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Investigations of the Two-Detector Covariance Method for the Measurement of Coupled Reactor Kinetics Parameters

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Abstract

For the measurement of point kinetics parameters of zero power reactors many different methods have been widely used and described in the literature. These include Rossi-α, probability distribution, variance-over-mean, frequency analysis, and polarity correlation of neutron detector signals. Up to now kinetics parameters of a coupled two-point reactor model have been measured only by two noise analysis techniques: Rossi-α (correlation functions) and frequency analysis (power spectral density, coherence functions) using two detectors to obtain space-dependent neutron signals.

This paper describes for the first time investigations of two-detector covariance measurements and their application to the determination of coupled kinetics parameters. This technique is compared with the Rossi-α method by theoretical considerations and measurements using identical detector signals in both analysis techniques. Results were also compared to those measured previously at the same reactor by frequency analysis.

Theoretical formulas for the Rossi-α experiment and variance and covariance measurements in a symmetrical two-point reactor model were derived. The material properties and neutron lifetimes for each reactor zone were assumed equivalent. The transport time of neutrons from one zone to the other was neglected. The parameters of the model include the decay constant of prompt neutrons and the coupling reactivity of the two core zones. These parameters were determined by least-squares fitting the theoretical curves to the experimental data.

The measurements have been performed at the Argonaut Reactor Karlsruhe (ARK) with a symmetrical two-slab core loading for different subcritical levels. Variances and covariances of neutron counts from detectors placed in both core regions were measured in two different ways.

In the first method the two-dimensional probability distribution of the number of counts from the detectors was measured using a small digital computer. The computer was used to control the gating times of two specially designed counting registers and to calculate the first and second moments. From that the covariances were calculated in real time and displayed on a CRT as a function of the counting time interval.

In the second method the variance and the covariance of the detector pulses were directly measured without using the probability distributions. For this purpose a new analyser based on a special calculational algorithm was developed. This analyser can be built easily with low hardware expense.

The results obtained by the Rossi-α and variance method agreed quite well. In contrast to point reactor parameter determination, where Rossi-α and variance techniques are equivalent, the variance and covariance method appear superior to the Rossi-α method in applications to coupled reactors. Kinetics parameters of a two-node reactor model can be determined more accurately by the variance technique.

The effect of dead-time losses in the signal channels or in the analyser on the measured variances was also studied. The variance technique was found to be more sensitive to dead-time than the Rossi-α technique. Therefore variance measurements at fast reactors yield incorrect results except special fast electronics is used in the pulse channels.

Bestimmung kinetischer Parameter eines gekoppelten Reaktors durch Messung der Kovarianz zweier Detektorsignale

Zusammenfassung

Während am Ein-Zonen-Reaktor mehrere Rauschmethoden zur Bestimmung der punktkinetischen Parameter eingesetzt wurden, ist an gekoppelten Reaktoren bisher nur die Rossi-α-Technik und die Frequenzanalyse zur Bestimmung kinetischer Parameter nach einem Zweipunkt-Modell verwendet worden. In diesem Bericht wird die Messung der Varianzen und der Kovarianz zweier Detektorsignale und ihre Verwendung zur Bestimmung der Parameter gekoppelter Reaktoren behandelt. Die Methode wird theoretisch und experimentell (bei Verwendung identischer Detektorsignale für beide Methoden) mit der Rossi-α-Technik und außerdem mit früheren Ergebnissen aus einer Frequenzanalyse am selben Reaktor verglichen.

Zur Ableitung der theoretischen Formeln für das Rossi-α-Experiment und Varianz- bzw. Kovarianzmessungen in einem symmetrischen Zwei-Punkt-Reaktor wird angenommen, daß die Transportzeit der Neutronen von einer Zone in die andere vernachlässigt werden kann. Durch Anpassung der theoretischen Kurven an die gemessenen Daten können die Zerfallskonstante des prompten Neutronenflusses und die Kopplungskonstante gewonnen werden.

An der symmetrischen Zwei-Zonen-Ladung des Argonaut Reaktor Karlsruhe (ARK) wurde bei verschiedenen unterkritischen Zuständen die Kovarianz auf zwei Arten bestimmt. Einmal wurde mit Hilfe eines Kleinrechners die zweidimensionale Wahrscheinlichkeitsverteilung der in einem Zeitintervall gezählten Neutronenimpulse gemessen, aus der dann die Varianzen und Kovarianzen berechnet wurden.
Im zweiten Fall wurden Varianz und Kovarianz direkt, also nicht über die Wahrscheinlichkeitsverteilung, gemessen. Dazu wurde ein neuer Analysator entwickelt, der mit Hilfe eines speziellen Algorithmus die Varianz bzw. Kovarianz auf einfachere Weise und mit geringerem hardware-Aufwand unmittelbar aus den Signalen ableitet.

Die Ergebnisse der Rossi-α und Varianzmessungen stimmen gut überein. Während beim Punktreaktor die Bestimmung der Reaktorparameter mit den beiden Methoden gleichwertig ist, können mit der Varianz-Kovarianz-Methode die Parameter des gekoppelten Reaktors genauer bestimmt werden als mit der Rossi-α-Technik.

Die Untersuchung von Totzeiten in den Signalkanälen oder im Analysator ergab, daß die Varianz-Methode empfindlicher auf Totzeitverluste reagiert als die Rossi-α-Methode. Vor allem bei schnellen Reaktoren wird daher die Varianz-Technik falsche Ergebnisse liefern, es sei denn, daß besonders schnelle elektronische Bauteile benutzt werden.

1. Introduction

Because of the assumed separability of the space and time variables of the neutron flux in reactors of relatively small size, the neutron flux fluctuations caused by the stochastic nature of the fission process are essentially the same at all positions in such zero power reactors. They differ only in their amplitudes which are proportional to the space-dependent steady-state mean value of the neutron flux. Time delays and phase shifts between flux variations at any two space points within the reactor are negligible. The neutron flux and its fluctuations at different positions are completely coupled.

A large number of measurements using different techniques of noise analysis /1,2,3,4,5,6,7/ have confirmed the predictions of the point reactor theory of neutron noise /8,7/. The theoretical formulas are not explicitely space-dependent. The spatial dependence of the magnitude of neutron flux fluctuations is taken into account implicitely by the detector sensitivity defined as the ratio of the average counting rate of the detector to the mean total fission rate in the reactor. Therefore there is basically no difference between one- and two-detector experiments with respect to the reactor-kinetics parameters measured in small single-zone zero power reactors. The only advantage of two-detector cross-correlation measurements in such reactors is that a lower detector efficiency and band-limited signal channels can be used together /9,10,11/.

