KfK 3104
Mai 1981

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METHOD FOR ANALYSIS OF INELASTIC ALPHA PARTICLE SCATTERING

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ISSN 0303-4003

ABSTRACT

The analysis of inelastic scattering of strongly interacting particles from nuclei suffers from the model dependence introduced when specifying the form factors (coupling potentials) e.g. in the frame work of a vibrational model. Moreover, the relation between strength and shape of the coupling potentials and corresponding quantities of the transition densities is not well established, and the interpretation in terms of transition rates and nuclear moments introduces additional uncertainties. In the present studies of inelastic scattering of 104 MeV alpha-particles from ^{50}Ti and ^{52}Cr we remove the constraints due to pre-chosen forms of the coupling potentials by applying the Fourier-Bessel method which is more flexible. It allows quite general shapes and provides realistic estimates of the uncertainties of the potentials and derived quantities (moments). Using the identities between the integral moments of a folded potential distribution and of the underlying nuclear matter distribution isoscalar transition rates have been derived.

Eine Methode zur Analyse der inelastischen Streuung von Alpha-Teilchen

Die Analyse der inelastischen Streuung stark wechselwirkender Teilchen mit Atomkernen leidet gewöhnlich unter der Modellabhängigkeit, die durch einfache mathematische Formen für die Formfaktoren (die Form der Kopplungspotentiale) - z.B. im Rahmen des Vibrationsmodells als Ableitung des Diagonalpotentials - eingeführt wird. Darüberhinaus ist die Relation zwischen Kopplungspotential und Übergangsdichte nicht klar. Die Deutung der Ergebnisse im Hinblick auf Übergangsraten und Momente der Materieverteilung bringt daher zusätzliche Unsicherheiten. In den vorliegenden Untersuchungen der inelastischen Streuung von 104 MeV Alpha-Teilchen von ^{50}Ti und ^{52}Cr werden die Einschränkungen durch vorgewählte Formen der Kopplungspotentiale durch Anwendung der Fourier-Bessel-Methode vermieden. Diese Methode ist recht flexibel, erlaubt recht allgemeine Formen und liefert realistische Abschätzungen der Unsicherheiten. Unter der Annahme, daß ein Faltungsmodell Potential und Übergangsdichte verknüpft und unter Gebrauch mathematischer Identitäten für die Momente gefalteter Verteilungen werden aus den Resultaten Werte für isoskalare Übergangsraten gewonnen.

1. INTRODUCTION

There is a considerable interest in comparing transition rates of collective nuclear excitations induced by different kinds of particles. Although the correlation between nuclear and electromagnetic excitation is well established and readily understood by the overall similarity of the transition operators, there remain interesting differences. In the picture of the collective model the shapes of the charge - and total matter distribution of a particular nucleus may differ so that electromagnetic and hadronic probes may feel different (permanent or dynamical) deformations thus revealing different collectivities of protons and neutrons. Moreover, due to different spin - and isospin structures of the interactions of different particles with nuclei, we have to expect that, in general, the transition strengths are dependent on the external field inducing the transitions. With those aspects in mind Madsen et al. [1] have discussed electromagnetic, (p,p') and (n,n') excitations and have related possible differences to the isovector part of the transition amplitudes. The inelastic scattering of alpha-particles involves only the isoscalar component, so that the corresponding rates might be an interesting source of independent information.

Basic difficulties, however, are present in all analyses of nuclear excitations since inevitably reaction models are involved and the interesting matrix elements are not specified in a model-independent way. All procedures (see ref. 2) proposed for the extraction of isoscalar transition rates from inelastic alpha-particle scattering are implicitely[3] or explicitely[4] based on the assumption that the (real) form factor $V_{if}\ (r_\alpha)$ (coupling potential) can be understood as the result of a folding of the transition density $\rho_{if}(r) = \langle i|\delta(\vec{r}-\vec{r}')|f\rangle$ with an effective alpha-particle-bound nucleon interaction V_{eff}

$$V_{if}\ (\vec{r}_\alpha) = \int \rho_{if}(\vec{r})\ V_{eff}(|\vec{r}-\vec{r}_\alpha|)\ d^3r \qquad (1)$$

An explicit folding model approach requires the specification of the transition density in addition to the alpha-particle-bound-nucleon interaction. If not taken from a microscopic structure model the collective model has been invoked for parametrizing ρ_{if}, e.g., as derivatives of the ground state density distribution ("vibrational model"). Even when explicit folding model calcu-

lations are avoided and only the coupling potentials are determined (lateron related to ρ_{if} by rescaling the strength: Bernstein procedure [3]), the analysis relies on the shapes of V_{if} as provided by an extended optical potential having various simple forms for the radial shape of the potential and its derivatives. Although the parametrization of the transition densities and potentials as provided by the collective model appears to be rather consistent [3,5] with the dependence of these quantities near the nuclear surface as given by microscopic structure calculations, the simple functional forms introduce strong constraints on the radial shape of ρ_{if} and V_{if}, respectively. These constraints may affect the extracted values of the isoscalar transition rates, they may distort the uncertainties and spoil systematic studies of differences between proton and neutron collectivities. It is, therefore, quite important to overcome such deficiencies and to have a method which is additionally able to give realistic estimates of the uncertainties of the extracted results.

