2.28)$$

The average time for the interval between t_1 and t_2 will thus be in this case:

$$\bar{t}_2 = \left[\frac{(m+1)\epsilon_1}{K\sigma_2^n} \right]^{1/(m+1)} + \frac{t_2 - t_1}{2} \quad (2.29)$$

2.7. Determination of the plastic strain increments corresponding to a stress state

The determination of the equivalent plastic strain increments requires the knowledge of the curves of the equivalent plastic strain as a function of the equivalent stress for various temperatures.

These curves are in fact a property of the material and the trend of these curves can thus present significant differences from material to material.

Two types of curves of $\sigma_{eq} - \epsilon_{pl eq}$ can be used in the programme:

2.7.1. First plastic law

This law assumes simply that the equivalent plastic strains are equal to zero for equivalent stresses smaller than the plastic limit. Beyond this limit, the equivalent stress remains constant and equal to this plastic limit, whatever the equivalent strain may be. Such an idealized behaviour of the materials is often used for theoretical analyses of plastic flow [11].

This plastic law can be represented on a diagramme $\sigma/(\epsilon_{pl} + \epsilon_{el})$ by two straight lines: a inclined one with a slope equal to the Young's modulus and an horizontal one, as shown on figure 5a.

The plastic limit is considered to be a function of the temperature. In the programme, this function is given in a table.

For each point of calculation, the programme interpolates in this table to determine the plastic limit.

The plastic strains increments are determined as follows:

If A is the representative point at the beginning of a time interval and if $\sigma_{eq 1}$ is the equivalent stress at the end of the time interval determined with the method of para 2.5, the point B on figure 5a is considered to represent the state at the end of the interval. The equivalent strain in this point is the same as for the point C, intersection of the horizontal $\sigma_{eq 1}$ and a straight line of slope equal to the Young's modulus and going through A.

The equivalent plastic strain increment is then equal to the strain difference between points A and B.

If this difference is negative the strain increment is made equal to 0.

This means, in other words, that, if the stress at the end of the interval is smaller as the value at the beginning, the representative point lies on the straight line A C and below the point A.

2.7.2. Second plastic law

The curve of the equivalent plastic strain as a function of the equivalent stress is assumed to be an exponential (fig. 5b) of the form:

$$\epsilon_{eq} = P \sigma_{eq}^Q \quad (2.30)$$

where P and Q are functions of the temperature. The parameters P and Q are determined in the programme in such a way that the points $(\sigma_{0.2}, \epsilon = 0.2)$ and $(\sigma_{rp}, \epsilon_{rp})$ lie on the curve:

$$Q = \frac{\ln \epsilon_{rp} - \ln 0.2}{\ln \sigma_{rp} - \ln \sigma_{0.2}} \quad (2.31)$$

$$P = e^{\left[\frac{\ln \sigma_{rp} \ln 0.2 - \ln \sigma_{0.2} \ln \epsilon_{rp}}{\ln \sigma_{rp} - \ln \sigma_{0.2}} \right]} \quad (2.32)$$

where $\sigma_{0.2}$ is the equivalent stress giving 0.2 % plastic equivalent strain and σ_{rp} and ϵ_{rp} are the rupture stress and strain.

The values of $\sigma_{0.2}$, σ_{rp} and ϵ_{rp} are input data of the programme. They are given in a table as a function of the temperature. The programme interpolates in this table the value of $\sigma_{0.2}$, σ_{rp} and ϵ_{rp} for each point of calculation and then calculates the values of P and Q.

The equivalent plastic strain at the end of the time interval is calculated for each point with the equivalent stress for this time. The equivalent plastic strain increment is determined as the difference between the end value and the beginning value. If this increment is negative its value is made equal to 0 for the same reason as hereabove (para 2.7.1.).

2.8. Convergence method

When the creep or plastic equivalent strain increments corresponding to the stress state are calculated, they are compared to the first guess assumed at the beginning of the process. If a sufficient agreement is reached, the calculation of the next time interval can be started. If, however, the agreement is not sufficient, a correction is to be brought on the guess of the equivalent strain increments, in order to get a better agreement at the next iteration.

The precision of this process is very important in what concerns the rapidity of convergence of the calculation. It is thus very important, in order to reduce the calculation time to use a method allowing to reach the convergence with a small number of iterations. On the other hand, the method must be sufficiently general to permit the use of any creep or plastic law.

A method, responding to these criteria has been settled up from the Newton-Raphson theory of solution of a system of non-linear equations [12],[13].

For each of the $n+1$ points of calculation, one gives at the beginning of each iteration a value δ_i of the equivalent permanent strain increment and one gets at the end of the iteration an other value of the increment equal to δ_{ci} .

At the convergence one must have:

$$\delta_{ci} = \delta_i \quad \text{for } i=1, n+1 \quad (2.33)$$

Each value of δ_{ci} can in fact be considered as a non linear function of the $n+1$ variables δ_j :

$$\delta_{ci} = f_i(\delta_1, \dots, \delta_j, \dots, \delta_{n+1}) \quad \text{for } i=1, n+1 \quad (2.34)$$

One has thus to solve the following system of equations:

$$f_i(\delta_1, \dots, \delta_j, \dots, \delta_{n+1}) - \delta_i = 0 \quad \text{for } i=1, n+1 \quad (2.35)$$

One knows a first approximation of the solution of this system of equations. This first approximation is given by the values δ_i^* taken for the first iteration.

One can now calculate corrections $\Delta\delta_i$ to these first values. By developing the function $f_i(\delta_j)$ in Taylor's serie limited at the linear term, one gets:

$$f_i(\delta_j) = f_i(\delta_j^*) + \sum_{j=1}^{n+1} (\Delta\delta_j) \left[\frac{\partial f_i}{\partial \delta_j} \right]_{\delta_j=\delta_j^*} \quad \text{for } i=1, n+1 \quad (2.36)$$

The equations (2.35) hereabove become then:

$$\sum_{j=1}^{n+1} (\Delta\delta_j) \left[\frac{\partial f_i}{\partial \delta_j} \right]_{\delta_j=\delta_j^*} - \Delta\delta_i = \delta_i^* - f_i(\delta_j^*) \quad \text{for } i=1, n+1 \quad (2.37)$$

These $n+1$ equations constitute a system of $n+1$ linear equations with the $n+1$ unknowns $\Delta\delta_j$. This system can easily be solved on a computer with the help, for instance, of the subroutine SIMQ [10]. The solution of this system gives us the corrections on the δ_j . The values of the strain increments to be introduced at the beginning of the next iteration are thus worth:

$$\delta_j^{**} = \delta_j^* + \Delta\delta_j \quad \text{for } j=1, n+1 \quad (2.38)$$

In the equations (2.37) hereabove, appear the values of the first partial derivatives. In the programme, these partial derivatives are determined by finite differences:

$$\left[\frac{\partial f_i}{\partial \delta_j} \right]_{\delta_j = \delta_j^*} = \frac{1}{h} \left[f_i(\delta_1^*, \dots, \delta_j^* + h, \dots, \delta_{n+1}^*) - f_i(\delta_1^*, \dots, \delta_j^*, \dots, \delta_{n+1}^*) \right] \quad (2.39)$$

2.9. Subdivision of the time interval

It has been observed that the convergence becomes very difficult when the permanent strain increments are too large. For this reason and also in order to reach a greater accuracy in the results, one has limited the length of the time steps. This is made automatically in the programme. The criterium used is that the equivalent permanent strain increments during a time step must be smaller than or equal to the equivalent elastic strain for the same point of calculation at the end of this time step. The time intervals specified in the input data can thus be automatically divided in subintervals to satisfy the criterion hereabove.

At the beginning of each time interval, the programme calculates a guess of the stresses at the end of the time interval by making the assumption that the permanent strains increments are equal to zero during the time interval. From this guess of the stresses, one calculates with a creep or plastic law a guess of the permanent strain increments. This guess is compared to the elastic strains. If necessary, the time interval length is divided and the variables functions of the time are calculated for this new value of the time.

2.10. Method of interpolation

It is mentioned hereabove on several occasions that interpolations are made in a table of properties. These interpolations are performed in the programme with a special subroutine allowing linear or parabolic interpolations. The values of the temperature in the table must be given in increasing order. The subroutine searches first between what points lies the value of the temperature T_x for which a property is to be determined. Assume that T_x lies between T_j and T_{j+1} . Then, the interpolated value of the property A_x , in the case of parabolic interpolation, will be expressed by:

$$A_x = A_j + (T_x - T_j) \frac{(bm - an)(T_x - T_j) + (an^2 + bm^2)}{mn(m + n)} \quad (2.40)$$

with

$$a = A_j - A_{j-1} \quad (2.41)$$

$$b = A_{j+1} - A_j \quad (2.42)$$

$$m = T_j - T_{j-1} \quad (2.43)$$

$$n = T_{j+1} - T_j \quad (2.44)$$

Using the same definitions as hereabove, one gets for a linear interpolation:

$$A_x = A_j + (T_x - T_j) \frac{b}{n} \quad (2.45)$$

3. Short description and directions for use of the programme

3.1. Brief description of the various subroutines included in the programme CRASH

The programme CRASH, in its version 2B, is constituted of a main programme and 21 subroutines. The total amount of cards is about 2000. The programming language is FORTRAN IV.

