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Abstract

The quasistatic (or prompt jump) treatment has been used to investigate the effect of bandlimited white reactivity noise on the kinetic behaviour of an uncontrolled critical point reactor. This treatment is valid for noise with a corner frequency $\omega_c << \beta/l$. Expressions for the first and second moments of neutron and delayed neutron precursor populations have been obtained on two levels of approximations. The possibility of linearising the equations is discussed. In conformity with the results of the influence of noise with $\omega_c >> \beta/l$, the mean square deviations developed by the system are found to be linear in time and of very small magnitude. This suggests that a critical uncontrolled reactor could be considered as almost stationary and ergodic.

Quasistatische Behandlung des Einflusses von bandbegrenztem weißen Reaktivitätsrauschen auf einen ungeregelten kritischen Punktreaktor

Zusammenfassung

Die Auswirkung von bandbegrenztem weißen Reaktivitätsrauschen auf das Zeitverhalten eines ungeregelten kritischen Punktreaktors wird in der quasistatischen Näherung untersucht. Diese Näherung ist gültig für Rauschen mit einer Eckfrequenz $\omega_c << \beta/l$. Die ersten und zweiten Momente der Verteilung von Neutronen- und Vorläufer-Anzahlen wurden für zwei Stufen der Approximation ermittelt. Ferner wird die Möglichkeit der Linearisierung der Gleichungen diskutiert. Die, in der Zeit linearen, mittleren quadratischen Abweichungen des Systems vom Ausgangszustand sind sehr klein. Dies stimmt mit früheren Ergebnissen über den Einfluß von Rauschen mit $\omega_c >> \beta/l$ voll überein. Ein kritischer ungeregelter Reaktor kann somit als nahezu stationär und ergodisch behandelt werden.

Contents

1. Introduction

The safe operation of a power reactor requires the permanent availability of the safety channel as an integral unit. This, in turn, supports demands for on-line testing of the various components of the safety equipment, continuously, during normal operation. Noise analysis techniques bear the potential of providing nondisturbing methods for monitoring the dynamic behaviour of reactors operating at power. This was pointed out by Thie /1/ and others at three conferences /2-4/ on reactor noise. Fry /5/ demonstrated the potential usefulness and the application of noise analysis for diagnosing incipient failure of components in HFIR and MSRE reactors. This illustrated, in special cases, the way, the noise method could become useful for monitoring power reactors to increase their availability and safety.

Recently Edelmann /6/ described two methods for in situ testing of neutron and temperature instrumentation of power reactors. In these methods, random excitations present in a power reactor, are used as dynamic test input to the integral signal channel. The information about the integral signal channel is then obtained, from the fluctuations of the available signals, using noise analysis techniques.

Surveillance of power reactors, in general, relies largely on checking the deviations of neutron detector or derived signals from set values. For fixing tolerance bands and confidence levels, an adequate assessment of the influence of inherent reactivity noise on the reactor is required. As external controls are mostly intermittent and internal feedback effects usually much slower than inherent reactivity

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noise phenomena, the critical reactor without a controller is a starting point of more than mere academic interest. Absence of control also yields limiting values for the expected asymptotic deviations of the reactor.

The problem of a white noise excited reactor has been treated previously /7-10/, but no directly usable information could be obtained from these results. In a recent paper /11/, we had investigated the dynamic behaviour of such a critical system possessing an additive reactivity noise, assumed to be band limited white, with a corner frequency ω_c greater than the largest time constant β/l of the system. With reasonably assumed parameters, characteristic for a reactor system, it was shown that the mean square deviations developed by the system are of very small magnitude and, hence, the system could be treated as stationary ergodic for further studies. This much desirable result is, however, to be confirmed for lower corner frequencies of the reactivity noise, which, for an equal noise amplitude leads to a much stronger (reactivity, neutron, precursor)-coupling.