However in large reactors, due to finite neutron velocity and migration length, one would expect time delays and a certain degree of decoupling between flux fluctuations at different positions. This would lead to local effects and space-dependent results in noise analysis experiments, as clearly demonstrated in /12/. Discrepancies were also found between point reactor theory and noise measurements in compound reactor geometries and reflected cores /13,14,15/.

Space-dependent theories predicted significant deviations from point kinetics /16,17/.

In general, the energy dependence of the neutron flux also has to be taken into account in the theoretical treatment of reactor noise. The general problem of space and energy dependent reactor noise has been attacked in several different approaches, including modal and nodal expansions of the neutron field /18,19,20/. The models developed in most theories are of a complicated nature and not directly amenable to application in practical experiments.

A reactor consisting of two loosely coupled separate core regions can be described by a two-node or two-point reactor model originally introduced by Baldwin /21/. From this model simple expressions can be derived for the quantities measured with the different techniques of noise analysis in zero power reactors. With these formulas coupled kinetics parameters can be obtained directly from experimental results by least-squares fitting, avoiding numerical calculations of space- and time-dependent neutron fields.

This has been shown by Albrecht and Seifritz for the two-node coherence function /22,23/ and by Dragt for correlation functions of ionisation chamber signals from a two-slab reactor /24/. Later Viehl and Seifritz /25,26/ published theoretical and experimental investigations on power spectral densities of neutron signals from an unsymmetric coupled reactor. Two-node and four-node models have been used also by Penland and Hanauer /15/ and Penland /20/ to describe the effect of reflectors on power spectral densities measured at the Oak Ridge Research Reactor and Pool Critical Assembly, respectively.

In this work a simple one-group two-node theory for Rossi-α, variance and covariance measurements with two detectors in a reactor composed of two weakly coupled core regions is developed. Theoretical formulas were verified by experimental results obtained at the two-slab Argonaut reactor at Karlsruhe.

Two prompt neutron decay modes, coupling and total reactivity were determined for this reactor.

A study on coupling effects in a thermal two slab reactor was published by Dalfes and Türkcan /27/ also. They used a more general nodal theory for interpretation of correlation functions only and applied it successfully to experiments at the Argonaut-type Low Flux Reactor (LFR) at Petten with two-slab and annular core configurations. Reactor parameters were determined from a correlation matrix equation instead of a least squares fitting procedure.

Variances and Covariances of neutron counts from two detectors have been used for the first time by Harris et al./28,29/ to study spatial coupling effects of neutron flux. They measured and calculated a modified coefficient of correlation derived from covariances and variances of counts from detectors in a long very weakly coupled seed-blanket reactor and a well-coupled cylindrical reactor. It was found that the correlation coefficient can be used to check dynamic reactor models.

However, direct information about kinetic parameters such as coupling coefficient, time constants and reactivity could not be determined in this case because the theoretical model did not use them explicitely. These parameters have been deduced directly from the variance and covariance measurements and are reported in this paper for the first time.

Investigations of coupled kinetics using noise analysis techniques at coupled fast /30/ and fast-thermal systems /31,32,33/ have been published by Mihalczo, Murley, Edelmann et al. and Dalfes et al., respectively. Moderator-reflected fast assemblies were studied by Borgwaldt et al. using a modal synthesis method /34/.

2. Two-node theory for Rossi-α, variance and covariance measurements for symmetrical systems

In the common theory of noise analysis experiments /8/ a general formula has been derived for the results measured with different techniques, characterized by their specific network response functions. Its specialized form for the point reactor model is

$$R_{ij} = \overline{r_i r_j} = \bar{r}_i \delta(T) \delta_{ij} + \bar{r}_i \bar{r}_j$$

$$+ \frac{F W_i W_j k^2 D}{\ell^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A_1(\omega) A_2(\omega) \left| H(\omega) \right|^2 \qquad (1)$$

$$i,j = 1,2$$

where

r_i	actual output of signal channel i
$\bar{r}_i$	mean value of r_i
δ_{ij}	Kronecker's symbol
W_i	detector efficiency in counts per fission
F	total fission rate of one node
$a_n(t)$	network response functions, n=1,2
$A_n(\omega)$	Fourier transform of $a_n(t)$
k	effective neutron multiplication factor
$D = \frac{\overline{\nu(\nu-1)}}{\bar{\nu}^2} \approx 0.8$	a nuclear material parameter /35/

ℓ prompt neutron lifetime

$H(\omega)$ source transfer function of point reactor .

From equ.(1) only the correlation functions measured in Rossi-α experiments with one and two detectors will be derived directly. Variance and covariance functions can be more easily obtained in a different way. Therefore equ.(1) is given in a form which is true only for Rossi-α experiments: the first term on the right-hande side does not appear in the general formula and has been added only to account for trivial coincidences produced in an auto correlation experiment at zero delay.

It will be shown that equation (1) also holds for noise measurements in a symmetrical two-node reactor system if the point reactor transfer function is replaced by an equivalent function of the two-point reactor. This pseudo transfer function is derived from the two-node model used by Albrecht and Seifritz /22/. It is described by the block diagram in figure 1. Its major properties are:

(1) Each node has a distinct input $S_i(\omega)$ which is a neutron source in our case and a distinct output $N_i(\omega)$ which is the neutron population, i=1,2.

(2) A perturbation of one node by the other only arises due to the output of the perturbing node affecting the input of the perturbed node.

(3) The nodes have identical transfer functions $G(\omega)$ (point reactor source transfer function)

(4) The coupling between nodes is identical, i.e. coupling in both directions is described by the same coupling function $K(\omega)$.

(5) The sources to the two nodes are uncorrelated and statistically the same. That is, they have the same mean value, auto power spectral density and their cross

power spectral density is zero ($S_i S_i^* = S_j S_j^*$, $S_i S_j^* = S_j S_i^* = 0$) .

Then the auto and cross power spectral densities of the outputs are

$$N_i N_i^* = |N_i|^2 = |G|^2 \frac{|S_i|^2 (1+|K|^2)}{|1-K^2|^2} \tag{2}$$

$$N_i N_j^* = N_j N_i^* = |G|^2 \frac{2\,Re(K)\,|S_i|^2}{|1-K^2|^2} \tag{3}$$

$$i,j = 1,2 \;;\; i \neq j$$

wherein

$$N_i N_j^* = \lim_{T\to\infty} \frac{N_{iT}(\omega)\, N_{jT}(\omega)}{T} \quad .$$

It has been assumed that the Fourier transforms of the input and output signals also exist in the limiting case of a infinite sample length T, otherwise $N_{iT}\, N_{jT}^*$ has to be replaced by its expectation or mean value /36/.