There are rather similar problems [6] in standard optical model analyses of elastic scattering when using prechosen functions for the shape of the potentials. In recent years, a significant progress has been made by introducing so-called model-independent techniques describing the optical potential (usually its real part) by a Fourier-Bessel series or other flexible sets of adequate functions[7]. In the present paper we apply a variant of the Fourier-Bessel (FB) method to the description of the coupling potentials for inelastic scattering of 104 MeV alpha-particles from ^{50}Ti and ^{52}Cr. For these cases sufficiently accurate experimental data are available[8], both of elastic and inelastic scattering, extending to large scattering angles, which is an important feature when introducing of the more advanced methods into the analysis where an increased sensitivity to details of the shapes of the potentials is observed. The transition moments and transition radii derived from the resulting potentials can be translated into corresponding quantities of the nuclear density distribution by use of Satchler's theorem[9] for radial moments of folding integrals, thus enabling a reliable and quantitative analysis in terms of "model independent" transition rates.

2. METHOD AND PROCEDURES

The present studies emphasize the methodical aspects which will be demonstrated by applying the procedures to 104 MeV alpha-particle scattering from the ground- and the first 2^+ states of ^{50}Ti and ^{52}Cr (recently measured by the Karlsruhe group[8]).

DWBA or coupled channels calculations of the differential cross sections require the specification of the diagonal part $U_{Diag}(r_\alpha)$ of the complex interaction potential as well as of the non-diagonal part U_{coupl} providing the coupling of different nuclear states. Usually, in the framework of the collective (vibrational) model both parts are deduced from an extended optical potential $U(\vec{r}_\alpha)$ deformed by the angular dependence of the half-way radius, e.g.

$$R = R_o \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu} (\hat{r}_\alpha)\right) \tag{2}$$

and expanded into powers (t) of $\sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu} (\hat{r}_\alpha)$. The coupling potentials[10] between various channels (where DWBA is restricted to coupling to the ground state) enter by matrix elements

$$\langle lI|U_{coupl}|I'l'\rangle = \sum_L \left[A(l,I,l'I',L,J) \sum_t u_L^{(t)} (r) \langle I||Q_L^{(t)}||I'\rangle\right] \tag{3}$$

where l, l' denote the partial waves of the incoming and scattered particle, I, I' the spin of the nuclear states, J the total channel spin and L the multipolarity. The factor A(l,I,l'I'; L,J) is purely geometrical and the reduced matrix elements of the operators $Q_L^{(t)}$ (operating only on target coordinates and built up by products $(\alpha_\lambda \otimes \alpha_{\lambda'})_L$ coupled to multipolarity L) represent the strength of the transition, while $u_L^{(t)}$ are the radial form factors*. The sum of products

$$\sum_t u_L^{(t)} (r_\alpha) \langle I||Q_L^{(t)}||I'\rangle \tag{4a}$$

*The standard DWBA procedure takes into account only the first order term (t=1) in eq. 3 which is consistent with the neglect of multiple excitation. As in a further step of simplication strength and shape of the optical potential is approximated by the result extracted from elastic scattering alone, the standard DWBA extracts only the transition strength $\langle 0^+||Q_L^{(1)}|I=L\rangle$ from inelastic scattering data.

corresponds - apart from a factor $(2I'+1)^{-1/2}$ - to the transition density of multipolarity L (there written in the special expansion of the vibrational model) and, for example,

$$<0^+||Q_L^{(1)}||I> = \beta_L^{(pot)} \delta_{LI} \qquad (4b)$$

is just the familiar "deformation" parameter. The superfix (pot) is introduced since we have to distinguish between deformation of the potential $U(\vec{r}_\alpha)$ and of the nuclear matter distribution $\rho(\vec{r})$.

In general the real as well as the imaginary part of $U(\vec{r}_\alpha) = V(\vec{r}_\alpha)+i\ W(\vec{r}_\alpha)$ are assumed to be nonspherical, and it has been shown[4] that complex coupling is important for inelastic alpha-particle scattering. In the following procedure $W(\vec{r}_\alpha)$ will be conventionally described by a Woods-Saxon form (WS) with geometry (R_W, a_W) independent from the real potential and deformed by

$$R_W = R_{OW} \quad (1 + \sum \alpha_{LM} Y_{LM} (\hat{r}_\alpha)),$$

handled in the way of a second order (t=2) vibrational model. The real part, however

$$v_L(r_\alpha) = \sum_t v_L^{(t)} (r_\alpha) <I||Q_L^{(t)}||I'> \qquad (5)$$

of the coupling potential is replaced by a more flexible parametrization, similar to the Fourier-Bessel method in elastic scattering analyses[6]. It consists of adding to a conventional form (say, the familiar derivative of a Woods-Saxon) an extra-potential given by a Fourier Bessel series*:

$$v_L(r_\alpha) = \beta_L^{(pot)} \cdot (v^o_{coupl}(r_\alpha) + \sum_{n=1}^{N} b_n\ j_1\ (q_n \cdot r_\alpha)) \qquad (6a)$$