The aim of the present analysis is, therefore, to calculate the moments of state variables of an uncontrolled critical point reactor, which is perturbed by reactivity noise with a corner frequency $\omega_c \ll \beta/l$, where β is the total delayed neutron fraction and l the mean prompt neutron life time. In contrast to the previously treated opposite case ($\omega_c \gg \beta/l$), the mathematical analysis, for the present case, could be developed in the quasi-static (or prompt jump) approximation. For the sake of clarity and simplicity only, a single group of delayed neutrons is considered here. In order to check the possibility of linearising the equations, we obtain the expressions for the moments of state variables in the following two cases

a) By linearising the equations as obtained from the quasistatic approximation.

b) By solving the exact equations as obtained from the quasistatic approximation.

It is found that the linearised equations give results, which are correct over a wide range of observation times. The mean square deviations of the system are again found to be very low, confirming the previous result, that the system could be treated as stationary ergodic for further studies.

2. Theory

2.1 The System's Stochastic Equations

We consider a critical point reactor and consider only one group of precursors. Neglecting feedback terms, the point reactor kinetic equations are

$$\frac{d}{dt} N(t) = \frac{\rho(t)-\beta}{l} N(t) + \lambda C(t) \qquad (1)$$

$$\frac{d}{dt} C(t) = \frac{\beta}{l} N(t) - \lambda C(t), \qquad (2)$$

where

$N(t)$ = neutron population,

$C(t)$ = delayed neutron precursor population,

β = total delayed neutron fraction,

l = prompt neutron life time,

λ = average delayed neutron decay constant

and $\rho(t)$ denotes the reactivity fluctuations present in the system. We make the following assumptions about the stochastic process $\rho(t)$:

a) $\rho(t)$ is a stationary Gaussian process with zero mean, i.e.

$$\langle\rho(t)\rangle = o \tag{3a}$$

b) the values of $\rho(t)$ at two different times t and $t+\tau$ are exponentially correlated, i.e.

$$\phi_{kk}(\tau) = \langle\rho(t)\rho(t+\tau)\rangle$$

$$= \langle\rho^2\rangle \quad e^{-\omega_c|\tau|}, \tag{3b}$$

where ω_c^{-1} is the mean correlation time and $\langle\rho^2\rangle$ denotes the mean square amplitude of reactivity fluctuations. The power spectral density function of reactivity noise is given by the Fourier transform of $\phi_{kk}(\tau)$, i.e.

$$\bar{\Phi}_{kk}(\omega) = \frac{\langle\rho^2\rangle}{\pi\omega_c} \frac{1}{1+(\omega/\omega_c)^2} \quad . \tag{4}$$

The value of this function depends largely on the ratio ω/ω_c. For $\omega << \omega_c$, $\bar{\Phi}_{kk}(\omega)$ is constant, meaning that the stochastic process has equal power in equal frequency intervals up to the corner frequency ω_c. For $\omega > \omega_c$, the power density drops very steeply towards zero. Such a stochastic process is called band limited white noise. In this analysis we assume $\omega_c << \beta/l$.

2.2 Quasistatic Approximation

For the case $\omega_c << \beta/l$, which is realistic for fast reactors, we may assume that the neutrons are at all times in equilibrium with the (precursor, reactivity)- state. With this quasistatic (or prompt jump) approximation and for low excitation amplitudes $|\rho| << \beta$, Eqs. (1,2) can be transformed to yield

$$N(t) \simeq (\lambda l/\beta)\ (1+\rho(t)/\beta)\ C(t) \tag{5}$$

$$\frac{d}{dt} C(t) = (\lambda/\beta)\ \rho(t)\ C(t) \tag{6}$$

As in these equations $\rho(t)$ is a stochastic variable the resulting $N(t)$, $C(t)$ are also stochastic variables, for which only the moments of their common probability distribution can be derived.

2.3 Moments of Neutron and Precursor Populations

The moments of neutron and precursor numbers, N(t) and C(t) respectively could either be found by first linearising Eq. (6) or by solving it directly. We shall discuss both methods.

