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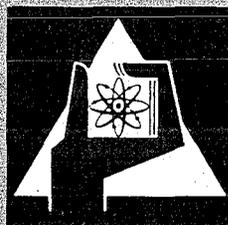
On the Kinetics Equations for Fast Reactors

P.F. Zweifel

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On the Kinetics Equations
for Fast Reactors

by

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ABSTRACT

The analysis of reactor kinetics experiments is, for historical reasons,
based on models more appropriate to thermal than to fast systems. We
propose that kinetics experiments analyses be carried out using only
the ordinary multigroup operators, and that no effort be made to intro-
duce parameters such as neutron lifetime, effective delayed neutron
fraction, etc. The resulting analysis is simpler and less likely to
produce confusion. Period measurements and pile oscillator experiments
are discussed, with variational principles being suggested for the
analysis.

ZUSAMMENFASSUNG

Die Auswertung von Reaktorkinetikexperimenten erfolgt - aus historischen
Gründen - unter Verwendung von Modellen, die sich eher für thermische als
für schnelle Anordnungen eignen. Es wird vorgeschlagen, Auswertungen von
Kinetikexperimenten lediglich mit den gewöhnlichen Multigruppenoperatoren
durchzuführen und nicht zu versuchen, Parameter wie z.B. Neutronenlebens-
dauer und effektiven Anteil verzögerter Neutronen einzuführen. Eine solche
Auswertung ist einfacher und weniger verwirrend. Für die Analyse von
Periodenmessungen und Pileoszillator-Experimenten werden Variationsprin-
zipien vorgeschlagen.

CONTENTS

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I.   Introduction

It is perhaps too well known to require comment that the simplified form
of the reactor kinetics equations which are valid for thermal reactors
cannot be applied to fast systems. As an example, the idea of a reactivity--
density transfer function is common in thermal reactors,[1] in which the
density can be measured by a 1/v-detector. Since there is no convenient way
to measure the density in a fast assembly, if the concept of transfer function
is to be applied, a different transfer must be used, e.g. reactivity-power.
Even then, the calibration of the oscillating reactivity in a pile oscillator
experiment requires some care.

The situation is similar with regard to other measurements in fast systems--
period measurements, rod drops, etc. The usual (i.e. thermal system)
analysis of these experiments is based, in effect, on the inhour equation
of point reactor kinetics which is only valid for thermal systems. Fischer[2]
has extended the inhour equation to a general system described by a
multigroup operator, his analysis involving "flux-adjoint" weighting of
various kinetic parameters. His procedure actually is a special, but
particularly practical, case of the general transport-theoretical, energy
dependent kinetic equations obtained, for example, in Reference 1.

Fischer's procedure is, in fact, correct, and should, if applied properly,
lead to a satisfactory analysis on the time behaviour of fast reactors.
However, we feel that the attempt to retain point reactor kinetics equations
in the "standard" form is ill advised, introducing confusion and diffi-

culty into the analysis of kinetics phenomena. For this reason, we present
an analytical description of the period measurement based strictly on
multigroup calculations. It is, in effect, equivalent to Fischer's treat-
ment, since his method requires multigroup calculations to be carried
out in order to obtain the "flux-adjoint" weight functions. The main merit
of our treatment is that it begins to discard the "conventional" form of
the kinetics equations and, as a result, leads to a considerably simplified
analysis. We then go on to apply a similar method to pile oscillator
experiments, using the so-called Rossopolous Variational Principle. (The
analysis of period measurements, incedentally, is based on the Ritz
Variational Principle.) The results obtained there are, we believe, new,
and should lead to a much improved analysis of pile oscillator experiments.
(In the same section we consider the transfer function for fast systems.)

Before considering these topics (in Secs. III and IV) we discuss, in Sec. II,
some general properties of the kinetics operator and its adjoint.

II. The Reactor Kinetics Operator and its Adjoint

Consider first a one-speed diffusion equation

$$(-\underline{\nabla} \cdot D\underline{\nabla} + \Sigma_a) \phi(\underline{r}) = S(\underline{r}), \tag{1}$$

which we write in the form

$$L \phi = S . \tag{2}$$

The linear operator L shall be referred to as the "diffusion operator".