The quantities j_1 are spherical Bessel functions, $q_n = n\pi/R_{cut}$ and R_{cut} is a suitably chosen cut-off radius beyond which the extra-potential vanishes.

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*Model independent analyses [11] of inelastic electron scattering introduce the charge density of multipolarity L as a Fourier-Bessel series with j_L. This representation is based on the approximate validity of the Born approximation, which is obviously a sitatuion different from alpha-particle scattering. In the present work the choice of j_1 in eq. 6 is just a matter of convenience because the form factor apparently has to vanish at $r_\alpha = 0$.

After fixing the $\beta_L^{(pot)} \cdot V^o_{coupl}$ term by a first guess or a shape resulting from a best-fit with $b_n \equiv 0$, the correction coefficients b_n are determined by a least-square fit to the data. Within the framework of the FB procedure the mean-square uncertainty of $v_L(r_\alpha)$ at the distance r_α is given by

$$[\delta v_L(r_\alpha)]^2 = \sum_{m,n} \langle \delta b_m \, \delta b_n \rangle_{av} j_1(q_m r_\alpha) j_1(q_n r_\alpha) \qquad (7)$$

with $\langle \delta b_m \delta b_n \rangle_{av}$ being the correlation matrix between the coefficients b_n (see e.g. ref. 6 for more details of the error determination). Two important points of the procedure should be emphasized:

a. the method considers shapes of $v_L(r_\alpha)$ which are more general than those provided by the vibrational model. The residual model dependence is due to the finite number N of Fourier-Bessel terms and due to the specific choice of the cut-off radius R_{cut}. In the course of the analyses N and R_{cut} are varied (see below).

b. The relation and coupling betweeen the diagonal part of the potential $V^o_{diag}(r_\alpha)$ and $v_L(r_\alpha)$ as given by the collective model is abandonned. Even with $b_n \equiv 0$ in eq. 6 the shape of $V_{coupl}(r_\alpha)$ is considered to be independent from $V_{diag}(r_\alpha)$. In this case β_L looses its traditional meaning as a surface deformation parameter, and only the complete product in eq. 6a is of relevance. A priori, a derivative form is chosen for $V^o_{coupl}(r_\alpha)$

$$V^o_{coupl} = R^{tr}_{ov} \cdot \frac{\partial V_{trans}}{\partial R_{ov}} \qquad (6b)$$

as suggested by the usual collective model, but with a shape of V_{trans} not necessarily identical to that of V_{diag}. In order to guarantee general requirements for the transition densities [20] the extreme inner part is parametrized by a polymial with zero value at r = 0 and smoothly adjusted to expression (6b) at an inner radius R_i(=2 fm).

Principally, the above method can be used in DWBA as well as in coupled channels calculations. In view of the simplifications in DWBA and in order to avoid effects due to the neglect of multiple excitation, the present results are based on coupled channels calculations. Thereby we generally fit elastic and inelastic scattering simultaneously.

The real diagonal part V_{diag} has been recently studied in extensive elastic scattering analyses using model independent techniques[12,13]. In particular, it has been shown[13] that the radial shape is very well approximated by a squared Woods-Saxon form (WS2) with parameter values which prove to be fairly well determined by the elastic scattering. Small readjustments due to coupling to the excited states have been taken into account by results of coupled channels calculations on the basis of the usual vibrational model (see line 2 of tab. 1). Therefore, for sake of simplicity the shape of $V_{diag}(r_\alpha)$ has not been varied simultaneously with the b_n-coefficients of $v_L(r_\alpha)$. Only the imaginary part has been readjusted when fitting the cross sections. Complex coupling is taken into account via the deformation $\beta_L^{(pot)}$ of the imaginary part. Coulomb excitation is included by a deformed Coulomb potential. All calculations used a modified version of the coupled channels code ECIS[14].

3. RESULTS

Table 1 presents the parameters of the real potentials, of the squared Woods-Saxon potential shape taken either from the analysis of elastic scattering only - ref. 12 - or from a coupled channel analysis on the basis of the usual vibrational model. Also shown are parameters of V_{trans}, the first derivative of which serves as a first guess to the FB coupling potential. We reiterate that V_{trans} and V_{diag} have independent geometries and that the factorization of the transition potential by eq. 6a is only a reminder to the vibrational model. Therefore, after fixing $\beta_L^{(pot)} \cdot R_V^{tr} \partial V_{trans}/\partial R_v^{tr}$ by a first fit (with $V_o^{trans} = V_o^{diag}$ so that $\beta_L^{(pot)}$ for itself becomes consistent with the features of the vibrational model), $\beta_L^{(pot)}$ has been kept fixed in the further procedure when varying the b_n coefficients.