2.3.1 Linearisation

In this section we assume that the fluctuations in the precursor population are small, so that Eq. (6) can be linearised. This assumption of linearisation is justified from the results obtained by exactly solving Eq. (6).

The linearised Eq. (6) is written as

$$\frac{d}{dt} C(t) \approx \frac{\lambda C_o}{\beta} \rho(t) \tag{7}$$

where

$$C_o = C(o) \approx \langle C(t) \rangle \tag{8}$$

The solution of Eq. (7) is written, directly, as

$$C(t) = C_o \left[\frac{\lambda}{\beta} \int_o^t \rho(u) du + 1 \right] \tag{9}$$

Taking the ensemble average and using Eq. (3a), Eq. (8) is immediately confirmed.

For the neutron number, from Eq. (5), we get

$$N(t) \approx (\lambda l/\beta)\left[C(t)+(\rho(t)/\beta)C_o\right] \qquad (10)$$

Taking the ensemble average and using Eqs. (3a) and (8)

$$\langle N(t)\rangle = (\lambda l/\beta)C_o \qquad (11)$$

Starting from an initial equilibrium between neutron and precursor populations, i.e.

$$\lambda C_o = (\beta/l)N_o = (\beta/l)N(o) \qquad (12)$$

we have $\langle N(t)\rangle = N_o$ (13)

From Eq. (9), we take the expressions for C(t) at two times t and s $(t \geq s \geq o)$, multiply and take the ensemble average, to obtain the following expression for the autocorrelation function of the precursor number:

$$\langle C(s)C(t)\rangle = C_o^2\left[\frac{\lambda^2}{\beta^2}\int_o^t\int_o^s \langle\rho(u)\rho(v)\rangle du dv + 1\right] \qquad (14)$$

Substituting for the reactivity autocorrelation function from Eq. (3b)

$$\langle C(s)C(t)\rangle = C_o^2\left[\frac{\lambda^2\langle\rho^2\rangle}{\beta^2}\int_o^t\int_o^s e^{-\omega_c|u-v|} du dv + 1\right] \qquad (15)$$

Defining x = u-v and y = u+v as new variables, a first integration over y leads to

$$<C(s)C(t)> \;=\; C_o^2 \left[\frac{\lambda^2 <\rho^2>}{\beta^2} \left(\int_o^t (t-x)\, e^{-\omega_c x} + \int_o^s (s-x)\, e^{-\omega_c x} - \int_o^{t-s} (t-s-x)\, e^{-\omega_c x}\, dx \right) + 1 \right] \qquad (16)$$

After a second integration we, finally, obtain

$$<C(s)C(t)> \;=\; C_o^2 \left[D\left(2s + \frac{e^{-\omega_c s}}{\omega_c} + \frac{e^{-\omega_c t}}{\omega_c} - \frac{e^{-\omega_c (t-s)}}{\omega_c} - \frac{1}{\omega_c} \right) + 1 \right] \qquad (17)$$

where

$$D = \frac{\lambda^2 <\rho^2>}{\omega_c \beta^2} \qquad (18)$$

Using Eq. (10), the autocorrelation function of the neutron population can be written as

$$<N(s)N(t)>$$

$$= \frac{\lambda^2 l^2}{\beta^2} \left[<C(s)C(t)> + \frac{C_o}{\beta} \left(<\rho(s)C(t)> + <C(s)\rho(t)> \right) + \frac{C_o^2}{\beta^2} <\rho(s)\rho(t)> \right] \qquad (19)$$

To evaluate the crucial term $\langle\rho(s)C(t)\rangle$, we multiply Eq. (9) by $\rho(s)$ and average. We obtain

$$\langle\rho(s)C(t)\rangle = \frac{\lambda C_o}{\beta} \int_0^t \langle\rho(\tau)\rho(s)\rangle \, d\tau$$