By dividing the output power spectral densities by the input power spectral density one obtaines two real functions which can be taken as the square modulus of two transfer functions:

$$|H_a(\omega)|^2 = |G(\omega)|^2 \frac{1+|K(\omega)|^2}{|1-K^2(\omega)|^2} \tag{4}$$

$$|H_c(\omega)|^2 = |G(\omega)|^2 \frac{2\,Re(K(\omega))}{|1-K^2(\omega)|^2} \tag{5}$$

With these transfer functions the results of neutronic noise experiments for the two-node reactor can be calculated from equ.(1) using the proper network response functions. However, different transfer functions have to be used for auto and cross-correlation experiments in order to account for the spatial dependence of two-node reactor results.

For this purpose, the coupling function $K(\omega)$ has to be calculated first. It is derived from the two-node kinetics equations /21/. Neglecting delayed neutrons and the delay in neutron coupling, one finds /22/ for a symmetrical system

$$K(\omega) = \frac{\varepsilon}{\Lambda} \cdot \frac{1}{\alpha + i\omega} \tag{6}$$

with

ε coupling reactivity between nodes

Λ prompt neutron generation time

$\alpha = \frac{\beta - \rho}{\Lambda}$ prompt neutron decay constant of a single node without coupling

ρ reactivity of one node

β effective delayed neutron fraction

The transfer function $G(\omega)$ of each node derived from the point kinetics equations without delayed neutrons is

$$G(\omega) = \frac{1}{\alpha + i\omega} \tag{7}$$

The network response functions for the Rossi-α experiment and their Fourier transforms are

$$a_1(t) = \delta(t)$$

$$a_2(t) = \delta(t - T)$$

$$A_1(\omega) = 1 \tag{8}$$

$$A_2(\omega) = e^{-i\omega T} \tag{9}$$

Substituting equs.(8),(9) and (4) with (6) and (7) into equ.(1), the auto correlation function of the counting rate of a neutron counter placed in one node of the reactor results:

$$R_{ii}(T) = \bar{r}_i \cdot \delta(T) + \bar{r}_i^{\,2} + \frac{F W_i^2 k^2 D}{4 \ell^2} \left(\frac{1}{\alpha_1} e^{-\alpha_1 T} + \frac{1}{\alpha_2} e^{-\alpha_2 T} \right) \tag{10}$$

with $$\overline{r}_i = W_i F \quad , \; i=1,2 \quad .$$

In a similar way the crosscorrelation function for two detectors, one in each node, is calculated from equs.(1), (5), (8) and (9):

$$R_{ij}(\tau) = \overline{r}_i \cdot \overline{r}_j + \frac{FW_iW_jk^2D}{4\ell^2}\left(\frac{1}{\alpha_1}e^{-\alpha_1\tau} - \frac{1}{\alpha_2}e^{-\alpha_2\tau}\right) \tag{11}$$

The decay constans α_1 and α_2 depend on the decay constant α of the two nodes without coupling and the coupling reactivity ε divided by the prompt neutron generation time

$$\alpha_1 = \alpha - \frac{\varepsilon}{\Lambda} = \frac{\beta-\rho-\varepsilon}{\Lambda} \tag{12}$$

$$\alpha_2 = \alpha + \frac{\varepsilon}{\Lambda} = \frac{\beta-\rho+\varepsilon}{\Lambda} \quad . \tag{13}$$

The correlation functions consist of a slower, α_1, and a faster, transient decay mode α_2. If α_2 is much larger than α_1, i.e. in the case of strong coupling, the correlation functions will be influenced by the transient only for short delay times. On the other hand, with very small coupling the difference between the two decay modes will become negligible.

Equation (11) also holds for a crosscorrelation experiment with detectors in the same region when the two exponential terms within the brackets are added as in equ.(10). Similar results were also found by Dragt /24/ in a different, somewhat more complicated derivation.

The results obtained for the two-node reactor differ from those of the one-point model by the existence of two exponential terms instead of only one in the auto and crosscorrelation functions. These terms are the same as in the point model except for the different decay constants and coefficients which depend on the degree of coupling as can be seen from equs.(12), (13). Increasing the coupling increases the difference between auto and crosscorrelation results as well as the difference between both of them and point reactor results. When the coupling between the nodes becomes negligible, i.e. ε=0, equ.(10) reduces to the corresponding equation of the point reactor. However in the crosscorrelation function (11) the correlated part vanishes and only the product of signal mean values remains.

Formulas for the variance and covariance of detector counts can be derived from equ.(1) in the same way as for auto and crosscorrelation functions when the network response functions and their Fourier transforms are taken to be

$$a_1(t)=a_2(t)=\begin{cases}\frac{1}{T} & 0\le t\le T\\ 0 & \text{otherwise}\end{cases}$$

$$A_1(\omega)=A_2(\omega)=\frac{1-e^{-i\omega T}}{i\omega T}\quad .$$

Then
$$r_i(t)=\frac{n_i(t-T,t)}{T}\tag{14}$$

$$\overline{r_i}=W_i F=\frac{\overline{n_i}}{T}\tag{15}$$

with $n_i(t-T,t)$ the number of counts from detector i in the time interval $(t-T,t)$ and $\overline{n_i}$ its mean value.

However, if the correlation functions $R_{ij}(T)$ are known, variance and covariance formulas can be easily obtained by integrating these functions /8/:

$$\overline{n_i n_j}(T)=\int_0^T dt_1\int_0^T dt_2\, R_{ij}(t_2-t_1)\tag{16}$$

From this measured quantities can be calculated according to equ.(1) as follows:

$$\sigma_{ij}(T)\overset{df}{=}\frac{\overline{n_i n_j}(T)-\overline{n_i}\cdot\overline{n_j}-\overline{n_i}\delta_{ij}}{\sqrt{\overline{n_i}\cdot\overline{n_j}}}=\frac{1}{\sqrt{\overline{n_i}\cdot\overline{n_j}}}\int_0^T dt_1\int_0^T dt_2\, C_{ij}(t_2-t_1)\tag{17}$$

where C_{ij} denotes the time dependent (correlated) part of the correlation functions arising from the integral in (1). Actually, it is not necessary to perform this calculation because $C_{ij}(T)$ from equs. (10), (11) are the sum and difference of two correlation functions similar to those valid for the point-reactor model the integral (16) of which is known to be proportional to

$$\frac{2}{\alpha^2}\left(1-\frac{1-e^{-\alpha T}}{\alpha T}\right)$$

Therefore it follows immediately that the variance and co-variance distributions are given by

$$\sigma_{ii}(T)=\frac{W_i k^2 D}{2\ell^2}\left[\frac{1}{\alpha_1^2}\left(1-\frac{1-e^{-\alpha_1 T}}{\alpha_1 T}\right)+\frac{1}{\alpha_2^2}\left(1-\frac{1-e^{-\alpha_2 T}}{\alpha_2 T}\right)\right] \tag{18}$$

$$\sigma_{ij}(T)=\frac{\sqrt{W_i W_j}\,k^2 D}{2\ell^2}\left[\frac{1}{\alpha_1^2}\left(1-\frac{1-e^{-\alpha_1 T}}{\alpha_1 T}\right)-\frac{1}{\alpha_2^2}\left(1-\frac{1-e^{-\alpha_2 T}}{\alpha_2 T}\right)\right] \tag{19}$$