Fig. 1 displays the experimental data and the calculated differential cross sections, and Fig. 2 compares the obtained FB coupling potentials with the best-fit derivative shapes corresponding to the V_{trans} parameters quoted in Table 1. The value of R_{cut} and N have been increased up to convergence of χ^2/F and of the errors of the integral quantities (radial moments). The error bands of the FB potentials result from averaging several single fits with different choices of the cut-off radius R_{cut} and of the number of FB terms. This procedure is expected to remove the remaining model dependence (see ref. 15). It is interesting to see how well the derivative

form approximates the resulting FB potentials. This result may be regarded as an a-posteriori-justification for using "vibrational model" shapes for the particular transitions considered here.

Table I Parameters of the real potentials used in the analysis

	V_o (MeV)	R_v (fm)	a_v (fm)	$\beta_2^{(pot)}$	χ^2/F Tot	0^+	2^+	Shape and Procedure
^{50}Ti V_{diag}	147.2	5.190	1.20	-	-	1.8	-	$(WS)^2$ elastic scattering
V_{diag}	144.0	5.249	1.19	0.122	3.2	2.4	3.9	$(WS)^2$ CC-calc.
V_{trans}	144.0	5.066	1.26	0.146	3.1	2.4	3.8	$(WS)^2$ Deriv.
V_{trans}	N=11-13	R_{cut}=11-13		0.146	2.8	2.3	3.3	$(WS)^2$ Deriv.+FB
V_{diag}	156.9	5.1	1.2	-	-	1.7	-	$(WS)^2$ el. sc.
^{52}Cr V_{diag}	154.0	5.2	1.2	0.130	2.1	1.7	2.7	$(WS)^2$ CC-calc.
V_{trans}	154.0	5.2	1.2	0.130	2.1	1.7	2.7	$(WS)^2$ Deriv.
V_{trans}	N=11-13	R_{cut}=11-13		0.130	2.1	1.7	2.7	$(WS)^2$ Deriv.+FB

4. ANALYSIS OF RADIAL MOMENTS

The analysis described in the preceding sections results in"model independent" transition potentials and the problem remains to interpret them in terms of the transition densities of the probed nucleus. There are simple mathematical relationships between the radial moments of two folded distributions when one is generated by folding a scalar function into the other[9] as assumed for V_{if} by eq. (1). Applying these relationships to the nonspherical optical potential we are able to deduce the transition moments of the nucleon distribution from the coupling potentials [2,16] via

$$\int v_L(r_\alpha)\, r_\alpha^{L+2}\, dr = \int \rho_L(r)\, r^{L+2}\, dr \circ J_o(V_{eff}) \qquad (8)$$

The alpha-bound-nucleon interaction which need not to be specified, enters (implicitely) by its volume integral $J_o(V_{eff})$. This quantity is related to the volume integrals of the monopole (diagonal) parts of the potential and of the density distribution, respectively,