Hereafter, follows a brief description of the various subroutines constituting the programme:

MAIN PROGRAMME:

the main programme makes first the initialisations used during all the job. At the beginning of a new problem, it puts the permanent strains equal to zero. At the beginning of a new time step, the values of the stresses and strains at the beginning of the interval are made equal to the values at the end of the previous interval.

Moreover, the main programme calls the main subroutines of the programme.

DATA:

reads and prints the data for each problem. It detects also the possible errors in the input data.

TIMING:

the purpose of this subroutine is to determine, for each calculation time, the values of the time dependent variables. From the maximum length of the time subinterval calculated in the subroutine DIVIDE (see para 2.9), the subroutine TIMING determines the values of the time dependent variables for the considered calculation time. The subroutine is divided in two main parts, the first one being used when the time dependent functions are expressed as parabolas, the other one being used when these functions are given by points or are cyclic functions (para 2.2).

PARAB:

is an auxiliary routine called from TIMING. It calculates the values of the various functions dependent of the time when they are parabolic (para 2.2.1).

CYCLE:

This is also an auxiliary subroutine for TIMING. Its determines the values of the functions dependent of the time when they are cycled (see para 2.2.3).

DIVIDE:

determines the maximum length of the time subintervals in order to avoid convergence difficulties (see para 2.9)

INIT:

is used for the division of the initial interval between -1 and 0 hr, needed in the case of plastic calculations as explained in para 2.4.

CORROS:

determines the value of the outer radius for each calculation time, in the case where corrosion occurs on the outer surface. The values of the stresses and strains for the outer calculation point are also corrected with the help of CORF.

CORF:

This is a small auxiliary routine used in CORROS.

CALC:

determines the first guess of the permanent strain increments (para 2.4). In the case where the swelling is taken into account, it controls if a contact is established or not and makes then the choice between the boundary conditions to be used (para 1.6 and 1.7).

TEDI:

calculates, for each calculation time, the temperature distribution in the sheath, following the method of para 2.3. When the temperature distribution is determined, this routine performs the various interpolations in the table of the properties, in order to assess these properties for each point of calculation.

INTER:

this auxiliary routine, called in TEDI, performs the interpolations in the table of the properties with the method explained in para 2.10.

ITER:

performs the iterations on the permanent strain increments. This subroutine calculates the partial derivatives $\partial f_i / \partial \delta_j$ and determines the corrections to the permanent strains increments (see para 2.8). The errors on these increments are also calculated here and compared with the convergence criterion.

STRAIN:

the boundary conditions are determined in this subroutine by using the method of para 1.6 or 1.7. The stress and total strain states compatible with the boundary conditions are then calculated for a given state of permanent strain.

CREEP:

calculates the equivalent creep strain increments according to the para 2.6.

PLAST:

the equivalent plastic strain increments are assessed here with the formula of para 2.7.

EQSSR:

this auxiliary routine calculates the equivalent stress or strain from the three components of the stress or strain by the equations of Mises or Tresca (para 1.4.1).

SIMQ:

this IBM subroutine [10] allows to solve a system of linear equations. It is used to solve the boundary conditions equations in STRAIN and to determine the corrections on the permanent strains increments in ITER.

PRINT:

prints the results of the calculations at the end of each time step.

MAX:

small auxiliary routine allowing to determine the maximum element of an array.

PAGE:

this routine counts the lines to be printed and takes a new page when it is necessary.

TIMAX:

allows to terminate the calculation when a given permanent strain is reached.

The figure 6 shows a diagramme illustrating how the various sub-routines are interdependent from each other.

3.2. Description of the input data

The input data for CRASH-2B are given with format specification. All the floating-point numbers are specified with the format E11.5, whereas the integers have a format I3. When formats are specified, values equal to 0 may remain blank.

A list of all the input data, with their location on the cards, their format, their meaning and their dimension, is given in the table 3.1 hereafter.

The following remarks may be done about the input data:

1. A data deck is constituted of a card number 1, followed of a series of cards decks each representing one problem. The card 1 is thus read only once per run.
2. Each problem is constituted of the following cards:
 - one card n^o 2
 - one card n^o 3

- one card n^o 4
- one card n^o 5a
- a serie of cards n^o 5b and c, the number of which is equal to the value of MM on cards 5a. The card 5b and c are omitted if MM is equal to 0. In this case, the programme makes use of the same properties as for the previous problem. When MM is >0, its minimum value is 2 or 3 respectively if linear or parabolic interpolation is performed.
- Then follow the data concerning the time dependent variables. Following the value given to ITIM (1,2 or 3) on card 3, a series of cards of type respectively 6, 7 or 8 is given.
 - i) If ITIM = 1 on card n^o 3, the following cards are given:
 - one card n^o 6a
 - one card n^o 6b
 - one card n^o 6c
 - one card n^o 6d
 - one card n^o 6e
 - one card n^o 6f
 - one card n^o 6g
 - one card n^o 6h
 - one card n^o 6i

The last four ones must only be given if IFZ = 1 on card n^o 3. In the other case (IFZ=2), they must be omitted.

- ii) If ITIM = 2, a series of cards n^o 7a and b is given. The value of TIME on the first card 7a must be equal to 0. For each calculation time, two cards, one card 7a and one card 7b are given. The calculation proceeds until a value of TIME equal to 0 is encountered. The easiest way to terminate a problem is thus to give two blank cards at the end of the series of cards 7. The number of time values is limited to 100.
- iii) If ITIM = 3 on card 3, cycling of the time dependent variables is performed. The variables PUL (power per unit length), TCS (coolant or sheath average temperature), PB (outer pressure), ARF (radial strength coefficient of the fuel) and AZF

(axial strength coefficient of the fuel) are reproduced equal to themselves from cycle to cycle. The variables PA (inner gas pressure), RF (fuel radius) and EZF (axial fuel strain) follow a linear increase or decrease in each part of any cycle, with a rate equal to the rate of the respective part of the first cycle (see para 2.2.3). The following cards are to be given:

- one card n^o 8a
- a series of cards n^o 8b and c, the number of which is equal to NCA on card 8a (maximum value allowed is 10). These cards allow to describe the first cycle. The calculation proceeds until the given number of cycles on card 8a is reached.

Table 3.1. List of the input data

Card	Datum fortran name	columns	format	d e s c r i p t i o n
1 (see re- marks 1 and 2)	IDAY	1 to 3	I3	day } these three data are used to print the date on the month } listing of the results year }
	IMONTH	4 to 6	I3	
	IYEAR	7 to 9	I3	
	NPRØM	10 to 12	I3	number of problems to be calculated
	DDEQD	13 to 23	E11.5	strain increment used for the calculation of the partial derivatives in the convergence process. If blank, the programme takes the nominal value 5×10^{-6} (equal to the value h in para 2.8)
	EPSIL1	24 to 34	E11.5	maximum error allowed at the convergence in the iterative process to determine the thermal conductivity. If blank, the programme takes the nominal value $.1^{\circ}\text{C}$ (see para 2.3)
	EPSIL2	35 to 45	E11.5	maximum error allowed at the convergence in the iterative process to determine the creep strains increments. If blank, the programme takes the nominal value $2 \cdot 10^{-6}$ (see para 2.8)
	TETI	46 to 56	E11.5	time (hrs) from which test printing is required. If no test printing is required, this value and the following ones on the card 1 remain blank
	ITEST1	57 to 59	I3	If >0, test printing 1 is obtained
	ITEST2	60 to 62	I3	If >0, test printing 2 is obtained
ITEST3	63 to 65	I3	If >0, test printing 3 is obtained	
ITEST4	66 to 68	I3	If >0, test printing 4 is obtained	
ITEST5	69 to 71	I3	If >0, test printing 5 is obtained	

Table 3.1. (continued)

Card 2	TITLE	1 to 80	20A4	Title of the problem. Maximum 80 free alphanumeric characters allowed
Card 3	N	1 to 3	I3	number of radial subdivisions in the thickness of the tube (maximum 20 divisions)
	IFZ	4 to 6	I3	if 1, the loads on the sheath are: a) inner and outer pressure, axial force due to these pressures and temperature gradient b) fuel swelling if any in radial and axial direction
	LEQ	7 to 9	I3	if 2, only the loads mentioned sub a) are taken into account if 1, the equivalent stress-strain is determined with the Mises theory if 2, the equivalent stress-strain is determined with the Tresca theory (see para 1.4.1)
	LCP	10 to 12	I3	- if 1, creep calculations are performed - if 2, plastic calculations are performed
	LCR	13 to 15	I3	In the case where LCP=1 (creep calculations), - if 1, use is made of the Norton creep law: $\dot{\epsilon} = k \sigma^n$ (para 2.6.1) - if 2, use is made of the creep law: $\dot{\epsilon} = k \sigma^n t^m$ (para 2.6.2) In the case where LCP = 2 (plastic calculation), - if 1, the stress-plastic strain relation is given by two straight lines (para 2.7.1) - if 2, the stress-plastic strain relation is an exponential curve (para 2.7.2)
	LTSH	16 to 18	I3	- if 1, time hardening creep rule is used (para 2.6.2) - if 2, strain hardening creep rule is used (para 2.6.2) This datum is only used when LCP = 1 and LCR = 2

