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Sum rules for baryon form factors of second-class currents

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ABSTRACT

By means of high energy constraints we relate the second-class current vertices $\langle \Delta' | J_\mu^{II} | \Delta \rangle$ and $\langle \Delta' | J_\mu^{II} | N \rangle$ with the $\langle N' | J_\mu^{II} | N \rangle$ vertex. (N denote the nucleon and Δ the $\Delta(1232)$-resonance.) For this purpose superconvergence sum rules are derived for a second-class vector ($J_\mu^{II} = V_\mu^{II}$) and axial vector current ($J_\mu^{II} = A_\mu^{II}$). This report extends the previous work (ref. |1|) to second-class form factors.

Summenregeln für Baryon-Formfaktoren der Ströme zweiter Art

ZUSAMMENFASSUNG

Mit Hilfe von Hochenergiebedingungen lassen sich die Matrixelemente $<\Delta'|J_{\mu}^{II}|\Delta>$ und $<\Delta'|J_{\mu}^{II}|N>$ der Ströme zweiter Art mit dem Matrixelement $<N'|J_{\mu}^{II}|N>$ verknüpfen. (N bezeichnet das Nukleon und Δ die Δ(1232)-Resonanz.) Zu diesem Zweck werden Superkonvergenzsummenregeln für Vektor- ($J_{\mu}^{II} = V_{\mu}^{II}$) und Axialvektorströme ($J_{\mu}^{II} = A_{\mu}^{II}$) zweiter Art aufgestellt. In diesem Bericht wird das Verfahren der vorangegangenen Arbeit (Ref. |1|) auf Formfaktoren zweiter Art erweitert.

CONTENTS

I. INTRODUCTION

This is a technical report in which we quote explicitly sum rules for the form factors of the second-class current vertices $\langle N'|J_\mu^{II}|N\rangle$, $\langle \Delta'|J_\mu^{II}|N\rangle$ and $\langle \Delta'|J_\mu^{II}|\Delta\rangle$ with N the nucleon and Δ the Δ(1232) resonance. It completes the appendix C of |1| - furthermore denoted by I - by the corresponding sum rules of second-class currents.

Second-class currents are destinguished from first-class currents in the following way: Assume J_μ^o represents the neutral member (I_3=0) of an isospin current multiplet (I=0,1,...), then

$$J_\mu^o \;=\; \underset{(-)}{+}\; (J_\mu^o)^+ \qquad \text{(I.1)}$$

defines a first- (second)-class current. There is an alternate definition by means of the G-parity operator originally due to S. Weinberg |2| (see also P. Langacker |3|).

$$G\, J_\mu^o\, G^{-1} \;=\; \underset{(+)}{-}\; (-1)^I\, \eta_p\, J_\mu^o \qquad , \qquad \text{(I.1')}$$

which is equivalent to (I.1) as long as the TCP theorem holds (η_p = +1(-1), the parity of the vector (axial vector) current). For example in the usual decomposition of the nucleon V-A current

$$\bar{u}(p')\big[\gamma_\mu F_1^V + \frac{i}{2m}\sigma_{\mu\nu}(p'-p)^\nu F_2^V + \underline{\frac{1}{m}(p'-p)_\mu F_3^V}$$

$$+ \gamma_\mu\gamma_5 F^A + \frac{1}{m}(p'-p)_\mu\gamma_5 F^P + \underline{\frac{1}{m}(p'+p)_\mu\gamma_5 F_3^A}\big]u(p) \quad , \qquad \text{(I.2)}$$

$$F_1^V(0) = 1 \;,\qquad F^A(0) = -1.26,$$

the underlined terms represent the second-class contributions.

Second-class currents cannot be excluded a priori in weak interactions. Nevertheless they are mostly disregarded in theoretical quark and gauge models. On the other side the experimental verification of second-class currents in nuclear β-decay is an open question and quite controversial as the recent development has shown |4|.

In |5| and |6| (see also I) we have used successfully pole term sum rules to determine the first-class matrix elements $<\Delta'|J^I_\mu|N>$ and $<\Delta'|J^I_\mu|\Delta>$ in terms of the experimentally known form factors of $<N'|J^I_\mu|N>$. In this report we summarize the corresponding sum rules for second-class currents, which are evaluated in |7|. There it is shown that in the framework of our approximation second-class contributions do not exist for small momentum transfers.

We now make a few comments on the derivation of the sum rules to give some background information which will facilitate the orientation in the following chapters. More details are found in |5|, |6| or I. Only isoscalar and isovector currents are considered. The sum rules are derived for the superconvergent parity conserving and regularized t-channel helicity amplitudes $G^{1,2}_\Lambda(s,t,u)$ of |8| for the peripheral processes

$$\Delta + J^{II}_\mu \rightarrow \Delta + \pi \quad , \tag{I.3}$$

$$N + J^{II}_\mu \rightarrow \Delta + \pi \quad , \tag{I.4}$$

$$N + J^{II}_\mu \rightarrow N + \pi \quad . \tag{I.5}$$

The reactions (I.3 - I.5) are initiated by a vector or axial vector field $J^{II}_\mu = V^{II}_\mu, A^{II}_\mu$.

