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Investigation of Spectral Synthesis and Few Group Methods for Fast Reactor Transients

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Abstract

In this report two methods of calculating fast reactor transients economically are considered: the energy synthesis method and the few group schemes. Two transients of interest for safety studies were considered for a simple representative 1-d fast reactor model. These were a partial voiding transient and a complete voiding and fuel slumping transient.

The energy synthesis method gave excellent agreement both statically and transiently with the exact 26 groups results. Only 3 trial functions were used: the unperturbed core and blanket and the perturbed core averaged flux spectra. The weighting functions were their corresponding adjoints.

In few group schemes normal flux weighted collapsing and bilinear (flux-adjoint) collapsing were considered. The flux weighted procedure did not really give satisfactory results with even up to 12 groups. However the bilinear flux-adjoint schemes give satisfactory results with only 6 to 8 groups when the unperturbed flux and perturbed adjoint spectra (or vice versa) were used. For cases where the reactivity effects are not so sensitive (away from one dollar), acceptable results were obtained with just the unperturbed fluxes and adjoints. With bilinear collapsing it is necessary to consider discontinuities arising at interfaces between regions where the flux and adjoint-spectra are different. This requires the current conditions to be modified at such interfaces, while the normal few group flux continuity is sufficient.

Untersuchungen der Spektralsynthese-Methode und der Wenig-Gruppen-Methoden für Transienten Schneller Reaktoren

Zusammenfassung

In diesem Bericht werden zwei Methoden zur Berechnung von Transienten in schnellen Reaktoren untersucht: die Methode der Energiesynthese und die Wenig-Gruppen-Methode.

Zwei im Hinblick auf Sicherheitsstudien interessante Transienten werden in einem einfachen, repräsentativen 1-dimensionalen Modell eines schnellen Reaktors genauer untersucht: ein Transient für teilweisen Verlust des Kühlmittels und ein zweiter Transient für vollständigen Kühlmittelverlust und Niederschmelzen der Brennstäbe (Brennstoff-Verdichtung).

Die Methode der Energiesynthese liefert eine ausgezeichnete Übereinstimmung sowohl im statischen als auch im transienten Verhalten mit dem als Referenzfall angenommenen 26-Gruppenergebnis. Benutzt wurden lediglich 3 Versuchsfunktionen. Diese wurden im ungestörten Reaktorkern und Brutmantel und im gestörten Kern mit einem gemittelten Flußspektrum berechnet. Als Wichtungsfunktionen wurden die entsprechenden adjungierten Lösungsfunktionen verwendet. Bei den Wenig-Gruppen-Methoden wurde die Energiezusammenfassung mit der normalen Flußwichtung und mit der bilinearen Wichtung (Fluß/adjungiert) untersucht. Das Verfahren mit Flußwichtung liefert keine zufriedenstellenden Ergebnisse bis herauf zu 12 Energiegruppen. Dagegen ergeben die Verfahren mit bilinearer Wichtung befriedigende Ergebnisse für nur 6 bis 8 Energiegruppen, wenn man den ungestörten Fluß und das gestörte adjungierte Spektrum verwendet (oder umgekehrt). In Fällen, bei denen die Reaktivitätseffekte nicht so empfindlich sind (etwas von 1 $ entfernt), erhält man hinreichende Ergebnisse unter der Verwendung der ungestörten Flüsse und ihrer adjungierten.

Bei der Energiezusammenfassung und bilinearer Wichtung müssen Unstetigkeiten genauer untersucht werden, die an Grenzflächen zwischen Gebieten auftreten, in denen sich die adjungierten Spektren unterscheiden. Die Stetigkeitsbedingungen für den Strom müssen an solchen Grenzflächen modifiziert werden, während die üblichen Stetigkeitsbedingungen für die Wenig-Gruppen-Flüsse verwendet werden können.

I. INTRODUCTION

In many fast reactor transients it is necessary to explicitly consider spatial and spectral variations in the flux during the course of the transients. Normal static models for fast reactors consider from 2o to over 2oo energy groups in 2 or 3 dimensions. To use such detail routinely in transient calculations would be impractical because of the high costs involved. Therefore approximations are needed, to reduce the dimensionality of the problem for transient analyses by either decreasing the number of energy groups, or space dimensions or both.

Spectral synthesis methods give a way of effectively reducing the number of groups to be considered in the spatial and time dependent calculations. These methods were originally developed for static problems (1,2,3) and have been carried over to transient problems (4,5,6,7,8). The basic idea is to expand the space energy-time dependent flux as a linear combination of known spectral functions multiplied by unknown spatial and time dependent combining or mixing functions. These expansions are then substituted into the system equations requiring these to be satisfied in a weighted integral sense gives reduced equations for the unknown combining coefficients. These equations can be derived either variationally or with the direct weight and integrate technique (4,5,8). Recently Stacey (7) has had good success with these methods for representative voiding and poisoning transients in a 1-d fast reactor.

Another approach is to just reduce the number of groups to be considered to a reasonable number 6-8 by using a group collapsing or condensation scheme. However, in actual fact, these condensation methods are really discontinuous synthesis methods with single trial functions and few group spatially dependent combining coefficients (4,5,8). The difference in the various collapsing schemes lies in the choice of the weighting function. When it is taken as unity, the normal flux-weighted, few group constants are obtained. When it is taken as the adjoint flux, the Pitterle (9) bilinear scheme results, which should lead to better reactivity predictions. As always with discontinuous trial and weighting functions, special consideration must be given to the internal

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boundary conditions at interfaces where the functions are discontinuous. This is true of the bilinear scheme, and even now there is arbitrariness as to what to do at these interfaces (12,13).

In this study both the energy synthesis method and various group collapsing schemes were investigated. The framework of the study was the simple, 1-d representative fast breeder power reactor model used by Stacey in his synthesis study (7), using his partial voiding transient and also a full voiding with fuel slumping transient. For these 1-d studies, the program RAUMZEIT (14) was modified to handle up to 26 energy groups (the standard), modified interface conditions and synthesis.

The results are encouraging and indicate that the methods should be pursued further. Synthesis gives excellent agreement with the exact 26 group results with only three trial functions. Eight and six group bilinear collapsing particularly with a combination of initial flux spectra and final adjoint spectra (on vice versa) give comparable results. Further studies will consider more complicated, multizoned 1-d reactors before proceeding to 2 and 3 dimensional reactors.

In the next two sections, the synthesis and group collapsing equations are derived. Then the calculational model is given for the transients and results described.

II. ENERGY SYNTHESIS METHOD

A. GENERAL REMARKS

The basic idea behind the various synthesis approximation techniques is to satisfy the original system equations (including external and internal boundary conditions) in some approximate weighted integral sense.

The synthesis techniques were originally introduced in the variational context by Selengut of reactor analysis (15). Since then these methods have found with applications for expanding flux spectral and spatial dependence especially in thermal reactors and now increasingly in fast reactors. Recently Stacey (8) showed that the energy synthesis method was able to quickly and accurately calculate fast reactor transient where spatial and spectral efforts were important.

The methods involve assuming the solution as an expansion of the product of known spectral functions and unknown spatial and time dependent coefficients. These are substituted into the system equations, which are then required to be satisfied approximately in a weighted integral sense. This yields the reduced equations for the unknown functions. These approximate equations can be obtained either by this direct weight and integrate procedure or by variational formulations, with the same results (4,5,8).

For reactor transient problems, the G multigroup diffusion equations may be written as *

$$\nabla \cdot D \nabla \vec{\Phi} - \Sigma \vec{\Phi} + (1-\beta) \chi_p \cdot F^T \vec{\Phi} + \Sigma \lambda_m \chi_m C_m = \tau \frac{d\vec{\Phi}}{dt} \quad (1)$$

$$\beta^m F^T \cdot \vec{\Phi} - \lambda_m C_m = \frac{d}{dt} C_m \quad (2)$$

$$m = 1, \ldots M$$

where

$\vec{\Phi}(r,t)$	Gx1 column vector of the multigroup flux
C_m	scalar delayed neutron precursor density for type m
Σ	GxG matrix of removal and scattering cross sections
F	Gx1 column vector of group Nu-fission cross section
D	GxG diagonal matrix of diffusion coefficients
τ	GxG diagonal matrix of inverse group velocities
χ, χ_m	Gx1 column vector of prompt and delayed fission neutron spectra, respectively
λ_m, β_m	delayed neutron precursor decay rate and yield per fission, respectively

together with the external boundary conditions of zero flux and current and the internal interface continuity conditions of flux and current.

$$\vec{\Phi} \,|_+ = \vec{\Phi} \,|_- \quad (3)$$

$$D \nabla \vec{\Phi} \,|_+ = D \nabla \vec{\Phi} \,|_- \quad (4)$$

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* In this study, the delayed χ's were generally taken equal to the prompt χ and β-eff was used. When RAUMZEIT was later modified to allow the use of different delayed and prompt χ's, the results were checked and all the conclusions obtained here regarding the various group collapsing and synthesis schemes were verified. In addition, it was found that using either β-eff and the same delayed and prompt χ's or the real β's and different χ's gave the same results. These results are shown in Appendix E.

An expansion of the form

$$\Phi = \sum_{n=1}^{N} \psi_n \, \phi_n \tag{5}$$

may be assumed for the multigroup flux, where for each trial function n

ψ_n is a Gxg matrix giving the energy dependence assumed with multigroup flux, e.g.

$$\begin{array}{cc} & \text{Gxg} \\ \psi_n = & \begin{bmatrix} \psi^n & o \ldots & \\ \psi_2^n & o \ldots & \\ o & \psi_3^n & \\ & \psi_4^n & \\ & \psi_5^n & \\ & & \psi_{G-1}^n \\ & & \psi_G^n \end{bmatrix} \end{array}$$

ϕ_n is a gx1 column vector of the n-th combining coefficient e.g.

$$\begin{array}{cc} & \text{gx1} \\ \phi_n = & \begin{bmatrix} \phi_n^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi_n^g \end{bmatrix} \end{array}$$

The g allows different spatial and time dependent combining coefficient for g broad groupings of the spectral trial function, i.e. it permits the many groups to be combined in g few broad groups.

To obtain the reduced equations Equation (5) is substituted into Equations (1) and (2). Then the flux equation is multiplied by a series of n weighting matrices ω_i^T to yield the M+m vector equations in the M+m unknown vector ϕ_n and C_m.

$$\sum_{n=1}^{N} \omega_i^T \{\nabla \cdot D \nabla \psi_n \phi_n - \Sigma \psi_n \phi_n + (1-\beta) \chi_p F^T \psi_n \phi_n + \sum_m \lambda_m \chi_m C_m - \tau \frac{d}{dt} \psi_n \phi_n\} = o \qquad (6)$$

$$\sum_{n=1}^{N} \beta^m F^T \psi_n \phi_n - \lambda_m C_m - \frac{d C_m}{dt} = o \qquad (7)$$

where ω_i is a Gxg matrix giving the energy dependence of the i-th weighting function of the equation

Similarlythe flux and current continuity conditions at interfaces must be satisfied in a weighted integral sense

$$\sum_{n=1}^{N} {}^F\omega_i^T \{\psi_n \phi_n|_+ - \psi_n \phi_n|_-\} = o \qquad (8)$$

$$\sum_{n=1}^{N} {}^j\omega_i^T \{D \nabla \psi_n \phi_n|_+ - D \nabla \psi_n \phi_n |_- \} = o \qquad (9)$$

$$i = 1, \ldots, N$$

where ${}^F\omega_i$ is a Gxg matrix with the energy dependence of the i-th weighting function for the flux continuity condition

${}^j\omega_i$ is a Gxg matrix with the energy dependence of the i-th weighting function for the current continuity condition.

