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Safeguards Statements Based on Relevant Components of Material Unaccounted For (MUF)

R. Avenhaus, D. Gupta, H. Singh

GESELLSCHAFT FÜR KERNFORSCHUNG M. B. H.

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R. Avenhaus, D. Gupta, H. Singh +)

Gesellschaft für Kernforschung mbH., Karlsruhe

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+) on delegation from the Bhabha Atomic Research Center, India

Zusammenfassung

Es besteht allgemeine Übereinkunft, daß die Materialbilanz eine der wichtigsten Maßnahmen zur Überwachung spaltbaren Materials ist, da sie Zahlen produziert. Das Ergebnis der Materialbilanz für eine kerntechnische Anlage während einer Inventurperiode ist eine Aussage am Ende der Inventurperiode, ob die Differenz zwischen dem Buch- und dem realen Inventar (Material Unaccounted For, MUF) signifikant ist, d.h. auf eine Entwendung hindeutet, oder nicht. Daher kommt der Analyse des MUF besondere Bedeutung zu.

Die vorliegende Arbeit besteht aus zwei Teilen. Im ersten Teil werden die denkbaren Komponenten des MUF (zufällige und systematische Fehler, Prozeßverluste usf.) zusammengestellt, und es werden mit Hilfe historischer Daten Aussagen über die relativen Größenordnungen dieser Komponenten gemacht. Im zweiten Teil werden die möglichen Aussagen der Inspektionsbehörde über den Wert des MUF analysiert. Dabei wird dem Problem der Fortpflanzung der Fehler 1. Art im Falle einer Folge von Inventurperioden besondere Beachtung geschenkt. Die Relationen zwischen den das Problem charakterisierenden Parametern: Fehler erster und zweiter Art, kritische Masse, Zahl der Inventurperioden und Varianz des MUF werden mit Hilfe von Nomographen illustriert.

Abstract

It is accepted generally that material accountancy is one of the most important safeguards measures as it produces numbers. The result of the material accountancy during one inventory period is a statement at the end of the inventory period whether or not the difference between the book inventory and the physical inventory (Material Unaccounted For, MUF) is significant. Therefore, the analysis of MUF is of central importance.

This paper consists of two parts: In the first part the possible components of MUF (random and systematic errors, process losses etc.) are collected. With the help of historical data statements are made on the relative orders of magnitude of these components. In the second part the possible statements on MUF of the safeguards authority are analyzed, in the case of one inventory period as well as in the case of a sequence of inventory periods. In the latter case special attention is given to the problem of propagation of errors of the first kind. The relations between the relevant parameters error first and second kind, critical mass, number of inventory periods and variance of MUF are illustrated with the help of nomographs.

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R. Avenhaus, D. Gupta, H. Singh+)

Institut für Angewandte Reaktorphysik
Kernforschungszentrum Karlsruhe
Federal Republic of Germany

1. Introduction

According to Article 30 of INFCIRC/153 [1], the technical conclusion of the Agency's verification activities shall be a statement .... of the amount of material unaccounted for over a specific period, giving the limits of accuracy of the amounts stated. However, a safeguards organization has to be in a position to know whether the MUF and the standard deviation of MUF are significant or not. In other words, the safeguards organization should be able to decide - in case the MUF is found to be other than zero with some value of standard deviation - whether these values can be explained by the known operating conditions in a facility or whether further information is required to explain them. For this purpose the safeguards organization requires a formalized model with the help of which it can arrive at a decision of this nature. In such a model different components of the MUF and a number of statistical quantities are required as input data.

In a recent publication [2] some published data on MUF were analysed mainly to understand the behaviour of the MUF data and to discern its various components.

The present paper has been divided into two parts. In the first part an effort has been made at the beginning to formalize the relation between all conceivable components of MUF which may be considered to be relevant. Some of the results in [2] are then discussed with a view to find out those components which contribute most to the actual values of MUF. On the basis of this analysis a number

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+) on delegation from the Bhabha Atomic Research Center, India of components considered initially for formalization has been eliminated to keep the subsequent treatment perspicuous. The second part deals with statements which an inspection organization can make on MUF after coming to a decision. With the help of some monographs the variation possibilities of a particular decision have been analysed with reference to a single inventory and a sequence of inventories.

2. Analysis of MUF-Components

2.1 Components of MUF

Under a diversion free condition, the difference between the book and the physical inventory (MUF) may be considered to consist of measurement errors a and process losses b.

Thus,

$$MUF = a + b \qquad (2.1)$$

Both these components have random and systematic parts.

The measurement errors may basically be composed of three parts, the reproducibility a_{r_1}, the systematic errors a_{r_2} which are of random origin (calibration errors) and biases a_s which will have a fixed value for a given laboratory and a given instrument, so that

$$a = a_{r_1} + a_{r_2} + a_s \qquad (2.2)$$

The process losses which may consist of unmeasured process losses which have the facility and the hidden process inventories which remain in the facility may have random parts b_r and systematic parts b_s, too:

$$b = b_r + b_s \qquad (2.3)$$

Both the two components a and b may consist of one part which is proportional to the feed F of one campaign, and an absolute part which is independent of F.

$$a = a^1 + a^2 F; \; b = b^1 + b^2 F \qquad (2.4)$$

Thus one obtains in the diversion-free case the following list of MUF-contributions:

$$MUF = a_{r_1}^1 + a_{r_2}^1 + a_s^1 + b_r^1 + b_s^1 + (a_{r_1}^2 + a_{r_2}^2 + a_s^2 + b_r^2 + b_s^2)F \qquad (2.5)$$

By definition, the expectation value E(MUF) of the MUF, if one assumes all contributions to be mutually independent, is given by

$$E(MUF) = E(a_s^1 + b_s^1 + (a_s^2 + b_s^2)F) = a_s^1 + b_s^1 + a_s^2 \cdot EF + b_s^2 \cdot EF \qquad (2.6)$$

The variance of MUF is given by

$$var(MUF) = var(a_{r_1}^1 + a_{r_2}^1 + b_r^1 + (a_{r_1}^2 + a_{r_2}^2 + b_r^2)F)$$

$$= var\ a_{r_1}^1 + var\ a_{r_2}^1 + var\ b_r^1 + E^2F(var\ a_{r_1}^2 + var\ a_{r_2}^2 + bar\ b_r^2)$$

(Terms with products of variances are neglected here.) (2.7)

If one consideres the relative MUF which is defined as MUF divided by feed, one has instead of (2.6,7)

$$E(\frac{MUF}{F}) = \frac{Ea_s^1}{EF} + \frac{Eb_s^1}{EF} + Ea_s^2 + Eb_s^2 \qquad (2.8)$$

$$var(\frac{MUF}{F}) = var(\frac{a_{r_1}^1}{F}) + var(\frac{a_{r_2}^1}{F}) + var(\frac{b_r^1}{F}) + var\ a_{r_1}^2 + var\ a_{r_2}^2 + var\ b_r^2 \qquad (2.9)$$

It may be seen that the biases of the measurement errors and the systematic part of the process losses contribute to the expectation value of the MUF, whereas the random and systematic parts of the measurement errors and the random parts of the process losses contribute to the variance of the MUF.