The asymptotic values for large counting intervalls are

$$\sigma_{ii}(\infty)=Q\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right) \tag{20}$$

$$\sigma_{ij}(\infty)=Q\left(\frac{1}{\alpha_1^2}-\frac{1}{\alpha_2^2}\right) \tag{21}$$

with

$$Q=\frac{\sqrt{W_i W_j}\,k^2 D}{2\ell^2}\ .$$

The behaviour of equs. (17) and (18) depends on both decay constants within the whole range of counting intervalls. It is expected therefore that variance and covariance measurements can be used more successfully than Rossi-α measurements with one and two detectors to determine α_1, α_2, and the coupling reactivity.

3. Experimental set-up and measurement techniques

The measurements were performed at the Argonaut Reactor Karlsruhe (ARK) with a two-slab core configuration at different levels of subcriticality. The main features of this reactor are well-known. The special two-slab core which was used for the experiments reported here is described in /23,37,38/.

Two BF_3 counters connected in parallel were used as neutron detectors in each slab. They were placed at symmetrical positions at the outer periphery of the active core zones to be sensitive only to neutrons from the adjacent core zone. Conventional electronics equipment was used to obtain normalized fast signal pulses suitable for tape recording and analysis. The dead-time of the two signal channels was 1 µs. Noise analysis of the signals was mainly performed off-line using a tape recorder although the newly developed analysers for Rossi-α and variance measurements were especially designed for real-time analysis.

The major interest of this work was concerned with variance and covariance measurements and their potential to determine coupled kinetics parameters. Rossi-α measurements were performed on the same records for comparison of the two techniques and their results with respect to their sensitivity to coupling effects in neutron noise. In this context special attention was paid to the influence of dead-time losses on the results in both cases.

Rossi-α measurements were performed with a new type of time analyser developed to overcome the problems with conventional analysers arising from dead-time losses in their trigger channel. Resulting discrepancies between theory and experiment have been studied in theoretical and experimental investigations /39,40,41,42,43,44/. The new analyseris described in /43,44,32/ where modified Rossi-α techniques and background problems are also discussed. The analyser - in principle a very flexible and versatile digital version of the delayed coincidence analyser used in the first Rossi-α experiments by Orndoff /45/ - measures the true correlation function of pulse signals. This is accomplished by using a shift register as a delay line providing time cycle superposition capability.

For the measurment of variance and covariance of detector counts two different techniques were applied. In the first technique a small digital computer coupled to a gated twin counting register was used to measure simultaneously the one- and two-dimensional probability distributions of counts arriving during time intervals of variable length T in one or two pulse channels. The computer automatically controlled the gating times of the counting registers between 1 µs and 1s as specified by software input. Between successive counting intervals a break of 10 µs was needed for data transfer from the counting registers to the computer memory and reset. After counting a specified number of counting intervals of same length T, the first and second moments of the probability distribution were calculated and variances and covariances were derived according to equ.(17). Then the measurement continued with the next specified counting time interval. Probability distributions and variance curves were displayed on a CRT during measurements. In off-line measurements of tape recorded data the same record was used for analyis with different counting intervals. Start and rewind of the tape recorder was also controlled by the computer. Thus, the measurements could be performed completely automatically.

In the second method variance or covariance of detector pulses was directly measured without using the probability distributions. For this purpose a new analyser based on a special calculational algorithm was developed /38/. This method has the advantage that a relatively small amount of hardware is necessary. Fig.2 shows the block diagram of the apparatus.

Within the time interval T there are two sequences of n_1, n_2 pulses in channel 1 resp. 2. The variance and covariance is defined by

$$\sigma_{k\ell} = \frac{\overline{n_k n_\ell} - \overline{n_k} \cdot \overline{n_\ell}}{\sqrt{\overline{n_k} \cdot \overline{n_\ell}}}$$

$k \neq \ell$ covariance

$k = \ell$ variance

with

$$\overline{n_k} = \frac{\sum_{i=1}^{N} n_{ki}}{N}$$

$$\overline{n_k \cdot n_\ell} = \frac{\sum_{i=1}^{N} n_{ki} n_{\ell i}}{N}$$

N = total number of time intervals T .

Accordingly the analyser must multiply n_{1i} and n_{2i} for each time interval T and add these products. The mean number of pulses $\overline{n}_k$ will be obtained by counting simultaneously the pulses of each channel during the total measuring time. Fig.3 shows how the multiplication process is performed.

If it is assumed that no pulses appear simultaneously in the two channels the pulse sequences n_1 and n_2 can be divided into sub-sequences n^*_{1j}, n^*_{2k}. During a subsequence there is no pulse in the other channel. Each sub-sequence n^*_{1j}, n^*_{2k} is multiplied by the total number of pulses which have arrived from the beginning of the time interval T_i in the other channel. These partial results are summed and the desired result $n_1 \cdot n_2$ is obtained.

The multiplication is realized by the apparatus shown in Fig.2. The two sequencies are counted in two binary registers B_1 and B_2. The addition of the actual content of one register to the content of the corresponding normal pulse counters $Z_1 \ldots Z_8$ is initiated by each pulse arriving in the other channel.

This corresponds to the multiplication process explained above: Each sub-sequence is multiplied by the sum of pulses which have arrived since the beginning of the time interval T in the other channel. The sum of the contents of the counters Z_i multiplied by the value 2^{i-1} of the corresponding binary register output is the required result $n_1 \cdot n_2$. Before the beginning of the next time interval, the binary registers are reset (but not the counters Z_i). At the end of N time intervals T the sum of products $\sum_{j=1}^{N} n_{1j} n_{2j}$ is stored in the counters Z_i.

The afore-mentioned condition that no pulses appear simultaneously in the two channels is non-realistic. Therefore an electronic separation of pulses is done by a special non-coincidence logic /38/. If two pulses are separated less than the maximum switching time of the registers one pulse is delayed by this device.

The new analyser is suitable for two-or one-detector experiments. For variance measurements input 1 and 2 of the analyser are connected.