Table 3.1. (continued)

	ITEM	19 to 21	I3	- if 1, the coolant temperature is given (para 2.3) - if 2, the mean sheath temperature is given (para 2.3)
	IINT	22 to 24	I3	the interpolation in the table of properties is parabolic, if 1 is given or linear, if 2 is given (see para 2.10)
	ILPR	25 to 27	I3	if 0, only a summary of the table of stress and strain is printed for each time (only the maximum values) if 1, the complete table is printed if >1, one line out of the given value is printed
	ITIM	27 to 30	I3	if 1, the variables functions of the time are parabolic (para 2.2.1) if 2, these variables are given by points (para 2.2.2) if 3, these variables are cyclic functions of the time (para 2.2.3)
	ILIM	31 to 33	I3	if 1, the calculation proceeds up to a given maximum value of the time if 2, the calculation proceeds until a given permanent strain is reached. If this is not reached for the given maximum time, a special message is printed and the programme shifts to the next problem
Card 4	RA	1 to 11	E11.5	cold inner radius of the tube (mm)
	RB	12 to 22	E11.5	cold outer radius of the tube (mm)
	HTC	23 to 33	E11.5	heat transfer coefficient between coolant and sheath ($W/cm^2 \text{ } ^\circ C$) This value must only be given when ITEM=1. For ITEM=2 it may be blank
	FPS	34 to 44	E11.5	Fraction of the heat generated in the sheath (-)
	CØRRA	45 to 55	E11.5	Corrosion rate (mm/hr); if blank, no corrosion occurs
	ECMAX	56 to 66	E11.5	Maximum permanent strain (-)(May be blank for ILIM=1)

Table 3.1. (continued)

Card 5a	MM	1 to 3	I3	number of temperature values in the table of the properties. If 0, the programme uses the same values as for the previous problem. This value must thus not be equal to 0 for the first problem. If different of 0, the minimum value of MM is 3 or 2, respectively if a parabolic interpolation (IINT = 1) or a linear one (IINT = 2) is asked
Card 5b	TT(J)	1 to 11	E11.5	Temperature value, for which the following physical properties are given ($^{\circ}\text{C}$):
	AKT(J)	12 to 22	E11.5	heat conductivity of the sheath ($\text{W}/\text{cm}^{\circ}\text{C}$)
	ALF(J)	23 to 33	E11.5	thermal expansion coefficient of the sheath between 0°C and the given temperature ($-/^{\circ}\text{C}$)
	YMT(J)	34 to 44	E11.5	Young's modulus of the sheath (kg/mm^2)
	AMUT(J)	45 to 55	E11.5	Poisson's ratio of the sheath (-)
	AKN(J)	56 to 66	E11.5	K parameter for the two creep laws
	ANN(J)	67 to 77	E11.5	n parameter for the two creep laws
Card 5c	AMN(J)	1 to 11	E11.5	m parameter of the creep law $\dot{\epsilon} = k\sigma^n t^m$ These last three values are only used when creep calculations are performed (LCP=1)
	PS2(J)	12 to 22	E11.5	- Yield strength of the material (kg/mm^2), if LCR = 1 - $\sigma_{0.2}$ of the material (kg/mm^2), if LCR = 2
	PRS(J)	23 to 33	E11.5	rupture stress of the material (kg/mm^2)
	PRE(J)	34 to 44	E11.5	rupture strain of the material (-) These last two values must only be given when LCR=2. For LCR=1 they may be blank
				The number of cards 5b and 5c must be equal to the MM value

Table 3.1. (continued)

Card 6a (see re- mark 2)	TIMEM	1 to 11	E11.5	maximum calculation time (hr)
	DTIME	12 to 22	E11.5	length of time interval (hr)
	NPR	23 to 25	I3	if equal to 0 a printing is obtained for the time 0. and for the last time; if >0 a printing is obtained for each time out NPR
Card 6b	PULO	1 to 11	E11.5	Coefficients in the equation of the power per unit length. The power per unit length is calculated for each time with the expression: $q' = (PULO) + (PUL1)t + (PUL2)t^2$ where q' is in W/cm and t in hr
	PUL1	12 to 22	E11.5	
	PUL2	23 to 33	E11.5	
Cards 6c	TCS0	1 to 11	E11.5	Coefficients of the equation of the coolant or sheath average temperature: $T_{cs} = (TCS0) + (TCS1)t + (TCS2)t^2$ with T_{cs} in $^{\circ}C$ and t in hr
	TCS1	12 to 22	E11.5	
	TCS2	23 to 33	E11.5	
Card 6d	PA0	1 to 11	E11.5	Coefficients of the equation of the inner pressure: $p_a = (PA0) + (PA1)t + (PA2)t^2$ with p_a in kg/cm^2 and t in hr
	PA1	12 to 22	E11.5	
	PA2	23 to 33	E11.5	
Card 6e	PBO	1 to 11	E11.5	Coefficients of the equation of the outer pressure: $p_b = (PBO) + (PB1)t + (PB2)t^2$ with p_b in kg/cm^2 and t in hr
	PB1	12 to 22	E11.5	
	PB2	23 to 33	E11.5	
Card 6f	RFO	1 to 11	E11.5	Coefficients of the equation of the free fuel radius: $r_f = (RFO) + (RF1)t + (RF2)t^2$ with r_f in mm and t in hr
	RF1	12 to 22	E11.5	
	RF2	23 to 33	E11.5	
Card 6g	ARFO	1 to 11	E11.5	Coefficients of the equation of the radial fuel strength coefficient: $\alpha_r = (ARFO) + (ARF1)t + (ARF2)t^2$ with α_r in kg/mm^3 and t in hr
	ARF1	12 to 22	E11.5	
	ARF2	23 to 33	E11.5	

Table 3.1. (continued)

Card 6h	EZFO	1 to 11	E11.5	Coefficients of the equation of the fuel axial strain: $\epsilon_{zf} = (EZFO) + (EZF1)t + (EZF2)t^2$ with ϵ_{zf} in (-) and t in hr
	EZF1	12 to 22	E11.5	
	EZF2	23 to 33	E11.5	
Card 6i	AZFO	1 to 11	E11.5	Coefficients of the equation of the axial fuel strength: $\alpha_z = (AZFO) + (AZF1)t + (AZF2)t^2$ with α_z in kg and t in hr The cards 6f to 6i inclusive must only be given when IFZ=1. For IFZ=2 they must be omitted
	AZF1	12 to 22	E11.5	
	AZF2	23 to 33	E11.5	
Card 7a (see re- mark 2)	TIME	1 to 11	E11.5	Time (hr), at which the functions depending of time (q' , t_{cs} , P_a , P_b , r_f , α_{ef} , ϵ_{zf} , α_{zf}) are given
	PUL	12 to 22	E11.5	Power per unit length (W/cm)
	TCS	23 to 33	E11.5	Coolant or sheath average temperature ($^{\circ}\text{C}$)
	PA	34 to 44	E11.5	Inner pressure (kg/cm^2)
	PB	45 to 55	E11.5	Outer pressure (kg/cm^2)
	RF	56 to 66	E11.5	Free fuel outer radius (mm). May be blank if IFZ = 2
	ARF	67 to 77	E11.5	Radial fuel strength constant (kg/mm^3). May be blank if IFZ = 2
Card 7b	EZF	1 to 11	E11.5	Free fuel axial strain (-) May be blank if IFZ = 2
	AZF	12 to 22	E11.5	Axial fuel strength constant (kg). May be blank if IFZ = 2
	KPR	23 to 25	I3	if 0, the results for this time are not printed if 1, the results are printed The maximum number of cards 7a und 7b is 100

Table 3.1. (continued)

Card 8a (see re- mark 2)	NCYC	1 to 3	I3	Number of cycles asked
	NCA	4 to 6	I3	Number of cards of type 8b and c necessary to describe a cycle (maximum value is 10)
	NCPR	7 to 9	I3	If 0 the first and last times only are printed If >0 a printing is obtained for each cycle out of NCPR at the positions where KPR > 0 on cards 8c
Card 8b Card 8c	are identical to cards 7a and b. Their number is equal to NCA. They describe the first cycle of the series			
(This section of the table is empty)				

Table 3.2.: Fields occupied by the various data on the input cards

	10	20	30	40	50	60	70	80					
1	IDAY	IMONTH	IYEAR	NPR0M	D D E Q D	EPSIL 1	EPSIL 2	T E T I	ITEST1	ITEST2	ITEST3	ITEST4	ITEST5
2	TITLE												
3	N	IFZ	LEQ	LCP	LCR	LTSH	ITEM	IINT	ILPR	ITIM	ILIM		
4	RA	RB	HTC	FPS	CORRA	ECMAX							
5a	MM												
5b	TT(J)	AKT(J)	ALF(J)	YMT(J)	AMUT(J)	AKM(J)	ANN(J)						
5c	AMN(J)	PS2(J)	PRS(J)	PRE(J)									
6a	TIMEM	DTIME	NPR										
6b	PULO	PUL1	PUL2										
6c	TCS0	TCS1	TCS2										
6d	PA0	PA1	PA2										
6e	PB0	PB1	PB2										
6f	RF0	RF1	RF2										
6g	ARF0	ARF1	ARF2										
6h	EZF0	EZF1	EZF2										
6i	AZF0	AZF1	AZF2										

Table 3.2. (continued)

	10	20	30	40	50	60	70	80
7a	TIME	PUL	TCS	PA	PB	RF	ARF	
7b	EZF	AZF	KPR					
8a	NCYC	NCA	NCPR					
8b	see	7a						
8c	see	7b						

3.3. Description of the output

The output possibilities of the programme are too large to be described here in details. Only the most usual output will be explained here.