We shall represent the <u>scalar product</u> of two functions, $\phi(\underline{r})$ and $\psi(\underline{r})$
as $(\phi(\underline{r}), \psi(\underline{r}))$, or simply (ϕ,ψ). In one-speed theory

$$(\phi,\psi) = \int \phi(\underline{r}) \, \overline{\psi}(\underline{r}) \, d^3r, \tag{3}$$

where $\bar{\psi}$ means the complex conjugate of ψ.[*] In multigroup theory this definition of scalar product must be extended to include a summation over groups, i.e.

$$(\phi,\psi) = \sum_{i=1}^{N} \int \phi_i \, (\underline{r}) \, \bar{\psi}_i(\underline{r}) \, d^3r, \tag{4}$$

where the subscript i refers to the i-th group, and N is the total number of groups. The integration, incidentally, is taken over the reactor volume.

The adjoint operator, L^*, is defined by the relation

$$(L\phi,\psi) = (\phi,L^*\psi) \tag{5}$$

for every ϕ belonging to the <u>domain</u> of L and every ψ belonging to the domain of L^*. (The domain of an operator is the space in which it operates.) Denoting domain of L by the symbol $D(L)$, we say that an operator L is <u>formally</u> self-adjoint if $L = L^*$; it is <u>self-adjoint</u> if, in addition, $D(L) = D(L^*)$.

The domain of a differential operator is generally specified by the boundary conditions. Let us derive the boundary conditions for L, re-stricting ourselves, for simplicity, to the case of a single space dimension x. We also define μ, the cosine of the angle between a neutron's velocity vector and the x-axis. Then, in diffusion theory, the neutron angular density $\bar{\psi}$ is represented by the sum of two terms [3]

$$\bar{\psi}(x,\mu) = \frac{1}{2} \phi(x) + \frac{3}{2}\mu \, J \, (x), \tag{6}$$

where $\phi(x)$ is the total flux and $J(x)$ is the net current. The <u>partial</u> <u>currents</u>, $J_\pm$, can be defined as follows:

---

[*] Usually the operators and functions of reactor physics are real, and the complex conjugation sign can be dropped. As we shall see, however, in Sec. IV, this is not always the case.

$$J_+(x) = \int_0^1 \mu \, \overline{\psi} \, (x,\mu) \, d\mu \, , \tag{7a}$$

$$J_-(x) = \int_{-1}^0 \mu \, \overline{\psi} \, (x,\mu) \, d\mu \, . \tag{7b}$$

$J_+(x)$ represents the number of neutrons crossing a unit area at x in the +x direction, while $-J_-(x)$ represents the number crossing in the -x direction. A straightforward integration yields the results

$$J_+ = \frac{1}{4} \, \phi(x) + \frac{1}{2} \, J(x), \tag{8a}$$

$$J_- = - \frac{1}{4} \, \phi(x) + \frac{1}{2} \, J(x). \tag{8b}$$

At a free surface, i.e. an interface between a diffusing medium and vacuum, there are of course no neutrons reentrant into the diffusing medium in any direction $-1 \le \mu < 0$. Since it is impossible to impose this condition exactly in diffusion theory, the somewhat weaker condition is applied that the _net_ number reentrant, i.e. J_-, should vanish. This gives as the boundary condition at a point x_s of the surface

$$J \, (x_s) = \frac{1}{2} \, \phi \, (x_s) \tag{9}$$

or using Fick's law

$$J \, (x) = - \frac{\lambda_{tr}}{3} \frac{d\phi}{dx} \tag{10}$$

we find

$$\frac{d\phi(x_s)}{dx} = \frac{3}{2\lambda_{tr}} \, \phi(x_s). \tag{11}$$

From this, it is easily seen that the flux linearly "extrapolates" to zero at a point $\frac{2}{3} \lambda_{tr}$ from the surface. This distance is commonly called the linear extrapolation length, and the normal boundary condition is that $\phi(x_s + \frac{2}{3}\lambda_{tr}) = 0$. However, transport theory yields a somewhat better result for the extrapolation length, and it is customary to take

$$\phi \, (x_s + 0.71 \, \lambda_{tr}) = 0 \qquad\qquad (12)$$

as the diffusion theory boundary condition.