The various synthesis (or collapsing) schemes arise from the choice of the ψ_n (as well as the g broad groupings) and the ω_i. The weightings for the differential equations and interface conditions do not have to be the same. In fact for the discontinuous synthesis case with different expansion and weighting functions in different regions, this is impossible and a number of possibilities exist.

The next section discusses the continuous spectral (or energy) synthesis method.

B. ENERGY SYNTHESIS

In the energy synthesis considered here the same expansion functions and weighting functions are used throughout the reactor. The flux expansion functions are considered to be spectra functions obtained elsewhere, e.g. from zero dimensional or space dependent calculations. The weighting functions are taken as the corresponding adjoints. The expansion then is a Gx1 vector of the many group flux spectra times a space and time dependent combining coefficient, i.e.

$$\Phi = \sum_i^N \psi_i \; \phi_i(r,t)$$

where

$$\psi_i = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \\ \psi_b \end{bmatrix}_i \qquad \omega_i = \begin{bmatrix} \psi_1^* \\ \psi_2^* \\ \\ \\ \\ \psi_b^* \end{bmatrix}_i$$

and

ψ_i, ψ_i^* are the ith multigroup flux and adjoint spectra respectively

The equations for the flux and precusors then become in matrix form

$$\begin{aligned} &\nabla \cdot \tilde{D} \nabla \vec{\phi} - \tilde{\Sigma} \vec{\phi} - (1-\beta) \tilde{\chi}_p \tilde{F}^T \vec{\phi} \\ &+ \sum_m \lambda_m \tilde{\chi}_m C_m = \tilde{\tau} \frac{d}{dC} \vec{\phi} \\ &\dot{C}_m = \beta_m \tilde{F}^T \cdot \vec{\phi} - \lambda_m C_m \end{aligned} \qquad (11)$$

where

$\vec{\phi}$ is a Nx1 vector of the spatially dependent combining coefficients ϕ_i

and the corresponding cross section matrix and vector elements are given as

$$
\begin{aligned}
\tilde{D}_{ij} &= \omega_i^T D\psi_j \\
\tilde{\Sigma}_{ij} &= \omega_i^T \Sigma \psi_j \\
\tilde{F}_i &= F^T \cdot \psi_i \\
\tilde{\chi}_p^i &= \omega_i^T \cdot \chi_p \\
\tilde{\chi}_i^m &= \omega_i^T \cdot \chi^m \\
\tilde{\tau}_{ij} &= \omega_i^T \tau \psi_j
\end{aligned}
\tag{12}
$$

At the internal interfaces, all the trial function expansions and weighting functions are continuous, therefore regardless of the weighting in the flux continuity equation, the flux continuity condition is

$$\vec{\phi}_+ = \vec{\phi}_-$$

For the current continuity condition it is reasonable to weight with the weighting function that appears at the interfaces. Since these are the same throughout the reactor, there is no ambiguity and the resulting current continuity in matrix form is

$$D \nabla \phi \,\big|_+ = D \nabla \phi \,\big|_- .$$

These are the same conditions and equations that one obtains for spectral synthesis using a variational functional with continuous flux and adjoint spectral trial functions (4,8).

In applying the method, a decision must be made as to how many trial functions are to use, and also which regions or conditions they should represent. When only a few trial function are required, the method offers great savings. This is demonstrated in the Results' Section where for the sample problem only 3 trial functions are needed, and are simply the static unperturbed and perturbed spectra.

III. GROUP COLLAPSING METHODS

A. GENERAL REMARKS

There are two main differences between group condensation schemes and the energy synthesis method discussed above. The first is that the many groups are reduced not to one broad group but to g broad groups. This is evidenced by the fact that ω_i and Φ_i become Gxg matrices and the ϕ_i become gx1 vectors instead of scalars. Secondly, different trial and weight functions are used in different parts of the reactors, rather than using the same ones throughout. However, only one expansion is used in each region. This can cause problems at interfaces where both trial and weighting functions are discontinuous.

There are two common condensation schemes. One is the normal flux weighting scheme and the other is the bilinear or flux-adjoint weighting scheme discussed by Pitterle (9). In the flux weighting scheme, the elements of the ω matrices are unity, whereas in the bilinear scheme they give an adjoint spectrum dependence. The interface conditions are only a problem with the bilinear scheme, since the weighting function as well as the trial functions may be discontinuous at an interface. In the normal scheme, the weighting function is the same (unity) throughout the reactor and introduces no discontinuities at the interfaces. Both methods are described below.

B. NORMAL FLUX WEIGHTED CONDENSATION

In both the flux and usual bilinear group collapsing schemes (8)*, the single expansion function in each region for the flux is the same, and has the form of a Gxg matrix

$$\psi = \begin{bmatrix} \psi_1/\sum\limits_{i\varepsilon 1}\psi_i & o & \cdots & o \\ \psi_2/\sum\limits_{i\varepsilon 1}\psi_i & o & \cdots & o \\ \cdots & \cdots & \cdots & \cdots \\ o & \psi_j/\sum\limits_{i\varepsilon 2}\psi_i & \cdot & o \\ \cdots & \cdots & \cdots & \cdots \\ o & o & \cdot & \psi_G/\sum\limits_{i\varepsilon g}\psi_i \end{bmatrix} \tag{12}$$

and

$$\phi = \begin{bmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_g \end{bmatrix} \tag{13}$$

where the

ϕ_i is the ith broad group flux integral in each of the g broad groups

ψ_i is the ith multigroup flux spectra

The rows in each column indicate the group collapsing scheme, on which multigroup are combined to form each of the g broad groups. Also the broad group ϕ_i actually represent the physical broad group flux, i.e. the sum of the many group fluxes in the broad group.

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* Another bilinear weighting scheme is given in Appendix C. It has a different assumed flux energy dependence

The difference between the normal and bilinear weighting schemes lies in the choice of the weighting matrix. For the usual scheme, the ω matrix has the same form as the ψ matrix except that the non-zero elements are unity. In effect this means that the many group equations are simply added up according to the new broad group structure, i.e. the Gxg matrix looks like

$$\omega = \omega_I \equiv \begin{bmatrix} 1 & o & & o \\ 1 & o & & o \\ \cdot & \cdot & \cdots & \cdot \\ o & 1 & \cdots & o \\ \cdot & \cdot & \cdots & \cdot \\ o & o & \cdots & 1 \end{bmatrix} \tag{14}$$

which is identified as ω_I, the unity weighting matrix.

The resulting diffusion theory equation in each region is given as

$$\nabla \tilde{D} \nabla \phi - (\tilde{\Sigma}_{tot} - \tilde{\Sigma}_s)\phi + (1-\beta)\, \tilde{\chi}_p \tilde{F}^T \phi + \sum_m \lambda_m \tilde{\chi}_m C_m = \tilde{\tau} \frac{d\phi}{dt} \tag{15}$$

$$\beta_m \tilde{F}^T \phi - \lambda_m C_m = \frac{d\, C_m}{dt}$$

where ϕ is the vector of broad group fluxes and now the gxg cross section matrices and vector elements are given as

$$\tilde{D}_{kk} = \frac{\sum_{i\epsilon k} D_i \psi_i}{\sum_{i\epsilon k} \psi_i}$$

$$\tilde{F}_k = \frac{\sum_{i\epsilon k} F_i \psi_i}{\sum_{i\epsilon k} \psi_i} \tag{16}$$

$$\tilde{\Sigma}_{TOT_{kk}} = \frac{\sum_{i\epsilon k} \Sigma_{tot_i} \psi_i}{\sum_{i\epsilon k} \psi_i}$$

$$\tilde{\Sigma}_{s_{I\to J}} = \frac{\sum_{i\epsilon I} \sum_{j\epsilon J} \psi_i \Sigma_{i\to j}}{\sum_{i\epsilon I} \psi_i}$$

$$\tilde{\chi}_{p_k} = \sum_{i\epsilon k} \chi_{p_i}$$

$$\tilde{\chi}_{m_k} = \sum_{i\epsilon k} \chi_{m_i}$$

which are the definitions of the normally used broad or few group constants.

Since the same weighting function is used throughout the reactor and it is simply the unity weighting matrix, it can be used at the interfaces without ambiguity, resulting in the normal few group flux and current continuity conditions

$$\begin{aligned} \phi|_+ &= \phi|_- \\ \tilde{D}\, \nabla\, \phi|_+ &= \tilde{D}\, \nabla\, \phi|_- \end{aligned} \tag{17}$$

These are the normal few group equations and have also been derived without problems from variational principles (4,8). These normal diffusion codes solve these equations. The problem lies in choosing the number of reduced groups and their structures, as well as the flux spectra to use in the expansion.

C. BILINEAR CONDENSATION

With the normal flux weighted condensation scheme, the difference in importance of the various multigroups in the collapsed broad groups is not accounted for. Using adjoint on bilinear weighting accounts for that and from first order perturbation theory should give better reactivity predictons (9). The flux expansion* is the same as before but now the weighting matrix is taken to represent the average adjoint structure in the collapsed broad groups, i.e.

$$\omega = \begin{bmatrix} \psi_1^*/\frac{1}{U_1} \sum_{i\varepsilon 1} \psi_i^* \Delta u_i & o & \cdots & \\ \psi_2^*/\frac{1}{U_1} \sum_{i\varepsilon 1} \psi_i^* \Delta u_i & o & \cdots & \\ \cdots & o & \cdots & \\ o & \psi_j^*/\frac{1}{U_2} \sum_{i\varepsilon 2} \psi_i^* \Delta u_i & \cdots & \\ \cdots & \cdots & \cdots & \\ o & o & \cdots & \psi_G^*/\frac{1}{U_g} \sum_{i\varepsilon g} \psi_i^* \Delta u_i \end{bmatrix} \qquad (18)$$

where

ψ_i^+	is the i-th multigroup average adjoint flux
Δu_i	is the lethargy width in the i-th multigroup
$U_I = \sum_{i\varepsilon I} \Delta u_i$	is the lethargy width in the I-th broad group.

Carrying out the substitution and weighting as before, Equations (15) again result for the few group fluxes and precursors in each region,

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* Another formulation is given in Appendix C. It is more complicated and does not really work as well, because the interfaces are not properly treated. That given here is essentially the Pitterle scheme (7).

with the exception that the few group gxg cross section matrices and vectors are now given by

$$\tilde{D}_{kk} = \frac{\sum_{i\epsilon k} \psi_i^* D_i \psi_i}{\frac{1}{U_k} \sum_{i\epsilon k} \psi_i^* \Delta u_i \sum_{i\epsilon k} \psi_i}$$

$$\tilde{F}_k = \frac{\sum_{i\epsilon k} F_i \psi_i^*}{\Sigma \psi_i}$$

$$\tilde{\Sigma}_{Tot_k} = \frac{\sum_{i\epsilon k} \psi_i^* \Sigma T_i \psi_i}{\frac{1}{U_k} \sum_{i\epsilon k} \psi_i^* \Delta u_i \sum_{i\epsilon k} \psi_i} \qquad (19)$$

$$\tilde{\Sigma}_{I\to J} = \frac{\sum_{i\epsilon I} \sum_{j\epsilon J} \psi_j^* \Sigma_{i\to j} \psi_i}{\frac{1}{U_j} \sum_{j\epsilon J} \psi_j^* \Delta u_j \sum_{i\epsilon I} \psi_i}$$

$$\tilde{\chi}_{p_k} = \frac{\sum_{i\epsilon k} \chi_i \psi_i^*}{\frac{1}{U_k} \sum_{i\epsilon k} \psi_i^* \Delta u_i}$$

$$\tilde{\chi}_{m_k} = \frac{\sum_{i\epsilon k} \chi m_i \psi_i^*}{\frac{1}{U_k} \sum_{i\epsilon k} \psi_i^* \Delta u_i}$$

Notice that when the ψ_i^* are taken as unity, the normal flux weighted group constants result.

