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Abstract

A method is presented to calculate the first and second moments of neutron and precursor populations for a critical reactor system described by point kinetic equations and possessing inherent reactivity fluctuations. The equations have been linearised on the assumption that the system has a large average neutron population and that the amplitude of reactivity fluctuations is sufficiently small. The reactivity noise is assumed to be band limited white with a corner frequency higher than all the time constants of the system. Explicit expressions for the exact time development of the moments have been obtained for the case of a reactor without reactivity feedback and with one group of delayed neutrons. It is found that the expected values of the neutron and delayed neutron precursor numbers tend asymptotically to stationary values, whereas the mean square deviations increase linearly with time at an extremely low rate.

Das asymptotische Verhalten eines kritischen, ungeregelten Punktreaktors

Zusammenfassung

Für ein kritisches Reaktorsystem mit inhärenten Reaktivitätsschwankungen wird, im Rahmen des Punktreaktormodells, eine Methode zur Berechnung der 1. und 2. Momente der Neutronen- und Vorläuferanzahlen entwickelt. Unter der Annahme, daß die mittlere Neutronenanzahl im System hoch und die Amplitude der Reaktivitätsschwankungen hinreichend klein sind, werden die Systemgleichungen linearisiert. Als Reaktivitätsschwankung wird bandbegrenztes weißes Rauschen angesetzt, mit einer Grenzfrequenz oberhalb aller Zeitkonstanten des Systems. Für den Fall eines ungeregelten Reaktors mit einer Gruppe verzögerter Neutronen lassen sich geschlossene Ausdrücke für die Zeitabhängigkeit der Momente angeben. Es zeigt sich, daß die Erwartungswerte für die Neutronen- und Vorläuferanzahlen asymptotisch stationäre Werte anstreben, während die mittleren quadratischen Abweichungen extrem langsam, linear mit der Zeit ansteigen.

Contents

1. Introduction

Due to the stochastic nature of neutron chain processes, the number of neutrons present in a reactor fluctuates. The main objective of the reactor noise theory is to definitely predict the dynamic behaviour of a reactor during normal operation and to predetermine the thresholds for the correlation functions and other quantities of relevant stochastic signals monitored in the system for surveillance purposes. The reactor noise theory has been formulated from various approaches in the last few years. Borgwaldt and Stegemann /1/ developed a common basic formula for the description of the neutronic noise analysis experiments in zero power nuclear reactors using a very rigorous approach. The theory was extended to study the neutron noise in a reactor with an external control loop by Borgwaldt /2/. Langevin technique has been applied by various authors to formulate the space and time dependence of the reactor noise /3-9/. The reviews of Seifritz and Stegemann /10/ and Uhrig /11/ indicate that the theory of zero power noise is fully understood but for some mathematical details which remain to be solved. On the contrary, the theory of noise phenomena in power reactors is still in its infancy. In a power reactor, inherent fluctuations in reactivity are present, induced mainly by fluctuations of the coolant and fuel temperatures through the temperature coefficient of reactivity. The temperature fluctuations arise mainly due to the stochastic nature of the transport phenomena such as heat transfer etc. Power reactor noise is investigated intensively at several places. The summary of this work, particularly from the point of view of theory, could be found in the review articles by Saito /12/ and in the book of Williams /13/.

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The aim of the present paper is to show, explicitly, that a reactor system operating at high power level at criticality will develop mean square deviations increasing linearly with time if it is uncontrolled, i.e. without a restoring force. Reactivity fluctuations present in such a system may be of either sign, but the amplitude of the induced neutron density fluctuations is very small as compared to the mean neutron density. In such a case the reactivity fluctuations $\delta k(t)$ may be assumed equivalent to an external fluctuating neutron source, the time average of which vanishes over a macroscopic interval.

The spatial effects inside the reactor could also cause changes in reactivity and power but the one point black box type treatment has the advantage of simplicity and in most cases yields kinetic answers that are correct within experimental accuracy. For simplicity we, therefore, take this approach and represent the system by point model kinetic equations. The system can then be assumed to be perturbed by a noise source $N_o(\delta k(t)/l)$, where $\delta k(t)$ represents reactivity fluctuations, l the average neutron life time and N_o is the average neutron density.