It is assumed that in forward direction, $t\approx 0$, the high energy behaviour of $G^{1,2}$ is determined by the leading t-channel Regge-trajectory

$$G_\Lambda^{1,2}(s,t,u) \underset{s\to\infty}{\sim} s^{\alpha_{eff}(t)-m} \quad , \quad t \approx 0 \text{ , fixed} \quad . \qquad (I.6)$$

In (I.6) m denotes the maximal helicity flip; m is a function of the helicities Λ of all particles. Superconvergence means $\alpha_{eff}-m < -1$. If $\alpha_{eff}-m+1 < -n$, n integer ≥ 0, the following sum rules are valid:

$$\int_{s_o}^{\infty} ds'(s')^n \text{ Im } G_\Lambda^{1,2}(s',t,u') - \int_{u_o}^{\infty} du'(\Sigma-t-u')^n \text{ Im } G_\Lambda^{1,2}(s',t,u') = 0 \qquad (I.7)$$

with s+t+u = Σ. In practice usually n=0. If m=3 also n=1 is possible, in which case we speak of a moment sum rule. The effective Regge-trajectories are specified in table 1.

TABLE 1: Leading trajectories of the t-channel

particles		$I_t = 0$		$I_t = 1$		$I_t = 2$	
a	$\bar{c}$	nat. parity	unnat. parity	nat. parity	unnat. parity	nat. parity	unnat. parity
$V_\mu^{II}(I=1)$	π	$\alpha_{eff}(0)<1$	η	ρ	B	exotic	
$V_\mu^{II}(I=0)$	π	-	-	A_2	π	-	
$A_\mu^{II}(I=1)$	π	ω	$\alpha_{eff}(0)<0$	A_2	A_1	exotic	
$A_\mu^{II}(I=0)$	π	-	-	ρ	B	-	

$\alpha_{eff}^{I_t=2}(0) < 1$, $\alpha_\rho(0) \approx \alpha_{A_2}(0) \approx \alpha_\omega(0) \approx 0.5$

$\alpha_\pi(0) \approx \alpha_{A_1}(0) \approx 0.0$, $\alpha_\eta(0) \approx \alpha_B(0) < 0$

The sum rules (I.7) are saturated by the N- and Δ-intermediate contributions in the zero width approximation $\Gamma_\Delta = 0$, so that we obtain the pole term sum rules

$$\sum_{i=N,\Delta} \{\mathrm{Res}_{s'=m_i^2}\left[s'^n G_\Lambda^{1,2}(s',t,u')\right] - \mathrm{Res}_{u'=m_i^2}\left[(\Sigma-t-u')^n G_\Lambda^{1,2}(s',t,u')\right]\} = 0 \qquad \text{(I.8)}$$

These sum rules and some of their first derivatives with respect to t at t=0 are listed in the sections III and IV (we call them t^o- and t^1-sum rules, respectively). Sum rules for higher derivatives yield no information since with increasing number of derivatives more and more of the low partial amplitudes get truncated. Among the t^1-sum rules we consider only those as reliable which converge very fast, i.e.

$$|s^n\, G_\Lambda^{1,2}(s,t,u)| \;<\; |s|^{-2} \quad , \quad n = 0,1 \quad , \quad s \to \infty \qquad \text{(I.9)}$$

Furthermore we restrict ourselves only to sum rules with helicity flip $m \geq 2$ for the reactions (I.3) and (I.4). For $m < 2$ the sum rules are not useful on account of their complicated kinematical structure.

II. NOTATIONS

The sum rules in the following sections III and IV are compiled for $G_\Lambda^{1,2}$ amplitudes with definite isospin in the t-channel $I_t = 0,1,2$. The definitions of the vertices, phase conventions and other technicalities are taken from |5|. Each sum rule contains the complete N- and Δ-contribution. The abbreviations $L_{N(\Delta)}$(No) have been introduced to denote the N(Δ)-contribution of the lefthand side of equation (No); analogously L(No) denotes the complete lefthand side of (No).

For the reaction (I.3) we destinguish three classes of sum rules: (1) sum rules of the helicity flip m=3 amplitudes, (2) moment sum rules (n=1) of the m=3 amplitudes, (3) sum rules of the m=2 amplitudes. Only the crossing antisymmetrical amplitudes lead to non-trivial sum rules in the cases (1) and (3) and the crossing symmetrical amplitudes in the case (2). The amplitudes of the reaction (I.4), $N + J_\mu^{II} \to \Delta+\pi$, have no definite crossing properties. To separate first-class sum rules from second-class ones, one has to combine the sum rules for (I.4) with the corresponding equations for the reaction $\Delta + J_\mu^{II} \to N+\pi$. For the reaction (I.5) again only the crossing antisymmetrical amplitudes lead to non-trivial sum rules.

The following abbreviations are used:

$$IS = -\frac{8}{15}\left(\frac{2}{3}\right) \qquad \text{(II.1)}$$

for the isospin $I_t = 0(2)$.

$$IS_1 = 2(-2) , \quad IS_2 = -\frac{5}{9}\left(\frac{1}{9}\right) , \quad IS_3 = -\frac{5}{12}\left(-\frac{1}{12}\right) , \qquad \text{(II.2)}$$

for $I_t = 1(2)$. The masses are denoted by

$$\text{pion: } m_\pi = 0.1396 \text{ GeV} , \quad \text{nucleon: } m = 0.9383 \text{ GeV},$$
$$\Delta(1232)\text{-resonance: } M = 1.232 \text{ GeV} \quad . \qquad \text{(II.3)}$$

Finally the following kinematical symbols will be used

$$M_+ = M+m , \quad M_- = M-m , \quad MDN2 = M^2+m^2 - Mm-m_\pi^2 \quad , \qquad \text{(II.4)}$$

$$y_N = -[2M_+M_- + m_\pi^2 + K^2], \quad y_\Delta = -[m_\pi^2 + K^2] , \tag{II.5}$$

$$g_{12} = 2M^2 g_1 + m_\pi^2 g_2 , \quad \overline{g_2} = \frac{g_{12}}{M^2} - 3g_2 , \tag{II.6}$$

$$f(G_i,G_j) = 4\,\overline{g_2}\,G_i - \left(\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2}\right) G_j . \tag{II.7}$$