With bilinear weighting at interfaces between different regions the adjoint or weighting function spectra as well as the trial function or flux spectra will also be discontinuous as opposed to the normal flux weighted case. Therefore, the choice of the weighting functions for the interface conditions is not quite so straightforward.

For the current condition, it is not unreasonable to use the averaged adjoint spectra as weighting function at the boundary, i.e.

$$\frac{1}{2}\,[\omega_+^T + \omega_-^T]\ \{D\ \nabla\ \psi\ \phi\ |_+ - D\ \nabla\ \psi\ \phi\ |_-\} = o \qquad (2o)$$

which results in

$$\frac{1}{2}\,[\tilde{D}_- + S_+]\ \nabla\ \phi\ |_- = \frac{1}{2}\,[D_+ + S_-]\ \nabla\ \phi\ |_+ \qquad (21)$$

where $\tilde{D}_+$ and $\tilde{D}_-$ are the bilinearly collapsed diffusion coefficient, at the positive and negative side of the interfaces, respectively and the S matrices are given by the cross products

$$S_{kk_-} = \frac{\sum_{i\varepsilon k} \psi_{i+}^* D_{i-} \psi_{i-}}{\frac{1}{U_k} \sum_{i\varepsilon k} \psi_{i+}^* \Delta u_i \sum_{i\varepsilon k} \psi_{i-}}$$

$$S_{kk_+} = \frac{\sum_{i\varepsilon k} \psi_{i-}^* D_{i+} \psi_{i+}}{\frac{1}{U_k} \sum_{i\varepsilon k} \psi_{i-}^* \Delta u_i \sum_{i\varepsilon k} \psi_{i+}} \qquad (22)$$

Notice that when the weighting functions are continuous this gives the normal current continuity condition.

If the effects of the discontinuities in the weighting functions are ignored, and the weighted currents on each side are merely equated, i.e.,

$$\omega_+ D \nabla \psi \phi|_+ = \omega_- D \nabla \psi \phi|_- \qquad (23)$$

then the more familiar ,usual condition is obtained

$$\tilde{D}_+ \nabla \phi_+ = \tilde{D}_- \nabla \phi_- \qquad (24)$$

where the diffusion coefficient is defined as in Equation 19. The results show that this is not as good as using the S matrices.

The S matrix formulation is the same as that Terney (1o) and Stacey (8) obtain from variational formulations by assuming that the variations in the flux combining coefficients at the interface are equal. There are ambiguities in the variational approach when both the weighting (adjoint) and trial (flux) functions are discontinuous at the same interface because, as is well known the problem becomes overdetermined (4,8,1o,11,12). One way to alliviate the over-determination is to equate variations at the interface, which gives these results obtained here. Another approach is to not allow both flux and adjoint functions to be discontinuous at the same interfaces (4). To do this artificial overlap regions are added where the flux or adjoint from the adjoint region is used together with these for the region in question. This approach was tried and the results are given in Appendix D. Unfortunately, it turns out that they are sensitive to the width of the overlap region, which apparently must be a least 1-2 diffusion lengths thick, and seems to act merely as the buffer to diminish the effects of the discontinuity. The approach also has the disadvantage of requiring extra regions. Buslik (11,12) has tried to alliviate this problem by adding the boundary conditions to the variational functional with Lagrange Multipliers which should be representative of the values of the adjoint current and flux at the interface. Recently, Stacey (13) has proposed not just equating the variations at the interface but postulating some relationship between them.

The end results of all approaches is to decide somewhat arbitrarily on a weighting function to use in requiring the internal continuity conditions to be satisfied. Here with the substitute and integrate procedure, the choice was simply to use the average adjoint or weighting spectra at the interface to introduce the S matrices.

For the flux continuity the choice is again not clear. In fact, there are 3 reasonable approaches. The first and probably physically most logical is just to require continuity of the few group fluxes, i.e.

$$\vec{\phi}_+ = \vec{\phi}_- \tag{25}$$

This implies that the weighting matrix is

$$F_\omega = \omega_I \tag{26}$$

given in Equation (14). Equation (25) may be written in the more general form as

$$\phi_+ = R_1 \ \phi_-$$

where R_1 is the unit matrix. This condition requires continuity of the few group flux and the choice is to use the averaged flux adjoint or weighting function at the interface, i.e.,

$$\frac{1}{2} \ (\omega_+^T + \omega_-^T] \ \{\psi \ \phi|_+ - \psi \ \phi|_-\} = 0 \tag{27}$$

where the ω_+ and ω_- are those given by Equation (18) for each side of the interface.

With the end result that

$$\phi_+ = R_2 \; \phi_- \tag{28}$$

where R_2 is a diagonal gxg matrix given by

$$R_2 = \left[\omega_+^T \; \psi_+ + \omega_-^T \; \psi_+\right]^{-1} \; \left[\omega_+^T \; \psi_- + \omega_-^T \; \psi_-\right] \tag{29}$$

A third alternative is to weight the flux continuity condition with the averaged adjoint current spectra at the interface, i.e.

$$\left[\omega_+^T \; D_+ + \omega_-^T \; D_-\right] \; \{\psi_+ \; \phi_+ - \psi_- \; \phi_-\} = o \tag{3o}$$

where the ω's are again by equation (18). This yields

$$(D_+ + C_-) \; \phi_+ = (D_- + C_+) \; \phi_- \tag{31}$$

where the cross product diffusion coefficients matrices C are given by

$$C_{+_k} = \frac{\sum_{i \epsilon k} \psi_{i+}^{*} \; D_{i+} \; \psi_{i-}}{\frac{1}{U_k} \sum_{i \epsilon k} \psi_{i+}^{*} \; \Delta u_i \sum_{i \epsilon k} \psi_i}$$

$$C_{-_k} = \frac{\sum_{i \epsilon k} \psi_{i-}^{*} \; D_{i-} \; \psi_{i+}}{\frac{1}{U_k} \sum_{i \epsilon k} \psi_{i-}^{*} \; \Delta u_i \sum_{i \epsilon k} \psi_i} \tag{32}$$

$$\phi_+ = R_3 \; \phi_- \tag{33}$$

where

$$R_3 = \left[D_+ + C_-\right]^{-1} \; \left[D_- + C_+\right] \tag{34}$$

This is the result Stacey (8) Terney (1o) obtain variationally when the adjoint current variations are equated at the interfaces.

The actual best choice of boundary conditions can really not be determined beforehand and must probably be resolved by actual comparison of the various approximations with the exact many group calculations.

The various possibilities given here were investigated numerically with the model given in the next section. The results indicate that using S matrix is necessary but that the usual flux continuity condition is sufficient when the S + R matrices are used, the diffusion theory finite difference equations at the interface are different than normal and the usual programs must be modified. Appendix A gives the modified diffusion theory equations and the alterations which were made to RAUMZEIT to handle tham.

IV. TEST PROBLEM

In order to economically test the syntheses and various group collapsing procedures, Stacey's (7) one-dimensional, representative fast breeder power reactor model was used. The reactor consisted of a single core zone of half width 175 cm and a blanket of width 5o cm. It is depicted in Figure 1 together with the material concentrations in the two zones.

Two transients were considered which are of interest for safety analysis. The partial voiding transient (identical to Stacey (7)) was initiated by linearly decreasing the Na concentration in the 5o cm of the core by 5o % in o.5 sec. To crudely mock up the Doppler effect and keep the total inserted reactivity about 5o % the B-1o concentration in that region was increased by 2o % in the time interval o.1 to o.5 sec. The second transient was a full voiding of the central region coupled with the addition of fuel and B-1o to mock up fuel slumping and the doppler effect. It was initiated by voiding the central 5o cm of Na in 2o millisec. The fuel concentration was linearly increased 5o % and the B-1o to .oo117 in the 2 to 2o millisec time interval.

Transient and static calculations were run with the finite difference, 1-d space time program, RAUMZEIT (14). It was modified to handle up to 26 groups, synthesis, and the modified interface conditions needed when bilinear weighting is used. The basis of comparison were the 26 group results obtained for both static and dynamic calculations. The 26 group cross sections were obtained from the Karlsruhe cross section set (16), and were used to obtain the collapsed group cross sections through the NUSYS system (17).

The static eigenvalues and reactivities for the 26 group, 1-d RAUMZEIT calculations are given in Table I. The transient power distributions initially and at the end of each transient are shown in Figure 2. The power distribution at the end of the partial voiding transient is the same as the initial power distribution, while that at the end of the fuel slumping transient refers to the addition of more fuel. In, both cases, the static and transient power distributions are essentially identical. This means that the spatial flux effects, are not so important for these transients. However, the reactivity effects must be calculated well so that the amplitude or total power changes are accurately predicted.

The total powers, power factions and moments for both transients are given in Tables II and III. The results are also plotted on Figures 3 and 4. In addition, the point kinetics results for both cases are shown in the figure*.

The point kinetics reactivity input was obtained by linearly interpolating between the reactivities obtained for each of the static calculations at the 3 perturbations. For the partial voiding transient the results are not too bad, with errors in the total power 5 - 1o % during the transient. For the fuel slumping transient, however, the powers are too high by up to factors of 1o with point kinetics. This indicates that it is not sufficient to use the end point reactivities and the interpolated in between values but that one must have a good reactivity trace or table as input. In other words, more 26 g static calculations or improved perturbation theory calculations are needed. When a better reactivity trace was used, point kinetics was capable of giving acceptable powers values.

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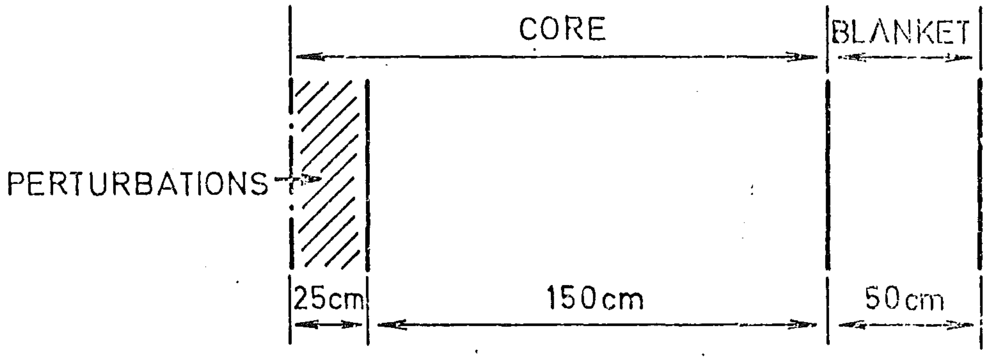
* The point kinetics were calculated with a KFK version of the AIREK III. code (19).