Remark:

It is possible that some other operating conditions and usages contribute to the MUF, as for example write offs, or transfers from one inventory period to the other. However, it is difficult to study them theoretically; therefore, it is assumed here that these contributions, if they are significant, are classified in a second action level.

The formalization procedures could be considerably simplified if before proceeding further, answers to the following two questions could be obtained:

(i) What components of MUF, listed in eq. (2.5) are important?

(ii) What is the distribution of MUF or relative MUF?

One statement can be made without further analysis: As a material balance consists always of sums of batch data, the contribution of the reproducibility of the measurement errors can be neglected in comparison to those of the calibration errors and the biases. Thus one has instead of eq. (2.5,8,9), if one writes a_r instead of a_{r_2}

$$MUF = a_r^1+a_s^1+b_r^1+b_s^1+(a_r^2+a_s^2+b_r^2+b_s^2)F \tag{2.10}$$

$$E(\frac{MUF}{F}) = \frac{Ea_s^1}{EF} + \frac{Eb_s^1}{EF} + Ea_s^2 + Eb_s^2 \tag{2.11}$$

$$var(\frac{MUF}{F}) = var(\frac{a_r^1}{F}) + var(\frac{b_r^1}{F}) +var\ a_r^2 + var\ b_r^2 \tag{2.12}$$

However, for small campaigns the contribution of the reproducibility may not be negligible, and has to be analysed carefully.

The question of the order of magnitude of the other MUF-components and the question of the MUF-distribution can be answered only on the basis of extensive analyses of historical data. Since such an analysis of more than 200 historical values of MUF has been made recently /_ 2_/, particularly in respect of these two questions, the results of this analysis are summarized below.

2.2 Order of Magnitude of MUF-Components; Distributions of MUF and Relative MUF

2.2.1 Measurement eroors vs process losses

In Table 1 a list of presently attainable systematic measurement errors is given.

In Tables 2A,B,C,D and Table 3 a list of MUF-values is given for different cases; they consist of MUF-values from a single reprocessing plant (2A), from a single unknown plant (2B), from a group of facilities handling U-235 in purified form, i.e. not reprocessing facilities (2C), from a group of facilities handling Pu in purified form (2D), and from a known type of facility and material (3). The data presented in these tables indicate that

(i) in Tables 2B,C,D and 3 the standard deviations are comparable to the standard deviations of the systematic errors alone.

(ii) In Table 3 80 % of the MUF-values can be explained by the systematic errors of measurement alone.

(iii) There is always a small but positive mean value of MUF.

From these facts one can conclude: The largest contribution to MUF comes from systematic errors of measurements. However, as the positive mean value of MUF indicates, there is a small contribution of process losses which cannot be neglected. This cannot come from measurement biases as they have to be positive or negative by nature.

2.2.2 Feed-dependent components vs feed-independent components

As the variance of the relative MUF-values appears to be much more stable than the variance of the absolute MUF-values and as the measurement errors contribute mainly to the total MUF-variance, one can conclude that the feed independent parts of the systematic errors which contribute to the variance of MUF are small compared to the feed dependent parts - that means one can neglect the terms a_r^1 and var $\left(\frac{a_r^1}{F}\right)$ in eqn. (2.10) and (2.12).

Furthermore the mean value of the relative MUF is much more stable than the mean value of the absolute MUF. Therefore, one can also neglect the feed independent systematic part of the process losses, that means the terms b_s^1 and $\frac{b_s^1}{EF}$ in eqn. (2.10) and 2.11).

Besides, it can be argued further that in most cases, the total integrated amounts of feed during the campaigns considered here are large, so that the random part of the feed-independent process losses as well as the feed-independent systematic measurement errors can also be neglected. This means that one can neglect the terms a_s^1 and b_r^1 in eq. (2.10).

Therefore one has instead of (2.10,11,12)

$$\frac{MUF}{F} = a_r + a_s + b_r + b_s \qquad (2.13)$$

$$E(\frac{MUF}{F}) = a_s + b_s;\ var(\frac{MUF}{F}) = var\ a_r + var\ b_r \quad (2.14)$$

2.2.3 Distribution of absolute MUF vs distribution of relative MUF

The analysis of the distribution of the MUF in [2] with the help of the 'k-statistics' [5] showed that:

(i) Both the absolute as well as the relative MUF are normal distributed, if they are considered facilitywise; they are not normal distributed if MUF-values of different types of facilities are considered together.

(ii) The relative MUF is 'better' normally distributed than the absolute MUF in the sense of the k-statistics.

One can therefore conclude: For the purpose of making statements on the significance of MUF the relative MUF, i.e. MUF divided by feed, is the most appropriate quantity; it is normally distributed with an expectation value and a variance which are given by the formulae (2.14).

3. Inspector's Statement on MUF

3.1 General

At the end of one inventory period, when the value of MUF for that inventory period is established, the safeguards organization has to decide whether the MUF is significant or not, in other words, he has to decide whether the MUF can be explained by measurement errors and process losses or whether a second action level has to be started to obtain further clarification.

For this purpose a two step procedure is proposed in the following:

The first step consists of a trial by the organization to explain the MUF with the help of the measurement errors alone. That means, it fixes an error first kind probability α and calculates the significance threshold x_a for the relative MUF according to the formula

$$1-\alpha = \phi(\frac{x_a}{\sigma_a}) \quad (3.1)$$

where σ_a is the standard deviation of the relative systematic error for the inventory period considered and ϕ is the Gaussian distribution function - according to chapter 2 the relative MUF can be assumed to be normally distributed.

If the relative MUF is smaller than x_a, the safeguards organization will be justified in accepting the MUF value as normal.

Note: The described test is a one-sided test. One can also construct a two sided test which means that the safeguards organization will not be justified in taking the MUF value as normal if it is found to be smaller than $-x_a$ and greater than x_a. In this case the relation between α and x_a is given by

$$1-\frac{\alpha}{2} = \phi\left(\frac{x_a}{\sigma_a}\right) \qquad (3.2)$$

For reasons given later, the one-sided test is used here. If the relative MUF is $>x_a$, the safeguards organization as a second step, fixes a new significance threshold according to the formula

$$1-\alpha = \phi\left(\frac{x-\mu}{\sigma}\right) \qquad (3.3)$$

where σ is the standard deviation and μ the average value of the relative MUF, as given by a collection of historical MUF-data and which is larger than x_a. Only if the MUF value is still greater than $x+\mu$, a second action level is started by the safeguards organization.

The reasons for this proposed procedure are the following:

As discussed in the second chapter, the greater part of the MUF-standard deviations are comparable to the standard deviations of the systematic errors, in one set of MUF-values 80% of the values could be explained with measurement errors alone; this therefore can be considered as the normal case.

The standard deviation of the systematic errors can be obtained by one or more of the following three possibilities

(i) Collection of historical data of standard deviations for one plant.

(ii) Comparison of the results of measurements with the same type methods in similar plants.

(iii) Performance of an interlabtest in the case of a new plant when no historical data are available.