4. Results and Conclusions

Tape records of detector signals at 7 different subcritical states of ARK between -5 ¢ and -4 $ were analysed (each 600 sec long). The interesting parameters α_1, α_2 and Q were determined by least-squares fitting the theoretical curves (10,11) and (18,19) to the results from the Rossi-α and variance-measurements, respectively. Fig.4 shows two typical results of variance and covariance measurements. In tab.1 the fitted parameters are listed for different subcritical levels of ARK. For comparison the same tape records were analysed by the Rossi-α-method. Fig.5 and tab.2 show the correlation functions and the results for the significant parameters.

From measurements and calculations the following statements can be made:

a) In contrast to point reactor parameter measurements, application of the variance method to determine kinetic parameters in coupled reactors is superior to the Rossi-α-method. The coupling between neutron flux fluctuations in both core zones influences all measuring points of the variance, but mainly the asymptotic values for large counting time intervals. Therefore fitting of 3 parameters to the measured variances will give better results than the Rossi-α-method. There coupling influences the correlation function at very short delay-times only. The fast transient α_2 is resolved badly and so the additional more difficult fitting of 4 parameters will be burdened with considerable errors.

b) There is no difference between the results for the kinetic parameters in the covariance and variance measurements.

c) The Rossi-α-method with one detector (autocorrelation) gives better results in two-slab-cores than two-detector (crosscorrelation) measurements. This can be explained by the fact, that the transport time of neutrons between the two core-zones was neglected in the theory. This time is about 0.28 msec /23/ and is therefore in the range of the channel width of the time analyser. Hence the value of the fast transient α_2 which influences only the first channels can be distorted. This effect is not observed in the variance method, because the fast transient influences essentially the asymptotic part of the curves. There the corresponding time intervals are much larger than the transport time of neutrons.

d) The fitted value of the near critical parameter α_2 is too small. Therefore the computed coupling reactivity ε is also too small. We can understand this error if we look at the absolute values of α_1 and α_2. Near critical, $\alpha_2 >> \alpha_1$ and thus the predominance of the first terms in equ.(10,11) and (18,19) is influenced only weakly by the second terms, which are weighted additionally by $\frac{\alpha 1}{\alpha 2}$ and $(\frac{\alpha 1}{\alpha 2})^2$, respectively. Therefore the fitting of parameters at near critical is expected to be less accurate.

Dead-time losses

The two methods are rather sensitive to dead-time losses in the counting-tubes or the connected electronic networks. In /43/ the influence of dead-time on the amplitude of the correlation function measured by the Rossi-α-method has been studied. The influence of dead-time losses on variance-measurements was now investigated by analysis of neutron detector signals with artificially introduced variable dead-time after each passed pulse. Fig.6 shows the results of variance analysis for various dead-times. The mean counting rate was about 520 counts/sec, corresponding to a mean time between pulses t_p of about 1.9 msec. A dead-time of $\tau=0.01 \cdot t_p$ has a considerable influence on the measured variance of a pulse sequence. The dead-time losses can be described satisfactory up to values of τ = 100 µsec = $0.05 \cdot t_p$ by a formula given in /46/. With the aid of this formula it is possible to estimate the correct kinetic parameters at one-detector point reactor experiments if dead-time τ is known.

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Tabelle 1

	COVARIANCE			AUTOVARIANCE eastern core zone		
ρ [$]	α_1 [sec^{-1}]	α_2 [sec^{-1}]	$\frac{\varepsilon}{\Lambda}$ [sec^{-1}]	α_1 [sec^{-1}]	α_2 [sec^{-1}]	$\frac{\varepsilon}{\Lambda}$ [sec^{-1}]
-3.9	187.3	393.1	102.9	180.2	385.1	102.4
-3.7	175.9	387.4	105.7	167.3	385.7	109.2
-2.5	133.5	359.2	112.8	126.4	359.7	116.6
-1.7	103.8	338.0	117.1	98.6	333.9	117.6
-0.8	69.8	294.3	112.2	68.8	298.7	114.9
-0.28	47.9	227.5	89.8	47.5	227.7	90.1
-0.05	39.5	179.1	69.8	39.6	173.7	67.0

Tabelle 2

	CROSSCORRELATION			AUTOCORRELATION eastern core zone		
ρ [%]	α_1 [sec^{-1}]	α_2 [sec^{-1}]	$\frac{\varepsilon}{\Lambda}$ [sec^{-1}]	α_1 [sec^{-1}]	α_2 [sec^{-1}]	$\frac{\varepsilon}{\Lambda}$ [sec^{-1}]
-3.9	183.1	676.6	246.7	179.9	391.4	105.7
-3.7	178.1	480.2	151.0	169.9	384.6	107.3
-2.5	148.8	451.3	151.2	124.9	359.2	117.1
-1.7	103.9	506.1	201.1	96.0	261.6	82.8
-0.8	82.7	308.3	112.8	67.9	297.9	115.0
-0.28	54.6	355.7	150.5	47.0	226.0	89.5
-0.05	45.9	394.1	174.1	37.0	151.8	57.4

α_2 and ε from crosscorrelation measurements not reliable (cf. 15 f.)