FIG - 1

TRANSIENT-MODEL



DENSITIES ($\#/cm^3 \times 10^{24}$)

MATERIALS	CORE	BLANKET
^{239}Pu	0.00098	0.00029
^{238}U	0.00697	0.01074
^{23}Na	0.010187	0.00648
^{56}Fe	0.01600	0.01500
^{10}B	0.00050	———
^{16}O	0.01750	0.02231

I Partial Voiding Transient (~ 50 ¢)

0 - 0.50 sec	50% Na removed
0.10 - 0.50 sec	B-10 added

II Slumping Transient (~ $ 1.15)

0 - 20 ms	all Na removed 50% more fuel added
2 - 20 ms	B-10 added

FIGURE 2

Transient Power Distributions

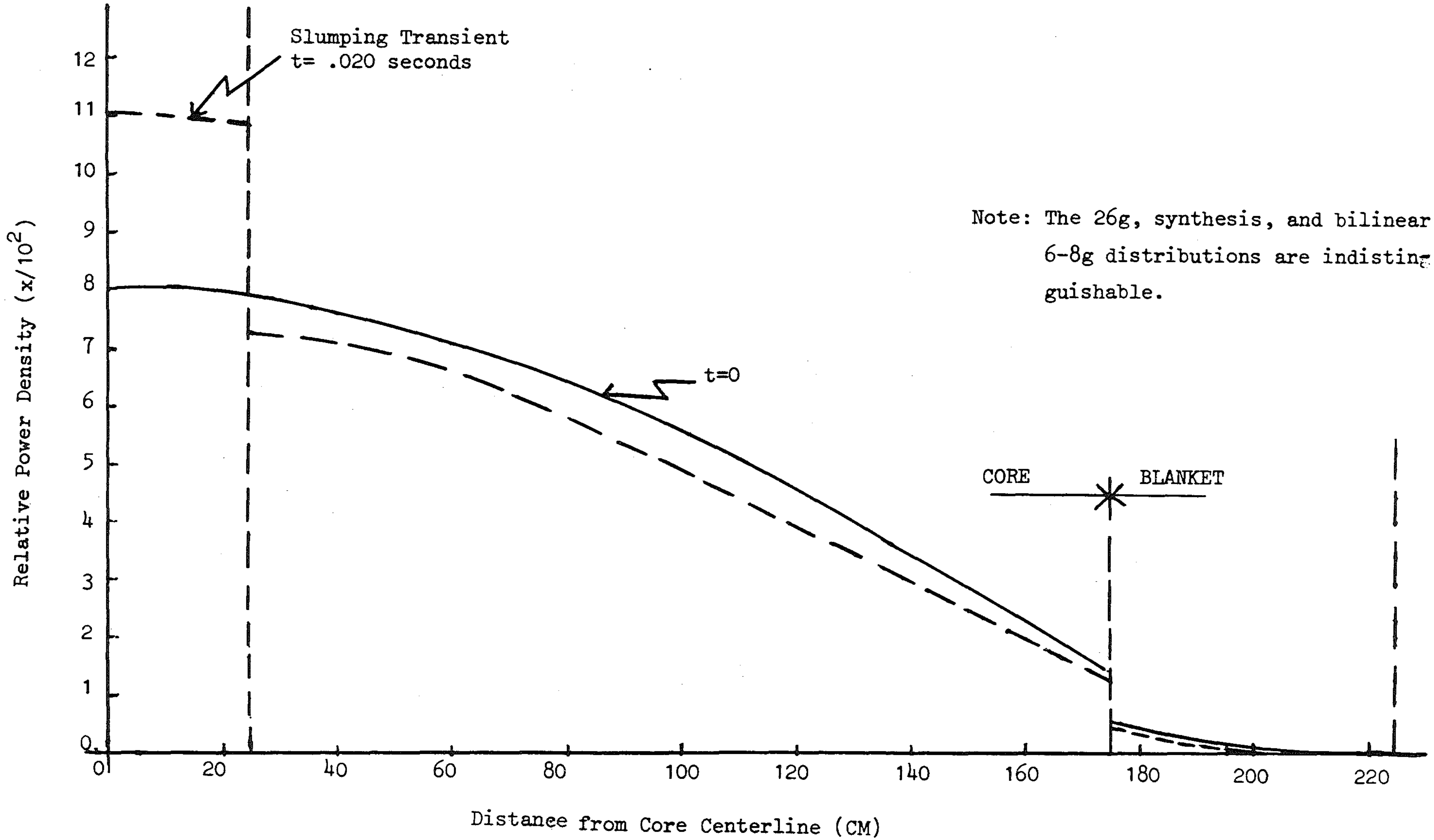
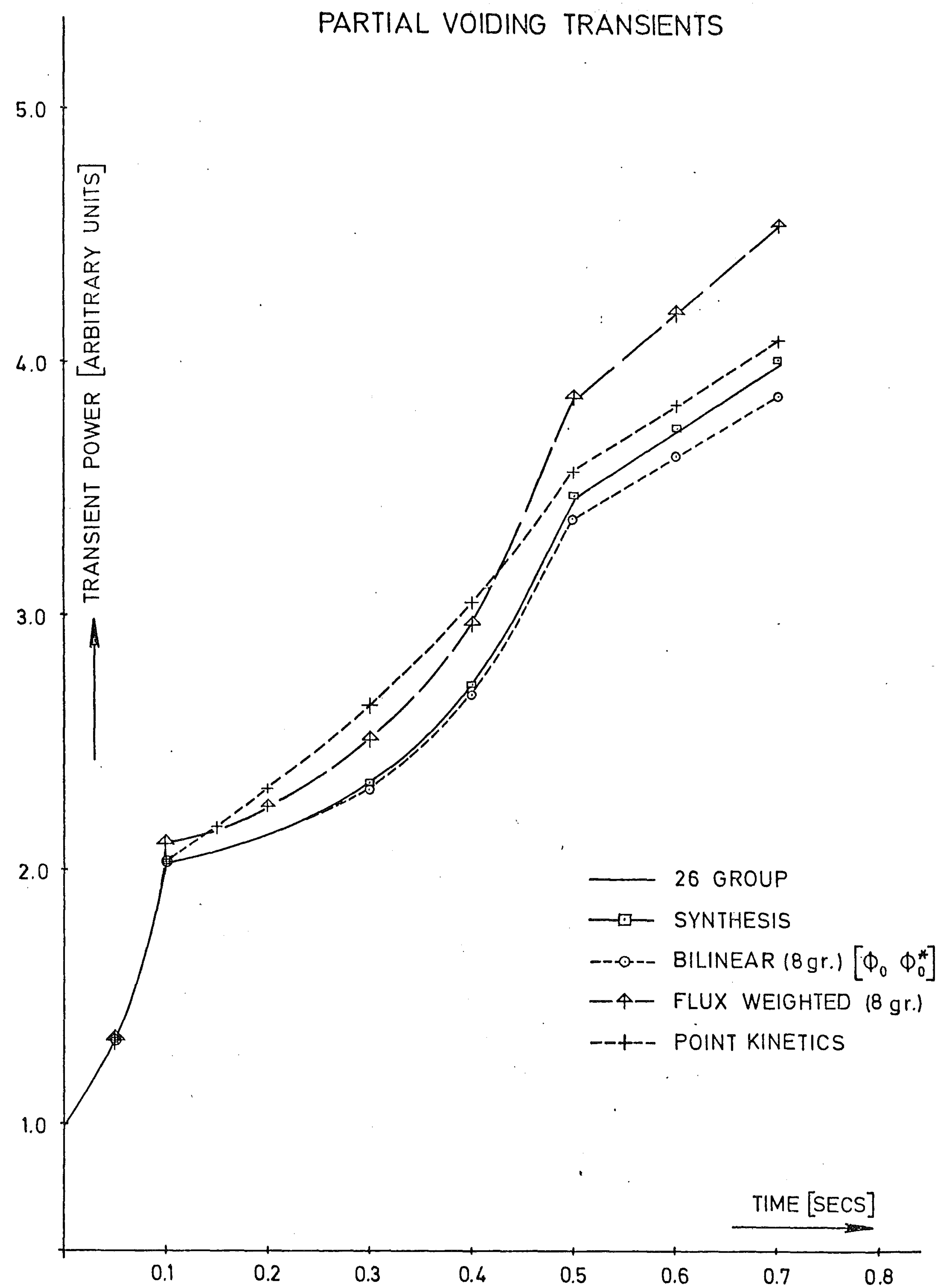


FIG-3


PARTIAL VOIDING TRANSIENTS
TRANSIENT POWER [ARBITRARY UNITS]
5.0
4.0
3.0
2.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
TIME [SECS]
26 GROUP
SYNTHESIS
BILINEAR (8 gr.) [Φ_0 Φ_0^*]
FLUX WEIGHTED (8 gr.)
POINT KINETICS

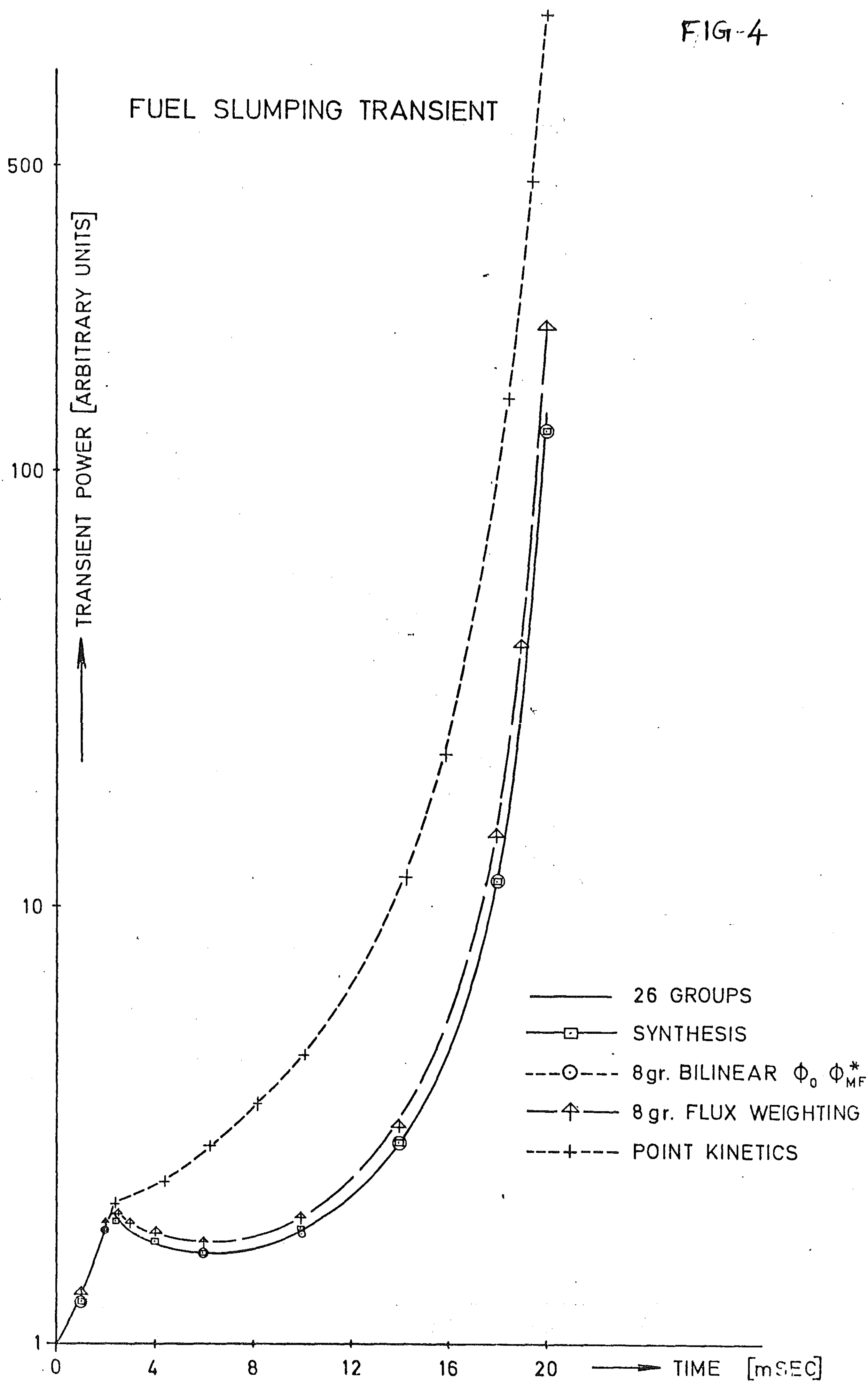


FIG-4
FUEL SLUMPING TRANSIENT
TRANSIENT POWER [ARBITRARY UNITS]
500
100
10
1
0
4
8
12
16
20
TIME [mSEC]
26 GROUPS
SYNTHESIS
8gr. BILINEAR Φ_0 Φ^*_{MF}
8gr. FLUX WEIGHTING
POINT KINETICS