The process losses should be taken into consideration only in a second step, as historical data on them are scarce and are much more difficult to establish (data on systematic errors could be obtained in principle with one interlabtest). In the case of new facilities such historical data do not exist at all. However, in these cases one could eventually consider historical data from similar plants.

3.2 The Case of one Inventory Period

For clarity of presentation, it has been assumed here that the MUF consists of systematic measurement errors alone which can be described by a variance, and possibly of a diversion. However, the calculations can very easily be generalized to the cases in which biases of measurements and process losses have to be taken into consideration.

For a given inventory period the error first kind probability α and the significance threshold x_a are related to each other according to eqn. (3.1). The probability of detection p (which is one minus the error second kind probability β) in case the fraction M of the feed is assumed to be diverted, is given by

$$p(M) = 1-\beta(M) = \phi\left(\frac{M}{\sigma} - \frac{x_a}{\sigma}\right) \tag{3.4}$$

or with (3.1)

$$p(M) = \phi\left(\frac{M}{\sigma} - U_{1-\alpha}\right); \quad U_{1-\alpha} = \phi^{-1}\left(\frac{x_a}{\sigma}\right) \tag{3.5}$$

Here, ϕ^{-1} is the inverse function of the Gaussian distribution function.

Note: The probability of detection in the case of the two sided test is given by

$$p(M) = \phi\left(\frac{M}{\sigma} - U_{1-\frac{\alpha}{2}}\right) + 1-\phi\left(\frac{M}{\sigma} + U_{1-\frac{\alpha}{2}}\right) \tag{3.6}$$

As this formula is more complicated than that forthe one sided test, the one sided test is used here although the following calculations can also be performed with the two sided test. It is also in the sense of a safeguards organization which is interested in the value of a MUF which is too large and not in that which is negative.

The relation between the four quantities p,M,σ and α is given as a nomograph in Fig. 1. This kind of representation is especially useful if one desires to fix any three of the parameters occuring in (3.5) and to determine the remaining one.

The use of the nomograph is illustrated with an example. Let p = 95 % and α = 5 %; following the dashed line, a value of $\frac{\alpha}{M}$ = 3.3 is obtained. This means that an amount = 3.3 times the value of the standard deviation (in %) of the systematic error for the inventory period considered, can be declared as diverted for the chosen values of p and α. If now one chooses a value of M = 2 % of the feed as significant, i.e. above which an amount, if diverted should be detected, then the actual value of σ has to be (by following the dashed line up to the right hand side ordinate) 0.6 % of the feed.

It is also possible to perform sensitivity tests with regard to the four variables with the help of this nomograph; for example, one could investigate the dependence of M and α if p and σ were kept constant. For p = 95 % and α = 1 % (instead of 5 % as in the previous case), one gets a value of M = 2.5 % for the same σ = 0.6 %. Thus a reduction of the α value by a factor of 5 (which means only 1/5th the number of false alarms than before) causes an increase of M from 2 to 2.5 % only. It is to be noted that in the range of p values of 90-99 % (i.e. β = 10-1 %), the results are symmetrical with respect to M and σ ; i.e. the same value of M = 2.5 % would be obtained for the fixed value of σ = 0.6 % if the α value was kept at 5 % and the value of p was increased from 95 % to 99 %.

Since both the value of M and σ are normalized with respect to feed, the nomograph can be used for practically any absolute values of throughput in a facility. Also, by varying the values of p and α, the absolute values of significant amounts could be kept in the same range for a given σ, in facilities with different throughputs. For example, in one plant with a throughput of 100 kg in a campaign, an amount of or above 2 kg could be declared as significant with a systematic measurement error σ of $\pm$ 0.6 %, with p = 95 % and α = 5 %. The same amount of or above 2 kg can also be declared as significant in another facility with a throughput of 80 kg in a campaign, with the same systematic measurement error of $\pm$ 0.6 % for a p = 99 % and α = 5 %, or for a p = 95 % and α = 1 %.

It is to be noted that the probability paper is particularly suitable for this type of nomographs. The parameter α is linear in the p, $\frac{M}{\sigma}$ plane. Besides, for a given set of values for p M and σ the corresponding value of α can be obtained fairly easily. A straight line parallel to the other α line is drawn through the point at which perpendiculars drawn from the given values of p and $\frac{M}{\sigma}$ values meet. The point at which the straight line meets the p-axis is the value of α saught.

3.3 The Case of More Inventory Periods

The case of a sequence of n inventories in one year brings with it a number of new features which were already the subjects of a number of previous publications [6, 7, 8]. Since an exhaustive treatment of this problem runs into very difficult mathematical problems only a special case has been considered here to indicate in a way similar to that for a single inventory period, the relations between the different quantities involved.

The n MUF-values for the n inventory periods are given by

$$\begin{aligned} MUF(1) &= I_o^1+T_1-I_1 \\ &\vdots \\ MUF(n) &= I_o^n+T_n-I_n \end{aligned} \tag{3.7}$$

where I_o^i is the starting inventory for the i-th inventory period, T_i the algebraic sum of all throughput measurements in the i-th inventory period (i.e. receivings minus shipments) and I_i the ending physical inventory of the i-th inventory period.

The main problem arises in the choice of the starting inventory I_o^i and as shown later, of the amount assumed to be diverted in a single inventory period. If one assumes that the accuracy of the physical inventory taking is considerably better than that of the throughput measurements for that inventory period, it is reasonable to choose the ending physical inventory of the foregoing inventory period as the starting inventory of the following inventory period. In that case one has instead of (3.7)

$$\begin{aligned} MUF(1) &= I_o+T_1-I_1 \\ MUF(2) &= I_1+T_2-I_2 \\ &\vdots \\ MUF(n-1) &= I_{n-2}+T_{n-1}-I_{n-1} \\ MUF(n) &= I_{n-1}+T_n-I_n \end{aligned} \tag{3.8}$$

Let F_n be the feed per inventory period and F the feed per year, that is

$$F = nF_n \tag{3.9}$$

Further let σ_I be the standard deviation of the inventory taking and σ_T the standard deviation of the throughput measurements divided by the feed per inventory period, both expressed in percentage. The latter is assumed to be independent of the length of the inventory period (i.e. only systematic errors determine the accuracy). Then the variance of the relative MUF per inventory period is given by

$$\text{var}\left(\frac{MUF(i)}{F_n}\right) = 2\frac{\sigma_I^2}{F_n^2} + 2\sigma_T^2 \text{ for all } i=1....n \qquad (3.10)$$

(One has $2\sigma_T^2$ as it is assumed that the systematic errors of the input and output measurements are independent of one another.)

If one fixes for all inventory periods the same error first kind probability - this is reasonable as by the chosen starting inventory the situation in all inventory periods is the same - and if one assumes that in each inventory the amount M_i is diverted, where

$$M = \sum_{i=1}^{n} M_i \qquad (3.11)$$

is the sum of the material assumed to be diverted in one year, then the probability of detection $p(M_1....M_n)$ i.e. the probability to detect a diversion at least once is given by

$$p(M_1....M_n) = 1-p\left(\frac{MUF(1)}{T_1} \leq x \mid M_1 \quad .... \quad \frac{MUF(n)}{F_n} \leq x \mid M_n\right) \qquad (3.12)$$

Here , x is the significance threshold for each inventory period, it is related to the error first kind probability α by an equation equivalent to eqn. (3.1).