A part of such an output is reproduced at figure 7. The programme prints first all the input data, exactly as they are on the input cards. A part of them is then printed again in a more legible form. For each calculation time where a printing is asked, one gets then:

- the number of iterations to reach the convergence, the maximum error on the permanent strains and its location.
- the values of the various functions dependent of the time.
- a table of the stresses and strains for each of the calculation points in the thickness

In this table, one finds successively for each point: the cold radius, the radius after deformation, the temperature, the equivalent stress, the equivalent permanent strain, the three components of the stress, of the total strain, of the permanent strain, of the thermal strain and of the elastic strain.

In the table, a letter M is printed beside each maximum value.

A summary of the table with all the maximum values and their location is then given.

3.4. Warning and errors messages

The programme is able to detect some errors made by the user in the preparation of the input data or errors occurring during the calculations. A list of the various messages is given hereunder with more detailed explanations.

- DATA ERROR IN THE TABLE OF PROPERTIES

END OF JOB

explanation: the temperature values in the table of properties are not given in increasing order. This error causes the job to be terminated. The table of properties is printed.

users response: correct the table of properties

- MATERIAL PROPERTIES NOT FOUND

END OF JOB

explanation: the value of MM (number of temperature values in the table of properties on card 5a has been given equal to 0 for the first problem of a job. This error causes the job to be terminated.

users response: give a correct table of properties

- NUMBER OF PROPERTIES NOT SUFFICIENT FOR THE TYPE OF INTERPOLATION ASKED

END OF JOB

explanation: the number of temperature values in the table of properties is smaller than 2 or 3, respectively if linear or parabolic interpolation is used. This causes the job to be terminated. The table of properties is printed.

users response: give a sufficient value of properties

- DATA ERROR AT THE TIME xxxx.x HOURS

THE PROGRAMME SHIFTS TO THE NEXT PROBLEM

explanation: at the beginning of a problem for which the time dependent functions are given by points (ITIM=2) or are cyclic functions (ITIM=3), the programme has encountered a time value smaller as the preceding one. This error causes the programme to shift to the next problem.

users response: correct the input data

- TOO MUCH INPUT DATA

THE PROGRAMME SHIFTS TO NEXT PROBLEM

explanation: the number of calculation points for the functions dependent of the time is larger as 100, when ITIM=2. This message is also printed when the number of calculation points for the first cycle (ITIM=3) is larger as 10.

users response: In both cases, reduce the number of calculation points to the admissible values.

- SINGULAR MATRIX ENCOUNTERED IN THE CALCULATION OF THE PERMANENT STRAIN INCREMENTS AT TIME `xxxx.x` HOURS AFTER `xx` ITERATIONS. THE PROGRAMME SHIFTS TO THE NEXT PROBLEM

explanation: the convergence process requires the solution of a linear equation system (see para 2.8). The matrix of this system may be singular for one of the following reasons:

- there is a data error in the geometry of the pin, if this message is printed during one of the first time intervals
- one of the functions dependent of the time is erroneous (This error occurs then for the time printed).
- one has given unrealistic properties of the materials
- a large extrapolation in the table of properties has been performed causing unrealistic values of properties to be calculated
- the iterative process does not converge due to too large calculation intervals (see para 2.9) or to a incorrect choice of the permanent strain difference DDEQD (on card 1) for the calculation of the partial derivatives.

This error causes the programme to shift to the next problem.

users response:

- correct the data errors if any
 - give more adequate values of the properties to avoid the extrapolation
 - use smaller time intervals
 - use an other value for DDEQD
- CONVERGENCE NOT REACHED AT `xxxx.x` HOURS (MAXIMUM ERROR IS `.xxxExx(xx)`)

explanation: the number of iterations in the convergence process is limited to 30. If the convergence criterion is not verified for this number of iterations, the message hereabove is printed. The programme begins then the calculation of the next time interval with incorrect values of the permanent strains. The maximum error on the permanent strain increments and its location are printed.

The causes of the convergence difficulties are the same as those of the previous error. This can also occur when use is made of a too small convergence criterion.

users response:

- see the previous error
- use a larger convergence criterion
- WARNING. VALUES OF THE MATERIAL PROPERTIES WERE EXTRAPOLATED AT THE FOLLOWING CALCULATION TIMES (HRS):

xxxx.x xxxx.x

This warning message appears each time that a table of stress and strain is printed, when between this printing and the previous one values of properties were extrapolated. As explained hereabove an extrapolation of the creep or plasticity properties can be the cause of a great inaccuracy particularly when this extrapolation is large.

3.5. Test printing

A printing of intermediate results is always possible thanks to the variables ITEST1, ITEST2, ITEST3, ITEST4 and ITEST5 on card 1 of the input data. A list of the test printed variables is given in the table 3.3. hereafter. As soon as one of these variables is larger than 0, the corresponding test printing is obtained for all the calculation times larger as the value of TETI on card 1.

This test printing facility was particularly usefull during the checking of the programme but can also be used to find data errors.

Table 3.3.: Test printing list

11		TIME	, HTC	, YM	, AMU	, TCØ	, TCS	, TCSE	, PUL	, PULE
		TCSP	, TCSS	, PULP	, PULS					
12	I	,R(I)	, TE(I)	, ETH(I)	, TAV(I)	, AKN1(I)	, ANN1(I)	, AMN1(I)	, PP(I)	, QP(I)
21	ICB	,TIME	, AZFA	, PA	, AL	, AM	, AN	, C1	, C2	, C3
22	I	,ECRS(I)	, DECR(I)	, ECRE(I)	, ECTS(I)	, DECT(I)	, ECTE(I)	, ECZS(I)	, DECZ(I)	, ECZE(I)
		ECEQS(I)	, DECEQ(I)	, ALR(I)	, AMR(I)	, ANR(I)				
23	I	,UE(I)	, ERE(I)	, ETE(I)	, SRE(I)	, STE(I)	, SZE(I)	, SEQE(I)		
24	I	,FSR(I)	, FST(I)	, FSZ(I)	, DECEQC(I)					
31	IT	,IERR	, KS	, TIME	, ERR					
32	I	,DECEQ(I)	, DECEQC(I)	, DECEQ(I)	, B(I)					
33		A1(1)	, A1(2)	, ...	, ...					
41	I	,SEQ(I)	, AKN1(I)	, ANN1(I)	, AMN1(I)	, TIMMØY	, DECEQC(I)			
42	I	,SEQE(I)	, PP(I)	, QP(I)	, ECEQS(I)	, ECEQE(I)	, DECEQC(I)	, DECEQ(I)	, SS	, SLP(I)
51	MDC	,DECM	, EEEQ	, DTIDIV	, FD					
52	KPR	,TIME	, DTIME	, TIMEP	, DTIMEP	, PUL	, TCS	, PA	, PB	, RF
		ARF	, EZF	, AZF						

4. Conclusions

The programme CRASH has a very general character:

- it takes into account practically all the possible loads on the sheath: outer coolant pressure, inner gas pressure and axial force resulting from these pressures, temperature gradient and fuel-sheath radial and axial interaction.
- these loads can be any functions of the time and three different methods allow to introduce them in the programme depending of their complexity.
- although two creep laws and two plastic laws only are programmed in the version described here, the programme can use any creep or plastic law.
- all the properties of the material can be any function of the temperature.

Notwithstanding its general character, the computing time of the programme is small enough to allow intensive parametric studies.

The possibilities of application of the programme are thus rather large: they cover the design of fuel pins as well as the analysis of experimental results of irradiation on possible sheath materials [14].

Such a general programme can thus be improved as soon as one has a better knowledge of the creep or plastic phenomenon. The main possibilities of improvement of CRASH could then be:

- the development and the programmation of a better creep or plastic law for the materials able to be used as sheath materials.
- to account the effects of irradiation on the creep and plasticity phenomena.
- the modifications of the material properties with the number of cycles when cycling occurs.
- to account the swelling of the material under irradiation.