Substituting from Eq. (3b) and evaluating the integral, we get for $s \leq t$

$$\langle\rho(s)C(t)\rangle = C_o \frac{\lambda\langle\rho^2\rangle}{\omega_c\beta} \left[2-e^{-\omega_c s} - e^{-\omega_c (t-s)} \right] \qquad (20)$$

Similarly, we have

$$\langle C(s)\rho(t)\rangle = C_o \frac{\lambda\langle\rho^2\rangle}{\omega_c\beta} \left[e^{-\omega_c (t-s)} - e^{-\omega_c t} \right] \qquad (21)$$

Substituting for various terms on the right hand side of Eq. (19) and using Eq. (12) we obtain the following expression for the autocorrelation function of the neutron population

$\langle N(s)N(t)\rangle$

$$= N_o^2 \left[D\left(2s + \frac{\lambda-\omega_c}{\omega_c\lambda} (e^{-\omega_c s} + e^{-\omega_c t}) - \frac{1}{\omega_c} e^{-\omega_c (t-s)} + \frac{2\omega_c-\lambda}{\omega_c\lambda}\right) + \frac{\langle\rho^2\rangle}{\beta^2} e^{-\omega_c (t-s)} + 1 \right] \qquad (22)$$

Proceeding in the same way, the expressions for cross-correlation functions are obtained. We have

$$\langle N(s)C(t)\rangle = N_oC_o\left[D\left(2s + \frac{e^{-\omega_c t}}{\omega_c} + \frac{\lambda-\omega_c}{\omega_c\lambda}\, e^{-\omega_c s}\right.\right.$$

$$\left.\left. - \frac{\lambda+\omega_c}{\omega_c\lambda}\, e^{-\omega_c(t-s)} + \frac{2\omega_c-\lambda}{\lambda\omega_c}\right) + 1\right] \qquad (23)$$

and

$$\langle C(s)N(t)\rangle = N_oC_o\left[D\left(2s + \frac{1}{\omega_c}\, e^{-\omega_c s} + \frac{\lambda-\omega_c}{\omega_c\lambda}\, e^{-\omega_c t}\right.\right.$$

$$\left.\left. - \frac{\lambda-\omega_c}{\omega_c\lambda}\, e^{-\omega_c(t-s)} - \frac{1}{\omega_c}\right) + 1\right] \qquad (24)$$

At time $s=t \gg 1/\omega_c$, the second moments of $N(t)$ and $C(t)$ can be reduced to the following simple form

$$\langle C^2(t)\rangle = C_o^2\left[2Dt+1\right] \qquad (25)$$

$$\langle N^2(t)\rangle = N_o^2\left[2D(t+1/\lambda) + \frac{\langle\rho^2\rangle}{\beta^2} + 1\right] \qquad (26)$$

$$\langle N(t)C(t)\rangle = N_oC_o\left[D(2t+1/\lambda) + 1\right] \qquad (27)$$

2.3.2 Exact Calculations

A direct integration of Eq. (6) yields

$$C(t) = C_o \exp\left[\frac{\lambda}{\beta}\int_o^t \rho(u)\,du\right] \tag{28}$$

Hence,

$$<C(t)> = C_o <\exp\left[\frac{\lambda}{\beta}\int_o^t \rho(u)\,du\right]> \tag{29}$$

The exponent in the above equation is a centred Gaussian random variable. For such variables X, a most useful lemma is

$$<e^X> = e^{\frac{1}{2}<X^2>} \tag{30}$$

Equation (29), giving the expectation (or mean) value for the precursor population then becomes

$$<C(t)> = C_o \exp\left[\frac{\lambda^2}{2\beta^2}\int_o^t\int_o^t <\rho(u)\,\rho(v)>du\,dv\right]$$

The double integral is evaluated in the same way as in Eq. (14). We obtain

$$<C(t)> = C_o \exp\left[D\left(t + \frac{e^{-\omega_c t}}{\omega_c} - \frac{1}{\omega_c}\right)\right] \tag{31}$$

where D is the same as given by Eq. (18). Substitution of Eq. (28) in Eq. (5) yields the following expression for the neutron population

$$N(t) = N_o(1+\rho(t)/\beta)\exp(\frac{\lambda}{\beta}\int_o^t \rho(u)du) \qquad (32)$$