If, as assumed, the physical inventory taking is much more accurate than the throughput measurements, one can factorize the expression (3.12) and obtain

$$p(M_1...M_n) = p\left(\frac{MUF(1)}{F_1} \leq x/M_1\right) \;..... \quad p(MUF(n) \leq x / M_n) \qquad (3.13)$$

This probability of detection depends strongly on the strategy of the operator i.e. his choice of the n-tupel $(M_1,...,M_n)$ i.e. the amount diverted in a single inventory period, with the boundary condition (3.11). If one assumes that the operator diverts the same amount per inventory period, $\frac{M}{n}$, then the probability of detection is given by

$$p(M) = 1- \left[\phi(U_{1-\alpha} - \frac{M}{n\sqrt{2\sigma_I^2+2\sigma_T^2 \cdot F_n^2}})\right]^n$$

or, with (3.9)

$$p(M) = 1- \left[\phi(U_{1-\alpha} - \frac{1}{\sqrt{2(n\frac{\sigma I}{F})^2+2\sigma_T^2}} \cdot \frac{M}{F})\right]^n \tag{3.14}$$

Eqn. 3.14 simplifies to

$$p(M) = 1- \left[\phi(U_{1-\alpha} - \frac{1}{\sqrt{2\sigma_T^2}} \cdot \frac{M}{F})\right]^n \tag{3.148}$$

if the accuracy for the inventory determination is neglected in comparison to that for the throughput. In this case the n probabilities of detection for the single inventory periods are independent.

The error first kind probability α' for the sequence of n inventory periods which is obtained by putting M = 0 in eqn. (3.14) is connected with the error first kind probability α for one inventory period by

$$1-\alpha' = (1-\alpha)^n \tag{3.15}$$

For $\alpha \ll 1$, one obtains

$$\alpha' = n\alpha \tag{3.16}$$

The relation (3.15) is illustrated in Fig. 2a.

It can be shown easily [7], that the mean number N of inventory periods between two false alarms is given by

$$N = \frac{1}{\alpha} \tag{3.17}$$

Correspondingly, the mean number of years N' between two false alarms is given by

$$N' = \frac{1}{\alpha'} \tag{3.18}$$

Therefore, the relation between N' and N is given by

$$N' = \frac{1}{1-(1-\frac{1}{N})^{n}} \qquad (3.19)$$

For $N \gg 1$ one obtains, corresponding to eqn. (3.16)

$$N' = \frac{N}{n} \qquad (3.20)$$

The relation (3.18) is shown in Fig. 2b.

One can either choose the error first kind probability for one inventory period (or the mean number of inventories per false alarm) and calculate the error first kind probability for one year (or the mean number of years per false alarm) or inversely.

Again, as in the case of one inventory period, the relation (3.13) between the different important quantities is represented as a nomograph. In order that the graphical representation does not become too complicated, the quantities $\sigma_I \cdot I/F$ and σ_T are kept constant. In Figure 3a two sets of the parameters inventory I, standard deviation σ_I, expressed as percent of inventory and feed F per year, which fulfill the condition $\sigma_I \cdot I/F$ = constant are given. The constants C_1 and C_2 are chosen in such a way that together with the chosen relative standard deviations σ_T of the throughput measurements the condition is fulfilled that the accuracy of the physical inventory taking is much better than the accuracy of the throughput measurements in the worst case i.e. n = 12. In Figure 3b the nomograph of equation (3.13) is given for the two sets of parameters

$$C_1 = \frac{\sigma_I \cdot I}{F} = 0.005\%, \quad 2\sigma_F = 0.1\ \% \text{ and } C_2 = \frac{\sigma_I \cdot I}{F} = 0.05\ \%, \quad 2\sigma_T = 1\%$$

An example is again given in the figure. For the case $C_1 = 0.005\%$, $2\sigma_T = 0.1$ % a probability of detection of 90 % is chosen. This gives for n = 1 inventory period per year and α = 1 % an amount of M = 0.36 % of the feed. This is the significant amount, which if diverted in the course of one year, can be detected with a probability of detection of 90 % (dashed path). If one now chooses n=12 inventory periods per year (this means according to eq. (3.16) α'=12 %, as one has to keep the error first kind probability α' per year constant in order to have a common basis for the comparison) one obtains the corresponding value of M = 0.04 % of the feed for the same probability of detection of 90 % (dashed point dashed path). Thus one sees that in increasing of the effort with respect to the inventory taking by a factor of 12 brings a factor of nine with respect to the amount to be detected.

In Fig. 4 the simplified eqn. 3.14a has been presented as a nomograph. Because of the elimination of the accuracy for inventory measurements, the measurement accuracy σ_T can be introduced as a parameter. A similar example is given to illustrate the use of the nomograph. It is to be noted that no significant change in the M/F values are obtained by using this simplified nomograph instead of that given in Fig. 3b. For p = 95%, n=1, α=1 % and σ_T = 0.5% one obtains M/F =2.8%; for p = 95%, n=12, α'=12 % and σ_T = 0.5 % M/F value is reduced to 0.3 %. Again, an increasing of the effort with respect to the inventory taking by a factor of 12 brings a factor of nine with respect to the amount to be detected.

4. Conclusions

In the present report an effort has been made to formalize the relation between the different components of MUF, to determine the more important of these components on the basis of an analysis of available MUF data and to analyse the relevant parameters which influence statements of a safeguards organization with regard to a possible diversion. In summarizing the results, a number of conclusions can be drawn. They are however, subject to the restrictions and boundary conditions discussed in this paper.

4.1 The basic number of components of MUF in a facility appears to be two namely, the measurement errors and unknown or unmeasured process losses and hidden inventories in that facility. A part of these components may be throughput dependent whereas, another part may be independent of the throughput. Both the components may have systematic and random constituants. Furthermore, the measurement and the process components may have a bias. The biases contribute to the expectation value and the rest of the components to the standard deviation of the MUF.

4.2 The data on MUF published sofar, reveal a number of interesting points. The relative MUF values normalized with respect to feed are better suited for safeguards purposes than the absolute values. They are composed mainly of feed dependent terms. They can mostly be explained by systematic errors in measurements which are supposed to be normal distributed. The contribution of the random errors in measurements may be negligible as it reduces rapidly with increasing number of measurements. Data specific to a particular type of facilities when normalized with respect to feed follow a normal distribution with a positive bias. The bias point to a feed dependent process loss. The systematic errors contribute mainly to the standard deviation of the distribution with a small contribution from the random variations of the feed dependent process losses.

4.3 Because of the dominating role played by the systematic errors in the composition of the MUF values, it is possible to develop a two step decision model for the preparation of statements on MUF. In the first step, the safeguards organization tries to explain the MUF with the help of systematic errors alone. For this purpose it fixes a threshold value of MUF with a given error first kind α. In case the actual values of MUF do not fall within this threshold, the organization sets a new threshold with the help of historical data which may be available for that type of a facility with the same value of α. Only in case the MUF values do not fall within this threshold also, a second action level is necessary to explain the high MUF values.