The phenomenon observed in the above system is then very similar to the Brownian motion of a free particle. Hence looking at the expression for the mean square displacement of the Brownian particle one could predict a linear increase of mean square deviations with time also in the present case. But the value of the scaling factor can not be guessed. As far as we know, the explicit expressions for the moments have not been obtained in the literature on reactor noise for the case considered here. As an illustration of the method described in this paper we obtain the exact expressions for the first and second moments of the neutron and precursor populations for the case of one group of delayed neutrons only. The reactivity noise is assumed to be band limited white with a corner frequency higher than all the time constants of the system. However, with some more effort the moments for all the six groups could be obtained.

2. Theory

2.1 The Reactor Kinetic Equation

The reactor kinetic equations for a critical reactor system can be written in the following matrix equation form

$$\frac{dX(t)}{dt} = AX(t) + R(t) \tag{1}$$

where

$$X(t) = \begin{bmatrix} N(t) \\ \vec{C}(t) \end{bmatrix} \tag{2a}$$

and

$$A = \begin{bmatrix} -\beta/l, \lambda_1, \lambda_2, \ldots, \lambda_6 \\ \beta_1/l, -\lambda_1, 0, \ldots, \lambda_6 \\ \beta_2/l, 0, -\lambda_2, \ldots, \lambda_6 \\ \beta_6/l, 0, 0, \ldots, \lambda_6 \end{bmatrix} \tag{2b}$$

In equations (2a) and (2b), N and $\vec{C}$ are the neutron and delayed neutron precursor densities respectively, λ_i the delayed neutron decay constant of the i^{th} group and β_i is the i^{th} group fraction of delayed neutrons from fission. Here, the system matrix does **not include reactivity feedback terms.**

R(t) in Eq. (1) contains the source term. If the reactivity fluctuations $\delta k(t)$ are assumed to be equivalent to an external noise source, we have

$$R(t) = \begin{bmatrix} N_o(\delta k(t)/l) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

Eq. (1) is a stochastic equation describing the process $X(t)$ in response to the input process $R(t)$. We postulate the following about the random input process $R(t)$:

a) The process $R(t)$ is stationary and Gaussian with zero mean.

b) It has an exponential correlation function i.e.

$$\langle R(t_1)\ R(t_2)\rangle \sim e^{-\omega_c|t_2-t_1|}$$

where ω_c is the corner frequency.

In view of the assumptions (a) and (b), $\delta k(t)$ has the following properties

$$\langle\delta k(t)\rangle = 0 \tag{4a}$$

$$\langle\delta k(t_1)\delta k(t_2)\rangle = \langle\rho^2\rangle\ e^{-\omega_c|t_2-t_1|} \tag{4b}$$

where $\langle\rho^2\rangle$ denotes the mean square amplitude of the reactivity fluctuations.

2.2 Solution of the Equation and Expressions for Moments

The solution of Eq. (1) can be written in the following form

$$X(t) = G(t,o)X(o) + \int_o^t G(t,t')R(t')dt' \tag{5}$$

where

$$X(o) = \begin{bmatrix} N(o) \\ \vec{C}(o) \end{bmatrix} = \begin{bmatrix} N_o \\ \vec{C}_o \end{bmatrix}$$

and G is the Green's function matrix i.e. a solution of the kinetic equation

$$\frac{dG}{dt} = AG \qquad\qquad t \geqslant o \tag{6}$$

with the initial condition

$$G(o,o) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

The i,j component of the Green's function matrix represents the expected number of particles of kind i (neutrons or precursors) at time $t > o$, starting with an initial situation of exactly one particle of kind j at time $t = o$.
From Eq. (6)

$$G(t,o) = e^{At} \qquad\qquad t \geqslant o \tag{8}$$

Hence Eq. (5) is written as

$$X(t) = e^{At}\, X(o) + \int_o^t e^{A(t-t')}\, R(t')dt' \tag{9}$$

Taking the ensemble average and using Eq. (4a) we get

$$\langle X(t)\rangle = e^{At}\, X(o) \tag{10}$$

The correlation matrix is obtained by taking the vector product X(t) at two different times t_1 and t_2 and averaging. Thus from Eq. (5) we have

$$\langle X(t_1)\; X'(t_2)\rangle$$

$$= \left\langle \left[e^{At_1} X(o) + \int_o^{t_1} e^{A(t_1-t')}\, R(t')dt' \right] \cdot \left[X'(o) e^{A't_2} + \int_o^{t_2} R'(t'') e^{A'(t_2-t'')} dt'' \right] \right\rangle \tag{11}$$

where ' on matrices and vectors denotes their transpose. On multiplication four terms are obtained, but due to the property (4a) the cross terms have zero contributions. Hence, we obtain

$$\langle X(t_1)X'(t_2)\rangle$$

$$= e^{At_1}X(o)X'(o)e^{A't_2}$$

$$+\int_o^{t_1}dt'\int_o^{t_2}dt''e^{A(t_1-t')}\langle R(t')R'(t'')\rangle e^{A'(t_2-t'')} \tag{12}$$