Taking the ensemble average

$$\langle N(t)\rangle = N_o\left[\left\langle \exp(\frac{\lambda}{\beta}\int_o^t \rho(u)du)\right\rangle + \left\langle \frac{\rho(t)}{\beta}\exp(\frac{\lambda}{\beta}\int_o^t \rho(v)dv)\right\rangle\right] \qquad (33)$$

The first average on the right hand side has already been evaluated. The method of evaluating the second average is given in the appendix. After some algebra, we obtain

$$\langle N(t)\rangle = N_o\left[1 + \frac{D}{\lambda}(1 - e^{-\omega_c t})\right] \cdot \exp\left[D(t + \frac{e^{-\omega_c t}}{\omega_c} - \frac{1}{\omega_c})\right] \qquad (34)$$

From Eq. (28), the expression for the autocorrelation function of C(t) is obtained as

$$\langle C(s)C(t)\rangle = C_o^2\left\langle \exp(\frac{\lambda}{\beta}\int_o^s \rho(u)du + \frac{\lambda}{\beta}\int_o^t \rho(u)du)\right\rangle \qquad (35)$$

Using lemma (30), we have

$$\langle C(s)C(t)\rangle = C_o^2 \exp\left[\frac{\lambda^2}{2\beta^2}\left(\int_o^s\int_o^s \langle\rho(u)\,\rho(v)\rangle\,dudv + \int_o^t\int_o^t \langle\rho(u)\,\rho(v)\rangle\,dudv + 2\int_o^t\int_o^s \langle\rho(u)\,\rho(v)\rangle\,dudv\right)\right]$$

The various integrals can be evaluated, yielding for $o \leq s \leq t$

$$\langle C(s)C(t)\rangle = C_o^2 \exp\left[D \left(3s+t+\frac{2}{\omega_c}\left(e^{-\omega_c s} + e^{-\omega_c t}\right) - \frac{e^{-\omega_c (t-s)}}{\omega_c} - \frac{3}{\omega_c}\right)\right] \qquad (36)$$

The autocorrelation function of the neutron population is obtained from Eq. (32). We get

$$\langle N(s)N(t)\rangle = N_o^2 \Big\langle (1+\rho(s)/\beta)(1+\rho(t)/\beta) \cdot \exp\left[\frac{\lambda}{\beta} \left(\int_o^t \rho(u)\,du + \int_o^s \rho(u)\,du \right)\right] \Big\rangle \qquad (37)$$

The evaluation of various averages is given in the appendix. The expression is, finally, evaluated as, for $o \leq s \leq t$

$$\langle N(s)N(t)\rangle = N_o^2 \left[1 + \frac{2D}{\lambda} \left(2-e^{-\omega_c s} - e^{-\omega_c t}\right) + \frac{\langle\rho^2\rangle}{\beta^2} e^{-\omega_c (t-s)} + \frac{D^2}{\lambda^2} \left(2\, e^{-\omega_c (t-s)} + 2\, e^{-\omega_c (2t-s)} - e^{-2\omega_c (t-s)} - 8e^{-\omega_c t} - 2e^{-\omega_c s} + 4\, e^{-\omega_c (s+t)} + 3\right)\right] \cdot \exp\left[D\left(3s+t + \frac{2}{\omega_c}\left(e^{-\omega_c s} + e^{-\omega_c t}\right) - \frac{1}{\omega_c} e^{-\omega_c (t-s)} - \frac{3}{\omega_c}\right)\right] \qquad (38)$$