4.4 The significant amount M i.e. the amount above which a diversion can be detected with a probability p, depends on four parameters namely, the values of the error first and second kind α and $\beta (1-\beta = p)$, the numbers of inventories n and the systematic errors of measurements for inventory and throughput, σ_I and σ_T respectively (expressed in percentage standard deviation) for the material balance period. For the case $\sigma_I \ll \sigma_T$ (which may be true for a majority of cases), the value of M expressed in absolute units, can be kept within a close spread over a wide range of throughputs in a particular type of facilities and a given σ_T, by choosing properly the values of α, β and n. This has the direct consequence that measurement errors (expressed as percentage of feed) can be kept at the same value for a large number of facilities of the same type but with varying throughputs. For very large throughputs, if the value of M is found to be excessively high, it can be reduced by increasing the number of inventories per year, but not linearly.

4.5 It is to be noted that the analysis of the dependence of M on different parameters mentioned in 4.4, refers only to the first step of the decision model, i.e. when the safeguards organization tries to explain the MUF values with the systematic errors of measurements alone. If the actual MUF value is found to be larger than the M obtained with a given set of α and β values, the safeguards organization has to test this MUF value for the same α and β values in the second step mentioned in 4.3.

Acknowledgement

The authors would like to thank W. Häfele for his interest in this work.

Table 1: Systematic Errors

Error Description	Relative Standard Deviation δ /%/			
	Values from an interlab test /3/		US values /4/	
	Pu	U	Pu	U
Input to CR				
Analytical	-	-	0.25	0.20
Volume	-	-	0.30	0.30
Sampling	-	-	0.20	0.20
Total for Input	2.7	1.4	0.44	0.41
Product from CR				
Analytical	-	0.20	0.30	0.10
Volume	-	-	0.10	0.20
Density	-	0.31	-	-
Sampling	-	-	0.20	0.10
Total for Product	0.25	-	0.37	0.24
Isotopic wt %				
U-234 (0.004 %)	-	8.02	-	-
U-235 (0.35 %)	-	1.24	-	-
U-236 (0.06 %)	-	2.51	-	-
Pu-238 (0.7%)	1.32	-	-	-
Pu-239 (70 %)	0.08	-	-	-
Pu-240 (23 %)	0.16	-	-	-
Pu-241 (5.5 %)	0.35	-	-	-
Pu-242 (1.5 %)	0.95	-	-	-
UF_6 Cylinder Measurement				
Netweight	-	-	-	0.1
Uranium Sampling	-	-	-	0.1
U-235 Sampling	-	-	-	0.03
Uranium Assay	-	-	-	0.15
U-235 Assay	-	-	-	0.30
Total Uranium	-	-	-	0.21
Total U-235	-	-	-	0.36

Table 2A: Normalized MUF (M_i) Values for a Reprocessing Facility [4]. (Normalized with Respect to Feed)

Serial No.	M_i (% of feed)	Serial No.	M_i (% of feed)	Serial No.	M_i (% of feed)
1	3.34	11	0.83	21	- 0.59
2	2.15	12	0.74	22	- 0.62
3	1.41	13	0.62	23	- 0.73
4	1.30	14	0.45	24	- 0.78
5	1.27	15	0.32	25	- 0.84
6	1.08	16	0.31	26	- 1.04
7	1.00	17	0.22	27	- 1.04
8	0.95	18	0.06	28	- 1.08
9	0.93	19	- 0.46	29	- 2.31
10	0.90	20	- 0.49	30	- 2.63

Mean value (μ): + 0.18

Standard deviation (σ): $\pm$ 1.25

Table 2B: Normalized MUF (M_i) Values for a Single Facility /‾5_7.
(Normalized with Respect to Beginning Inventory and Receipts.)

Serial No.	M_i (% of Input)	Serial No.	M_i (% of Input)	Serial No.	M_i (% of Input)
1	1.94	10	0.23	19	0.04
2	1.38	11	0.17	20	0.02
3	1.30	12	0.12	21	- 0.06
4	1.00	13	0.11	22	- 0.15
5	0.85	14	0.09	23	- 0.19
6	0.65	15	0.08	24	- 0.80
7	0.46	16	0.08	25	- 1.12
8	0.33	17	0.08	26	- 1.23
9	0.27	18	0.06		

Mean value (μ): + 0.22
Standard deviation (σ): ± 0.70

Table 2C: Normalized MUF (M_i) Values for Facilities Handling U-235, other than Reprocessing Plants /1,7,10_7. (Normalized with Respect to Input)

Serial No.	M_i (% of Input)	Serial No.	M_i (% of Input)	Serial No.	M_i (% of Input)
1	0.73	10	0.24	19	0.06
2	0.67	11	0.21	20	0.06
3	0.65	12	0.18	21	0.06
4	0.55	13	0.17	22	0.04
5	0.44	14	0.16	23	0.02
6	0.44	15	0.16	24	0.01
7	0.30	16	0.09	25	- 0.05
8	0.25	17	0.07		
9	0.25	18	0.07		

Mean value (μ): + 0.23

Standard deviation (σ): $\pm$ 0.22

Table 2D: Normalized MUF (M_i) Values for Facilities handling Pu and Pu-239, other than Reprocessing Plants /‾6,7_7.
(Normalized with Respect to Input.)

Serial No.	M_i (% of Input)	Serial No.	M_i (% of Input)
1	1.64	11	0.18
2	1.36	12	0.10
3	1.11	13	0.10
4	0.51	14	0.08
5	0.47	15	0.06
6	0.39	16	0.06
7	0.29	17	- 0.10
8	0.23	18	- 0.14
9	0.22	19	- 1.28
10	0.19		

Mean value (μ): + 0.14
Standard deviation (σ): $\pm$ 0.67

Table 3: Normalized Values of MUF (M_i) for Groups of Facilities. Types of Facilities and Material Used Unknown. (Small Number of Data/Facility)