All this is well known. Now let us consider the boundary condition obeyed by the neutron <u>importance</u>, or adjoint flux. Clearly, a neutron leaving the free surface has zero importance, since it will never return. Thus, if we represent by ϕ^* and J^* the importance flux and current, the proper physical boundary condition for the adjoint is

$$J^*_+ = 0. \qquad\qquad (13)$$

From Eq. (8a) we find then

$$J^*(x) = - \frac{1}{2} \, \phi^*(x). \qquad\qquad (14)$$

We now use the <u>adjoint</u> of Fick's law:

$$J^*(x) = + \frac{\lambda_{tr}}{3} \frac{d\phi^*}{dx} \, , \qquad\qquad (15)$$

to obtain, in analogy with Eq. (11)

$$\frac{d\phi^*(x_s)}{dx} = - \frac{3}{2\lambda_{tr}} \, \phi^*(x_s). \qquad\qquad (16)$$

In other words, the <u>flux and adjoint obey the identical boundary condition.</u> Thus, $D(L) = D(L^*)$. It is easy to verify, by partial integration, that $L = L^*$ (the boundary terms vanish by virtue of Eqs. (11) and (16)). Thus L is self-adjoint.

We now construct a multigroup operator out of the L's corresponding to various groups:

$$
M = \begin{pmatrix}
L_1 & 0 & 0 & . & . & . & . & . & . & . & 0 \\
\Sigma_{12} & L_2 & 0 & . & . & . & . & . & . & . & 0 \\
\Sigma_{13} & \Sigma_{23} & L_3 & 0 & . & . & . & . & . & . & 0 \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
. & & & & & & & & & & \\
\Sigma_{1N} & \Sigma_{2N} & . & . & . & . & . & . & . & . & L_N
\end{pmatrix} \qquad . \quad (17)
$$

(Here Σ_{ij} is the transfer cross section from group i to group j).

It would be tempting to construct the adjoint, M^*, by interchanging rows and columns of M (since each individual element of M is self-adjoint). However, we now give an argument for the invalidity of such a procedure. The point is that the multigroup diffusion operator is an approximation to the energy-dependent diffusion operator. It is not correct to take the adjoint of an approximation to some operator and assume it is the same as the approximation to the adjoint. In other words, the correct procedure to use in deriving M^* is not to take the adjoint of M, but rather to form a multigroup approximation to the energy-dependent adjoint equation. The net effect is that, whereas the group constants in M are obtained by averaging over a flux spectrum, the group constants in M^* must be obtained by averaging over an adjoint spectrum. Thus, the operator M^*, in addition to being transposed, has elements that differ somewhat from the elements of M due to the different weighting procedure. *

Numerical verification has been given by Kiefhaber [4].

However, the following simple example should be enough to convince the most

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* In the perturbation matrices, δk, introduced in Sec. III, the group constants must be obtained by bilinear weighting, i.e. the product of flux times adjoint. This follows since the multigroup perturbation formula is an approximation to an exact formula which involves the product of energy-dependent flux with energy-dependent adjoint. See Reference 4.

skeptical. Consider the one-speed, one dimensional neutron transport equation with isotropic scattering:

$$\mu \frac{\partial \Psi}{\partial x} + \Sigma_\tau \Psi = \frac{\Sigma_s}{2} \int_{-1}^{1} \Psi(x,\mu')d\mu' + S(x,\mu).$$

If we were to expand the angular density, as in Eq. (6)

$$\Psi(x,\mu) = \frac{1}{2} (\phi(x) + 3\mu J(x)) + \ldots \tag{19}$$

and insert into Eq. (18), we would obtain the P_1 approximation which, in one speed, is equivalent to diffusion theory:

$$\Sigma_a \phi + \frac{dJ}{dx} = S_0 \tag{20a}$$

$$\frac{1}{3} \frac{d\phi}{dx} + \Sigma_\tau J = S_1 , \tag{20b}$$

where $S_k = \int_{-1}^{1} \mu^k S(x,\mu)d\mu$. $\tag{21}$

In matrix form this becomes

$$\begin{pmatrix} \Sigma_a & \frac{d}{dx} \\ \frac{1}{3} \frac{d}{dx} & \Sigma_\tau \end{pmatrix} \begin{pmatrix} \phi \\ J \end{pmatrix} = \begin{pmatrix} S_0 \\ S_1 \end{pmatrix} . \tag{22}$$