The reduced (or centred) covariance matrix is given by

$$P_{red}(t_1,t_2) = \langle X(t_1)X'(t_2)\rangle - e^{At_1}X(o)X'(o)e^{A't_2}$$

$$= \int_o^{t_1}dt'\int_o^{t_2}dt''e^{A(t_1-t')}\langle R(t')R(t'')\rangle e^{A'(t_2-t'')} \tag{13}$$

In order to make our treatment more transparent we assume at this point that the reactivity noise is band limited white noise with corner frequency $\omega_c \gg \beta/1$. In this case the correlation function given by (4b) could be assumed to be of the following form when substituted inside the integrand

$$\langle\delta k(t_1)\delta k(t_2)\rangle = \frac{2\,\langle\rho^2\rangle}{\omega_c}\,\delta(t_2-t_1) \tag{14}$$

From Eqs. (3) and (4b) it is clear that

$$\langle R_m(t_1)R_n(t_2)\rangle = R_{mn}\,\delta(t_2-t_1) \tag{15}$$

where

$$R_{mn} = \frac{2\langle\rho^2\rangle}{\omega_c}\,\frac{N_o^2}{1^2} \qquad m=n=1 \tag{16}$$

$$= o \qquad \text{otherwise}$$

Substitution of Eqs. (15) and (16) in Eq. (13) gives

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\frac{N_o^2}{l^2}\int_o^{t_1}dt'\int_o^{t_2}dt''e^{A(t_1-t')}M\delta(t'-t'')e^{A'(t_2-t'')} \tag{17}$$

where

$$M = \begin{bmatrix} 1 & o \\ o & o \end{bmatrix} \tag{18}$$

For this centred correlation matrix we have to distinguish two cases

a) $o < t_1 \leqslant t_2$

For this case we have

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\frac{N_o^2}{l^2}\int_o^{t_1}e^{A(t_1-t')}M\,e^{A'(t_2-t')}dt' \tag{19}$$

Using the semigroup property of the Green's function matrix e^{At} the above equation can be written as

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\frac{N_o^2}{l^2}\int_o^{t_1}e^{A(t_1-t')}M\,e^{A'(t_1-t')}dt'\,e^{A'(t_2-t_1)} \tag{20}$$

b) $o < t_2 \leqslant t_1$

In this case

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\frac{N_o^2}{l^2}\int_o^{t_2}e^{A(t_1-t')}M\,e^{A'(t_2-t')}dt' \tag{21}$$

Again from the semigroup property we have

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\frac{N_o^2}{l^2}\,e^{A(t_1-t_2)}\int_o^{t_2}e^{A(t_2-t')}Me^{A'(t_2-t')}dt' \tag{22}$$

2.3 One Group of Delayed Neutrons: the exact expressions for the expected value of neutron and delayed neutron precursor populations and the centred moments.

In order to evaluate the exact expressions for $\langle N(t)\rangle$ and $\langle C(t)\rangle$ we must evaluate the matrix e^{At}. For a single group of delayed neutrons A takes the simple form of a two by two matrix. In this case, the characteristic equation of the matrix A is given by /14/

$$\omega^2 + (\lambda+\beta/l)\omega = o \tag{23}$$

where $\beta = \sum_{i=1}^{6} \beta_i$ is the total fraction of delayed neutrons and λ is the mean delayed neutron decay constant, defined by

$$\frac{\beta}{\lambda} = \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}$$

The roots of Eq. (23), i.e. the eigenvalues of A, are

$$\omega_1 = o \; ; \quad \omega_2 = -(\lambda+\beta/l)$$

We denote $(\lambda+\beta/l)$ by α. It is easily seen that

$$\alpha \approx \beta/l \tag{24}$$

This is obvious from the data, say for a light water reactor, where

$$\beta = 6.4 \times 10^{-3}$$
$$l = 10^{-5} \text{ sec}$$
$$\lambda = 0.075 \text{ sec}^{-1}$$

Hence $\frac{\beta}{l}$ = 640 sec^{-1} and λ in comparison is very small. From Eq. (24) α is recognised as the prompt neutron decay constant or Rossi alpha for a delayed critical system.