Serial No.	M_i	Ref.	Serial	M_i	Ref.	Serial	M_i	Ref.
1	5.86	5	46	0.40	11	91	0.07	5
2	5.74	5	47	0.39	5	92	0.07	11
3	2.96	5	48	0.36	5	93	0.06	11
4	2.44	5	49	0.35	5	94	0.06	5
5	2.44	5	50	0.35	5	95	0.06	5
6	2.22	5	51	0.34	5	96	0.05	5
7	2.14	5	52	0.31	5	97	0.05	5
8	2.05	5	53	0.30	5	98	0.05	11
9	1.80	11	54	0.30	5	99	0.04	11
10	1.78	5	55	0.29	5	100	0.04	5
11	1.62	5	56	0.27	11	101	0.03	5
12	1.52	5	57	0.22	5	102	0.02	5
13	1.43	5	58	0.21	5	103	0.01	5
14	1.35	5	59	0.21	5	104	0.01	5
15	1.24	5	60	0.21	5	105	0.01	5
16	1.23	5	61	0.21	5	106	0.01	5
17	1.20	5	62	0.19	5	107	-0.01	11
18	1.18	5	63	0.19	5	108	-0.02	5
19	1.18	5	64	0.18	5	109	-0.02	5
20	1.16	5	65	0.17	5	110	-0.03	5
21	1.14	5	66	0.15	5	111	-0.03	5
22	1.06	5	67	0.13	5	112	-0.05	11
23	1.00	5	68	0.13	5	113	-0.06	5
24	0.94	5	69	0.13	5	114	-0.06	5
25	0.94	11	70	0.13	5	115	-0.07	5
26	0.92	5	71	0.12	5	116	-0.09	5
27	0.90	11	72	0.12	5	117	-0.10	5
28	0.85	11	73	0.12	5	118	-0.13	5
29	0.82	5	74	0.11	5	119	-0.17	5
30	0.78	5	75	0.11	5	120	-0.32	5
31	0.69	5	76	0.11	5	121	-0.36	5
32	0.67	5	77	0.11	5	122	-0.38	11
33	0.64	5	78	0.11	11	123	-0.41	5
34	0.63	5	79	0.10	5	124	-0.62	5
35	0.62	11	80	0.10	5	125	-1.96	5
36	0.61	5	81	0.10	5	126	-3.22	5
37	0.60	5	82	0.09	5			
38	0.54	5	83	0.09	5			
39	0.53	5	84	0.09	5			
40	0.52	5	85	0.09	5			
41	0.49	5	86	0.09	5			
42	0.49	11	87	0.08	5			
43	0.49	11	88	0.07	5			
44	0.43	5	89	0.07	5			
45	0.42	5	90	0.07	5			

Mean value (μ): 0.48

Standard deviation (σ): $\pm$ 1.01

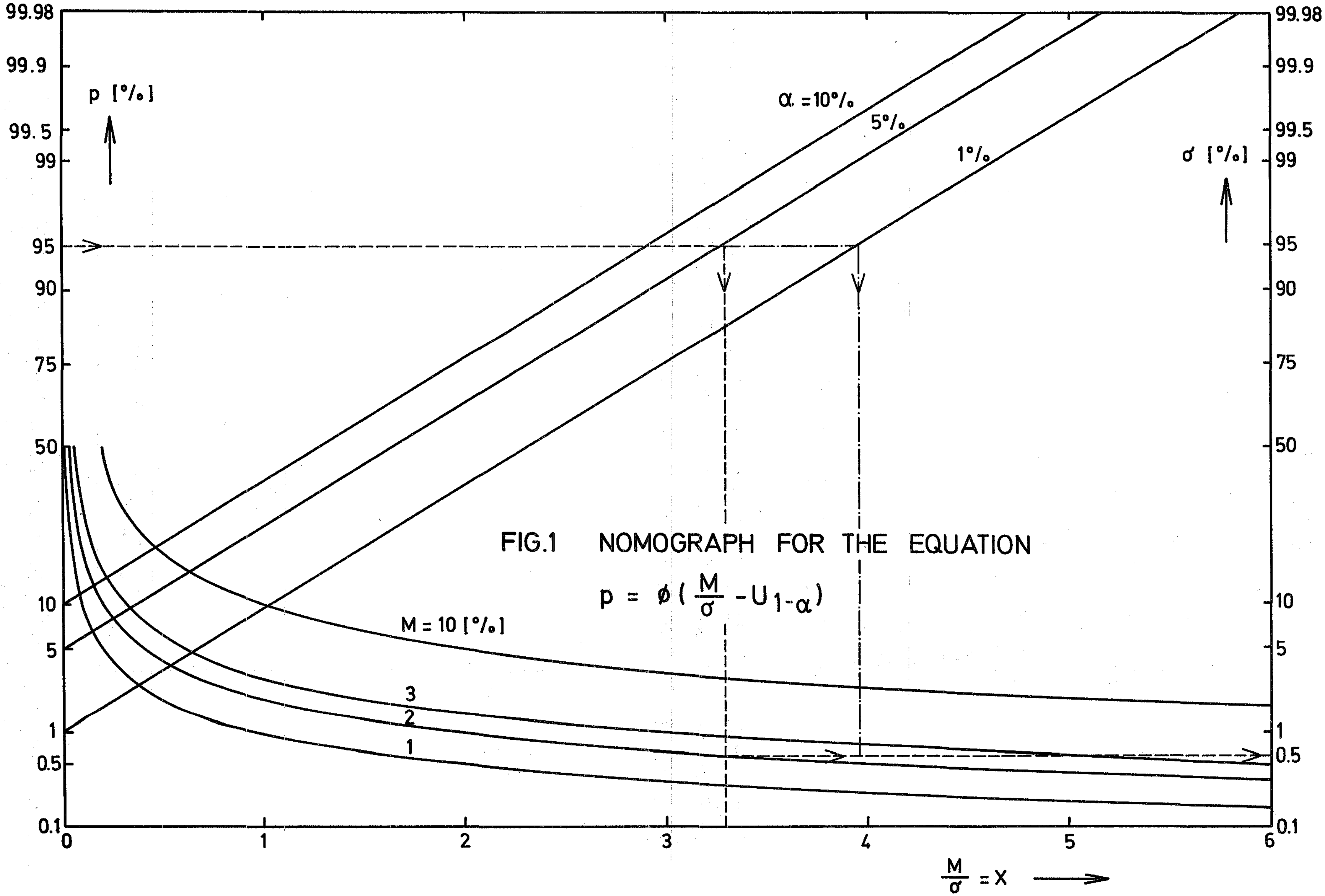

FIG.1 NOMOGRAPH FOR THE EQUATION

$$p = \phi\left(\frac{M}{\sigma} - U_{1-\alpha}\right)$$

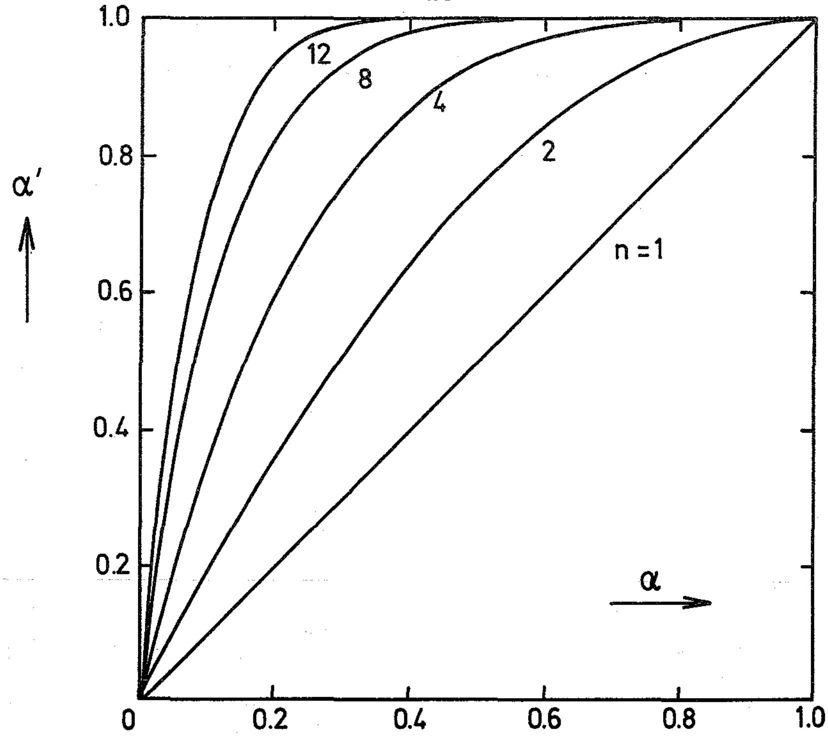

FIG. 2a RELATION BETWEEN ERROR FIRST KIND PROBABILITY α FOR ONE INVENTORY PERIOD, ERROR FIRST KIND PROBABILITY α' FOR ONE YEAR AND NUMBER n OF INVENTORY PERIODS PER YEAR