In the following we quote explicitly only the sum rules of the isovector current J_μ^{II}. The corresponding sum rules of the isoscalar current are given by the following substitutions: For all three reactions (I.3-I.5) only I_t=1 sum rules exist. They are obtained for the reaction (I.3) from the I_t=0,2 isovector sum rules by setting IS=0, since the $<\Delta'|J_\mu^{II}|N>$ - vertex does not contribute. The form factors of the $<\Delta'|J_\mu^{II}|\Delta>$ - vertex G_i^V are to be replaced by the isoscalar form factors G_i^S, |5|. For the second reaction (I.4) one has to put $IS_1 = IS_2 = 0$, $IS_3 = -1/12$ and to replace the isovector form factors of the $<N'|J_\mu^{II}|N>$- and $<\Delta'|J_\mu^{II}|\Delta>$-vertex by the corresponding isoscalar ones. There are no non-trivial sum rules for the last reaction (I.5).

For typographical reasons we denote in the sections III and IV the isospin in the t-channel by I instead of I_t.

The parity conserving, regularized helicity amplitudes $G^{1,2}_{\lambda_d\lambda_{\bar b},\lambda_{\bar c}\lambda_a}$ corresponds to the t-channel reaction $\bar\pi(q,S) + J_\mu^{II}(K,\lambda_a) \to B_2(p_2,\lambda_d) + \overline{B_1}(p_1,\lambda_{\bar b})$. For further details in particular the definition of the hadronic coupling constants g, g^*, (g_1,g_2) of the πNN, $\pi N\Delta$, $\pi\Delta\Delta$ vertices see the appendix of I or |5|. There one finds also the definition of the nucleon, N-Δ excitation and Δ form factors: F_i, C_i, G_i.

III. SUM RULES FOR THE VECTOR CURRENT V_μ^{II}

III.1 $\Delta V_\mu^{II} \to \Delta \pi$

III.1.1.1 Sum rules of the m=3, I=0,2 amplitudes

$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}$, $G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S0}$, $G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SS}$ are crossing symmetrical.

a) t^o-sum rules

$$G^{1V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}: \quad IS\cdot Mg^*\left(\frac{m}{2M} C_3^{V'} + C_4^{V'}\right) + \frac{1}{9}\frac{m^2}{M^2}\frac{g_{12}}{M^2} G_7^V = 0 \qquad \text{(III.1)}$$

b) t^1-sum rules

$$G^{1V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}: \quad IS\cdot Mg^* C_4^{V'} + \frac{2}{9}\frac{m^2}{M^2}\frac{g_{12}}{M^2} G_7^V = 0 \qquad \text{(III.2)}$$

III.1.1.2 Sum rules of the m=3, I=1 amplitudes

$G^{1V\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}$ is crossing symmetrical.

a) t^o-sum rules

$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}$ has no t^o-contribution.

$$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},SO}: \quad \{-\frac{4}{3} Mg^{*}[K^2(C_4^{V'}-C_5^{V'})-m^2C_6^{V'}] + \frac{4}{9}\frac{m^2}{M^2} g_{12}\ G_7^V\}(m_\pi^2-K^2) = 0 \qquad \text{(III.3)}$$

$$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},SS}: \quad -\frac{4}{3} Mg^{*}\ C_6^{V'} + \frac{1}{9}\frac{K^2}{M^2} f\ (G_5^V,G_6^V) - \frac{4}{9}(\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2})G_7^V = 0 \qquad \text{(III.4)}$$

b) t^1-sum rules

$$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}: \quad [-\frac{4}{3} Mg^{*}C_4^{V'} + \frac{2}{9}\frac{m^2}{M^2}(\overline{g_2}-3g_2)G_7^V](m^2-K^2)+\frac{2}{m_\pi^2-K^2} L(III.3) = 0 \qquad \text{(III.5)}$$

$$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},SO}: \quad [-\frac{4}{3} Mg^{*}C_4^{V'}+\frac{2}{9}\frac{m^2}{M^2}(\overline{g_2}-3g_2)G_7^V]K^2-\frac{2}{3}\frac{m^2}{M^2}(m_\pi^2-K^2)g_2G_7^V-\frac{1}{2}\frac{1}{m_\pi^2-K^2}L(III.3)=0 \qquad \text{(III.6)}$$

$$G^{2V\ I=1}_{\frac{3}{2}-\frac{3}{2},SS}: \quad g_2\ (G_7^V + \frac{K^2}{4M^2} G_6^V) = 0 \qquad \text{(III.7)}$$

III.1.2.1 Moment sum rules of the m=3, I=0,2 crossing symmetrical amplitudes

a) t^o-sum rules

$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}$ has no t^o-contribution.

$$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SO}: \quad -\frac{3}{4}\, IS \cdot y_N\, L_N(III.3) + y_\Delta\, L_\Delta(III.3) = 0 \qquad (III.8)$$

$$G^{2V,I=0,2}_{\frac{3}{2}-\frac{3}{2},SS}: \quad -\frac{3}{4}\, IS \cdot y_N\, L_N(III.4) + y_\Delta\, L_\Delta(III.4) = 0 \qquad (III.9)$$

b) t^1-sum rules

$$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}: \quad -\frac{3}{4}\, IS \cdot y_N\, L_N(III.5) + y_\Delta\, L_\Delta(III.5) = 0 \qquad (III.10)$$

$$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SO}: \quad -\frac{3}{4}\, IS \cdot \frac{y_N}{M^2} L_N(III.6) + \frac{y_\Delta}{M^2}\, L_\Delta(III.6) - \frac{3}{4}\, IS \cdot L_N(III.3) + L_\Delta(III.3) = 0 \qquad (III.11)$$

$$G^{2V\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SS}: \quad \frac{y_\Delta}{M^2} \frac{4}{3}\, L(III.7) - \frac{3}{4}\, IS \cdot L_N(III.4) + L_\Delta(III.4) = 0 \qquad (III.12)$$