Acknowledgements

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Nomenclature

Variables

a	sheath inner radius
b	sheath outer radius
C_1	integration constant
C_2	integration constant
C_3	integration constant
E	Young' modulus
F_h	fraction of the heat generated in the sheath
F_z	axial force on the sheath
F_{zf}	axial force on the fuel
h	heat transfer coefficient between coolant and sheath finite difference on the permanent strains increments (see para 2.8)
k	thermal conductivity of the sheath
K	coefficient appearing in the creep laws (see para 2.6)
L	integral defined by the equation (1.33)
m	coefficient of the second creep law (para 2.6)
M	integral defined by the equation (1.34)
n	coefficient of the creep laws (para 2.6)
N	integral defined by the equation (1.35)
p_a	inner gas pressure
p_b	outer coolant pressure
p_c	fuel-sheath contact pressure
P	coefficient of the second plastic law (para 2.7.2)
q	heat source in the sheath
q'	total linear power
Q	coefficient of the second plastic law (para 2.7.2)
r	radial coordinate
r_f	hot free fuel radius
t	time
T	temperature

T_{co}	coolant temperature
T_{cs}	coolant or sheath average temperature
u	radial displacement
u_a	radial displacement at the sheath inner surface
z	axial coordinate
α	sheath thermal expansion
α_r	radial fuel strength constant
α_z	axial fuel strength constant
$\gamma_{r\theta}$	shear strain component
γ_{rz}	shear strain component
$\gamma_{\theta z}$	shear strain component
δ_i	guess of the equivalent permanent strain increment at point i
δ_{ci}	calculated equivalent permanent strain increment at point i
δ_j^*	guess of the equivalent permanent strain increment at point j for the previous iteration
Δt	length of a time interval
$\Delta \delta_j$	correction to the guess of the equivalent permanent strain increment
ϵ_{eq}	equivalent strain
ϵ_r	radial component of the strain
ϵ_{rp}	equivalent rupture strain
ϵ_t	value defined by the relation (1.19)
ϵ_z	axial component of the strain
ϵ_{zf}	hot free fuel axial strain
ϵ_θ	tangential component of the strain
θ	angular coordinate
μ	Poisson's ratio
σ_{eq}	equivalent stress
σ_r	radial stress component
σ_{rt}	equivalent rupture stress
σ_z	axial stress component
σ_θ	tangential stress component
$\sigma_{0.2}$	equivalent stress, leading to 0.2 % permanent strain
Σ	summation sign

τ_{rz} shear component of the stress
 $\tau_{r\theta}$ shear component of the stress
 $\tau_{\theta z}$ shear component of the stress
 φ heat flux
 $\frac{\partial \dots}{\partial \dots}$ partial derivation sign

Subscripts

a at the inner sheath surface
av average
b at the outer sheath surface
c permanent (creep or plastic)
el elastic
f fuel
i relative to the i th point of calculation
r radial
rp rupture
th thermal
z axial
 θ tangential

Superscripts

- average
· time derivative
* relative to the previous time interval or to the previous iteration

Positive directions of the stresses in cylindrical coordinates

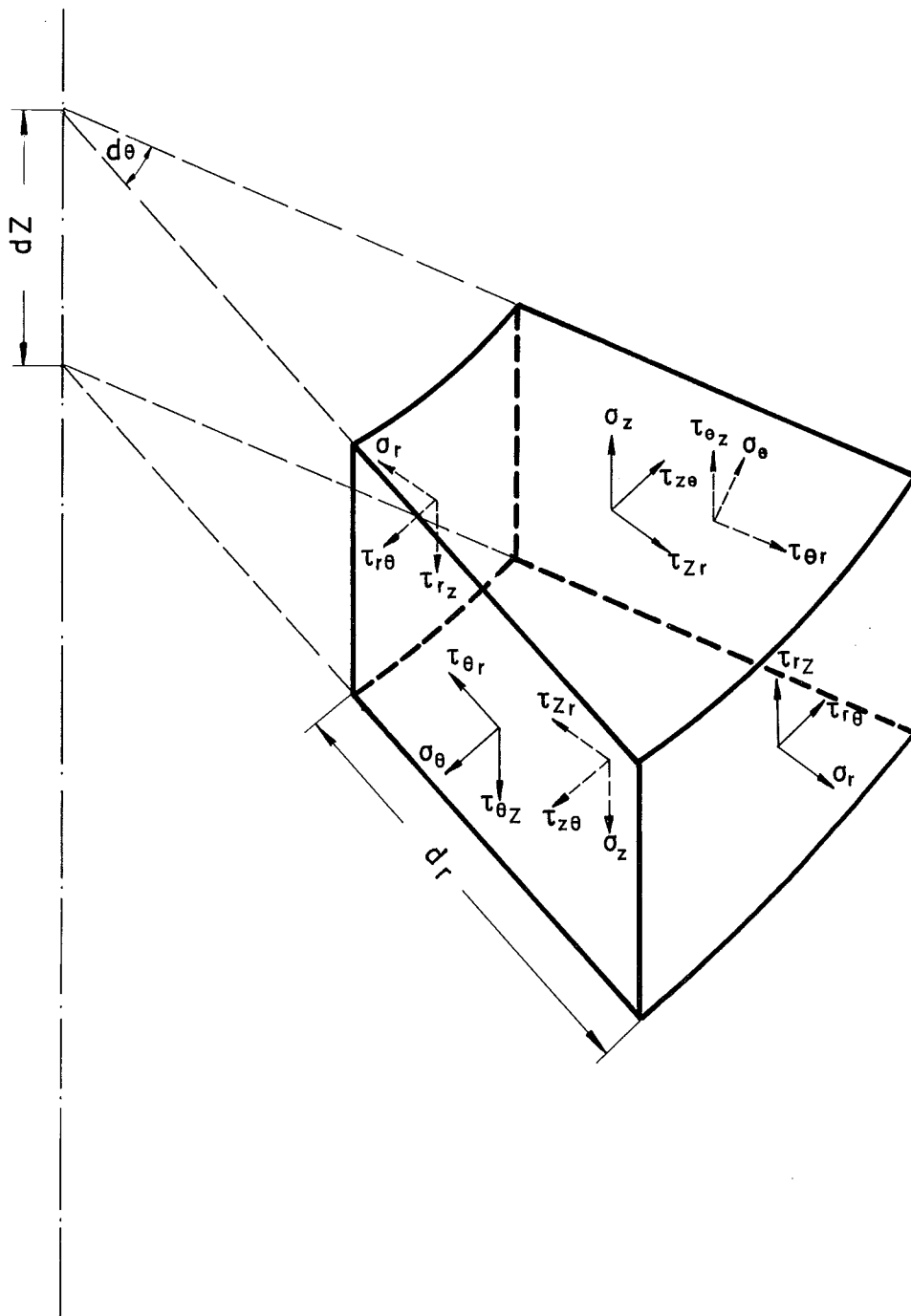
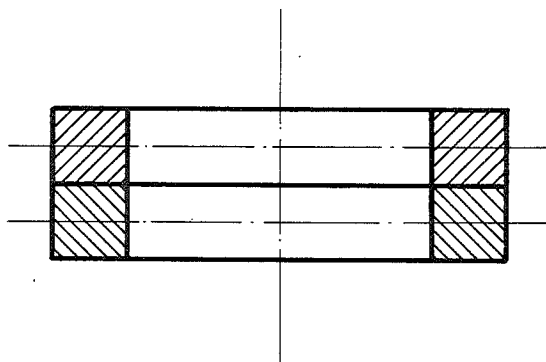


Figure 1

Axial cross section through two small elements of the sheath

a

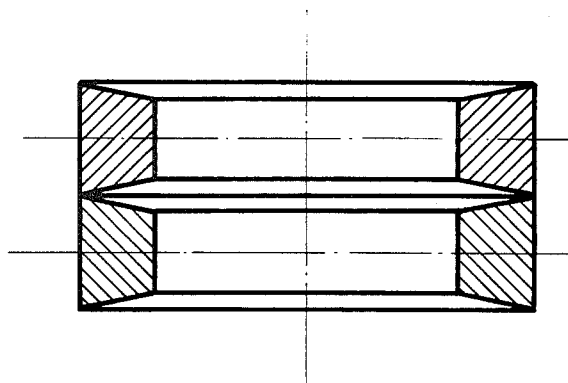
before deformation



b

after deformation

cross section not plane :
continuity not complied
with.