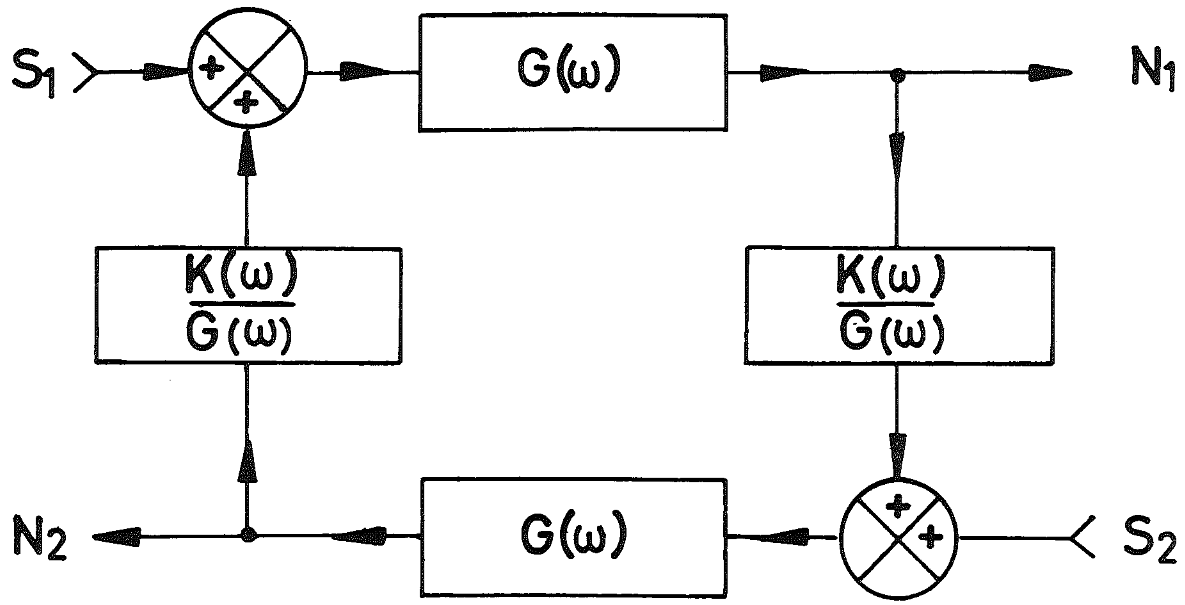
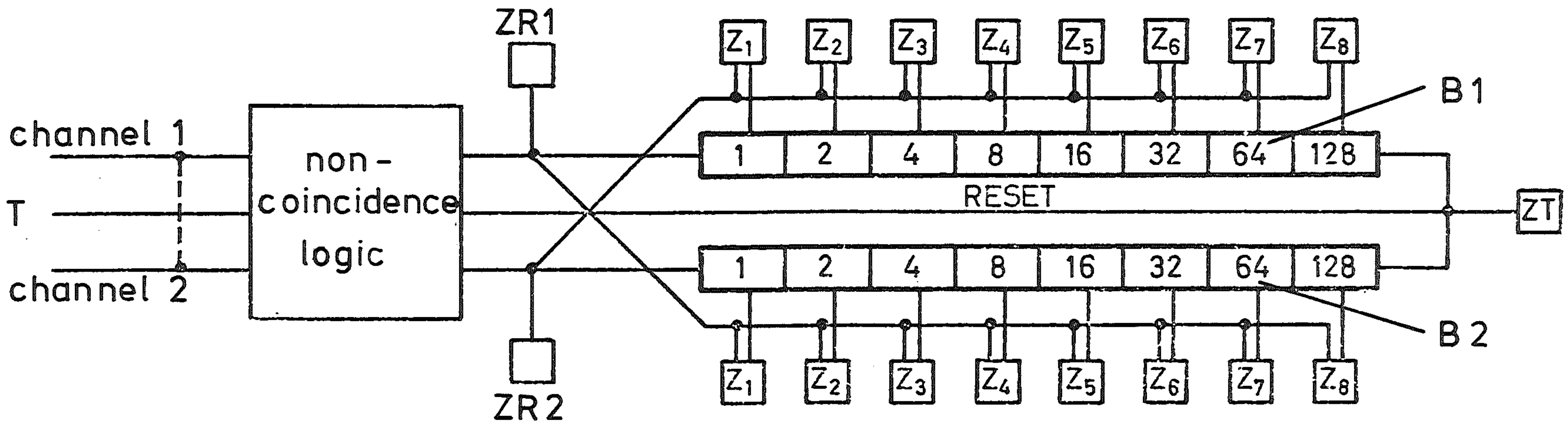


Fig.1 Block diagram of symmetrical two node system [22]



ZR 1, ZR 2	pulse counter to register the number of pulses in each channel