Using Sylvester's theorem /14/ one can write the Green's function matrix as

$$e^{At} = \begin{bmatrix} G_{11}(t) & G_{12}(t) \\ G_{21}(t) & G_{22}(t) \end{bmatrix} \tag{25}$$

where

$$G_{11}(t) = \frac{1}{\alpha}\left[\lambda + \frac{\beta}{l}e^{-\alpha t}\right] \tag{26}$$

$$G_{12}(t) = \frac{1}{\alpha}\left[\lambda - \lambda\, e^{-\alpha t}\right] \tag{27}$$

$$G_{21}(t) = \frac{1}{\alpha}\left[\frac{\beta}{l} - \frac{\beta}{l}\, e^{-\alpha t}\right] \tag{28}$$

$$G_{22}(t) = \frac{1}{\alpha}\left[\frac{\beta}{l} + \lambda e^{-\alpha t}\right] \tag{29}$$

Hence from Eq. (10) we have

$$\begin{bmatrix} \langle N(t)\rangle \\ \langle C(t)\rangle \end{bmatrix} = \begin{bmatrix} G_{11}(t) & G_{12}(t) \\ G_{21}(t) & G_{22}(t) \end{bmatrix} \begin{bmatrix} N_o \\ C_o \end{bmatrix} \tag{30}$$

Substituting for G_{11}, G_{12}, G_{21} and G_{22} we obtain the following expressions for the expected values of the neutron density and the delayed neutron precursor densities

$$\langle N(t)\rangle = \frac{(N_o+C_o)\lambda}{\alpha} + \frac{N_o(\beta/l)-\lambda C_o}{\alpha}\, e^{-\alpha t} \tag{31}$$

$$\langle C(t)\rangle = \frac{(\beta/l)(N_o+C_o)}{\alpha} + \frac{\lambda C_o-(\beta/l)N_o}{\alpha}\, e^{-\alpha t} \tag{32}$$

To evaluate the centred correlation matrix we introduce the matrix

$$B(t) = \int_0^t e^{A(t-t')} M\, e^{A'(t-t')} dt' \tag{33}$$

Substituting for the Green's function from Eq. (25) the above matrix can be easily evaluated. We obtain

$$B(t) = \begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix} \tag{34}$$

where

$$b_{11}(t) = \frac{1}{\alpha^2}\left[\lambda^2 t + \frac{2\lambda\beta}{l\alpha}(1-e^{-\alpha t}) + \frac{\beta^2}{2l^2\alpha}(1-e^{-2dt})\right] \tag{35}$$

$$b_{12}(t) = b_{21}(t) = \frac{(\beta/l)}{\alpha^2}\left[\lambda t - \frac{\lambda}{\alpha}(1-e^{-\alpha t})\right.$$

$$\left. + \frac{\beta}{2l\alpha}(1+e^{-2\alpha t}) - \frac{\beta}{l\alpha}e^{-\alpha t}\right] \tag{36}$$

$$b_{22}(t) = \frac{(\beta/l)^2}{\alpha^2}\left[t - \frac{3}{2\alpha}(1-\tfrac{4}{3}e^{-\alpha t}) - \frac{1}{2\alpha}e^{-2\alpha t}\right] \tag{37}$$

It should be mentioned here that the above expressions (35) to (37) are reduced to still simpler form if we substitute $\alpha = \beta/l$. We obtain

$$b_{11}(t) = \frac{l^2}{\beta^2}\left[\lambda^2 t + 2\lambda(1-e^{-\frac{\beta}{l}t}) + \frac{\beta}{2l}(1-e^{-\frac{2\beta}{l}t})\right] \tag{38}$$

$$b_{12}(t) = b_{21}(t) = \frac{1}{\beta}\left[\lambda t - \frac{\lambda l}{\beta}(1-e^{-\frac{\beta}{l}t}) + \frac{1}{2}(1+e^{-\frac{2\beta}{l}t}) - e^{-\frac{\beta}{l}t}\right] \tag{39}$$

$$b_{22}(t) = t - \frac{3\beta}{2l}(1-\tfrac{4}{3}e^{-\frac{\beta}{l}t}) - \frac{1}{2\beta}(1-e^{-\frac{2\beta}{l}t}) \tag{40}$$