c

plane cross section

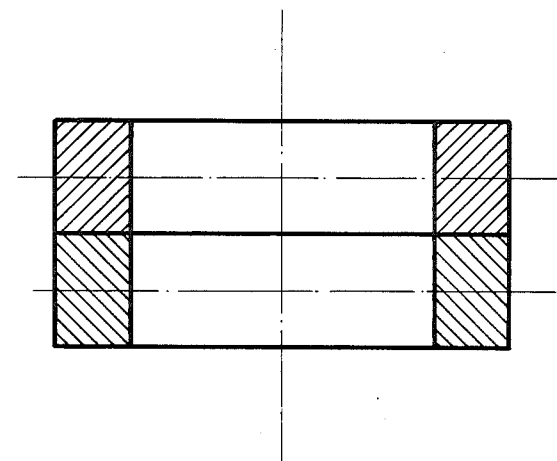


Figure 2

Radial and tangential stresses on an element of volume

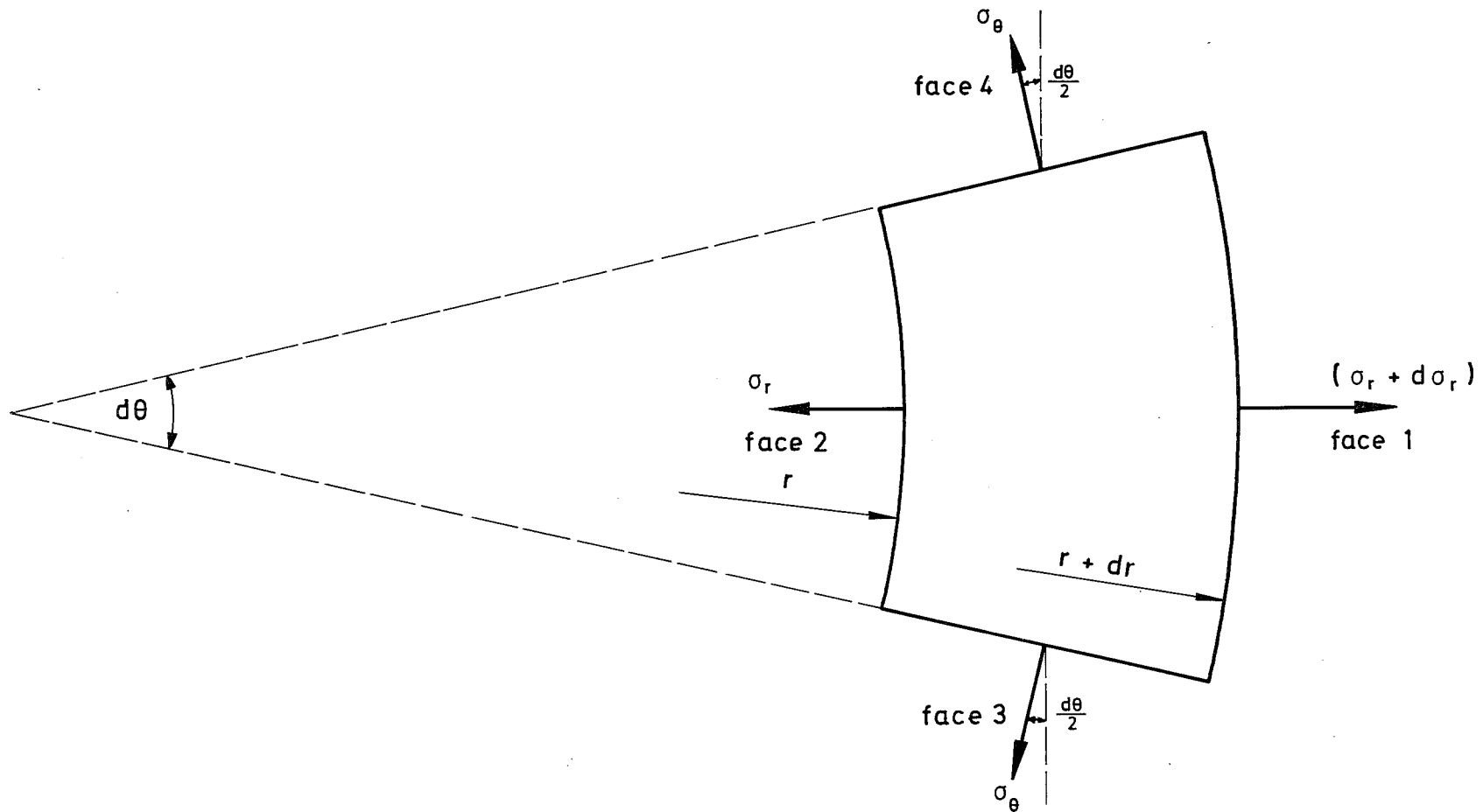
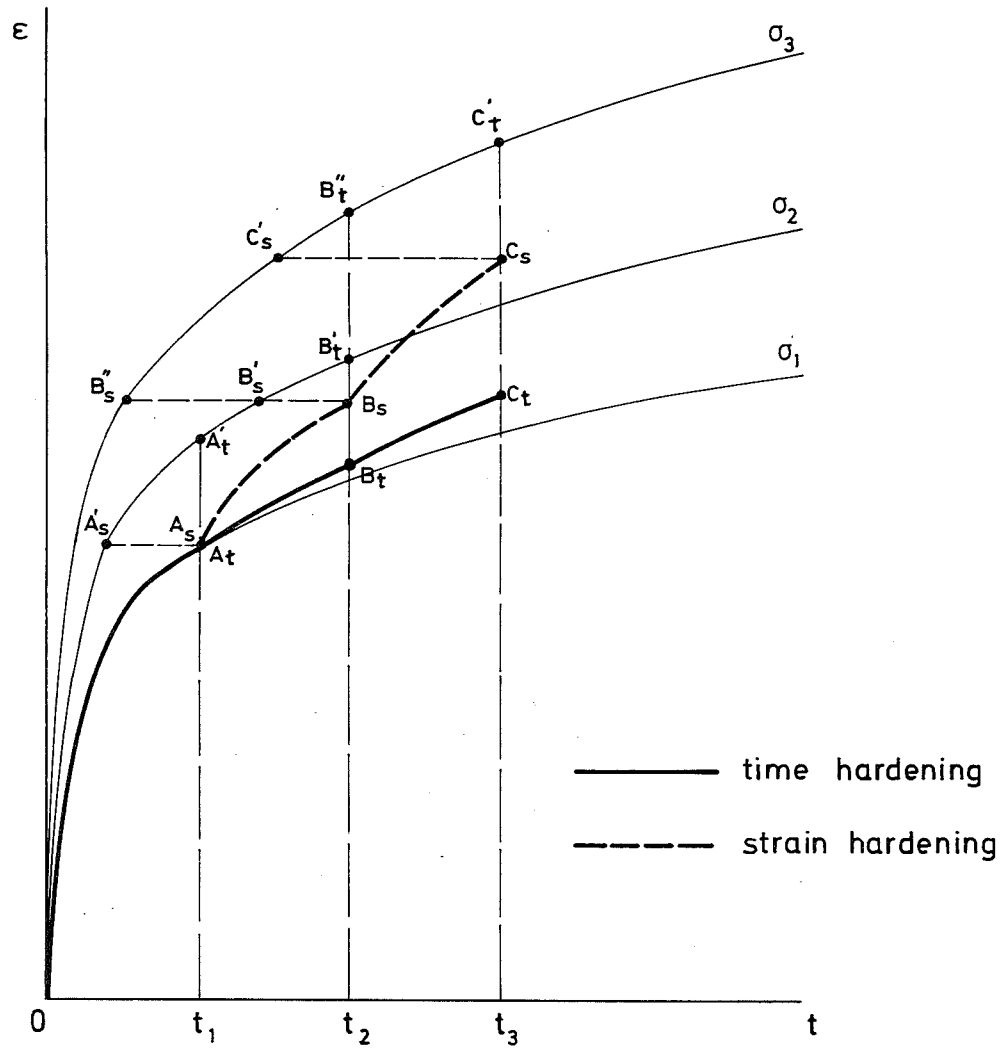