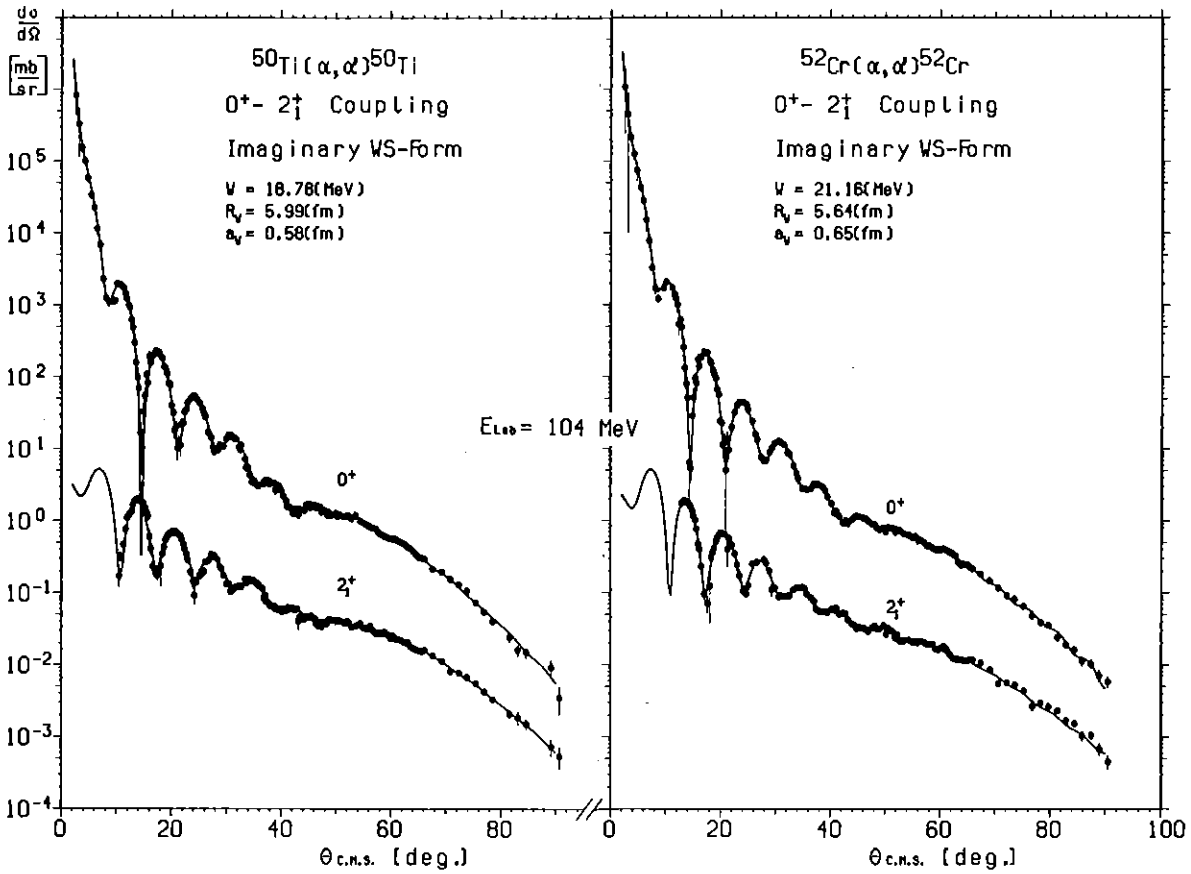

Fig. 1 Elastic and inelastic scattering of 104 MeV alpha-particles by ^{50}Ti and ^{52}Cr: Experimental cross sections and results of coupled channels calculations.

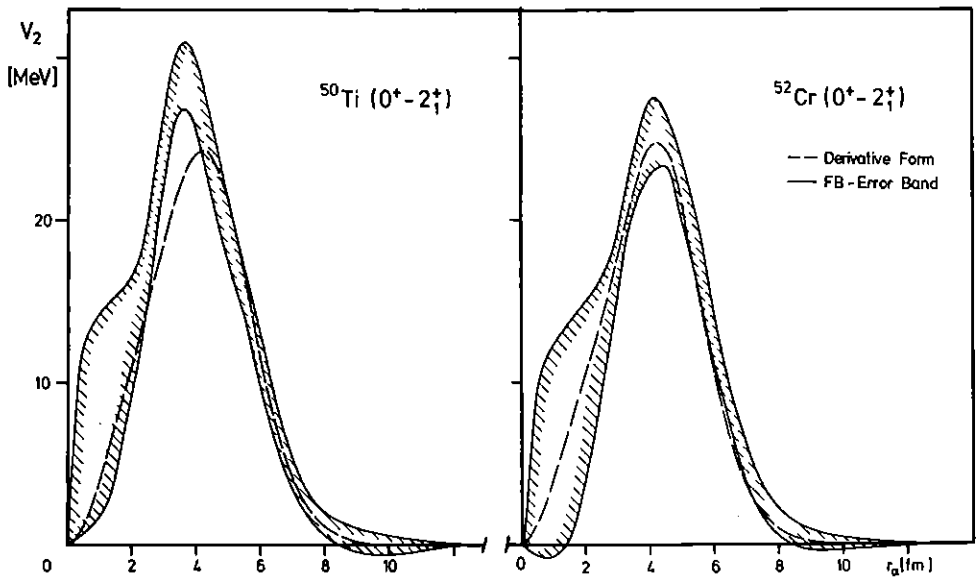

Fig. 2 Transition potentials for the $0^+ - 2^+$ transition in ^{50}Ti and ^{52}Cr induced by 104 MeV alpha-particles.

$$J_o(V_{diag}) = A \cdot J_o(V_{eff}) \quad . \tag{9}$$

With these relations we write the isoscalar transition rates*

$$B(IS,I_i \to I_f) = \frac{1}{2I_i+1}\left[\frac{Z}{A}\int \rho_L(r)\ r^{L+2}\ dr\right]^2 \tag{10}$$

directly as

$$B(IS,I_i \to I_f) = \frac{1}{2I_i+1}\left[\frac{Z}{J_o(V_{diag})}\int v_L(r_\alpha)\ r_\alpha^{L+2}\ dr_\alpha\right]^2 \tag{11}$$

Using another relation of folded potentials[9] the squared transition radius

$$R_{tr}^2(v_L) = \langle r^{L+2}\rangle / \langle r^L\rangle \tag{12a}$$

of the potential v_L can be related to the corresponding quantity of the transition density ρ_L by

$$R_{tr}^2(v_L) = R_{tr}^2(\rho_L) + \frac{1}{3}(2L+3)\langle r^2\rangle_{eff} \tag{12b}$$

For this the ms radius $\langle r^2\rangle_{eff}$ of the effective interaction potential has to be known. Assuming that the real part of the alpha-particle nucleus optical potential can be understood as a folded potential, $\langle r^2\rangle_{eff}$ is just the difference between the ms radii of the potential and of the underlying matter distribution.