TABLE I

STATIC 26 GROUP AND SYNTHESIS RESULTS WITH RAUMZEIT

EIGENVALUES AND REACTIVITIES:

Case		26 group	Synthesis Partial Voiding Model	Synthesis Fuel Slumping Model
Initial	k	.995005	.995007	.995007
10 % void	k	.996606	.996608	.996603
	ρ	.001601	.001601	.001596
	%		0	(-.3 %)
50 % void + B-10	k	.997042	.997049	.997037
	ρ	.002037	.002042	.002030
	%		(+.2 %)	(-.3 %)
100 % Void More Fuel + B-10	k	.998682	-	.998677
	ρ	.003677		.003670
	%			(-.2 %)

POWER FRACTIONS AND MOMENTS

Case		26 group	Synthesis Partial Voiding Model	Synthesis Fuel Slumping Model
Initial	(PF) Core 1	.1990	.1990	.1990
	(PF) Core 2	.7897	.7898	.7897
	(PF) Blanket	.0113	.0113	.0111
	Moment	71.2	71.2	71.2
50 % Void		.1998	.1998	.1998
		.7891	.7892	.7891
		.0111	.0110	.0110
		70.7	70.7	70.7
100 % Void More Fuel + B-10		.2730		.2730
		.7171		.7170
		.0099		.0098
		65.0		65.0

TABLE II

PARTIAL VOIDING TRANSIENT

Time (Seconds)	26 group	SYNTHESIS	FLUX WEIGHTED 6g	FLUX WEIGHTED 8g	BILINEAR WEIGHTED 6g (ORIG Ø	BILINEAR WEIGHTED 8g ORIG Ø*)
			TOTAL REACTOR POWER			
o	1.o	1.o	1.o	1.o	1.o	1.o
.o5	1.331	1.331	1.336	1.346	1.331	1.331
.1o	2.o37	2.o35	2.o6o	2.1o5	2.o36	2.o33
.2o	2.136	2.136	2.118	2.247	2.126	2.126
.3o	2.346	2.346	2.26o	2.5o7	2.315	2.326
.4o	2.732	2.736	2.528	2.915	2.659	2.694
.5o	3.46o	3.477	3.oo2	3.862	3.289	3.383
.6o	3.721	3.741	3.18o	4.198	3.517	3.628
.7o	3.984	4.oo8	3.359	4.542	3.147	3.877
Computing Time for Transient (Minutes)	38	.25	.8o	1.6	.8o	1.6
			POWER FRACTIONS AND MOMENTS			
Osec						
Core 1	.199o	.199o	.1988	.1989	.1989	.1989
Core 2	.7891	.7898	.7899	.7898	.7899	.7898
Blanket	.o113	.o113	.o113	.o113	.o112	.o113
Moment	71.2	71.2	71.2	71.2	71.21	71.21
.5o sec						
Core 1	.1998	.1998	.1994	.1999	.1999	.1999
Core 2	.7892	.7892	.7896	.7889	.7891	.7891
Blanket	.o11o	.o11o	.o11o	.o11o	.o11o	.o11o
Moment	7o.7	7o.7	7o.8	7o.7	7o.7	7o.7

TABLE III

FUEL SLUMPING TRANSIENT

MODEL TIME (sec)	26 group	Synthesis	FLUX WEIGHTED 6g	FLUX WEIGHTED 8g	BILINEAR WEIGHTED 6g or F -	BILINEAR WEIGHTED 8g or A	6g MF F-	8g or A	6g or F-	8g MFA
TOTAL REACTOR POWER										
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
.001	1.282	1.280	1.284	1.292	1.286	1.281	1.282	1.281	1.282	1.280
.002	1.845	1.839	1.856	1.891	1.845	1.842	1.846	1.839	1.846	1.839
.004	1.718	1.712	1.717	1.802	1.713	1.714	1.727	1.720	1.733	1.719
.006	1.624	1.616	1.592	1.716	1.611	1.617	1.637	1.631	1.641	1.626
.010	1.831	1.815	1.698	1.961	1.787	1.812	1.851	1.842	1.854	1.833
.014	2.877	2.856	2.352	3.160	2.696	2.812	2.918	2.901	2.905	2.868
.018	11.755	11.857	5.541	14.59	9.008	10.70	12.04	11.91	11.61	11.46
.020	134.37	135.1	16.71	210.8	61.5	102.5	140.0	137.4	122.6	122.4
Computing Time for Transient (Min.)	40	.3	.8	1.7	0.8	1.7				
POWER FRACTIONS AND MOMENTS										
0 sec										
Core 1	.1990	.1990	.1988	.1989	.1989	.1989	.1988	.1986	.1988	.1988
Core 2	.7897	.7899	.7899	.7898	.7899	.7898	.7900	.7901	.7900	.7899
Blanket	.0113	.0111	.0113	.0113	.0112	.0113	.0112	.0113	.0113	.0113
Moment	71.2	71.2	71.2	71.2	71.2	71.2	71.2	71.2	71.2	71.2
.020 sec										
Core 1	.2735	.2747	.2704	.2713	.2730	.2734	.2725	.2733	.2726	.2734
Core 2	.7167	.7156	.7198	.7169	.7172	.7168	.7177	.7169	.7176	.7168
Blanket	.0098	.0097	.0098	.0098	.0098	.0098	.0098	.0098	.0098	.0098
Moment	65.0	64.9	65.4	65.0	65.0	65.0	65.1	65.0	65.1	65.0

V. RESULTS

A. SYNTHESIS

The syntheis trial and weighting functions for the 2 transients were obtained from 26 group, 1-d static diffusion theory calculations. In each case only 3 functions were used. Two were the averaged spectra in the core and blanket for the unperturbed reactor.

The third trial function was taken as the averaged flux spectra in the central core region for the perturbed reactor condition, again obtained from a static, 1-d 26 group diffusion theory calculation. For all trial functions, the corresponding adjoints were taken as the weighting functions.

The static eigenvalue for the 26 group and synthesis models are given in table I for various reactor conditions. The agreement in reactivity worths is excellent. In addition the power distribution shapes also are in good agreement as shown in table I by the power fractions in each of the 2 core zones and the blanket, as well as the moment of the power measured from the center line*. Figures 2 show the power distribution which can not be distinguished from the 26 group static transient results.

The transient results bear out the promise of the static calculation. The total power as a function of time for both transients are tabulated in Tables II and III together with initial and final power fractions and moments. The agreement between the transient total powers calculated with synthesis and 26 groups at the end of the transient is within 1 %.

In addition, the power sharing is also well calculated as revealed in Tables II and III, as well as in Figure 2. Only in the fuel slumping transient is there appreciable spectral power shifting, but it is well calculated.

The remarkable time saving feature of synthesis for the transient calculation is also shown in Tables II and III. The time requirements with synthesis are equivalent to using a 3 group model. However, the accuracy obtained is on the order of that obtained with 6 to 8 or more groups.

---

* Power moments is defined as $\overline{\mathcal{X}} = \int \mathcal{X} \; \nu\Sigma_f(\mathcal{X}) \; \phi(\mathcal{X}) \; d\mathcal{X} / \int \nu\Sigma_f(\mathcal{X}) \; \phi(\mathcal{X}) \cdot d\mathcal{X}$

FEW GROUP RESULTS

FLUX WEIGHTED GROUP CONSTANTS

The simplest approach for obtaining few group constants is to use the flux weighting procedure. Many 1-d, static calculations were carried out within the NUSYS system to determine the best few group structures for various numbers of groups. The standard was to do well on static eigenvalues and reactivity worths for a variety of perturbations ranging from voiding to adding fuel and boron. In each case, the collapsing was done with the averaged 26 g spectra in each region obtained from the 26 g, 1-d diffusion theory calculation for the unperturbed reactor. It was found help-ful to select group structures so that the ratios of perturbed to unperturbed fluxes was nearly the same for each group in the new broad group. The best group structures are given in Table IV and Appendix B.

Table IV gives the static results for the best 6, 8 and 12 group structures for the cases with perturbations in the central core zone corresponding to the partial voiding and fuel slumpings of interest. There is fairly substantial improvement in going from 6 to 8 groups, but going from 8 to 12 groups brings less of a change. In most cases, the reactivity errors are less than a few percent. However, for these transients such accuracy is not good enough. The transient results are given in Tables II and III for 6 and 8 groups and are shown in Figures 2 and 3. Even going to 12 g gave unacceptable errors of more than 3o % in the total powers for the fuel slumping transient.

These results show the importance of calculating the reactivity extremely accurately for perturbations above 5o ¢ and in the neighborhood of $ 1. In fact for the partial voiding transient, the point kinetics results are even better than the 8g flux weighted result, because of the better reactivity values. Adding more regions as transition regions near the interfaces had no effect on calculating the reactivity worths of the perturbations. Therefore in order to gain acceptable accuracy more than 12 groups would have to be used when simple flux weighting is used for the collapsing. This originally motivated the attempt to see if bilinear weighting with only a few (6-8) groups would work. These results are given in the next section.

TABLE IV

NUSYS FEW GROUPS STATIC EIGENVALUES

Case	26 g	FLUX WEIGHTED 12g	8g	6g	BILINEARLY WEIGHTED UNPERTURBED SPECTRA 8g	6g
Base k	.995085	.995063	.995065	.995049	.995089	.995091
10 % Void k	.996684	.996697	.996712	.996668	.996697	.996698
Δk	.001599	.001634	.001647	.001619	.001610	.001609
% error		(2.0 %)	(+3.0%)	(1-3 %)	(7 %)	(5 %)
Partial						
Void k	.997119	.997166	.997213	.996942	.997123	.997089
Δk	.002034	.0021030	.002148	.001893	.002036	.001998
% error		(3.3 %)	(5-6 %)	(6-9 %)	(.1 %)	(-1.8%)
Total Void k	.998748	.998755	.998791	.998267	.998715	.998633
+ More Fuel Δk	.003663	.003693	.003726	.003218	.003628	.003542
+ Boron %error		(.8 %)	(+1.1%)	(-12.1%)	(-9 %)	(-3.3%)

The best few group structures were found to be

12 g: 1-3, 4, 5, 6, 7, 8-9, 10, 11-12, 13, 14-15, 16-17, 18-26
10 g: 1-3, 4, 5, 6, 7, 8-9, 10, 11-12, 13, 14-26
8 g: 1-3, 4-6, 7, 8, 9, 10, 11-12, 13, 14-26
7 g: 1-3, 4-6, 7, 8-9, 10, 11-13, 14-26
6 g: 1-3, 4-6, 7, 8-9, 10-13, 14-26

VI. BILINEARLY WEIGHTED GROUP CONSTANTS

The first attempt with bilinearly weighted cross sections was to use the initial or unperturbed averaged spectra. Both the flux and adjoint spectra in each region were obtained again from the 1-d, 26 group static calculation of the unperturbed reactor. The reduced group constants were obtained within the NUSYS system using the version of the condensation routine modified by Kiefhaber to handle the Pitterle method.