III.1.2.2 Moment sum rules of the m=3, I=1 crossing symmetrical amplitudes

a) t^0-sum rules

$$G^{1V\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}: \quad -\frac{4}{3}\, y_N\, Mg^* \left(\frac{m}{2M}\, C_3^{V'} + C_4^{V'}\right) + y_\Delta\, \frac{1}{9} \frac{m^2}{M^2} \frac{g_{12}}{M^2}\, G_7^V = 0 \qquad (III.13)$$

t^1-sum rules are not considered on account of the convergence criterion (I.9).

III.1.3 Sum rules of the m=2 amplitudes

We use the linear combinations

$A_1^{1V(\pm)}$, $A_1^{2V(\pm)}$, $A_0^{1V(\pm)}$ of I and

$$A_S^{1V(\pm)} := G^{1V}_{\frac{3}{2}-\frac{1}{2},ss} \pm G^{1V}_{\frac{1}{2}-\frac{3}{2},ss}$$

III.1.3.1 Sum rules of the m=2, I=0,2 amplitudes

$A_1^{1V(+)I=0,2}$, $A_1^{2V(-)I=0,2}$, $A_0^{1V(+)I=0,2}$, $A_S^{1V(+)I=0,2}$ are crossing symmetrical.

a) t^o-sum rules

$A_1^{1V(-)I=0,2}$, $A_0^{1V(-)I=0,2}$, $A_S^{1V(-)I=0,2}$ have no t^o-contributions.

$A_1^{2V(+)I=0,2}$: The sum rules are identical with (III.1).

b) t^1-sum rules

$$A_1^{1V(-)I=0,2}:\ \{IS\cdot Mg^*[(M_+ - \frac{m_\pi^2-K^2}{4M})m\, C_3^{V'} + M_+M_-\ C_4^{V'} + K^2C_5^{V'} + m^2C_6^{V'}]$$

$$+ \frac{1}{6}\frac{m^2}{M^2}(m_\pi^2+K^2)(\overline{g_2}-g_2)G_7^V + \frac{1}{9}\frac{m^2}{M^2}\frac{4M^2-m_\pi^2}{M^2}g_{12}G_7^V\}(m_\pi^2-K^2)=0 \qquad \text{(III.14)}$$

$$A_0^{1V(-)I=0,2}: \ \{IS\cdot Mg^{*}[\frac{m}{2M} K^2 C_3^{V'} - \frac{2M^2+2m^2-K^2-m_\pi^2}{4M^2}(K^2(C_4^{V'}-C_5^{V'})-m^2 C_6^{V'})]$$

$$+ \frac{1}{9}\frac{m^2}{M^2}\frac{8M^2+m_\pi^2}{M^2} g_{12} G_7^V - \frac{4}{3}\frac{m^2}{M^2} m_\pi^2 g_2 G_7^V\} (m_\pi^2-K^2) + \frac{2K^2}{m_\pi^2-K^2} L(III.14) = 0 \qquad (III.15)$$

$$A_S^{1V(-)I=0,2}: \ IS\cdot Mg^{*} C_6^{V'} + \frac{2}{9}\frac{K^2}{M^2}[\frac{2g_{12}}{M^2} G_5^V + (\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\frac{3g_2-\overline{g_2}}{2}) G_6^V] \qquad (III.16)$$

$$+ \frac{8}{9}(\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\frac{3g_2-\overline{g_2}}{2}) G_7^V - \frac{4}{3}\frac{m_\pi^2-K^2}{M^2} L(III.7) + \frac{1}{4M^2} L(III.9) = 0$$

III.1.3.2 Sum rules of the m=2, I=1 amplitudes

$A_1^{1V(-)I=1}$, $A_1^{2V(+)I=1}$, $A_0^{1V(-)I=1}$, $A_S^{1V(-)I=1}$ are crossing symmetrical.

a) t^o-sum rules

$A_1^{1V(+)I=1}$, $A_1^{2V(-)I=1}$ have no t^o-contributions.

$A_0^{1V(+)I=1}$, $A_S^{1V(+)I=1}$: The sum rules are identical with (III.3, III.4).

b) t^1-sum rules

$$A_1^{1V(+)I=1}: \quad [L(A13) - 2L(A5)]\ (m_\pi^2 - K^2) = 0 \qquad \text{(III.17)}$$

$$A_0^{1V(+)I=1}: \quad [-\frac{4}{3} Mg^* K^2 (\frac{m}{2M} C_3^{V'} + C_4^{V'}) - \frac{1}{9}\frac{m^2}{M^2}\frac{2MM_- - K^2}{M^2} g_{12} G_7^V](m_\pi^2 - K^2)$$

$$+ (m_\pi^2 - K^2) L(III.6) - K^2 L(III.3) - [\frac{m_\pi^2 + K^2}{m_\pi^2 - K^2} + (\frac{m}{2M} + \frac{m_\pi^2 - K^2}{4M^2})]\ L(III.3) = 0 \qquad \text{(III.18)}$$

$$A_S^{1V(+)I=1}: \quad -\frac{4}{3} Mg^* C_6^{V'} + \frac{1}{9}\frac{K^2}{mM} \{4(3g_2 - 2\overline{g_2}) G_5^V - [\frac{g_{12}}{M^2} - \frac{K^2}{M^2}(3g_2 - 2\overline{g_2})] G_6^V\}$$

$$- \frac{4}{9}\frac{M}{m}[\frac{g_{12}}{M^2} - \frac{K^2}{M^2}(3g_2 - 2\overline{g_2})] G_7^V - \frac{4}{3}\frac{M}{m}\frac{m_\pi^2 - K^2}{M^2} L(III.7) + \frac{1}{2}\frac{m_\pi^2 - K^2}{mM} L(III.4) = 0 \qquad \text{(III.19)}$$