Z_i , i=1,...,8	pulse counter
B 1, B 2	binary pulse counter
ZT	pulse counter to register the number of time intervals

Fig. 2 Block diagram of variance analyzer

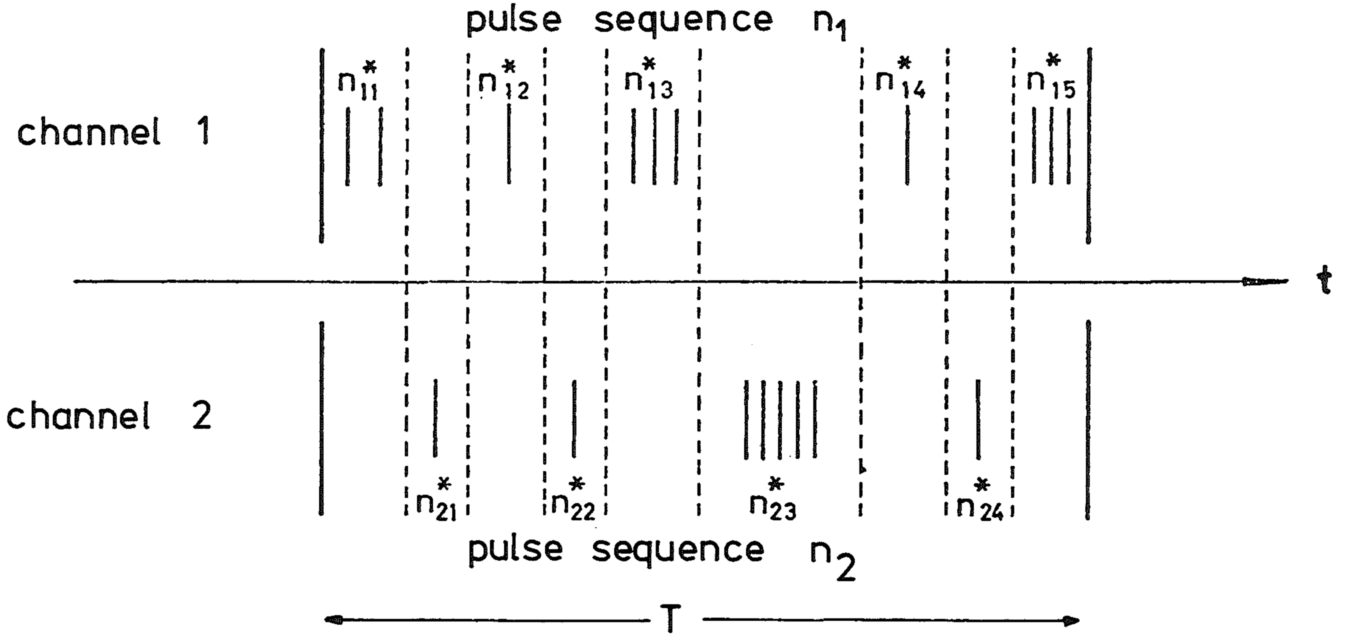
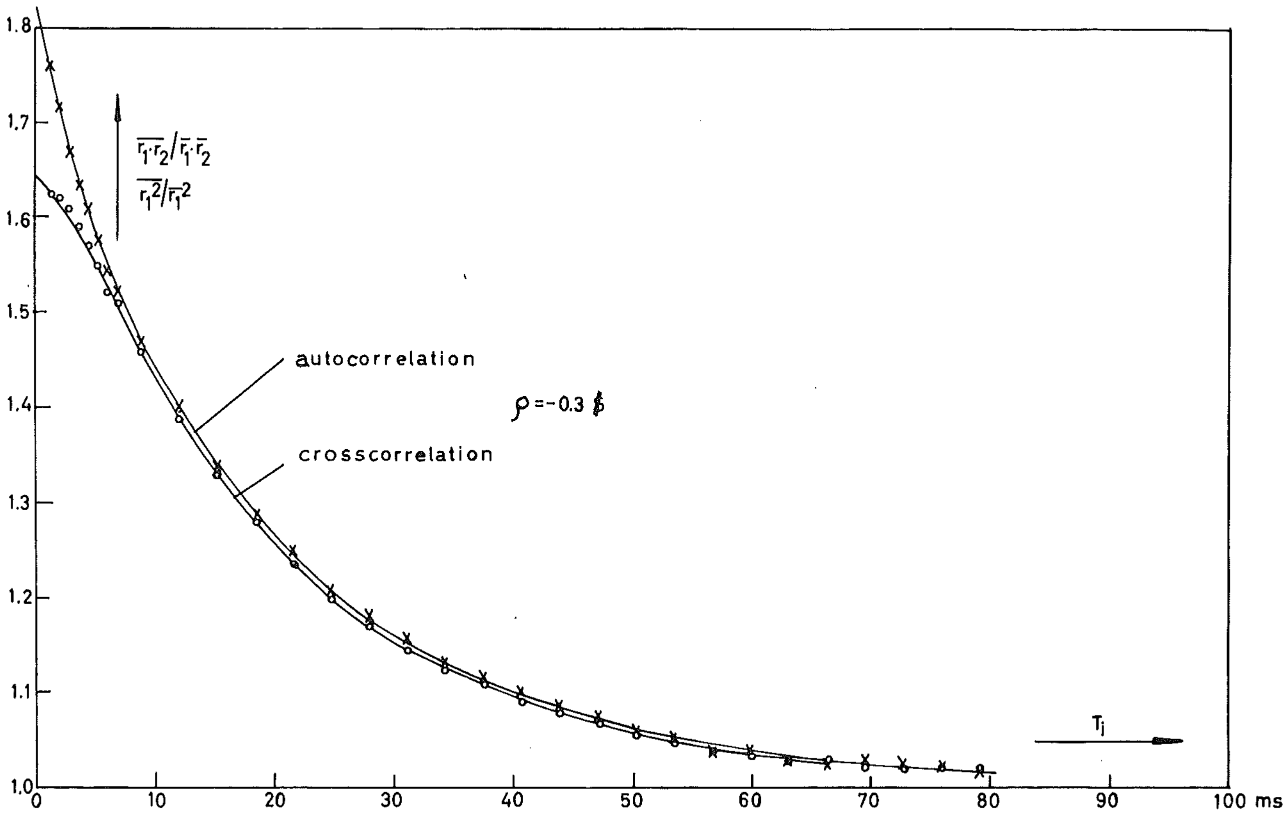