Figure 3

Strain curve in time and strain hardening

a) Increasing stress



b) Decreasing stress

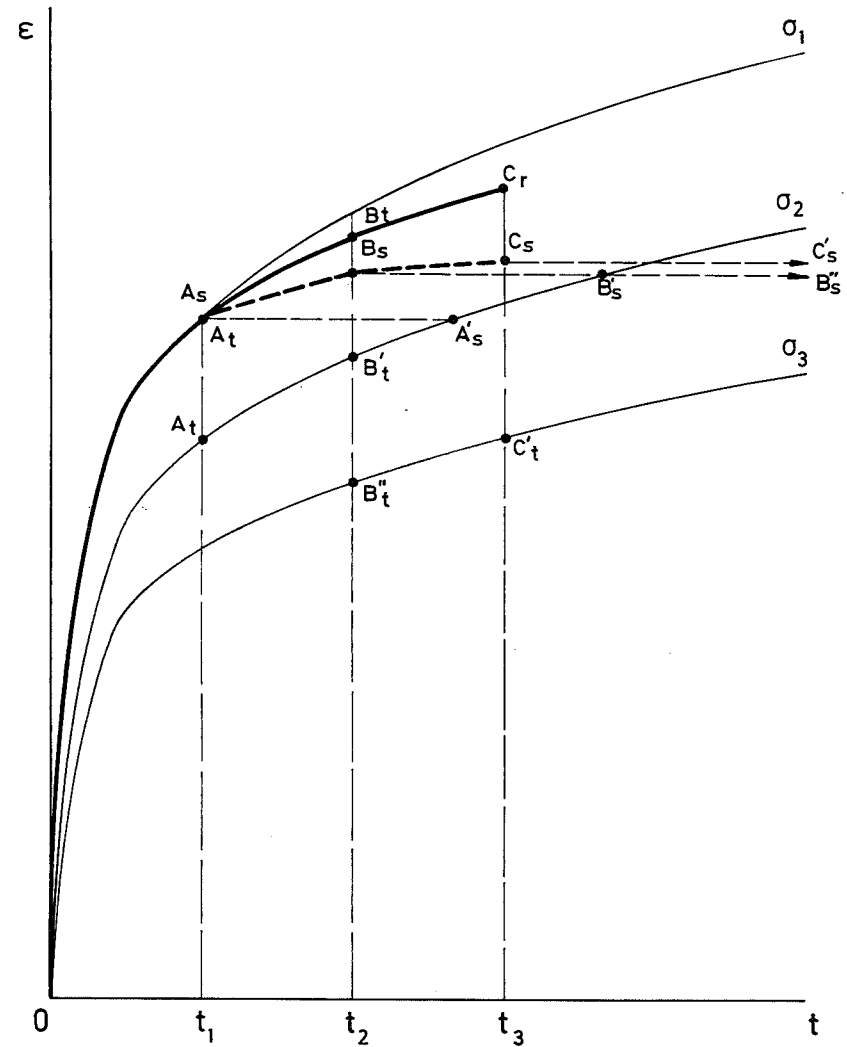
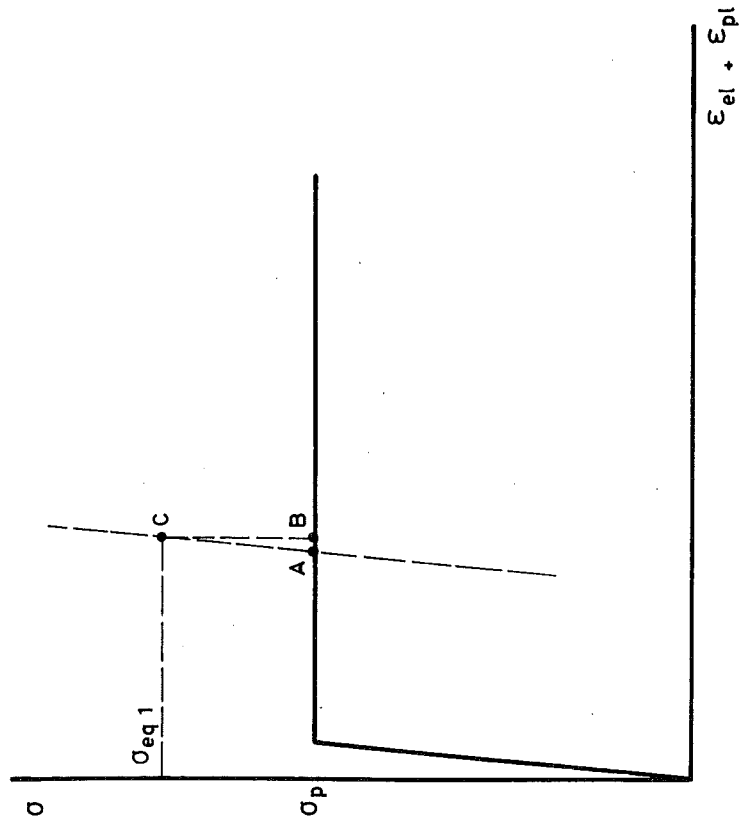


Figure 4

Plastic laws

a) First plastic law



b) Second plastic law

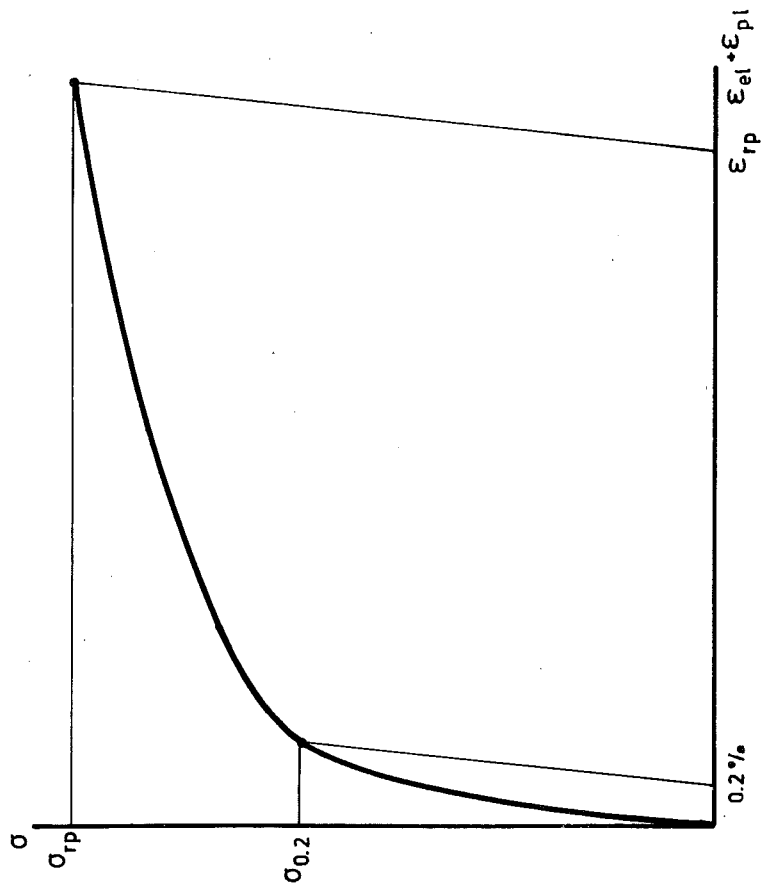


Figure 5

Interdependence of the various subroutines from each other

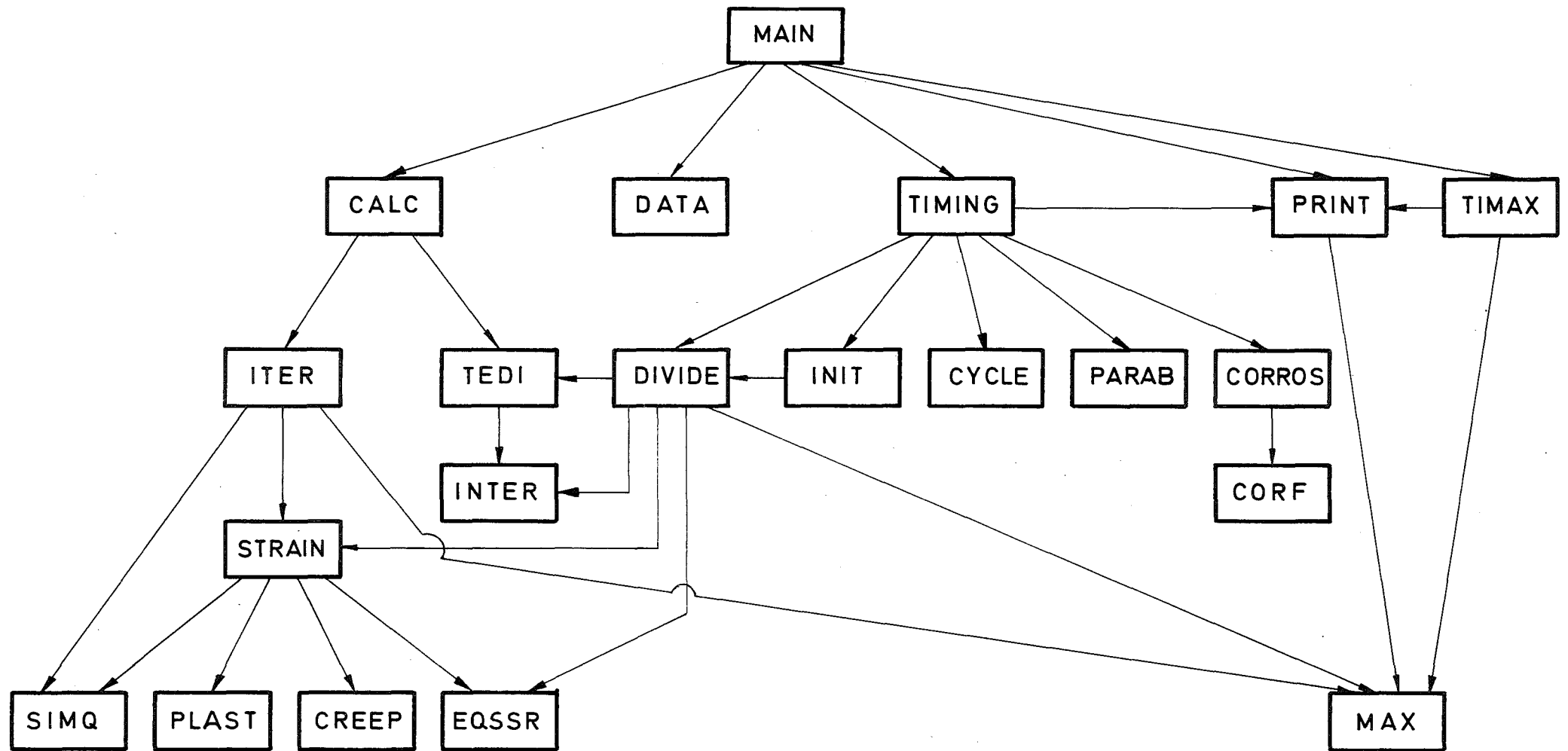


Figure 6

DATA.