Proceeding the same way, the expressions for the cross-correlation functions are obtained. We have

$$<N(s)C(t)> \;=\; N_oC_o\left[\,1 + \frac{D}{\lambda}\left(3 - 2e^{-\omega_c s} - e^{-\omega_c(t-s)}\right)\right]\cdot$$

$$\exp\left[D\left(3\,s + t + \frac{2}{\omega_c}\left(e^{-\omega_c s} + e^{-\omega_c t}\right)\right.\right.$$

$$\left.\left. - \frac{1}{\omega_c}\, e^{-\omega_c(t-s)} - \frac{3}{\omega_c}\right)\right] \qquad (39)$$

$$<C(s)N(t)> \;=\; N_oC_o\left[\,1 + \frac{D}{\lambda}\left(1 - 2e^{-\omega_c t} + e^{-\omega_c(t-s)}\right)\right]\cdot$$

$$\exp\left[D\left(3s + t + \frac{2}{\omega_c}\left(e^{-\omega_c s} + e^{-\omega_c t}\right)\right.\right.$$

$$\left.\left. - \frac{1}{\omega_c}\, e^{-\omega_c(t-s)} - \frac{3}{\omega_c}\right)\right] \qquad (40)$$

At times $s=t \gg 1/\omega_c$, the moments of neutrons and precursors are reduced to the following expressions:

$$<N(t)> \;=\; N_o\left(1 + \frac{D}{\lambda}\right) e^{Dt} \qquad (41)$$

$$<C(t)> \;=\; C_o\, e^{Dt} \qquad (42)$$

$$<N^2(t)> \;=\; N_o^{\,2}\left[\,1 + \frac{<\rho^2>}{\beta^2} + \frac{4D}{\lambda}\left(1 + \frac{D}{\lambda}\right)\right] e^{4Dt} \qquad (43)$$

$$\langle C^2(t)\rangle = C_o^2\, e^{4Dt} \tag{44}$$

$$\langle NC(t)\rangle = N_oC_o \left(1 + \frac{2D}{\lambda}\right) e^{4Dt} \tag{45}$$

2.4 Normalised Centred Covariances for $t \gg 1/\omega_c$

The expressions for the normalised centred covariances of neutrons and precursors can be easily obtained from their expressions for the first and second moments. At time $t \gg 1/\omega_c$ they reduce to very simple forms for the linearised case as well as for the exact calculations.

Case a Linearisation

$$\mu_{CC}(t,t) = \frac{\langle C^2(t)\rangle - \langle C(t)\rangle^2}{C_o^2}$$

$$= 2Dt \tag{46}$$

$$\mu_{NN}(t,t) = \frac{\langle N^2(t)\rangle - \langle N(t)\rangle^2}{N_o^2}$$

$$= 2D(t+1/\lambda) + \langle\rho^2\rangle/\beta^2 \tag{47}$$

$$\mu_{NC}(t,t) = \frac{\langle NC(t)\rangle - \langle N(t)\rangle\langle C(t)\rangle}{N_oC_o}$$

$$= D(2t + 1/\lambda) \tag{48}$$

Case b Exact Calculations

$$\mu_{NN}(t,t) = \left[1 + \frac{\langle\rho^2\rangle}{\beta^2} + \frac{4D}{\lambda}\left(1 + \frac{D}{\lambda}\right)\right] e^{4Dt} - \left[1 + \frac{D}{\lambda}\right]^2 e^{2Dt} \qquad (49)$$

$$\mu_{CC}(t,t) = e^{4Dt} - e^{2Dt} \qquad (50)$$

$$\mu_{NC}(t,t) = \left[1 + \frac{2D}{\lambda}\right] e^{4Dt} - \left(1 + \frac{D}{\lambda}\right) e^{2Dt} \qquad (51)$$

3. Discussion

Eqs. (41) and (42) show an apparent exponential increase in time in the expected values of neutron and precursor populations. This increase suggests that the fluctuations in reactivity act as a positive feedback to the system. However, it is to be noted that the value of the exponent in these equations is very small. For a quantitative estimate, we take $\lambda = .1$/sec, assume a reactivity noise amplitude $\langle\rho^2\rangle/\beta^2 \simeq 10^{-6}$ and the extremely low value of $\omega_c = \lambda$ for the corner frequency. This gives $D \approx 10^{-7}$/sec. Hence even for $t = 10^5$ secs (= 1 day), the exponent has a magnitude of .01 only. Hence, the increase in the mean neutron and precursor numbers, which is in fact due to nonlinearities, is negligibly small.