The adjoint transport equation is

$$- \mu \frac{\partial \Psi^*}{\partial x} + \Sigma_\tau \Psi^* = \frac{\Sigma_s}{2} \int_{-1}^{1} \Psi^*(x,\mu')d\mu' + S^*(x,\mu). \tag{23}$$

A similar P_1 expansion leads to an adjoint set

$$\begin{pmatrix} \Sigma_a & -\frac{d}{dx} \\ -\frac{1}{3} \frac{d}{dx} & \Sigma_\tau \end{pmatrix} \begin{pmatrix} \phi^* \\ J^* \end{pmatrix} = \begin{pmatrix} S_0^* \\ S_1^* \end{pmatrix} . \tag{24}$$

If we compare Eqs. (22) and (24), we note that the adjoint P_1 transport

operator is _not_ obtained from the P_1 flux operator by interchanging rows and columns and taking the adjoint of each element. The reason for this dicrepancy is that the P_1 equations are approximations to the exact transport equation. One cannot take the adjoint of an approximation by the usual rules and expect to get the same result as if he had approximated the exact adjoint. From now on, we assume the various group constants have been obtained as we recommend, and that adjoints can be formed for all operators which we discuss.

Let us now modify the multigroup operator M to include delayed neutron effects. M operates on a column vector where N elements ψ_1, ψ_2, ... ψ_N represent the N-group neutron fluxes. Consider an N + I dimensional vector

$$\Phi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \psi_N \\ C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_I \end{pmatrix} \tag{25}$$

Here C_1, C_2, ... C_I represent the delayed neutron precursors; as usual we denote by λ_i and β_i the decay constant and fractional yield of the i-th precursor group. Then the multigroup reactor kinetics equations can be combined into a single equation for Φ:

$$\frac{\partial \Phi}{\partial t} = V(F-A)\Phi, \tag{26}$$

where the matrices V, F, and A are defined as follows:

$$V = V_{ii}\delta_{ij}; \quad V_{ii} = v_i, \ 1 \leq i \leq N; \quad V_{ii} = 1, \ N + 1 \leq i \leq N + I. \tag{27}$$

In other words, V is a diagonal matrix where first N elements are the group

velocities v_i and whose last I elements are 1.

A is an extension of the multigroup matrix M defined in Eq. (17):

$$
A = \begin{pmatrix}
L_1 & 0 & 0 & . & . & . & . & . & . & 0 \\
\Sigma_{12} & L_2 & 0 & . & . & . & . & . & . & 0 \\
\Sigma_{13} & \Sigma_{23} & L_3 & 0 & . & . & . & . & . & 0 \\
. & & & & & & & & & \\
. & & & & & & & & & \\
. & & & & & & & & & \\
\Sigma_{1N} & \Sigma_{2N} & . & . & . & L_N & 0 & . & . & 0 \\
0 & 0 & 0 & . & . & 0 & \lambda_1 & 0 & . & 0 \\
0 & 0 & 0 & . & . & . & 0 & \lambda_2 & . & 0 \\
. & & & & & & & & & \\
. & & & & & & & & & \\
. & & & & & & & & & \\
0 & 0 & . & . & . & . & . & . & . & \lambda_I
\end{pmatrix} \tag{28}
$$

Similarly, the fission matrix F includes the effect of both prompt and delayed neutrons. We denote by χ_i^P the fraction of prompt fission neutrons emitted into the i-th group, and by χ_i^D the corresponding quantity for delayed neutrons. Then