III.2 $N V_\mu^{II} \to \Delta \pi$

III.2.1 Sum rules of the I=1,2 amplitudes

a) t^o-sum rules

$$G^{1V\ I=1(2)}_{\frac{3}{2}-\frac{1}{2},S1}: \quad NV_1^s + IS_1\ NV_1^u + IS_2\ DV_1^s + IS_3\ DV_1^u = 0 \qquad \text{(III.20)}$$

$G^{2V\ I=1(2)}_{\frac{3}{2}-\frac{1}{2},S1}$: The sum rules are identical with (III.20).

$$G^{1V\ I=1(2)}_{\frac{3}{2}-\frac{1}{2},SO}: \quad (NV^s_2 + IS_1 NV^u_2 + IS_2 DV^s_2 + IS_3 DV^u_2)\ \frac{m_\pi^2 - K^2}{2mM_+} + K^2 L(\text{III.20}) = 0 \qquad \text{(III.21)}$$

$$G^{1V\ I=1(2)}_{\frac{3}{2}-\frac{1}{2},SS}: \quad NV^s_3 + IS_1 NV^u_3 + IS_2 DV^s_3 + IS_3 DV^u_3 = 0 \qquad \text{(III.22)}$$

with

$$NV^s_1 = 0$$

$$NV^u_1 = g(C^{V'}_3 + \frac{M_-}{m} C^{V'}_4)$$

$$DV^s_1 = (\frac{m}{M}\frac{g_{12}}{M^2} + \frac{K^2}{M^2}\overline{g_2})C^{V'}_3 - \frac{M_-}{m}(\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2})C^{V'}_4 + \frac{K^2}{M^2}\frac{g_{12}}{M^2} C^{V'}_5 + \frac{m^2}{M^2}\frac{g_{12}}{M^2} C^{V'}_6$$

$$DV^u_1 = mg^* \, 2(3 + \frac{m}{M}\frac{MDN2}{M^2})\ G^V_7$$

$$NV^s_2 = 0$$

$$NV^u_2 = g\ [K^2(C^{V'}_4 - C^{V'}_5) - m^2\ C^{V'}_6]$$

$$DV^s_2 = 2\ \frac{mM_+}{M^2}\ \overline{g_2}K^2 C^{V'}_3 + (\frac{M_+^2 - Mm}{M^2}\ \frac{g_{12}}{M^2} + \frac{2M_+M_- - K^2}{M^2}\overline{g_2})\ K^2 C^{V'}_4 + (\frac{M_+^2 - Mm}{M^2}\ \frac{g_{12}}{M^2} + \frac{K^2}{M^2}\overline{g_2})(K^2 C^{V'}_5 + m^2 C^{V'}_6)$$

$$DV^u_2 = mg^*\ \frac{m}{M}\ 4(2M_+^2 + m_\pi^2)\ G^V_7$$

$$NV^s_3 = mg^*\ \frac{K^2}{m^2}\ F^V_3$$

$$NV^u_3 = g\ C^{V'}_6$$

$$DV^s_3 = -\ (\frac{M_+^2 - Mm}{M^2}\ \frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2})\ C^{V'}_6$$

$$DV^u_3 = -Mg^*[4\ \frac{MDN2}{M^2}\ \frac{K^2}{M^2}(G^V_5 + \frac{K^2}{4M^2}\ G^V_6 + G^V_7) + 4\ \frac{2M_+^2 + m_\pi^2}{M^2}(G^V_7 + \frac{K^2}{4M^2}\ G^V_6)]$$

b) <u>t^1-sum rules</u>

$$G^{1V\ I=1}_{\frac{3}{2}-\frac{1}{2},S1}: \quad (NV_4^s+2NV_4^u-\frac{5}{9}DV_4^s-\frac{5}{12}DV_4^u)(m_\pi^2-K^2)$$

$$+(NV_2^s+2NV_2^u-\frac{5}{9}DV_2^s-\frac{5}{12}DV_2^u)\frac{m_\pi^2+K^2}{m^2}-\frac{M_+}{m}K^2\,L(III.20,I=1) = 0 \qquad (III.23)$$

$$G^{1V\ I=1}_{\frac{3}{2}-\frac{1}{2},S0}: \quad NV_5^s + 2NV_5^u - \frac{5}{9}DV_5^s - \frac{5}{12}DV_5^u = 0 \qquad (III.24)$$

$$G^{1V\ I=1}_{\frac{3}{2}-\frac{1}{2},SS}: \quad NV_6^s + 2NV_6^u - \frac{5}{9}DV_6^s - \frac{5}{12}DV_6^u = 0 \qquad (III.25)$$