Fig. 3 Multiplication procedure



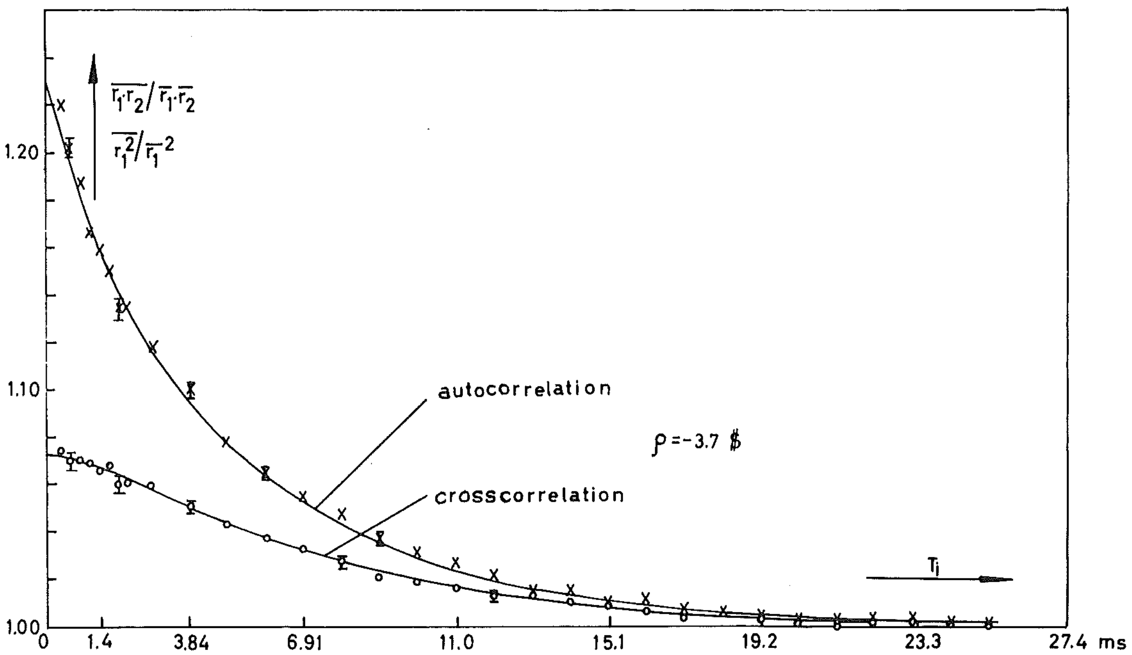


Fig. 4 Rossi-α-measurements at a two slab core loading of ARK

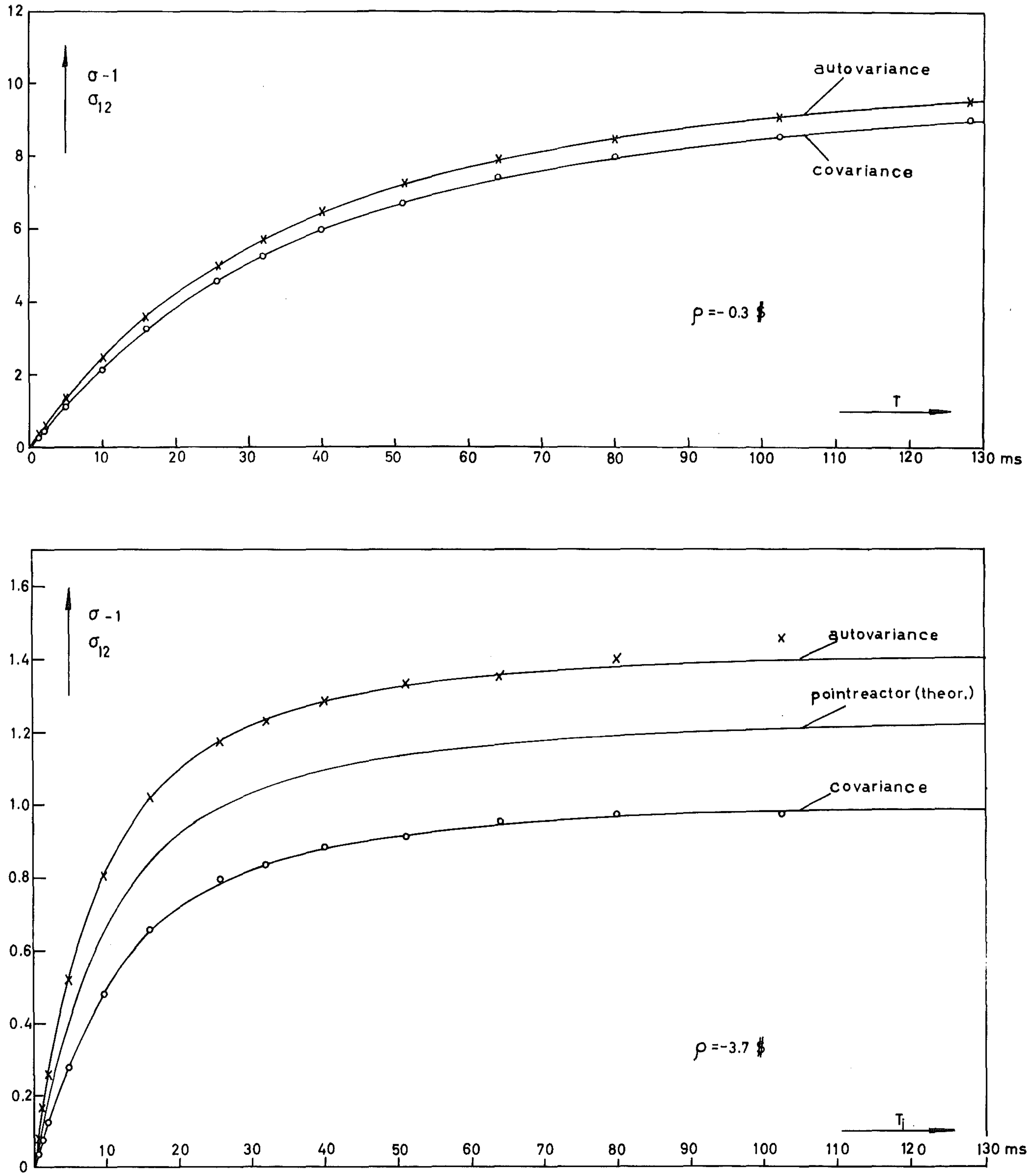


Fig.5 Variance measurements at a two slab core loading of ARK

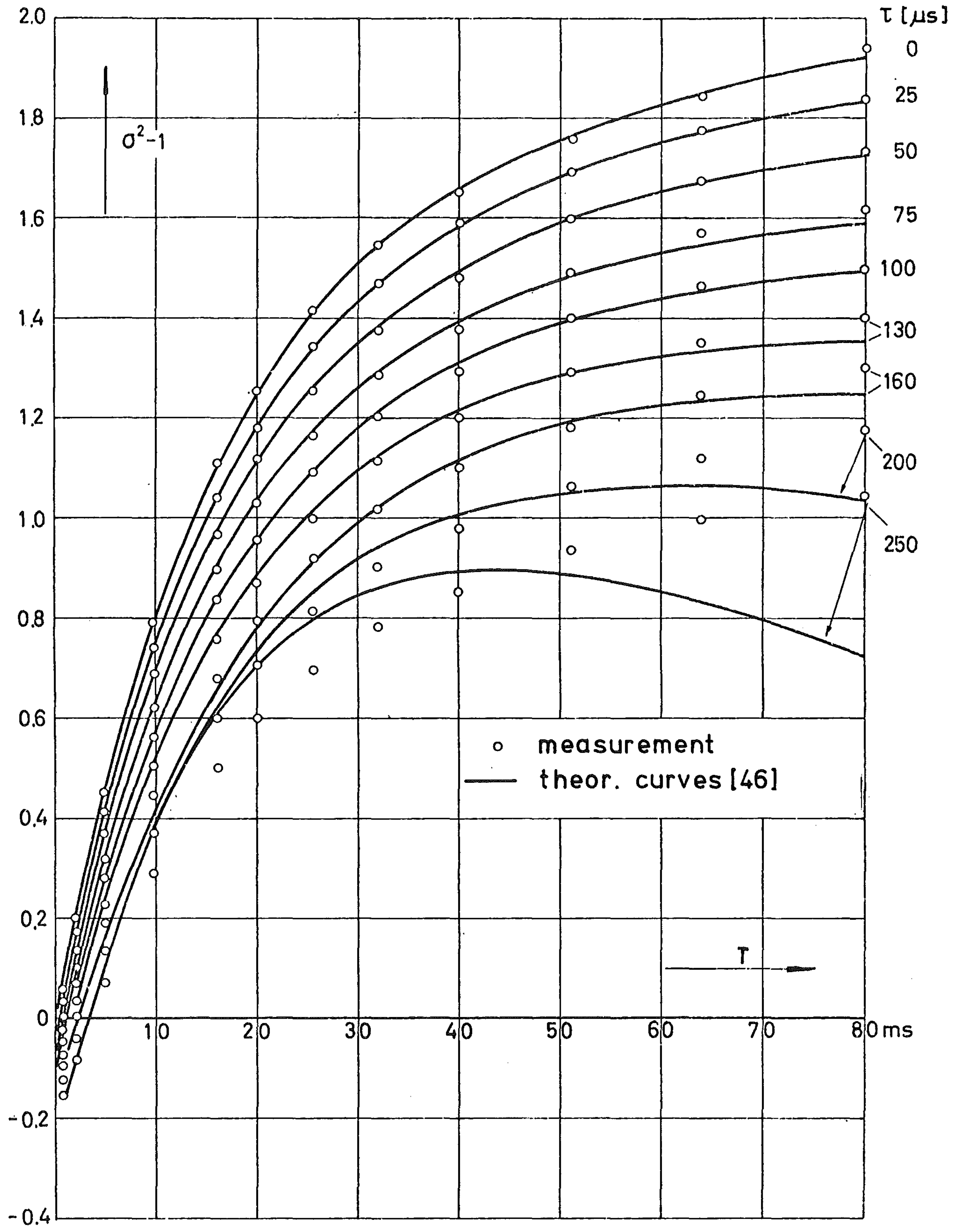


Fig. 6 Attenuation of variance by dead-time losses