The static eigenvalues obtained by NUSYS are shown in Table IV. Both the6 and 8 group reactivities are much improved over the flux weighted values. In fact the 8 group bilinearly weighted results are as good as or a little better than the 12 group flux weighted results. However, the reactivity for the fuel slumping perturbation still has errors which are too high.

The transient results are again given in Tables II and III and on Figure 2.

Table II shows that both the 6 and 8 g group constants obtained with bilinear weighting using the unperturbed flux and adjoint give acceptable results for the partial voiding transient, the 8 g results are too low by only 2 - 3 %, and even the 6 g results are only 5 - 6 %.

However, for the fuel slumping transient with a reactivity of the order of 1.15 the seemingly small errors in reactivity (1 ₵) are actually very important. The 8 group power at the end of the transient is 23 % too low which is probably, only marginally acceptable. The 6 group power is 54 % too low and clearly unacceptable.

Therefore, when the reactivity is in the neighbarhood of $ 1, it must be calculated with good accuracy. One approach would be to use more groups. When 12 groups are used the reactivity error for the fuel slumping case dropped to about .3 % and gave powers which were acceptable, only 6 or 7 % too low. However, continually adding groups is not so attractive an alternative, especially when the number must be greater than 12 or so. Another approach is to calculate the reactivity better by a wiser choice of flux and adjoint spectra. Using a combination of the initial flux and final adjoint spectra (or vice versa) is a way to do this. In a sense this approaches the synthesis method where use is made of knowledge or an estimate of the spectra at the beginning and at the end of the transient. For the 8 g bilinear scheme it is apparent that only a little improvement is needed.

When this approach is used the interface conditions between regions in which different adjoint and flux spectra are used become important. The studies with only the unperturbed flux and adjoint were originally made by ignoring the modified interface condition. That is, only the D matrices and not the S matrices were used for the current condition and the usual few group flux continuity conditions were used. But since the original adjoint in both core regions are nearly identical the S + D matrices at the important first interface were essentially the same only a barely discernible change in the results was observed when the S matrices were added. However, for the cases where the adjoint spectra was different in adjacent regions, namely, using the slumped fuel case adjoint appreciable efforts are apparent. Tables V + VI give the results for static eigenvalues and reactivities for the various 8 + 6 group cases using the version of RAUMZEIT modified to handle the new interface conditions.

The importance of the current interface conditions for 8 groups can be seen in Table V by comparing the columns with D's and those with S's. For the cases with the original adjoint there is essentially no difference in results since the adjoint is practically the same at the core interface, and the S + D matrices are nearly equal. When the original adjoint and final flux are used to get the few group constants, all the eigenvalues and reactivity are calculated extremely well. When the final adjoint and intial flux are used, however, the eigenvalues and reactivities deteriorate when the interface conditions are ignored. When the modified conditions are used, i.e. the S's are used, excellent agreement is again obtained. Using final flux and adjoint is always as bad as or a little worse than using the original flux and adjoint.

The other columns show the effect of using the various flux continuity conditions given in Section III as well as the improved current condition. There is no constant improvement in using R_2 and R_3 over the

the usual few group continuity condition (R_1 = UNIT MATRIX). The usual condition works as well as any of the others. Therefore, because of its simplicity and physical meaning, it is reasonable to use it for the flux continuity condition.

The results are the same for 6 groups as Table VI shows. When the adjoints are discontinuous, the modified current condition with the S matrices must be considered. For both cases when a mix of the final and initial flux and adjoint spectra are used is the agreement with the 26 of static eigenvalues and reactivities good.

The final and intermediate reactivities determined with the 8 group bilinear mixed mode scheme, including those with S matrices, gave really acceptable results. A disadvantage of using a few number of groups with mixed weighting is that both the interface conditions become more important and the intermediate reactivities are not calculated as well which can lead to erroneous transient results. In fact when only 1 group was used with the mixed modes the total overall static reactivity cases fairly well predicted but the partial perturbations had reactivity errors of 1o - 58 % leading to transient power errors of factors of 1/2 to 1o.

The 6 and 8 g transient results for the difficult fuel slumping transient are also given in Table III and Figure 4 for both cases with mixed fluxes and adjoints. All these results have the corrected current continuity condition and the usual flux continuity conditions. The 6 and 8 group models give essentially the same results and either could be used. This accuracy is in contrast to when only the original fluxes and adjoints are used for the weighting. Further, the accuracy for both mixed mode models is good and comparable to that with synthesis. Power distributions are shown in Figure 2 and are also good.

The results for the 2 mixed mode model are different but within 1o - 12 % of each other. This reflects both the differences in the two, spectra combinations and also the effect of the interfaces. The better results are those with the nearly continuous adjoint at the interface. However, for both combinations the results are acceptable, especially for the 8 group model.

TABLE V: 8 GROUP BILINEAR STUDY OF INTERFACE CONDITIONS

SPECTRA FOR COLLAPSING		$\phi_o\ \phi^+$				$\phi_o\ \phi_{MF}$				$\phi_{MF}\ \phi_o^*$				$\phi_{MF}\ \phi_{MF}^*$			
Case	26 g	D,R_1	S,R_1	S,R_2	S,R_3	D,R_1	S,R_1	S,R_2	S,R_3	D,R_1	S,R_1	S,R_2	S,R_3	D	S	S,R_2	S,R_3
Base k	.995005	.994994	.994996	.994965	.994986	.995032	.995033	.995003	.995053	.994994	.994997	.994967	.994988	.994973	.994971	.994941	.995001
10 % void	.996606	.996592	.996594	.996565	.996585	.996620	.996630	.996601	.996648	.996590	.996592	.996563	.996583	.996558	.996565	.996536	.996598
Δk	.001601	.001598	.001598	.001600	.001599	.001588	.001597	.001598	.001595	.001596	.001595	.001596	.001595	.001585	.001594	.001595	.001597
% error		-0.19	-0.19	-0.06	-0.12	-0.81	-0.25	-0.19	-0.37	-0.3	-0.37	-0.31	-0.37	-1.00	-0.44	-0.37	-0.25
Partial Void	.997042	.997009	.997011	.996982	.997002	.997017	.997072	.997044	.997090	.997028	.997030	.997002	.997021	.996990	.997043	.997015	.997082
	.002037	.002015	.002015	.002017	.002016	.001985	.002039	.002041	.002037	.002034	.002033	.002035	.002033	.002017	.002072	.002074	.002081
		1.07	1.07	-0.98	-1.03	-2.54	+0.09	+0.20	0	-0.2	-0.2	-0.1	-0.2	-0.98	+1.72	+1.81	+2.16
Total Void	.998682	.998630	.998632	.998608	.998625	.998592	.998698	.998674	.998714	.998673	.998675	.998653	.998667	.998595	.998700	.998677	.998744
with More	.003677	.003636	.003636	.003643	.003639	.003560	.003665	.003671	.003661	.003679	.003678	.003688	.003679	.003622	.003729	.003736	.003743
Fuel + B		-1.09	-1.09	-1.02	-1.03	-3.28	-0.32	-0.16	-0.44	+0.05	0	+0.24	+0.05	-1.5	+1.41	+1.68	-0.95

TABLE VI

6 GROUP BILINEAR STUDY OF INTERFACE CONDITIONS

		$\emptyset_o \; \emptyset_o^x$		$\emptyset_{MF} \; \emptyset_o^x$		$\emptyset_{MF}^x \; \emptyset_o$		$\emptyset_{MF}^x \; \emptyset_{MF}$	
Case	26 g	D	S	D	S	D	S		
Base	.995005	.995001	.995009	.995010	.995017	.995034	.995035	.994976	.994966
10% Void	.996606	.996605	.996612	.996612	.996619	.996629	.996639	.996570	.996570
	.001601	.001604	.001603	.001602	.001602	.001595	.001604	.001594	.001604
		(+0.19%)	(+.06%)	(+.06%)	(+.06%)	(-.37%)	(+0.19%)	(-.43%)	(+.19%)
50% Void+B	.997042	.996986	.996993	.997030	.997037	.997043	.997116	.997016	.997082
	.002037	.001985	.001984	.002020	.002020	.002009	.002081	.002040	.002116
		(-2.552)	(-2.60)	(.84%)	(-.83%)	(-1.37%)	(+2.16%)	(1.15%)	(+3.88%)
More Fuel	.998682	.998550	.998557	.998690	.998696	.998556	.998697	.998577	.998715
	.003677	.003549	.003548	.003680	.003679	.003522	.003662	.003601	.003749
		(-3.48%)	(-3.52%)	(+0.8%)	(.05%)	(-4.21%)	(-.41%)	(-2.07%)	(+1.96%)

CONCLUSIONS

Synthesis Methods

The results obtained here show that energy synthesis is a very promising techniques for treating fast reactor transient. In the 2 transients of interest only 3 trial (and Adjoint weightung) functions were needed to obtain excellent agreement with the "exact" 26 group results.

However, for more complicated reactors with more dimensions and representative regions it may be necessary to use more trial functions say 6-8 to span the spectra of the reactor and the perturbation. If too many energy trial functions are required, it may be necessary to use discontinuous synthesis methods and face the interface problem again. A basic trouble with the energy synthesis approach for real reactors is that a 2 or 3 dimensional but few group transient problem still must be solved.

Probably the most attractive alternative is to use a Kaplan type approach, where the problem will always be reduced to a one-dimensional synthesis in the z-direction by using either partially or fully collapsed planar trial functions.

Few Group Methods

With few groups methods, the importance of preserving the reactivity effects of a variety of perturbations (especially in the neighborhood of \$ 1) become apparent. For the normal flux weighting condensation scheme more than 12 groups would be needed to get acceptable accuracy.

By using bilinear flux-adjoint condensation schemes, the reactivity effects and transients are calculated much better. Using only 8 groups and the original Flux and adjoint spectra to do the collapsing gave both static and transient results better than the 12 group flux weighted case.

There are cases where using the initial spectra is not quite good enough. When a mixture of the initial flux and final adjoint spectra (or vice vasa) is used, excellent results are obtained also for these cases. This may be thought of as modifying either the flux or the adjoint function slightly to get a better reactivity consideration of the perturbation using this method enabled 6 or 8 groups to be used to get comporable accuracy as with synthesis.

With bilinear weighting it is necessary to account for the discontinuities which occur at the interfaces between regions where different flux and adjoint collapsing spectra are used. This is especially true of the adjoint and the effect it has on the current continuity condition. When the normal current condition at the interface is modified by using the averaged adjoint spectra at the interface, the problem appears to be resolved. The usual few groups flux continuity condition appears to be acceptable without modifications.

The methods will carry over directly to more complicated multidimensional reactors. For these problems however, the interface conditions and discontinuities may become even more important. In addition, they do require modifying the internal boundary conditions built into most finite difference codes.

Both the synthesis and bilinear weighting group collapsing schemes preserve reactivity effects. In addition, they also, in effect, preserve the effective delayed neutron fraction and generation time, since the approximate transient results agree with the exact results. Where the effect of the delayed neutrons having a different spectra is considered explicity, the results given here are maintained. These results are shown in Appendix E, and indicate that one can use either β_{eff} and the same χ's for delayed and prompt neutrons or the actual β's and the different χ's.