=====
 COLD OUTER RADIUS 3.000 MM COLD INNER RADIUS 2.600 MM HEAT TRANSFER COEF. 12.100 W/CM2 DC
 CORROSION RATE 0.00450 MU/HR
 MAX. ERR. ON AV. TEMP. 0.100 DC MAX. ERR. ON PLAST.STR. 0.200E-05 - HEAT FRACTION IN SHEATH 0.0300 -

OPTIONS USED.

=====
 IFZ =1 FUEL SWELLING IS TAKEN INTO ACCCUNT
 LEQ= 1 EQUIVALENT STRESS/STRAIN STATE DETERMINED BY THE MISES CRITERION
 LCP =2 PLASTICITY CALCULATIONS ARE PERFORMED
 LCR =2 THE STRESS-PLASTIC STRAIN CURVE IS EXPONENTIAL
 ITEM=1 THE COOLANT TEMPERATURE IS GIVEN
 IINT=2 LINEAR INTERPCLATION IS USED
 ITIM=3 CYCLING OF THE TIME DEPENDENT VARIABLES IS PERFORMED
 ILIM=1 A MAXIMUM RUNNING TIME IS GIVEN

TABLE OF THE PROPERTIES OF THE SHFATH MATERIAL.

TEMPE- RATURE DC	THERMAL CONDUCT. W/CM DC	LIN. EXPAN. COEFFICIENT -/DC	YOUNG'S MODULUS KG/MM2	POISSON'S RATIO	K	CREEP PARAMETERS			PLASTICITY PARAMETERS		
						N	M		A	B	C
600.00	0.21800	0.18500E-04	16000.	0.3000	0.0	0.0	0.0	0.0	20.00	40.00	0.25000
700.00	0.23400	0.18700E-04	15000.	0.3000	0.0	0.0	0.0	0.0	18.00	35.00	0.30000

STATE OF STRESS AND STRAIN AFTER 0.0 HOURS. (1TH CYCLE)

=====
 ITERATIONS NUMBER 5 - ERROR ON PLAST.STRAINS 0.123E-06 -(11)
 LINEAR POWER 500.00 W/CM COOLANT TEMPERATURE 600.00 DC AVERAGE TEMPERATURE 645.87 DC
 INNER PRESSURE (GAS) 0.0 KG/CM2 OUTER PRESSURE 0.0 KG/CM2 AXIAL FORCE 19.00 KG
 HOT FREE FUEL RADIUS 2.6320 MM RAD. FUEL STRENGTH 5000. KG/MM3 CONTACT PRESSURE 62.02 KG/CM2
 FREE FUEL AX. STRAIN 0.012100 - AX. FUEL STRENGTH 0.100E 09 KG
 YOUNG'S MODULUS 15541. KG/MM2 POISSON'S RATIO 0.3000 -

TABLE OF STRESSES AND STRAINS.

IN- DEX	COLD RADIUS MM	HOT RADIUS MM	TEMPE- RATURE DC	EQU. STRESS KG/MM2	EQU. PLAST. STRAIN	STRESS KG/MM2	TOTAL STRAIN	PLAST. STRAIN	THERMAL STRAIN	ELASTIC STRAIN
1	2.6000	2.6319	671.63 M	7.39	0.000002	R -0.62	0.012785 M	0.000002	0.012521 M	0.000262
						T -6.87	0.012260	-0.000001	0.012521 M	-0.000260
						Z -8.77	0.012100	-0.000002	0.012521 M	-0.000420
3	2.6800	2.7129	661.37	3.06	0.000000	R -0.74	0.012394	0.000000	0.012316	0.000078
						T -2.22	0.012270	0.000000	0.012316	-0.000047
						Z -4.25	0.012100	-0.000000	0.012316	-0.000217
5	2.7600	2.7939	651.28	2.60	0.000000	R -0.71	0.012023	-0.000000	0.012115	-0.000093
						T 2.22	0.012268	0.000000	0.012115	0.000153
						Z 0.21	0.012100	-0.000000	0.012115	-0.000016
7	2.8400	2.8748	641.35	6.30	0.000001	R -0.57	0.011667	-0.000001	0.011918	-0.000250
						T 6.45	0.012256	0.000000	0.011918	0.000338
						Z 4.59	0.012100	0.000000	0.011918	0.000182
9	2.9200	2.9557	631.57	9.94	0.000017	R -0.33	0.011318	-0.000017	0.011724	-0.000389
						T 10.31	0.012235	0.000011	0.011724	0.000501
						Z 8.74	0.012100	0.000006	0.011724	0.000370
11	3.0000	3.0366	621.92	12.74 M	0.000096 M	R -0.00	0.010947	-0.000096 M	0.011533	-0.000491 M
						T 13.26 M	0.012206	0.000054 M	0.011533	0.000619 M
						Z 12.15 M	0.012100	0.000042 M	0.011533	0.000525 M

Figure 7a

VALUES AND LOCATIONS OF MAXIMA.

IN-DEX	COLD RADIUS MM	HOT RADIUS MM	TEMPERATURE DC	EQU. STRESS KG/MM2	EQU. PLAST. STRAIN	STRESS KG/MM2	TOTAL STRAIN	PLAST. STRAIN	THERMAL STRAIN	ELASTIC STRAIN					
			671.63	1	12.74	11	0.000096	11							
						R -0.74	4	0.012785	1	-0.000096	11	0.012521	1	-0.000491	11
						T 13.26	11	0.012270	4	0.000054	11	0.012521	1	0.000619	11
						Z 12.15	11	0.012100	--	0.000042	11	0.012521	1	0.000525	11

STATE OF STRESS AND STRAIN AFTER 10000.0 HOURS. (5TH CYCLE)

ITERATIONS NUMBER	3		ERROR ON PLAST. STRAINS	0.125E-06	-(4)					
LINEAR POWER	500.00	W/CM	COOLANT TEMPERATURE	600.00	DC	AVERAGE TEMPERATURE	643.81	DC		
INNER PRESSURE (GAS)	50.00	KG/CM2	OUTER PRESSURE	0.0	KG/CM2	AXIAL FORCE	119.40	KG		
HOT FREE FUEL RADIUS	2.6430	MM	RAD. FUEL STRENGTH	5000.	KG/MM3	CONTACT PRESSURE	635.22	KG/CM2		
FREE FUEL AX. STRAIN	0.017100		AX. FUEL STRENGTH	0.100E 09	KG					
YOUNG'S MODULUS	15562.	KG/MM2	POISSON'S RATIO	0.3000						

TABLE OF STRESSES AND STRAINS.

IN-DEX	COLD RADIUS MM	HOT RADIUS MM	TEMPERATURE DC	EQU. STRESS KG/MM2	EQU. PLAST. STRAIN	STRESS KG/MM2	TOTAL STRAIN	PLAST. STRAIN	THERMAL STRAIN	ELASTIC STRAIN				
1	2.6000	2.6417	666.79	M	22.01	0.006701								
						R -6.85	M	0.004726	M	-0.006676	0.012425	M	-0.001022	M
						T 14.10	M	0.016048	M	0.002896	0.012425	M	0.000728	M
						Z 16.08		0.017099		0.003781	0.012425	M	0.000894	
3	2.6800	2.7221	656.49		22.29	0.006727								
						R -6.22		0.004512		-0.006693	0.012219		-0.001015	
						T 14.58		0.015707		0.002765	0.012219		0.000723	
						Z 17.31		0.017099		0.003929	0.012219		0.000951	
5	2.7600	2.8024	646.36		22.58	0.006770								
						R -5.61		0.004292		-0.006719	0.012018		-0.001007	
						T 15.03		0.015380		0.002645	0.012018		0.000717	
						Z 18.51		0.017099		0.004074	0.012018		0.001008	
7	2.8400	2.8828	636.37		22.87	0.006823								
						R -5.02		0.004068		-0.006752	0.011819		-0.001000	
						T 15.44		0.015064		0.002535	0.011819		0.000709	
						Z 19.67		0.017099		0.004216	0.011819		0.001063	
9	2.9200	2.9631	626.53		23.17	0.006887								
						R -4.46		0.003840		-0.006792	0.011624		-0.000992	
						T 15.80		0.014760		0.002435	0.011624		0.000700	
						Z 20.80		0.017099		0.004357	0.011624		0.001118	
10	2.9550	2.9982	622.26		23.30	M	0.006919	M						
						R -4.22		0.003738		-0.006813	0.011539		-0.000989	
						T 15.95	M	0.014630		0.002395	0.011539		0.000696	
						Z 21.28	M	0.017099		0.004418	0.011539		0.001141	M

VALUES AND LOCATIONS OF MAXIMA.

			666.79	1	23.30	10	0.006919	10							
						R -6.85	1	0.004726	1	-0.006813	10	0.012425	1	-0.001022	1
						T 15.95	10	0.016048	1	0.002896	1	0.012425	1	0.000728	1
						Z 21.28	10	0.017099	--	0.004418	10	0.012425	1	0.001141	10

Figure 7b