$$
F = \begin{vmatrix}
\nu_1 \Sigma_{f1} X_1^P (1-\beta) & \nu_2 \Sigma_{f2} X_1^P (1-\beta) & \cdots & \nu_N \Sigma_{fN} X_1^P (1-\beta) & \lambda_1 X_1^D \cdot \lambda_2 X_1^D & \cdots & \lambda_I X_1^D \\
\nu_1 \Sigma_{f1} X_2^P (1-\beta) & \cdots & \cdots & \nu_N \Sigma_{fN} X_2^P (1-\beta) & \lambda_1 X_2^D & \cdots & \lambda_I X_2^D \\
\vdots & & & & & & \vdots \\
\nu_1 \Sigma_{f1} X_N^P & \cdots & & & & \cdots & \lambda_I X_N^D \\
\beta_1 \nu_1 \Sigma_{f1} & \beta_1 \lambda_2 \Sigma_{f2} & \cdots & \beta_1 \nu_N \Sigma_{fN} & 0 & \cdots & 0 \\
\beta_2 \nu_1 \Sigma_{f1} & \cdots & & \beta_2 \nu_N \Sigma_{fN} & 0 & \cdots & 0 \\
\vdots & & & & & & \vdots \\
\beta_I \nu_1 \Sigma_{f1} & \cdots & & \beta_I \nu_N \Sigma_{fN} & 0 & \cdots & 0
\end{vmatrix}
$$

(29)

We denote by K the <u>reactor kinetics operator</u>

$$\frac{\partial \Phi}{\partial t} = K\Phi,$$ (30a)

$$K = V(F-M).$$ (30b)

As we have already pointed out, K^* is obtained not only by interchanging rows and columns of K, but also by <u>introducing individual elements into the transposed matrix which have been averaged over the adjoint spectrum.</u>

III. Analysis of Period Measurements; the In-hour Equation.

To solve Eq. (30a), we seek solutions of the form $\Phi(\underline{r},t) = A(\underline{r})e^{\omega t}$. This yields the eigenvalue equation

$$KA = \omega A.$$ (31)

The operator K has as its domain space the Hilbert space $L^2(\underline{r}) \times L^2(\underline{r}) \times \ldots \times L^2(\underline{r})$, the Cartesian product taken N+I times. Thus the spectrum of K will consist, in general, of a point spectrum, a continuous spectrum and, perhaps, even a residual spectrum. However, in all practical cases the multigroup kinetics operator K is further approximated by the use of finite differences to represent the derivatives. Thus, the actual K one deals with is an operator on a finite dimensional Hilbert space, and so its spectrum consists entirely of eigenvalues. The largest eigenvalue ω_0 is the inverse reactor period, $1/T$, and we wish to calculate it and to relate it to the reactor reactivity (i.e. to derive an in-hour equation).

This is most conveniently carried out through first-order perturbation theory. We note that for a critical reactor $\omega_0 = 0$. Let K_0 represent the kinetics operator of the critical reactor, and K_0^* the corresponding adjoint. Then

$$K_0\Phi_0 = 0$$ (32a)

and

$$K_0^*\Phi_0^* = 0,$$ (32b)

where we represent the adjoint flux by the symbol ϕ_0^*.
If

$$K = K_0 + \delta K, \tag{33}$$

where δK represents the insertion of a reactivity sample, then to first order

$$\delta\omega_0 = \omega_0 = \frac{(\phi_0^*, \delta K\phi_0)}{(\phi_0^*, \phi_0)}. \tag{34}$$

This perturbation expression is highly accurate, because it is based on a variational principle (the Ritz Variational Principle). [5] This means that the error in ω_0 is proportional to the product of the errors in flux and adjoint. Loosely speaking, if the spatial dependence of the asymptotic flux and adjoint in the perturbed reactor differ from the corresponding quantities in the critical reactor by 10% each, we should expect the above expression to give ω_0 to within 1% = 10% x 10%. Of course ϕ_0 and ϕ_0^* are to be computed by the usual multigroup codes. This method, incidentally, should be expected to yield the dominant eigenvalue with no problem <u>since it calculates the perturbation of that eigenvalue.</u>

We now wish to connect ω_0 with the reactivity. The simplest way to do this is to calculate, from Eq. (34), ω_0 as a function of an artificial perturbation,

$$\nu_i \longrightarrow \alpha\nu_i, \tag{35}$$

where α is some non-negative constant. As is well known this "perturbation" causes a reactivity

$$\rho = (1-\alpha). \tag{36}$$

In this way, a curve of ρ versus ω_0 could be plotted even before the period measurements are carried out. This curve represents the in-hour equation for the particular reactor configuration being used, and is applied to the analysis of all period measurements in that particular assembly.