FUTURE WORK

The work will be continued by applying these procedures to more complicated reactor models in both one and more dimensions. A 1-d reactor will be considered first, which has the characteristics of a more typical fast reactor prototype. This will be investigated to make sure that both the number of trial functions does not become excessive and that the modified interface conditions hold up.

A 2-d, r-z study should also consider a typical, prototype reactor. For this energy synthesis, Kaplan synthesis, and the bilinear collapsing schemes should be checked before any of the methods are excluded.

The results to date indicate that these approximate methods are useful, fast and accurate. They should find greater applications in actually solving fast reactor transients of interest.

ACKNOWLEDGEMENTS

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APPENDIX A

RAUMZEIT MODIFICATIONS

The Subroutine RAUM of RAUMZEIT solves a source problem

$$- D_n \nabla^2 \phi + H_n \phi = S \tag{A.1.}$$

which is put in a difference equation format

$$- \frac{D_n^2}{h_n^2} (\phi_{i+1} + \phi_{i-1} - 2\phi_i) - \frac{\rho D_n}{2rh_n} (\phi_{i+1} - \phi_{i-1}) + H_n \phi_i = S_i \tag{A.2.}$$

where subscript n is for the material, i for the point.
These equations are solved in a recursion relation

$$\phi_{i-1} = \alpha_i^{-1} (\phi_i + ß_{i-1}) \tag{A.3.}$$

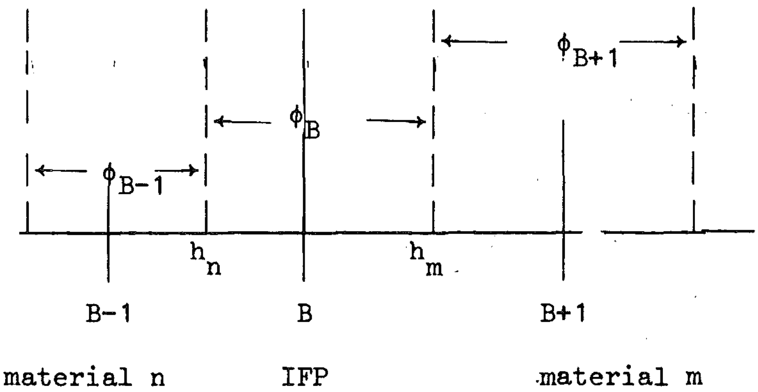
where

$$\alpha_{i+1} = \frac{2I + h^2 D_n^{-1} h_n - (1- \frac{h\rho}{2r}) \alpha_i^{-1}}{1 + \frac{h\rho}{2r}} \tag{A.4.}$$

$$ß_i = \frac{h^2 D^{-1} S_i + (1- \frac{h\rho}{2r}) \alpha_i^{-1} ß_{i-1}}{1 + \frac{h\rho}{2r}} \tag{A.5.}$$

At an interface point B one assumes the normal continuity realtions

$$D_n \nabla \phi_B^- = D_m \nabla \phi_B^+ \tag{A.6.}$$



and uses it to couple the difference equation for the half mesh boxes on either side of the interface. These difference equations are

$$- D_n \nabla \phi_B + \frac{D_n \phi_B^- - \phi_{B-1}}{h_n} + \frac{h_n H_n \phi_B^-}{2} = \frac{h_n S_{Bn}}{2} \tag{A.7.}$$

and

$$- D_m \frac{\phi_{B+1} - \phi_B^+}{h_m} + D_m \nabla \phi_B^+ + \frac{h_m H_m \phi_B^+}{2} = \frac{h_m S_{Bm}}{2} \tag{A.8.}$$

Solving for $D_n \nabla \phi_B^-$ and $D_m \nabla \phi_B^+$, and substituting in the current continuity relation and using the flux continuity

$$\phi_B^+ = \phi_B^- \equiv \phi_B \tag{A.9.}$$

one has for

$$\alpha_{B+1} = \frac{h_m D_m^{-1}}{2} \quad \frac{1 - \frac{h_n \rho}{2r}}{1 + \frac{h_n \rho}{2r}} \quad \left(h_n H_n + \frac{2D_n}{h_n} - \frac{2D_n}{h_n} \alpha_B^{-1}\right) + h_m H_m + \frac{2D_m}{h_m} \tag{A.10.}$$

and

$$\beta_B = \frac{h_m}{2} D_m^{-1} \quad \frac{1 - \frac{h_m \rho}{2r}}{1 + \frac{h_n \rho}{2r}} \quad \left(h_n S_{Bn} + \frac{2D_n}{h_n} \alpha_B^{-1} \beta_{B-1}\right) + h_m S_{B,m} \tag{A.11.}$$

With the modifications of the interface boundary conditions given in Section III, the current and flux continuity relation at the interface become

$$(D_n + S_L) \nabla \phi_B^- = (D_m + S_R) \nabla \phi_B^+ \tag{A.12.}$$

$$R\phi_B^- = \phi_B^+$$

These are used to get new recursion relationships at the interfaces By rewriting A.12. as

$$(D_n + S_L) D_n^{-1} D_n \nabla \phi_B^- = (D_m + S_R) D_m^{-1} D_m \nabla \phi_B^+ \tag{A.13.}$$

Substituting in (A.13.) for $D_n \nabla \phi_B^-$ and $D_m \nabla \phi_B^+$ from equations (A.7.) and (A.8.), and rearranging, α_{B+1} and β_B can be obtained

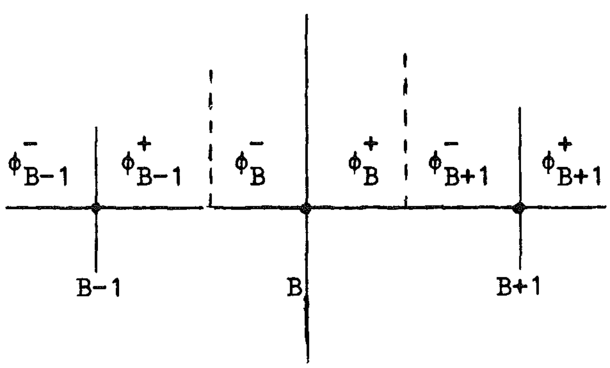
Let $$\frac{D_n + S_L}{2} \equiv \Delta_n$$

and $$\frac{D_m + S_R}{2} \equiv \Delta_m$$

Writing the recursion relation as

$$\phi_i^+ = \alpha_i^{-1} \left(\phi_{i+1} + \beta_i\right) \tag{A.14.}$$

and noting that at all other points $\phi_i^- \equiv \phi_i^+ \equiv \phi_i$ except at the interface



where $\phi_i^+ = R_i \, \phi_i^-$

the algorithem is in general

$$\alpha_{B+1} = \frac{h_m \Delta_m^{-1}}{2} \; \frac{1-\frac{h_m\rho}{2r}}{1+\frac{h_n\rho}{2r}} \left(h_n\Delta_n D_n^{-1} H_m + \frac{2\Delta_n}{h_n} - \frac{2\Delta_n}{h_n}\alpha_B^{-1}\right)$$

$$+ \frac{2\Delta_m}{h_m} W + h_m \Delta_m D_m H_m R \qquad (A.15.)$$

$$\beta_B = \frac{h_m \Delta_m^{-1}}{2} \; \frac{1-\frac{h_m\rho}{2r}}{1+\frac{h_n\rho}{2r}} \left(h_n\Delta_n D_n^{-1} \; S_{B_n} + \frac{2\Delta_n}{h_n}\alpha_B^{-1} \; \beta_{B-1}\right)$$

$$+ h_m \Delta_m D_m^{-1} R S_{B_m} \qquad (A.16.)$$

These two relations reduce to (A.1o.) and (A.11.), when $\Delta_m = D_m$; $\Delta_n = D_n$ and R=I, i.e. where there are no discontinuities.

These new relations were included in subroutine RAUM of RAUMZEIT to accomodate the extraterms arising out-of using discontinuous functions in bilinear collapsing of cross section libraries. When there are no discontinuities the results are the same as before.

APPENDIX B

BEST GROUP STRUCTURALS

The normal 26 group structure is given below as well as the selections of best few group structures.

gp	Energy	Range	12	1o	8	7	6
1	6.5 -	1o.5 MeV					
2	4 -	6.5	1	1	1	1	1
3	2	4					
4	1.4 -	2	2	2			
5	o.8	1.4	3	3	2	2	2
6	o.4	o.8	4	4			
7	o.2	o.4	5	5	3	3	3
8	o.1	o.2					
9	46.5	1oo keV	6	6	4	4	4
1o	21.5	46.5	7	7	5	5	
11	1o.o	21.5	8	8	6		
12	4.65	1o				6	5
13	2.15	4.65	9	9	7		
14	1.o	2.15					
15	465	1ooo eV	1o				
16	215	465					
17	1oo	215	11				
18	46.5	1oo		1o	8	7	6
19	21.5	46.5					
2o	10	21.5					
21	4.65	1o.o	12				
22	2.15	4,65					
23	1.o	2.15					
24	0.465	1.o					
25	0.215	0.465					
26	Thermal	group					

APPENDIX C

ALTERNATE BILINEAR FORMULATION

Another bilinear collapsing scheme was formulated and investigated. The ground rules were that the same weighting function in any given region was to be used for the differential equation, and the flux and current continuity conditions at the interface. The boundary conditions were to be obtained by setting the weighted current and flux in each region equal to that in the adjacent region. An additional requirement was that the boundary conditions should have the form of the usual conditions.

Algebrarially

$$(\omega^T \nabla D\psi\phi)|_+ = (\omega^T \nabla D\psi\phi)|_- \qquad (C.1.)$$

$$(\omega^T \psi \phi)|_+ = (\omega^T \psi \phi)|_- \qquad (C.2.)$$

where ω is the same weighting matrix used to obtain the reduced group diffusion equation and ω and ψ are chosen so that Eqs. C.1. and C.2. reduce to

$$\tilde{D} \nabla \phi |_+ = \tilde{D} \nabla \phi |_- \qquad (C.3.)$$

$$\phi |_+ = \phi |_- \qquad (C.4.)$$

where the D's are the appropriately defined few group diffusion coefficients in each region. Notice that this method again neglects the effects of discontinuities at the interfaces. Instead it just substitutes a new set of conditions which are to be satisfied and look like the usual conditions.

A choice of ψ and ω that makes this work is

$$\psi = \begin{bmatrix} \frac{\psi_1}{\sum_{i\epsilon 1} \psi_i \ \psi_i^*} \ \frac{\Sigma \ \psi_i^* \ \Delta U_i}{U_1} & & & \\ \frac{\psi_2}{\sum_{i\epsilon 1} \psi_i \ \psi_i^*} \ \frac{\Sigma \ \psi_i^* \ \Delta U_i}{U_1} & & & \\ & \ddots & & \\ & & & \frac{\psi_G}{\sum_{i\epsilon g} \psi_i \ \psi_i^*} \ \frac{\Sigma \ \psi_i \ \Delta\psi_i}{U_g} \end{bmatrix} \qquad (C.5.)$$

$$\omega = \begin{bmatrix} \frac{\psi_1^*}{\frac{1}{U_1} \sum_{i\epsilon 1} \psi_i^* \ \Delta U_i} & & & \\ \frac{\psi_2^*}{\frac{1}{U_1} \sum_{i\epsilon 1} \psi_i^* \ \Delta U_i} & & & \\ & \ddots & & \\ & & & \frac{\psi_G^*}{\frac{1}{U_1} \sum_{i\epsilon g} \psi_i^* \ \Delta U_i} \end{bmatrix} \qquad (C.7.)$$

where all the terms are defined as before and

$$\phi = \begin{bmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_g \end{bmatrix} \qquad (C.8.)$$

However, now the few group fluxes (the ϕ_i) are not the simple flux integral over the broad group, but rather the importance averaged flux integral.