with

$$NV_4^s = 0$$

$$NV_4^u = g\ (\frac{K^2}{m^2}C_5^{V'} + C_6^{V'})$$

$$DV_4^s = -\frac{K^2}{m^2}\frac{m}{M}(\frac{g_{12}}{M^2}+\frac{M_+}{M}g_2)C_3^{V'} - \frac{K^2}{m^2}\frac{M_+M_-}{M^2}g_2C_4^{V'} - (\frac{M_+^2-Mm}{M^2}\frac{g_{12}}{M^2}+3\frac{K^2}{M^2}g_2)(\frac{K^2}{m^2}C_5^{V'}+C_6^{V'})$$

$$DV_4^u = -Mg^*\ 2\ [2\frac{2M_+^2+m_\pi^2}{M^2}+\frac{K^2}{M^2}(3+\frac{MDN2}{M^2})]\ G_7^V$$

$$NV_5^s = 0$$

$$NV_5^u = g\ [\frac{K^2}{m^2}(C_4^{V'}+C_5^{V'}) + C_6^{V'}]$$

$$DV_5^s = -2\frac{K^2}{m^2}\frac{m}{M}\frac{g_{12}}{M^2}C_3^{V'} - 6\frac{K^2}{m^2}\frac{m_\pi^2}{M^2}g_2C_4^{V'} + (\frac{M_+^2-Mm}{M^2}\frac{g_{12}}{M^2}-\frac{K^2}{M^2}\overline{g}_2+3\frac{m_\pi^2+K^2}{M^2}g_2)[\frac{K^2}{m^2}(C_4^{V'}-C_5^{V'})-C_6^{V'}]$$

$$DV_5^u = -4\ Mg^*\ (2\frac{M_+^2+2m_\pi^2}{M^2}+\frac{K^2}{M^2}\frac{MDN2}{M^2})\ G_7^V$$

$$NV_6^s = NV_6^u = 0$$

$$DV_6^s = - g_2 \, C_6^{V'}$$

$$DV_6^u = - 4 \, Mg^* \, (G_7^V + \frac{K^2}{4M^2} G_6^V)$$

III.3 N V_μ^{II} → N π

III.3.1 t^o-sum rules of the m=1, I=0 amplitudes

$G^{2V\ I=0}_{\frac{1}{2}-\frac{1}{2},S1}$, $G^{2V\ I=0}_{\frac{1}{2}-\frac{1}{2},SO}$, $G^{2V\ I=0}_{\frac{1}{2}-\frac{1}{2},SS}$ are crossing symmetrical.

$$G^{2V\ I=0}_{\frac{1}{2}\ \frac{1}{2},S1} : \ mg^*\{[K^2 \ MDN2 - m \, M_+^2 M_- - m_\pi^2 \, m(2M+m)] \, C_3^{V'}$$

$$+ [MDN2(M_+^2 \frac{M}{m} - 2K^2) - M^2(M_+^2 + 2m_\pi^2)] \, C_4^{V'} \qquad \text{(III.26)}$$

$$+ [MDN2 \, (2 + \frac{M}{m}) - 3M^2] \, (K^2 C_5^{V'} + m^2 C_6^{V'})\}(m_\pi^2 - K^2) = 0$$

III.3.1.2 t^o-sum rules of the m=1, I=1 amplitudes

$G^{2V\ I=1}_{\frac{1}{2}\ \frac{1}{2},S1}$ is crossing symmetrical.

$G^{2V\ I=1}_{\frac{1}{2}-\frac{1}{2},S1}$ has no t^o-contribution.

$$G^{2V\ I=1}_{\frac{1}{2}-\frac{1}{2},SO}:\quad mg^*\{2\,\frac{M_+}{m}\,\frac{MDN2}{M^2}\,K^2C_3^{V'} + \left[(M_+^2+2M_+M_--K^2)\,\frac{MDN2}{M^2m^2} - \frac{M_+^2-m_\pi^2}{mM}\right]K^2C_4^{V'}$$

$$+\left[(M_+^2+K^2)\,\frac{MDN2}{m^2M^2} - \frac{M_+^2-m_\pi^2}{mM}\right](K^2C_5^{V'}+m^2C_6^{V'})\}(m_\pi^2-K^2) = 0 \qquad \text{(III.27)}$$

$$G^{2V\ I=1}_{\frac{1}{2}-\frac{1}{2},SS}:\quad g\,K^2F_3^V + \frac{1}{9}\,mg^*\left[(M_+^2-K^2)\,\frac{MDN2}{M^2} - \frac{m}{M}\,(M_+^2-m_\pi^2)\right]C_6^{V'} = 0 \qquad \text{(III.28)}$$

t^1-sum rules are not considered.

IV. SUM RULES FOR THE AXIAL VECTOR CURRENT A_μ^{II}

IV.1 $\Delta\ A_\mu^{II} \rightarrow \Delta\ \pi$

IV.1.1.1 Sum rules of the m=3, I=0,2 amplitudes

$G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}$, $G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SO}$, $G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SS}$ are crossing symmetrical.

a) t^o-sum rules

$G^{2A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}$ has no t^o-contribution.

b) <u>t^1-sum rules</u>

$$G^{2A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}: \quad IS\cdot Mg^{*}C_4^{A'} - \frac{2}{9}\frac{m^2}{M^2} f(G_5^A,G_6^A) + \frac{4}{9}\frac{m^2}{M^2}\frac{g_{12}}{M^2}(G_6^A+G_7^A) = 0 \qquad (IV.1)$$

IV.1.1.2 <u>Sum rules of the m=3, I=1 amplitudes</u>

$G^{2A\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}$ is crossing symmetrical.