From Eq. (41), it is also seen that in the limit of $t \to 0$, $\langle N(t)\rangle$ does not approach the assumed initial value N_o. The extra term $(D/\lambda)N_o$ is due to the use of the quasistatic approximation, which admits a discontinuity in the mean neutron number at a point of discontinuity in the reactivity.

In Eqs. (36), (38), (39) and (40) for the autocorrelation function, it is obvious that, because of the order of magnitude for D, a Taylor's expansion of the exponential could be truncated after the linear term, even for large values of s and t (say 10^5 secs). These expressions then become practically linear in time, and if one neglects the terms containing higher powers of D, they become exactly equal to their corresponding expressions obtained for the linearised case.

It is noted, from Eqs. (49) to (51), that, if we restrict the expansion of exponentials to linear terms, the asymptotic values for all the normalised centred covariances of precursors and neutrons are almost equal, increasing linearly in time according to the term 2Dt, D being defined by Eq. (18). This value of D is exactly the same as that previously found /11/ for the case of reactivity noise with $\omega_c \gg \beta/l$. This, therefore, confirms that the deviations developed by the system, due to inherent reactivity noise, are of extremely low magnitude. Hence, the system may, with a high degree of confidence, be approximated as stationary ergodic, for evaluating the effect of all other noise sources on the response of a reactor.

The main reasons for these low deviations, as discussed earlier /11/, could be

a) the unproved assumption of a well defined initial state,

b) the missing influence of other system parameters, which should be included in a more realistic evaluation.

First hand assessments show that the effect (a) of initial conditions on the drift of the system should be of greater importance than the asymptotic effect of band limited white reactivity noise. From the theory and application of the inhour equation /12/, one can show that the asymptotic development of the neutron population is given by

$$N(t) = A\, e^{\omega t}$$

where A is a constant and

$$\omega = \rho / l_{eff}; \quad l_{eff} = \sum_i \beta_i / \lambda_i$$

Taking delayed neutron parameters of U-235 /13/ a typical value for the effective neutron life time is

$$l_{eff} = 82 \text{ msec.}$$

Assuming an initial reactivity $\rho \approx 10^{-3}\ \beta$, we find that

$$\omega \sim .8\ 10^{-4}/\text{sec.}$$

Hence even for short observation intervals of $t \approx 10$ secs, the accumulated drift ($\approx \omega t$) is of the order of 10^{-3}. This is much larger than the previously investigated effect.

4. Conclusions

We have studied the response of a critical power reactor to the inherent reactivity noise, assumed to be band limited white. Since the corner frequency ω_c of the noise is in general much less than β/l in a power reactor, the analysis has been developed in the quasistatic approximation.

Considering a single group of delayed neutrons explicit expressions, for the exact time development of the first and second moments of the neutron and precursor numbers, have been derived. The validity of the linearised equations is also discussed and found applicable in most practical cases.

As a special application of the derived expressions, it has been found that the normalised centred covariances of neutrons and precursors increase linearly with time at a very low rate. This rate is exactly equal to that of the earlier investigated opposite case (for $\omega_c \gg \beta/l$). It is, therefore, confirmed that the mean square deviations developed by a reactor system due to band limited white reactivity noise are of very small order, and hence the system could, with a high degree of confidence, be approximated as stationary ergodic.