Because of the form of the operator K, it is actually not necessary to solve the complete set of equations (32) to determine ϕ_0 and ϕ_0^*, as Fischer has

already pointed out. In particular, the equations decompose, leading to

$$C_{i0} = \beta_i \sum_{K=1}^{N} \frac{\nu_K \Sigma_{fK} \Psi_{K0}}{\lambda_i} \tag{37a}$$

and

$$C_{i0}^* = \sum_{K=1}^{N} \chi_K^D \Psi_{K0}^*. \tag{37b}$$

Thus, if the fluxes and adjoint fluxes are calculated, the precursor and adjoint precursor concentrations can be found immediately. Then the complete vectors Φ_0 and Φ_0^* can be constructed in order to evaluate ω_0.

This concludes our discussion of period measurements. We next turn to the question of pile-oscillator experiments.

IV. Pile Oscillator Experiments

The usual analysis of pile oscillator experiments is based on point kinetics equations involving time derivatives of the neutron and precursor densities; the analysis is usually carried out in terms of reactor transfer functions.[1] For reasons outlined in Sec. I, we feel this procedure must be modified somewhat for fast reactors.

A pile oscillator may be thought of as a perturbation to the reactor kinetics operator with a sinusoidal time dependence. That is, we consider the kinetics operator

$$\frac{\partial \Phi}{\partial t} = K_0 \Phi + k e^{i\omega t} \Phi. \tag{38}$$

We suppose that $\Phi(t)$ can be written as a sum of the stationary distribution, Φ_0, of the critical reactor ($K_0 \Phi_0 = 0$) and a small perturbation, Φ_1:

$$\Phi = \Phi_0 + \Phi_1. \tag{39}$$

Then, substituting into Eq. 38 we find

$$\frac{d\Phi_1}{dt} = K_0\Phi_1 + ke^{i\omega t}\Phi_0 , \tag{40}$$

where a presumably small term, proportional to $k\Phi_1$, has been dropped. We now assume a solution to Eq. (40) of the form

$$\Phi_1 = \Psi e^{i\omega t} . \tag{41}$$

This gives the following equation for Ψ:

$$(K_0 - i\omega)\Psi = - k\Phi_0 . \tag{42}$$

In a pile oscillator experiment, one observes a reaction rate with a detector, first in the critical system and then in the perturbed system. In the critical system, the reaction rate can be written simply as a scalar product:

$$R_0 = (\Sigma_d, \Phi_0) . \tag{43}$$

However, the perturbed flux is not real, indicating a phase shift between perturbation and response. Writing

$$\Psi_1 = \Psi_{1R} + i\Psi_{1I} \tag{44}$$

we find easily, for the real part of Φ

$$\text{Re } \Phi = \Phi_0 + R\cos(\omega t+\delta) , \tag{45}$$

where

$$R = \sqrt{\Psi_{1R}^2 + \Psi_{1I}^2} , \tag{46a}$$

$$\delta = \tan^{-1} \frac{\Psi_{1I}}{\Psi_{1R}} . \tag{46b}$$

Then the perturbed time-dependent reaction rate is given by

$$R(t) = (\Sigma_d, \text{Re } \Phi). \tag{47}$$

(In an actual experiment the phase angle δ can also be measured, and compared to calculation.)

It is of some interest to write down the equation obeyed by Ψ_{1R} and Ψ_{1I}:

$$(K_0^2 + 1) \Psi_{1I} = -\frac{k}{\omega} \Psi_0 \tag{48a}$$

and

$$\Psi_R = \frac{K_0}{\omega} \Psi_I . \tag{48b}$$

Eqs. (48) can, of course, be solved directly to calculate the response to a given oscillation. To calibrate the reactivity one can use the identical procedure as in the period measurement analysis described in Sec. III. That is, a theoretical oscillation of

$$k_{ij} = \delta_{ij} \nu_i(\alpha-1), \quad 1 \leq i \leq N \tag{49a}$$

$$= 0 \qquad N + 1 \leq i \leq I \tag{49b}$$

corresponds to a <u>reactivity</u> oscillation $\rho = 1 - \alpha$. The appropriate reaction rates can then be calculated as a function of reactivity in order to calibrate the oscillator.