a) <u>t^o-sum rules</u>

$$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}: \quad [-\frac{4}{3} Mg^{*}C_3^{A'} + \frac{2}{9}\frac{m}{M}\frac{g_{12}}{M^2} G_7^A](m_\pi^2-K^2) = 0 \qquad (IV.2)$$

$$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},SO}: \quad \{-\frac{4}{3}Mg^{*}C_5^{A'}+\frac{1}{9}\frac{K^2}{M^2}[f(G_5^A,G_6^A)-2\frac{g_{12}}{M^2}G_6^A-4\overline{g_2}G_7^A]\}(m_\pi^2-K^2) - \frac{4M}{m}\frac{K^2}{m_\pi^2-K^2}L(IV.2)=0 \qquad (IV.3)$$

$$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},SS}: \quad -\frac{4}{3} Mg^{*}(C_5^{A'} + \frac{K^2}{m^2} C_6^{A'}) = 0 \qquad (IV.4)$$

b) <u>t^1-sum rules</u>

$$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}: \quad -\frac{4}{3}Mg^{*}C_5^{A'} + \frac{1}{9}\frac{K^2}{M^2}[f(G_5^A,G_6^A)-2\frac{g_{12}}{M^2}G_6^A - 4\overline{g_2}G_7^A] + \frac{M}{m(m_\pi^2-K^2)} L(IV.2)$$

$$+ \frac{m_\pi^2-K^2}{M^2}\{\frac{1}{3}Mg^{*}\frac{M^2}{m^2}C_4^{A'} + \frac{1}{18}[f(G_5^A,G_6^A)-2\frac{g_{12}}{M^2}G_6^A-4\overline{g_2}G_7^A+2\frac{g_{12}}{M^2}G_7^A]\} = 0 \qquad (IV.5)$$

$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},SO}$: $-\frac{4}{3}Mg^{*}[C_5^{A'} + \frac{K^2}{m^2} C_4^{A'}] - \frac{1}{9}\frac{K^2}{M^2}[f(G_5^A, G_6^A) - 2\frac{g_{12}}{M^2} G_6^A - 4\overline{g_2}\, G_7^A]$

$$-\frac{1}{9}\frac{K^2}{M^2}[4\frac{g_{12}}{M^2} G_7^A + 3g_2 \frac{m_\pi^2 - K^2}{M^2} G_6^A] = 0 \qquad \text{(IV.6)}$$

$G^{1A\ I=1}_{\frac{3}{2}-\frac{3}{2},SS}$: The N-exchange does not contribute anymore.

IV.1.2.1 Moment sum rules of the m=3, I=0,2 crossing symmetrical amplitudes

a) t^o-sum rules

$G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},S1}$: $-\frac{3}{4}$ IS $\cdot$ $y_N L_N$(IV.2) + $y_\Delta L_\Delta$(IV.2) = 0 (IV.7)

$G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SO}$: $-\frac{3}{4}$ IS $\cdot$ $y_N L_N$(IV.3) + $y_\Delta L_\Delta$(IV.3) = 0 (IV.8)

$G^{1A\ I=0,2}_{\frac{3}{2}-\frac{3}{2},SS}$: $IS \cdot Mg^{*}\, y_N\, (C_5^{A'} + \frac{K^2}{m^2} C_6^{A'}) = 0$ (IV.9)

t^1-sum rules are not considered.

IV.1.2.2 Moment sum rules of the m=3, I=1 crossing symmetrical amplitudes

a) t^o-sum rules

$G^{2A\ I=1}_{\frac{3}{2}-\frac{3}{2},S1}$ has no t^o-contribution.

b) t^1-sum rules

$$G^{2A\ I=1}_{\frac{3}{2}-\frac{3}{2},S1} : \quad -\frac{4}{3} Mg^{*} y_N C_4^{A'} - \frac{2}{9} \frac{m^2}{M^2} y_\Delta \ f(G_5^A, G_6^A) + \frac{4}{9} \frac{m^2}{M^2} \frac{g_{12}}{M^2} y_\Delta (G_6^A + G_7^A) = 0 \qquad \text{(IV.10)}$$

IV.1.3 Sum rules of the m=2 amplitudes

Again we use the linear combinations $B_1^{1A(\pm)}$, $B_1^{2A(\pm)}$, $B_0^{2A(\pm)}$, $B_S^{2A(\pm)}$ of I.

IV.1.3.1 Sum rules of the m=2, I=0,2 amplitudes

$B_1^{1A(-)I=0,2}$, $B_1^{2A(+)I=0,2}$, $B_0^{2A(+)I=0,2}$, $B_S^{2A(+)I=0,2}$ are crossing symmetrical.

a) t^o-sum rules

$B_1^{1A(+)I=0,2}$ and $B_1^{2A(-)I=0,2}$ have no t^o-contributions.

$B_0^{2A(-)I=0,2}$: $\{IS\cdot Mg^* \,[K^2 C_3^{A'} - \frac{1}{4}\frac{m}{M}(2M_+^2 - m_\pi^2 - K^2)\, C_5^{A'}]$

$$+ \frac{1}{36}\frac{m}{M}(K^2+m_\pi^2)\frac{K^2}{M^2}[f(G_5^A,G_6^A) - 2\frac{g_{12}}{M^2}G_6^A - 4\overline{g_2}\,G_7^A] \qquad \text{(IV.11)}$$

$$+ \frac{4}{9}\frac{m}{M}K^2\frac{g_{12}}{M^2}[G_5^A + \frac{K^2}{4M^2}G_6^A - \frac{1}{2}G_7^A]\}(m_\pi^2-K^2) = 0$$

$B_S^{2A(-)I=0,2}$: $IS \cdot Mg^* (2M_+^2 - m_\pi^2 - K^2)(C_5^{A'} + \frac{K^2}{m^2}C_6^{A'}) = 0$ (IV.12)

b) t^1-sum rules

$B_1^{1A(+)I=0,2}$: $L(IV.1)\,(m_\pi^2-K^2) + \frac{1}{2}\frac{m}{M}\frac{L(IV.7)}{m_\pi^2 - K^2} = 0$ (IV.13)

Further t^1-sum rules are not considered.