From Eqs. (20) and (22) the following expressions for the reduced correlation matrix are then obtained

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\,\frac{N_o^{\;2}}{l^2}\begin{bmatrix} b_{11}(t_1) & b_{12}(t_1) \\ b_{21}(t_1) & b_{21}(t_1)\end{bmatrix} e^{A'(t_2-t_1)} \tag{41}$$

for $o < t_1 \leq t_2$

$$P_{red}(t_1,t_2) = \frac{2<\rho^2>}{\omega_c}\,\frac{N_o^{\;2}}{l^2}\; e^{A(t_1-t_2)}\begin{bmatrix} b_{11}(t_2) & b_{12}(t_2) \\ b_{21}(t_2) & b_{22}(t_2)\end{bmatrix} \tag{42}$$

for $o < t_2 \leq t_1$

The components of the correlation matrix can be easily evaluated from the above expressions,but the results obtained are lengthy. For clear presentation we consider two cases:

case a) $t_1 = t_2 = t$

For this case, the following expressions for the mean square deviations are obtained from Eq. (40) or Eq. (41):

$$<\mu(t,t)>_{NN} = \frac{2<\rho^2>}{\omega_c}\,\frac{N_o^{\;2}}{l^2}\; b_{11}(t)$$

$$= \frac{2<\rho^2>}{\beta^2}\,\frac{N_o^{\;2}}{\omega_c}\left[\lambda^2 t + 2\lambda(1-e^{-\frac{\beta}{l}t}) + \frac{\beta}{2l}(1-e^{-\frac{2\beta}{l}t})\right] \tag{43}$$

$$<\mu(t,t)>_{NC} = \frac{2<\rho^2>}{\omega_c}\,\frac{N_o^{\;2}}{l^2}\; b_{12}(t)$$

$$= \frac{2<\rho^2>}{\beta^2}\,\frac{N_o^{\;2}}{\omega_c}\,\frac{\beta}{l}\left[\lambda t - \frac{\lambda l}{\beta}(1-e^{-\frac{\beta}{l}t}) + \frac{1}{2}(1+e^{-\frac{2\beta}{l}t}) - e^{-\frac{\beta}{l}t}\right] \tag{44}$$

$$<\mu(t,t)>_{CC} = \frac{2<\rho^2>}{\omega_c} \frac{N_o^{\ 2}}{l^2} \ b_{22}(t)$$

$$= \frac{2<\rho^2>}{\beta^2} \frac{N_o^{\ 2}}{\omega_c} \ \frac{\beta^2}{l^2} \Big[t - \frac{3\beta}{2l} \ (1-\frac{4}{3} \ e^{-\frac{\beta}{l}t})$$

$$- \frac{1}{2\beta} \ (1-e^{-\frac{2\beta}{l}t}) \Big] \qquad (45)$$

case b) $\quad t_2-t_1 \quad >> \frac{1}{\alpha}$

In this case the elements of the reduced correlation matrix reduce to the following form

$$<\mu(t_1,t_2)>_{NN} = \frac{2<\rho^2>}{\beta^2} \frac{N_o^{\ 2}}{\omega_c} \ \lambda(\lambda t_1+1) \qquad (46)$$

$$<\mu(t_1,t_2)>_{NC} = \frac{2<\rho^2>}{\beta^2} \frac{N_o^{\ 2}}{\omega_c} \ \frac{\beta}{l}(\lambda t_1+1) \qquad (47)$$

$$<\mu(t_1,t_2)>_{CN} = \frac{2<\rho^2>}{\beta^2} \frac{N_o^{\ 2}}{\omega_c} \ \frac{\lambda\beta}{l}(t_1 - \frac{1}{\beta}) \qquad (48)$$

$$<\mu(t_1,t_2)>_{CC} = \frac{2<\rho^2>}{\beta^2} \frac{N_o^{\ 2}}{\omega_c} \ \frac{\beta^2}{l^2}(t_1 - \frac{1}{\beta}) \qquad (49)$$

2.4 Asymptotic Values of the Moments

The asymptotic values of the moments are obtained by taking the limit of the corresponding expressions for infinite time