A somewhat more accurate calculation can be made. Recalling that Eq. (42) is only approximate (a "small" term has been dropped) we can use the solution to (42) along with the adjoint solution to

$$(K_0^* + i\omega) \Psi_1^* = \Sigma_d \tag{50}$$

in the Rossopolous Variational Principle. [6] This principle states that the following functional is stationary with respect to variations of the trial functions Ψ_1 and Ψ_1^*

$$J_s = (\Sigma_d, \Psi_1) + (\Psi_1^*, - k\Psi_0) + (\Psi_1^*, (K_0 - i\omega)\Psi_1) \qquad (51)$$

The stationary value of this functional gives the best estimate of (Σ_d, Ψ_1), i.e. the response of the detector to the flux perturbation. Then it follows, again with $\Psi_1 = Re^{i\delta}$, that

$$Re\ J_s = (\Sigma_d, R)\cos(\omega t + \delta) \qquad (52)$$

so that the __perturbation__ in the counting rate is given by $Re\ J_s$.

This use of the Rossopolous Variational Principle is optional, to obtain higher accuracy. If less accuracy were acceptable, then Eqs. (48) could be solved directly to obtain the response to a given oscillator perturbation with the reactivity being related as described there. Incidentally, since typical experiments are carried out with square waves, it would be necessary to analyze the square waves into its Fourier components, and to carry out a type of calculation described above for each Fourier component.

A __transfer__ __function__ is defined as the ratio of (Laplace Transform) input to output. Usually, the reactivity is taken to be the input, while the output is some type of reaction rate. For example, if the output is neutron density, the reaction rate is that of a 1/v detector. If the output, on the other hand, is power, then the reaction rate is (theoretically) that of a fission counter.

For the purpose of stability analysis, it does not make much difference which type of transfer function is measured, since if one oscillation is unbounded, all others based on the same fundamental quantity (in this case neutron flux) will also be unstable. In particular, for linear systems it follows that no theoretical calibration, as described above, is necessary in the experimental determination of the transfer function. To calculate the transfer function, it is only necessary to proceed as described earlier in this section, computing the response as a function of ω to whatever input the experiment has used.

The situation is somewhat different for the nonlinear case in which the response depends not only on the frequency but also on the amplitude of the input. In

such a case reactivity calibration as described above might be necessary to insure that the entire experiment was carried out at constant amplitude.

V. Conclusion

We have tried to present a simplified and practical technique for treating reactor kinetics measurements and calculations in fast reactors. We have avoided the definition of a large number of parameters such as effective β, neutron lifetime, and so forth. Rather, we depend on variational principles plus straight forward solution of the multigroup flux and adjoint equations. We strongly recommend the use of Kiefhaber's "Case d" weighting in obtaining the multigroup constants.

The way described here was suggested during some conversations with Drs. Peter McGrath and Erhard Fischer concerning the discrepancy between measured and calculated reactivity coefficients in fast critical experiments.[7] The author is grateful to Drs. McGrath and Fischer not only for suggesting the problem but for extensive and valuable conversations.

LITERATURE

[1] See, for example, G. Robert Keepin "Physics of Nuclear Kinetics"
Addison-Wesley, Reading, Mass. (1965).

[2] E.A. Fischer "Über die Bedeutung des Adjungierten Flusses in der Reaktor-
kinetik" Report KFK 508 (1967).

[3] George I. Bell and Samuel Glasstone, "Nuclear Reactor Theory"
Van Nostrand Rheinhold Company, New York (1970). Sec. 2.4.

[4] E. Kiefhaber "Comment on the Calculation of Neutron Lifetime and
Material Work" Report KFK 882 (1968). See Table 2 of this report.

[5] Reference 3, Sec. 6.4c

[6] Reference 3, Sec. 6.4b. Bell and Glasstone do not refer to the
method as the "Rossopolous Variational Principle", a name which is
actually in fairly common usage.

[7] W.W. Little, Jr. and R.W. Hardie, Nuc. Sci. Engineering, 115 (1969).