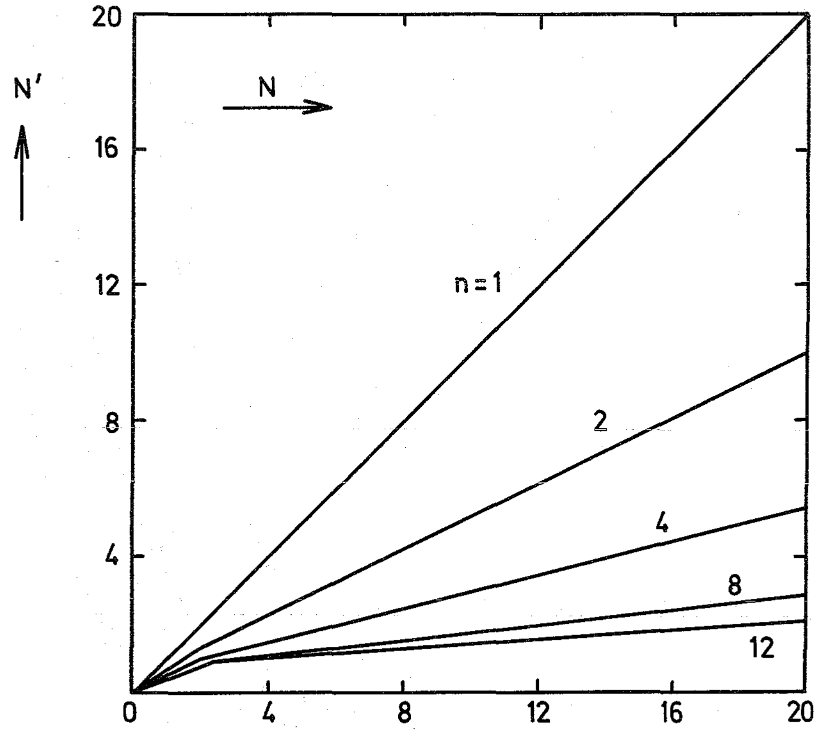

FIG. 2b RELATION BETWEEN MEAN NUMBER OF INVENTORIES N, MEAN NUMBER OF YEARS N', BETWEEN TWO FALSE ALARMS AND NUMBER n OF INVENTORIES PER YEARS

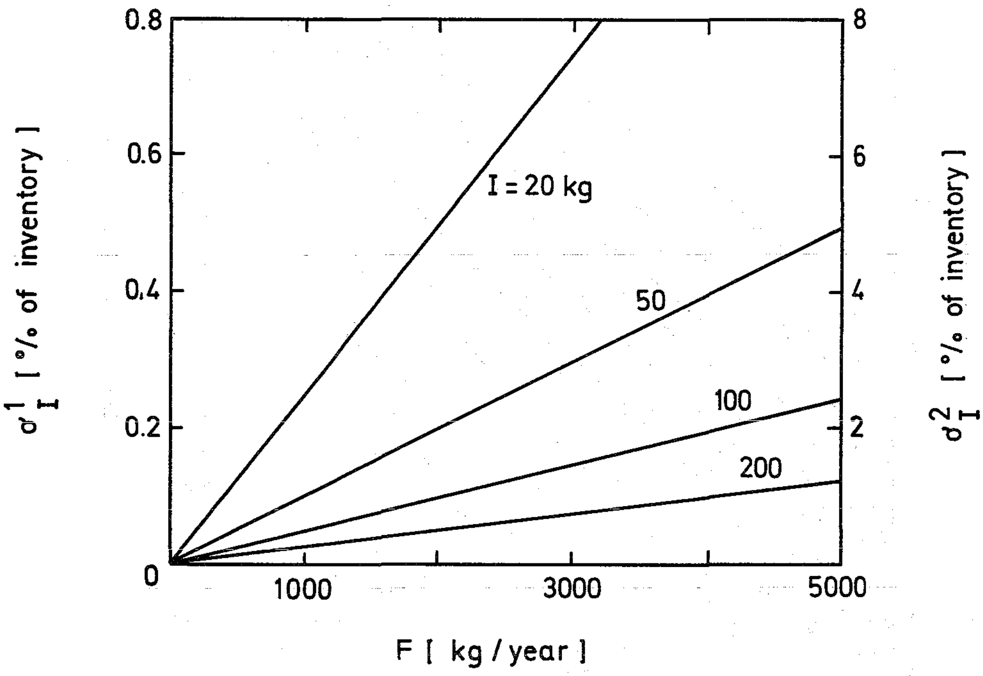

FIG. 3a SET OF PARAMETERS $\sigma_I^{1,2}$, F AND I WHICH FULFILL THE CONDITION $C_1 = \sigma_I^1 \cdot I / F = 5 \cdot 10^{-5}$ (LEFT VERTICAL AXIS) AND $C_2 = \sigma_I^2 \cdot I / F = 5 \cdot 10^{-4}$ (RIGHT VERTICAL AXIS)