For $t \to \infty$ we get

$$\langle N(t)\rangle \simeq \frac{\lambda(N_o+C_o)}{\alpha} \simeq \frac{\lambda l(N_o+C_o)}{\beta} \tag{50}$$

$$\langle C(t)\rangle \simeq \frac{(\beta/l)(N_o+C_o)}{\alpha} \simeq (N_o+C_o) \tag{51}$$

$$\langle \mu(t,t)\rangle_{NN} \simeq \frac{2\langle\rho^2\rangle}{\beta^2} \frac{N_o^2}{\omega_c} \lambda^2 t \tag{52}$$

$$\langle \mu(t,t)\rangle_{NC} \simeq \frac{2\langle\rho^2\rangle}{\beta^2} \frac{N_o^2}{\omega_c} \frac{\lambda\beta}{l} t \tag{53}$$

$$\langle \mu(t,t)\rangle_{CC} \simeq \frac{2\langle\rho^2\rangle}{\beta^2} \frac{N_o^2}{\omega_c} \left(\frac{\beta}{l}\right)^2 t \tag{54}$$

2.5 Special case of an Initial Equilibrium between Neutron and Precursor Populations

One of the most important cases in reactor systems is that which corresponds to an initial equilibrium between precursors and neutrons i.e. we initially have

$$\lambda C_o = \frac{\beta}{l} N_o \tag{55}$$

This situation is also characteristic for most practical cases, which are near equilibrium. At the start of normal reactor operation this situation might not be strictly true but it is reached very soon because the prompt neutron decay constant α is of the order of 10^{-3} secs. Also it is true, in all reactors, that the precursor population is very high compared to the neutron population, i.e. $N_o+C_o \simeq C_o$. Hence from Eqs. (50) and (51) the asymptotic values of the expected neutron and precursor numbers are for $t \to \infty$

$$\langle N(t)\rangle \simeq \frac{\lambda 1 C_o}{\beta} = N_o \tag{55}$$

$$\langle C(t)\rangle \simeq C_o \tag{56}$$

The normalized asymptotic values for the variances are then given by

$$\frac{\langle \mu(t,t)\rangle_{NN}}{N_o^{\,2}} \sim \frac{2\langle \rho^2\rangle}{\beta^2} \frac{\lambda^2}{\omega_c} t \tag{57}$$

$$\frac{\langle \mu(t,t)\rangle_{NC}}{N_o C_o} \sim \frac{2\langle \rho^2\rangle}{\beta^2} \frac{\lambda^2}{\omega_c} t \tag{58}$$

$$\frac{\langle \mu(t,t)\rangle_{CC}}{C_o^{\,2}} = \frac{2\langle \rho^2\rangle}{\beta^2} \frac{\lambda^2}{\omega_c} t \tag{59}$$

3. Discussion

From Eqs. (50) and (51) it is seen that the asymptotic values for the expected population of neutrons and delayed neutron precursors are constant values. It is obvious from Eqs. (31) and (32) that the asymptotic constant values are reached very soon because the second term decays with a very short relaxation time $\frac{1}{\alpha}$ (for a light water reactor $\frac{1}{\alpha} \simeq 10^{-3}$ sec). From Eqs. (52) to (54) one observes that the values of all the second order moments increase linearly with time. This result is expected because the system equation (1) is equivalent to the stochastic equation for the Brownian motion of a free particle /15/.

It is well known that the mean square displacement of a Brownian particle goes on increasing linearly with time, because there is no restoring force which prevents it from drifting. Hence it is also expected that the reactor system without a controller, as described by Eq. (1), will develop mean square deviations which increase linearly with time. One will have to apply some control to keep the reactor stationary.

From Eqs. (41) and (42) one can see that if measurements are performed at two times t_1 and t_2, the initial correlation amplitude depends linearly upon t_1 (if $t_2 > t_1$) and in the time interval (t_1,t_2) the correlation function relaxes according to the term $e^{A'(t_2-t_1)}$. The decay of the correlation function in the interval (t_1,t_2) obeys the dynamics of the reactor system mathematically described by the system's Green function $e^{A'(t_2-t_1)}$.