Table 2 presents the results which are based on such an implicit folding model interpretation (RMA) of the real potentials found in the analysis of the experimental data. The values of the ms radius and the volume integrals of the diagonal part have been adopted from FB analyses of the elastic scattering [12]. The value of $\langle r^2\rangle_{eff}^{1/2} = 2.6$ fm can be deduced from previous results[13] of elastic alpha-particle scattering on ^{40}Ca, for which the matter radius is believed to be known. Alternatively, neglecting small differences of $\langle r^2\rangle_m$ and of the ms radii of the charge distributions (known from ref. 17) we may extract $\langle r^2\rangle_{eff}$ from $\langle r^2\rangle_{diag}$ for every particular case (these values are given in table 2). There is an additional uncertainty. If the effective alpha-bound nucleon interaction is density-dependent and if the elastic and inelastic scattering probe different parts of the nucleus then $\langle r^2\rangle_{eff}$ may slightly differ. The transition rates are represented by the enhancement factors $G_2 = B(IS;\ 0^+ \to 2_1^+)/B_{sp}$ (using the radius parameter $r_o = 1.2$ fm in the definition of the single particle unit). In addi-

*The factor Z/A is introduced just for convenience[3] when comparing with electromagnetic transition rates.

Table 2 Volume integrals, rms and transition radii, and transition rates for the $0^+ \rightarrow 2_1^+$ transitions in ^{50}Ti and ^{52}Cr

Quantity	^{50}Ti	^{52}Cr	Procedure
$J_o(V_{diag})$ (MeV fm^3)	306 ± 6	303 ± 4	FB-analysis ref. 15
$\langle r^2 \rangle^{1/2}$ (V_{diag}) (fm)	4.47± 0.12	4.48 ± 0.09	
$\langle r^2 \rangle^{1/2}_{eff}$ (fm)	2.68± 0.16	2.61 ± 0.13	
$R_{tr}(v_2)$ (fm)	6.58± 0.14	6.91 ± 0.03	
R_{tr} (ρ_2) (fm)	5.1 ± 0.3	5.60 ± 0.15	RMA of FB-potentials
G_2(s.p.u.)	7.1 ± 0.3	8.8 ± 0.1	
G_2(s.p.u.)	6.5	8.0	RMA of $(WS)^2$ derivative
G_2(s.p.u.)	6.1	7.9	BP
G_2(s.p.u.)	6.3±0.4	11.6±0.5	Electromagn. ref. 18.

tion to the FB potentials the coupling potential v_2 in the usual $(WS)^2$ derivative form (just of V_{trans} in Table 1) have been considered in a radial moment analysis (RMA) as well as with the Bernstein (BP) procedure (slightly modified since the shape of V_{trans} is not identical with the shape of V_{diag}). We quote errors only in the cases of FB potentials (see appendix) as constraints in the shapes of v_L might underestimate the real uncertainties.

The original Bernstein procedure, which has been extensively used in the past for determination of isoscalar transition rates, is essentially based on a parametrization of V_{if} and ρ_{if} as a first derivative of the diagonal potential or of the ground state density with Woods-Saxon $(V_o \cdot f(r_\alpha, R))$

or Fermi ($\rho_o \cdot g(r,c)$) shapes, and the half-way radii R and c, respectively

$$v_L = \beta_L^{(pot)} \cdot R \cdot V_o \circ f'(r_\alpha) \tag{13a}$$

$$\rho_L = \beta_L^{(\rho)} \cdot c \cdot \rho_o \cdot g'(r) \tag{13b}$$

As the shapes of $f'(r_\alpha)$ and $g'(r)$ do not differ very much and as

$$V_o \tilde{\approx} \rho_o \cdot J_o(V_{eff})$$

eqs. 10 and 11 lead to the approximate identity

$$\beta_L^{(pot)} \cdot R \tilde{\approx} \beta_L^{(\rho)} \cdot c$$

The scaling relation (first suggested by Blair[19]) is used in the procedure introduced by Bernstein and can approximately provide values of the transition rates. But obviously these rates have to be regarded as model dependent. The Fourier-Bessel method, however, removes, the model dependence of the real coupling potentials and enables realistic estimates of the uncertainties of the extracted quantities. The analysis of radial moments avoids the difficulties of explicit folding model calculations and replaces doubtful and less justified scaling recipes.