IV.1.3.2 Sum rules of the m=2, I=1 amplitudes

$B_1^{1A(+)I=1}$, $B_1^{2A(-)I=1}$, $B_O^{2A(-)I=1}$, $B_S^{2A(-)I=1}$ are crossing symmetrical.

a) t^o-sum rules

$B_1^{1A(-)I=1}$ has no t^o-contribution

$B_1^{2A(+)I=1}$: The sum rule is identical with (IV.2).

$B_O^{2A(+)I=1}$: The sum rule is identical with (IV.3)

$B_S^{2A(+)I=1}$: The sum rule is identical with (IV.4)

t^1-sum rules are not considered.

IV.2 $N A_\mu^{II} \to \Delta \pi$

According to (I.9) only t^o-sum rules are considered.

IV.2.1 t^o-sum rules of the I=1,2 amplitudes

$$G^{1A\ I=1,2}_{\frac{3}{2}-\frac{1}{2},S1}: \quad NA_1^s + IS_1 NA_1^u + IS_2 DA_1^s + IS_3 DA_1^u = 0 \tag{IV.14}$$

$G^{2A\ I=1,2}_{\frac{3}{2}-\frac{1}{2},S1}$: The sum rules are identical with (IV.14).

$$G^{2A\ I=1,2}_{\frac{3}{2}-\frac{1}{2},S0}: \quad (NA_2^s+IS_1 NA_2^u+IS_2 DA_2^s+IS_3 DA_2^u)\ \frac{m_\pi^2-K^2}{m^2} - \frac{2M_+M_-}{m^2} K^2 L(IV.14)=0 \tag{IV.15}$$

$$G^{2A\ I=1,2}_{\frac{3}{2}-\frac{1}{2},SS}: \quad NA_3^s + IS_1 NA_3^u + IS_2 DA_3^s + IS_3 DA_3^u = 0 \tag{IV.16}$$

with

$$NA_1^s = mg^* F_3^A,$$

$$NA_1^u = \frac{g}{2}\left(\frac{2m}{M_+} C_3^{A'} + C_4^{A'}\right),$$

$$DA_1^s = \left(\frac{m}{M}\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2}\right)\frac{m}{M_+} C_3^{A'} + \frac{1}{2}\left[\left(1+\frac{K^2}{M^2}\frac{m}{M_+}\right)\frac{g_{12}}{M^2} - \frac{K^2}{M^2}\overline{g_2}\right] C_4^{A'} + \frac{m^2}{M^2}\frac{m}{M_+}\frac{g_{12}}{M^2} C_5^{A'},$$

$$DA_1^u = mg^* \frac{m}{M} [4 \frac{MDN2}{M^2} (G_5^A + \frac{K^2}{4M^2} G_6^A) + 3 \frac{m_\pi^2}{M^2} G_6^A + 2 (\frac{M_+M_-}{M^2} + \frac{(2M+m)m_\pi^2}{M_+ \; M^2}) G_7^A] \; ,$$

$$NA_2^s = mg^* \quad K^2 \; F_3^A \; ,$$

$$NA_2^u = g \quad m^2 \quad C_5^{A'} \; ,$$

$$DA_2^s = 2 \frac{mM_-}{M^2} \overline{g_2} K^2 C_3^{A'} + [\frac{M_-^2+Mm}{M^2} \frac{g_{12}}{M^2} + \frac{M_+M_-}{M^2} \overline{g_2}] K^2 C_4^{A'} + m^2 (\frac{M_-^2+mM}{M^2} \frac{g_{12}}{M^2} + \frac{K^2}{M^2} \overline{g_2}) C_5^{A'} \; ,$$

$$DA_2^u = - mg^* \frac{m}{M} K^2 \; [4 \frac{MDN2}{M^2} (G_5^A + \frac{K^2}{4M^2} G_6^A - G_7^A) + \frac{3m_\pi^2}{M^2} G_6^A] \; ,$$

$$NA_3^s = 0$$

$$NA_3^u = g \; (C_5^{A'} + \frac{K^2}{m^2} C_6^{A'}) \; ,$$

$$DA_3^s = (\frac{M_-^2+Mm}{M^2} \frac{g_{12}}{M^2} - \frac{K^2}{M^2} \overline{g_2})(C_5^{A'} + \frac{K^2}{m^2} C_6^{A'}) \; ,$$

$$DA_3^u = 0$$

IV.3 <u>$N A_\mu^{II} \rightarrow N \pi$</u>

Among the four m=1 amplitudes belonging to unnatural parity exchange in the t-channel $G^{2A\; I=0}_{\frac{1}{2}\;\frac{1}{2},S1}$ and $G^{2A\; I=1}_{\frac{1}{2}-\frac{1}{2},S1}$ are crossing symmetrical and $G^{2A\; I=0}_{\frac{1}{2}-\frac{1}{2},S1}$ has no t^o-contributions. The remaining I=1 amplitude $G^{2A\; I=1}_{\frac{1}{2}\;\frac{1}{2},S1}$ although superconvergent at t=0 will not be used because the A_1-exchange with $\alpha_{A_1}(0) \approx 0$ contributes. It is not to be expected that an approximation keeping only the low energy contributions gives a good result because of slow convergence for $t \approx 0$.

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