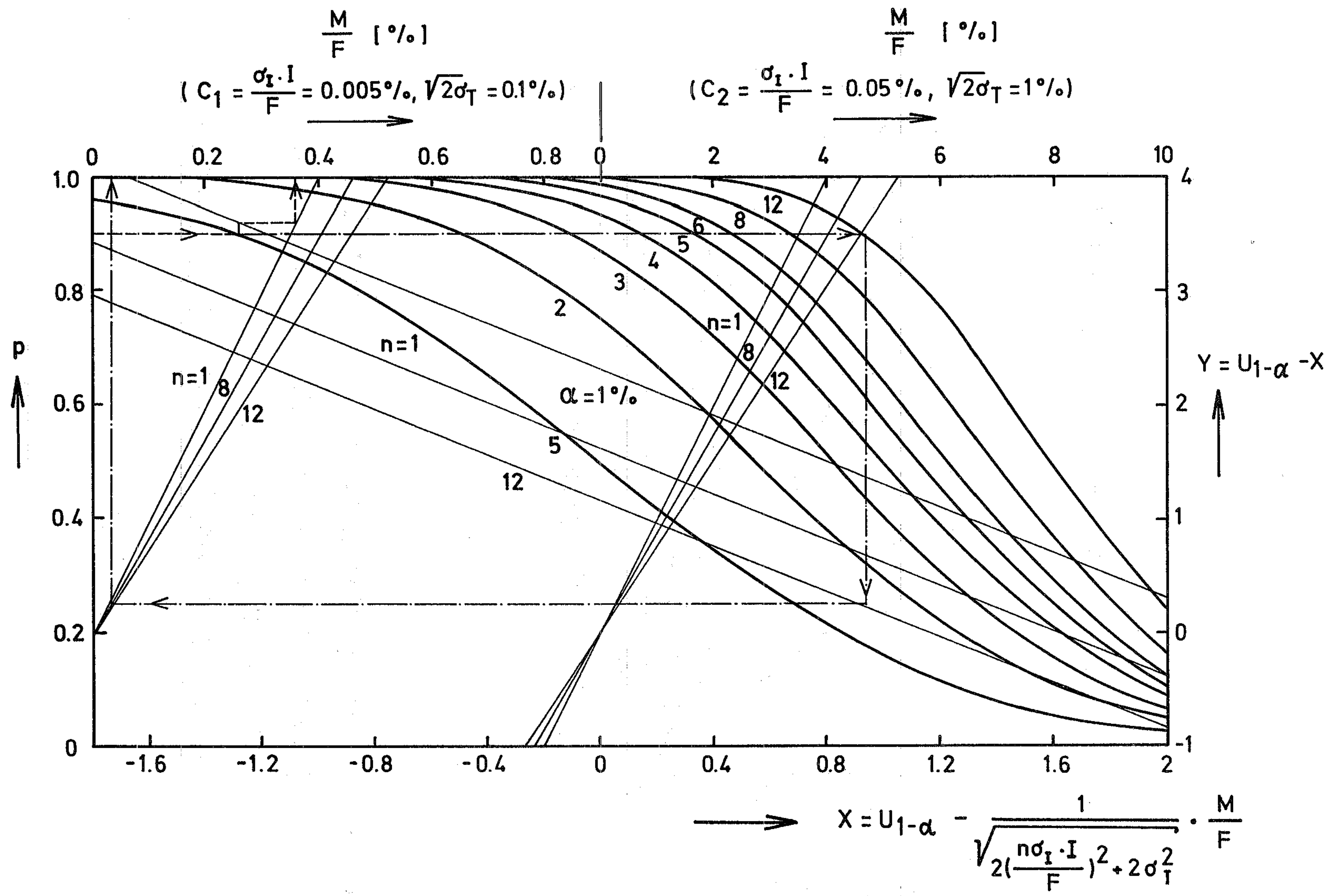

FIG. 3b NOMOGRAPH FOR THE EQUATION $p = 1 - \left[\emptyset \left(U_{1-\alpha} - \frac{1}{\sqrt{2\left(\frac{n\sigma_I \cdot I}{F}\right)^2 + 2\sigma_T^2}} \cdot \frac{M}{F} \right) \right]^n$

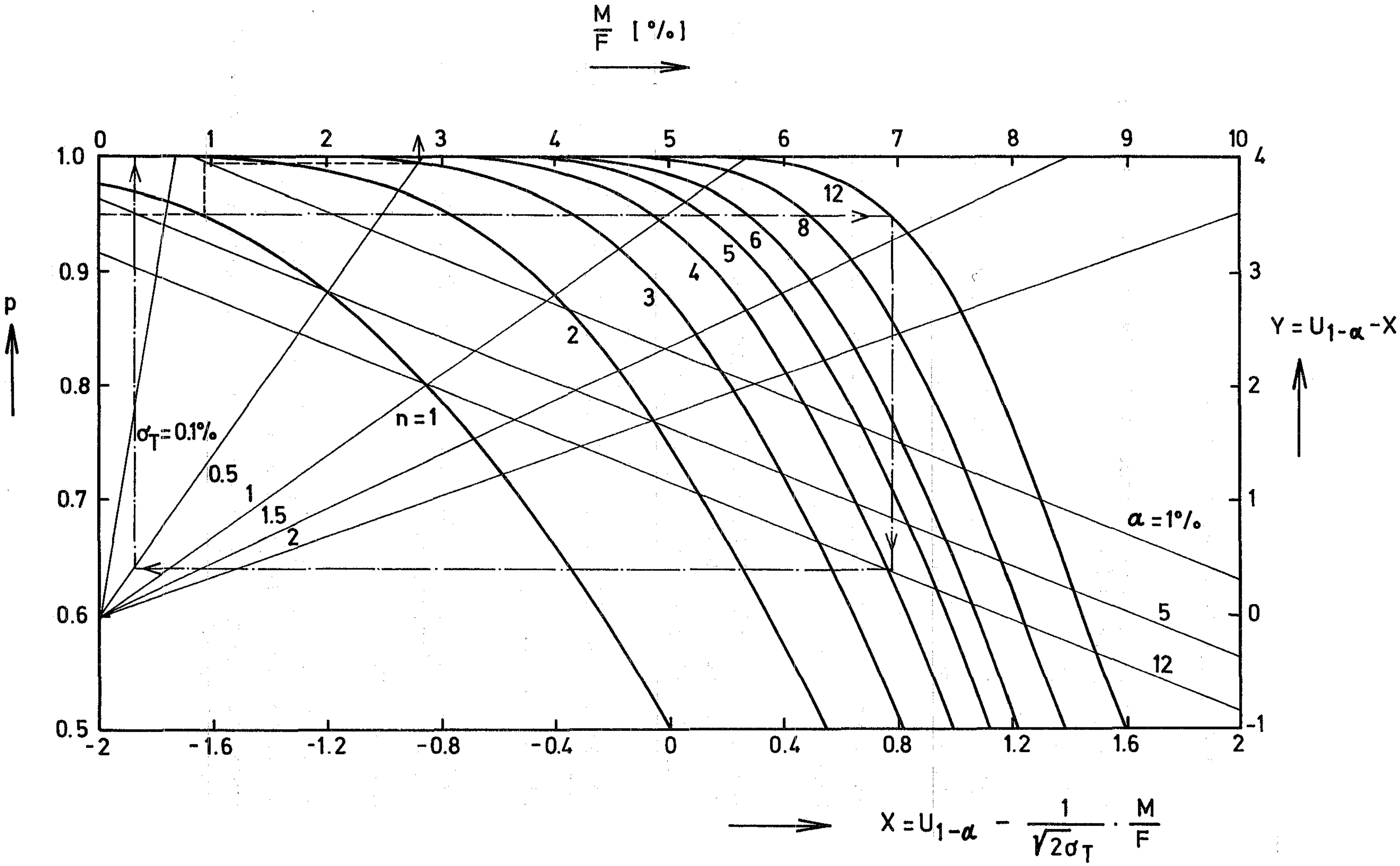

FIG.4 NOMOGRAPH FOR THE EQUATION (EXACT INVENTORY) $p = 1 - \left[\emptyset \left(U_{1-\alpha} - \frac{1}{\sqrt{2}\sigma_T} \cdot \frac{M}{F} \right) \right]^n$

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