5. CONCLUDING REMARKS

The two advantages of the Fourier-Bessel method in analyzing inelastic form-factors are the possibility to consider quite general shapes and the resulting realistic estimates of uncertainties. For the two cases considered here the method reveals that the experimental inelastic scattering cross sections, even measured to large angles determine mostly the surface region of the coupling potential. In that region it can indeed very well be parametrized by somewhat generalized collective model form-factors. Most likely, these are the reasons why the Fourier-Bessel method does not significantly improve the goodness of the fits (χ^2/F), once V^o_{coupl} is fixed as the derivative of a best-fit form of V_{trans}. The present results demonstrate what has been argued before on the basis of the overall-similarity between collective

model for factors and microscopically calculated form-factors and by inspecting the radial weight of the transition operators[19,21]. The finding that the values of the partial χ^2/F for the inelastic cross sections (see Table 1) cannot be reduced below 2 may be a consequence of some simplifications, in particular the neglect of multiple excitation from higher lying excited states. It is,of course, a matter of increasing complexity to incorporate consistently couplings to other excited states.

The observation that the surface region dominates the transition strengths suggests that a folding model with density-independent interaction is quite a reasonable description. Exploiting the identities[9,16] between the moments of a folded potential distribution and the underlying nuclear density distribution we avoid the problem of specifying the effective alpha-particle bound nucleon interaction and extract the isoscalar transition rates in a procedure (Radial Moment Analysis) which is less doubtful and much better justified than other methods widely used in literature.

We would like to thank Professor Dr. G. Schatz for his stimulating interest, Professor Dr. D.F. Jackson for useful comments and Mrs. K. Feißt for her help in the computer work.

REFERENCES

1 V.A. Madsen, V.R. Brown and J.D. Anderson,
Phys. Rev. Lett. 34, 1398 (1975).
Phys. Rev. C12, 1205 (1975).

2 H. Rebel, Z. Physik A277, 35 (1976).

3 A.M. Bernstein, Advances in Nuclear Physics, ed. M. Baranger and E. Vogt, New York, Plenum 1969 p.

4 H.J. Gils and H. Rebel, Z. Physik, A274, 259 (1975).

5 G. Bertsch and S.F. Tsai, Phys. Lett. 50B, 319 (1974).

6 E. Friedman and C.J. Batty, Phys. Rev. C 17, 34 (1978).

7 I. Brissaud and M.K. Brussel, J. Phys. 63, 481 (1977).
L.W. Put and A.M. Paans, Nucl. Phys. A 291, 93 (1977).
F. Michel and R. Vanderporten, Phys. Rev. C 16, 142 (1977).

8 R. Pesl, H.J. Gils, H. Rebel, E. Friedman, J. Buschmann, K. Feißt, S. Zagromski (in preparation).

9 G.R. Satchler, J. Math. Phys. 13, 1118 (1972).

10 T. Tamura, Rev. Mod. Phys. 37, 679 (1965).

11 B. Dreher, J. Friedrich, K. Merle, H. Rothhaas and G. Lührs, Nucl. Phys. A 235, 219 (1974).

12 E. Friedman, H.J. Gils, H. Rebel, and R. Pesl, Nucl. Phys. A to be published.

13 H.J. Gils, E. Friedman, H. Rebel, J. Buschmann, S. Zagromski, H. Klewe-Nebenius, B. Neumann, R. Pesl, and G. Bechtold, Phys. Rev. C 21, 1239 (1980).

14 G.W. Schweimer and J. Raynal, unpublished program description, Kernforschungszentrum Karlsruhe (1973), private communication.

15 H.J. Gils, E. Friedman, H. Rebel, J. Buschmann, S. Zagromski, H. Klewe-Nebenius, B. Neumann, R. Pesl, and G. Bechtold, KfK-Report 2838 (1979).

16 R.S. Mackintosh, Nucl. Phys. A 266, 379 (1976).

17 H.D. Wohlfahrt, in "What Do We Know about the Radial Shape of Nuclei in the Ca-Region?" Proc. Karlsruhe, Internat. Discussion Meeting, May 2-4, 1979, KfK-Report 2830, eds. H. Rebel, H.J. Gils, G. Schatz.

18 C.W. Towsley, D. Cline and R.N. Horoshko, Nucl. Phys. A 250, 381 (1979).

19 J.S. Blair, Proc. Intern. Conf. Nuclear Structure, Kingston, Canada (1960) (D.A. Bromley and E.W. Vogt eds.) The University of Toronto Press, Toronto, 1960.
N. Austern and J.S. Blair, Ann. Phys. 33, 15 (1965).

20 R.F. Verdier, Dissertation, Harvard University 1968,
R. Neuhausen, private communication.

21 D.F. Jackson, Nucl. Phys. A123 (1969) 273.

APPENDIX

Determination of errors of the Fourier-Bessel potentials and of various integral quantities.