Carrying out the usual substitute, weight and integrate procedure leads to the equations

$$\nabla\cdot\tilde{D}\nabla\phi - (\tilde{\Sigma}_{Tot} - \tilde{\Sigma}_s)\phi + (1-\beta)\,\tilde{\chi}_p\,\tilde{F}^T\,\phi + \sum_m \lambda_m\,\tilde{\chi}_m\,C_m = \tilde{\tau}\,\frac{d\phi}{dt} \qquad (C.9.)$$

$$\beta_m\,\tilde{F}^T\,\phi - \lambda_m\,C_m = \frac{dC_m}{dt} \qquad (C.1o.)$$

and at the interfaces

$$\phi_+ = \phi_-$$

$$\tilde{D}_+\nabla\,\phi_+ = \tilde{D}_-\nabla\,\phi_-$$

where

$$\tilde{D}_{kk} = \sum_{i\varepsilon k} \psi_i^{*}\,D_i\,\psi_i / \sum_{i\varepsilon k} \psi_i^{*}\,\psi_i$$

$$\tilde{F}_k = \sum_{i\varepsilon k} F_i\,\psi_i\,\sum_i \frac{\psi_i^{*}\,\Delta U_i}{U_k} / \sum_{i\varepsilon k} \psi_i^{*}\,\psi_i$$

$$\tilde{\Sigma}_{Tot_k} = \sum_{i\varepsilon k} \psi_i^{*}\,\Sigma_{Tot_i}\,\psi_i / \sum_{i\varepsilon k} \psi_i^{*}\,\psi_i$$

$$\Sigma_{I\to J} = \frac{\left(\sum_{i\varepsilon I} \psi_i^{*}\,\frac{\Delta U_i}{U_I}\right)\left(\sum_{i\varepsilon J}\sum_{i\varepsilon I} \psi_j^{*}\,\Sigma_{i\to j}\,\psi_i\right)}{\left(\sum_{i\varepsilon I} \psi_i^{*}\,\psi_i\right)\left(\sum_{j\varepsilon J} \psi_j^{*}\,\frac{\Delta U_j}{U_J}\right)}$$

$$\chi_k = \sum_{i\varepsilon k} \psi_i^{\,x} \chi_i / \sum_{i\varepsilon k} \frac{\psi_i^{\,x} \Delta U_i}{U_k} \quad \text{for prompt}$$

or delayed as the case may be.

Notice that when the ψ_i are unity the results are the same as for normal flux weighting. Further, the averaged group constants are really averaged over the product of the flux and adjoint spectra, rather than the product of the averages. The new few group flux is really not a pure flux but an importance weighted flux integral.

One-dimensional NUSYS calculations were carried out with this collapsing scheme for the various cases with 8 groups. Table C.1. shows a comparison of the results obtained with this formulation and those using the Pitterle scheme without accounting for the interface discontinuities. This means that the boundary conditions with the Pitterle scheme were just the usual current and flux continuity conditions and in fact are more or less comparable to those used in the new scheme.

The modified results are somewhat better than the usual Pitterle formulation. However, the improvement is not great enough. The remaining errors, especially in the cases of discontinuous adjoints, indicate that the interface conditions are not being properly treated as had been hoped.

Modifying the standard Pitterle method with the new interface conditions gives acceptable results as seen in the Results sections. Presumably the same improvement could be obtained here, by using modified interface conditions. But then one of the motivations for this formulation is lost. In addition, the somewhat unphysical interpretation of the few group flux would remain.

TABLE C.1.

COMPARISON OF PITTERLE (P) AND MODIFIED (M) BILINEAR SCHEMES 8 GROUP-NUSYS

Cases	$\phi_o \phi_o^x$		$\phi_o \phi_{MF}^x$		$\phi_{MF} \phi_o^x$		$\phi_{MF} \phi_{MF}^x$	
	P	M	P	M	P	M	P	M
INITIAL k	.995087	.995063	.995091	.995076	.995096	.995065	.995031	.995031
10% Void k	.996697	.996668	.996677	.996673	.996694	.996660	.996632	.996614
Δk	.001610	.001605	.001586	.001597	.001598	.001595	.001601	.001597
% error	+.7 %	+ .4 %	- .8%	+.1%	-.1%	-.3%	+ .1%	- .1%
Partial Void k	.997123	.997103	.997083	.997077	.997148	.997122	.997058	.997050
Δk	.002036	.002040	.001992	.002001	.002052	.002057	.002027	.002033
% error	+ .1%	+ .3%	-2.1%	-1.6%	+ .9 %	+1.1	-.3%	-.1%
Slumping k	.998715	.998706	.998649	.998656	.998757	.998731	.998649	.998662
Δk	.003628	.003643	.003558	.003558	.003661	.03666	.003618	.003645
%error	-.9%	-.6%	-2.9%	-2.3%	+.1%	.1%	-1-2%	-.5%

APPENDIX D

OVERLAPPING REGIONS

One of the suggestions for avoiding ambiguity problems at interface is to not allow the expansion and weighting functions to be discontinuous at the same interface (4). This was attempted here simply by allowing the flux spectra to be discontinuous at the region interfaces, but by requiring the adjoint spectra from the lefthand region to be used accross the interface a short distance into the righthand region. This is depicted in Figure D.1.

The usual Pitterle bilinear scheme was used where the group constants were calculated in each of regions (now 5 instead of 3), using the approximate adjoint and flux as shown in the Figure. The regular boundary conditions of continuous current and flux were used to determine if this were a way to solve the problem. Results were obtained as a function of width of the overlap regions, ranging from 2 cm in a region to, completely overlapping the reactor with the core adjoint. The results are given in Table D.1 for the cases with the final adjoint spectra. The cases which used the initial adjoint spectra are not shown since the spectra is essentially continuous at the first interface and overlapping had no noticeable effect.

The eigenvalues and reactivities are fairly sensitive to the width of the overlap region, which is not very satisfying. For the original flux case an overlap width of 1o cm or so (∿ 2 diffusion lengths) gives good results. For the final flux spectra the best results are obtained with about 5 cm or one diffusion length. By the proper choice of this width, the effects of discontinuities seem to be ameliorated. However, this can also be obtained by using the modified boundary conditions. The other disadvantage of this method is that involves adding more regions to the reactor model.

FIGURE D.1

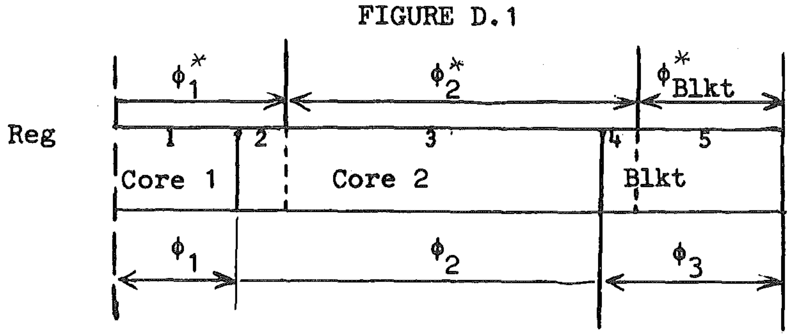


TABLE D.1

OVERLAPPING ADJOINT STUDIES

8 GROUPS NUSYS

Case	Orig. Flux-MF Adjoint					MF Flux - MF Adjoint				
Overlap	Base	2 cm.	6 cm.	10 cm.	All	Base	2 cm.	6 cm.	10 cm.	All
INITIAL k	.995091	.995095	.995096	.995103	.995113	.995031	.995032	.995038	.995043	.995045
10% Void k	.996677	.996683	.996694	.996698	.996708	.996632	.996637	.996647	.996650	.996652
Δk	.001586	.001588	.001598	.001595	.001595	.001601	.001605	.001609	.001607	.001607
% error	-.8%	-.7%	-.1	-.2%	-.3%	+.1%	+ .4 %	+ .6 %	+ .5 %	+ .5 %
Partial Void k	.997083	.997104	.997132	.997144	.997146	.997058	.997075	.997104	.997119	.997140
Δk	.001992	.002009	.002036	.002041	.002051	.002027	.002043	.002066	.002076	.002095
% error	- 2.1 %	-1.2 %	+.10	+.3%	+ .8 %	-.3%	+ .4 %	+ 1.6 %	+ 2.1 %	+ 3.0 %
Slumping k	.998649	.998682	.998729	.998756	.998780	.998649	.998683	.998730	.998755	.998778
Δk	.003558	.003587	.003633	.003653	.003667	.003618	.003651	.003692	.003712	.003733
% error	-2.9%	-2.1%	-.8%	-.3%	+ .1 %	-1.2%	-.3%	+ .8 %	+ 1.3 %	+ 1.9 %

APPENDIX E

β AND χ EFFECTS

Originally RAUMZEIT did not allow different delayed neutron and prompt neutron spectra, even though they are substantically different from one another as seen in Figure E.1. However this difference was accounted for by using a βeff instead of the actual (i.e. isotope averaged) β's.

When RAUMZEIT was modified, the provision for allowing different prompt and delayed χ's was added. The calculations were repeated using actual β's corresponding to the βeff (o.oo32) used previously and the different prompt and delayed spectra, for the two transients and the various group collapsing and synthesis schemes were tested. The conclusions reached before hold up, and the agreement between the various collapsing models, synthesis, and the exact results are maintained. The results for the fuel slumping transient are shown in Table E.1 for the original adjoint and final flux 8g model, the 26 group model and synthesis.

This means that the collapsing and synthesis scheme in addition to accurately predicting reactivity effects also account properly for βeff and the generation time. In fact, the agreement between the two calculational models indicate that using βeff and the same χ's is equivalent to using the actual β's and different χ's. The differences are only of the order of 1-2 %.

TABLE E.1

β AND χ EFFECT

Time millisec.	26 gr.		8 gr. $\emptyset_{MF}\ \emptyset_o^x$		6 gr. $\emptyset_{MF}\ \emptyset_o^x$		Synthesis	
	A	B	A	B	A	B	A	B
0	1.0		1.0	1.0	1.0		1.0	1.0
2	1.845	1.843	1.839	1.838	1.846	1.845	1.839	1.840
4	1.718	1.715	1.720	1.718	1.727	1.725	1.712	1.711
6	1.624	1.619	1.631	1.627	1.637	1.633	1.616	1.610
8	1.665	1.657		1.668		1.676		1.645
10	1.831	1.820	1.842	1.834	1.851	1.843	1.815	1.807
12	2.173	2.158		2.178		2.189		2.164
14	2.877	2.852	2.901	2.882	2.918	2.900	2.856	2.839
16	4.62	4.57	4.66	4.623		4.658	4.608	4.582
18	11.76	11.59	11.97	11.78	12.04	11.93	11.86	11.82
20	134.4	131.0	137.4	134.7	140.0	137.4	135.1	135.4

A: $\chi_p = \chi_d$ but β_{eff} is used

B: $\chi_p \neq \chi_d$ and actual β is used

FIGURE E.1

Prompt and Averaged Delayed Neutron Spectra