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Appendix

Eq. (37), term by term, is written as

$\langle N(s)N(t)\rangle$

$$= N_o^2 \left[\left\langle e^{X_1+X_2} \right\rangle + \left\langle \frac{\rho(t)}{\beta} e^{X_1+X_2} \right\rangle + \left\langle \frac{\rho(s)}{\beta} e^{X_1+X_2} \right\rangle + \left\langle \frac{\rho(s)\rho(t)}{\beta^2} e^{X_1+X_2} \right\rangle \right] \qquad \text{(A-1)}$$

where
$$X_1 = \frac{\lambda}{\beta} \int_o^t \rho(u)\,du \qquad \text{(A-2)}$$

$$X_2 = \frac{\lambda}{\beta} \int_o^s \rho(u)\,du \qquad \text{(A-3)}$$

X_1 and X_2 are centred Gaussian random variables.

We first of all evaluate the average

$$\left\langle \frac{\rho(s)\rho(t)}{\beta^2} e^{X_1+X_2} \right\rangle$$

Generalising a procedure used by Akcasu /10/, this average can be written in the form

$$\frac{\partial^2}{\partial\alpha_1\partial\alpha_2} \left\langle e^{\alpha_1\frac{\rho(s)}{\beta} + \alpha_2\frac{\rho(t)}{\beta} + X_1+X_2} \right\rangle \Bigg|_{\alpha_1,\alpha_2 = o} \qquad \text{(A-4)}$$

Using lemma (30), the equation (A-4) is written as

$$\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}\left[\exp\left\{\frac{1}{2}\left(\frac{\alpha_1^2}{\beta^2}<\rho^2(s)> + \frac{\alpha_2^2}{\beta^2}<\rho^2(t)> + <X_1^2>\right.\right.\right.$$

$$+ <X_2^2> + 2<X_1X_2> + \frac{2\alpha_1\alpha_2}{\beta^2}<\rho(s)\rho(t)>$$

$$+ \frac{2\alpha_1}{\beta}<\rho(s)X_1> + \frac{2\alpha_1}{\beta}<\rho(s)X_2> + \frac{2\alpha_2}{\beta}<\rho(t)X_1>$$

$$\left.\left.\left. + \frac{2\alpha_2}{\beta}<\rho(t)X_2>\right)\right\}\right]_{\alpha_1,\alpha_2 = o}$$

$$= \frac{1}{\beta^2}\Big[<\rho(s)\rho(t)> + (<\rho(t)X_1> + <\rho(t)X_2>)\cdot$$

$$(<\rho(s)X_1> + <\rho(s)X_2>)\Big]\; e^{\frac{1}{2}\langle(X_1+X_2)^2\rangle} \qquad \text{(A-5)}$$

The second and third terms of the right hand side of Eq. (A-1) can be evaluated by writing them in the form

$$\frac{\partial}{\partial\alpha}\left\langle e^{\alpha\frac{\rho(t)}{\beta} + X_1+X_2}\right\rangle\Bigg|_{\alpha = o}$$

and proceeding as above.

The first term is obtained directly using lemma (30).

Evaluating the various terms, the following expression for the autocorrelation function of the neutron population is obtained:

$$
\begin{aligned}
&\langle N(s)N(t)\rangle \\
&= N_o^2 \Big[1 + \frac{1}{\beta} \left(\langle \rho(s)X_1\rangle + \langle \rho(s)X_2\rangle + \langle \rho(t)X_1\rangle \right. \\
&\qquad \left. + \langle \rho(t)X_2\rangle \right) + \frac{1}{\beta^2} \langle \rho(s)\rho(t)\rangle + \\
&\qquad \frac{1}{\beta^2} \left(\langle \rho(t)X_1\rangle + \langle \rho(t)X_2\rangle \right) \left(\langle \rho(s)X_1\rangle \right. \\
&\qquad \left. + \langle \rho(s)X_2\rangle \right) \Big] \cdot e^{\frac{1}{2}\langle (X_1+X_2)^2\rangle} \qquad \text{(A-6)}
\end{aligned}
$$

Inserting the definitions (A-2),(A-3) into this expression and evaluating various integrals, one obtains Eq. (38).