The error analysis follows the procedures described in detail in refs. 6 and 15. The variances of the potentials $v_L(r_\alpha)$ are defined by eq. (7), rewritten as

$$\left[\delta v_L(r_\alpha)\right]^2 = 2 \cdot f_{min} \sum_{m,n=1}^{N} (M^{-1})_{mn}\, j_1(q_m r_\alpha) \cdot j_1(q_n r_\alpha) \qquad \text{(A-1)}$$

Here, f_{min} is the value of χ^2/F at the minimum and M^{-1} the covariance matrix which is obtained numerically by performing the χ^2 fit. In addition to the potentials themselves some integral quantities are of physical interest, in particular, the radial moments of the transition potentials

$$\langle r^K \rangle = \frac{\int_o^\infty v_L(r_\alpha)\, r_\alpha^{K+2} dr_\alpha}{\int v_L(r_\alpha) r_\alpha^2 dr_\alpha} = \frac{\langle r^K J_o \rangle}{J_o(v_L)} \qquad \text{(A-2)}$$

for $L \neq 0$ and $K = L, L+2$ and the transition radius

$$R_{tr}(v_L) = \left[\langle r^{L+2} \rangle / \langle r^L \rangle\right]^{1/2} \qquad \text{(A-3)}$$

The volume integral $J_o(v_L) = 4\pi \int_o^\infty v_L(r_\alpha)\, r_\alpha^2\, dr_\alpha$ of the transition potentials plays a minor part than $J_o(V_{diag})$ which has been shown to be an interesting characteristic quantity of the optical potential.
The B(IS) values are determined by the square of the moment

$$M_L(IS) = \frac{1}{4\pi} \frac{J_o(v_L)}{J_o(V_{diag})} \langle r^2 \rangle \qquad \text{(A-4)}$$

The determination of the uncertainties

$$\delta\langle r^K \rangle = \left[\frac{\delta\langle r^K J_o \rangle}{\langle r^K J_o \rangle} + \frac{\delta J_o(v_L)}{J_o(v_L)} \right] \cdot \langle r^K \rangle \qquad \text{(A-5)}$$

$$\delta R_{tr} = \frac{1}{2} \left[\frac{\delta\langle r^{L+2} \rangle}{\langle r^{1/2} \rangle} + \frac{\delta\langle r^L \rangle}{\langle r^L \rangle} \right] \cdot R_{tr} \qquad \text{(A-6)}$$

$$\delta M_L = \left[\frac{\delta \langle r^K J_o \rangle}{\langle r^K J_o \rangle} + \frac{\delta J_o(V_{diag})}{J_o(V_{diag})} \right] \cdot M_L \qquad \text{(A-7)}$$

requires the evaluation of the errors of the integrals

$$\int_o^\infty v_L(r)\ r^{K+2}\ d\vec{r}_\alpha ,$$

which enter by $\langle r^K J_o \rangle$ (J_o representing the special case for K=0).
A straight-forward application of the Gaussian error propagation law gives together with eq. (A-1)

$$\delta \langle r^K J_o \rangle = \left[\sum_{m,n=1}^{N} C_{mn} \cdot V_m^K \cdot V_n^K \right]^{1/2} \qquad \text{(A-8)}$$

The terms C_{mn} are defined by

$$C_{mn} = \langle \delta b_m \cdot \delta b_n \rangle = 2 \cdot f_{min} \cdot (M^{-1})_{mn}$$

and

$$V_n^K = \int_o^{R_{cut}} r^{K+2} \cdot j_1 \left(\frac{n\pi}{R_{cut}} \cdot r_\alpha \right) d\vec{r}_\alpha \qquad \text{(A-9)}$$

$$= \frac{4\, R_{cut}^{K+3}}{n^{K+3}\, \pi^{K+2}} \int_o^{n\pi} \rho^{K+2}\ j_1(\rho)\ d\rho$$

Here R_{cut} is the cut-off radius of the Bessel-functions. Finally, partial integration results in the expressions:

$$V_n^o = o \qquad \text{(A-10)}$$

$$V_n^K = (K+2) \cdot \left[(-1)^{n+1} \sum_{l=1}^{K} (-1)^{l+1} \frac{K!}{(K-k)!} (n\pi)^{K-k} \right] - Z \qquad \text{(A-11)}$$

for $K \neq o$

$k = 2l-2; \quad k \leq K$

$$Z = \begin{cases} (K+2) \cdot K! & \text{for } K \text{ even} \\ o & \text{for } K \text{ odd} \end{cases}$$

As in the present studies $V_{diag}(r_\alpha)$ is not described by a Fourier Bessel potential, we have adopted the results from a FB analysis of the elastic

scattering data (ref. 12) for estimating $\delta J_o(V_{diag})/J_o(V_{diag})$ in eq. A-6. In all cases the values and uncertainties finally quoted are averages of a large number of results with different cut-off radii R_{cut} and numbers N of Fourier-Bessel terms.