For the special case of an initial equilibrium between the neutron and precursor populations we see that the averaged asymptotic values of these number quantities remain equal to their initial values, at time t = o, and the initial correlation amplitudes build up exactly with the same speed (Eqs. 57 to 59). This suggests that our system consisting of neutrons and precursors may be imagined as a composite Brownian particle which has a diffusion coefficient equal to

$$D = \frac{\langle \rho 2 \rangle}{\beta^2} \frac{\lambda^2}{\omega_c}$$

For a light water reactor $\lambda \sim 10^{-1}\ \text{sec}^{-1}$ and $\frac{\beta}{l} \simeq 10^3\ \text{sec}^{-1}$. Hence, for the corner frequency of the reactivity noise one could take a value of the order of $10^4\ \text{sec}^{-1}$ $(\omega_c >> \frac{\beta}{l})$. If the reactivity fluctuations are assumed to have a mean square magnitude $\langle \rho^2 \rangle \gtrsim 10^{-6}\ \beta^2$, the value of D comes out to be of the order of $10^{-12}\ \text{sec}^{-1}$. Hence the mean square deviations will develop, approximately, at the rate of 10^{-12} per second. With this rate the system's root mean square drift would be less than 10^{-3} in a day. This relative drift is much smaller than expected.

It should be noted that this will be true for any reactor system, because even for reasonable lower values of the corner frequency ω_c and a higher reactivity noise amplitude the drift may differ at the most by one or two orders of magnitude. This low drift of the system could be caused by (a) the high value for the corner frequency of the reactivity noise, (b) the assumption that the initial system state is well defined, (c) neglecting other system parameters which should be included for a more realistic evaluation. The analysis with lower values of the corner frequency for the reactivity noise ($\omega_c << \beta/l$), which would be a realistic assumption for all fast reactor systems, could be more conveniently developed in the prompt jump approximation. This extension, which necessitates some reformulation, is now under development and will be published in a subsequent paper.

4. Conclusions

We have developed a method for calculating the moments of state variables for a reactor system excited by stochastic reactivity fluctuations. For a clear exposition explicit expressions for the first and second moments of the neutron and precursor populations have been obtained for one group of delayed neutrons only and under the assumption that the corner frequency of the reactivity noise is large compared to all time constants of the system. It should, however, be emphasized that other variables which, e.g., describe external feedback and temperature effects could be included into the system matrix and expressions for additional moments be obtained. This extension of our work is planned for the near future.

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References

/1/ H. Borgwaldt and D. Stegemann
"A Common Theory for Neutronic Noise Analysis Experiments in Nuclear Reactors"
Nukleonik, 7, 313 (1965)

/2/ H. Borgwaldt
"Neutronic Noise in a Reactor with an External Control Loop"
Nukleonik, 11, 76 (1968)

/3/ J.R. Sheff and R.W. Albrecht
"The Space Dependence of Reactor Noise. I. Theory, II. Calculations"
Nuclear Science and Engng., 24, 246 (1966); 26, 207 (1966)

/4/ M.M.R. Williams
"Probability Distributions in Simple Neutronic Systems via The Fokker Planck Equation"
Journal of Nuclear Energy, 23, 633 (1969)

/5/ W. Seifritz
"At Power Reactor Noise Induced by Fluctuations of the Coolant Flow"
Atomkernenergie, 16, 29 (1970)

/6/ M.M.R. Williams
"The Kinetic Behaviour of Simple Neutronic Systems with Randomly Fluctuating Parameters"
Journal of Nuclear Energy, 25, 563 (1971)

/7/ Karmeshu and N.K. Bansal
"Exact Calculations of First Moment in a Simple Neutronic System"
Nuclear Science and Engng., 57, 245 (1975)

/8/ Karmeshu and N.K. Bansal
"Stability of Moments in a Simple Neutronic System with Stochastic Parameters", Nuclear Science and Engng., 58, 321 (1975)

/9/ Z.A. Akcasu and M. Karasulu
"Nonlinear Response of Point Reactors to Stochastic Inputs"
Annals of Nuclear Energy, 3, 11 (1976)

/10/ W. Seifritz and D. Stegemann
"Reactor Noise Analysis"
Atomic Energy Review (IAEA), 9, 129 (1971)

/11/ R.E. Uhrig
"Random Noise Techniques in Nuclear Reactor Systems"
Ronald Press, N.Y. (1970)

/12/ K. Saito
"On the Theory of Power Reactor Noise, Parts I, II, III"
Annals of Nuclear Science and Engineering, 1, 31; 1, 109; 1, 209 (1974)

/13/ M.M.R. Williams
"Random Processes in Nuclear Reactors"
Pergamon Press (1974)

/14/ L.A. Pipes and L.R. Harvill
"Applied Mathematics for Engineers and Physicists"
McGraw Hill (1970)

/15/ G.E. Uhlenbeck and L.S. Ornstein
"On the Theory of the Brownian Motion"
Phys. Rev. 36, 823 (1930)