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NGFM

A Three-Dimensional Steady-State Multigroup Diffusion Theory Code in Rectangular Geometry, Based on Nodal Green's Function Method

M. Manolova
Institut für Neutronenphysik und Reaktortechnik
Projekt Schneller Brüter

Kernforschungszentrum Karlsruhe

KERNFORSCHUNGSZENTRUM KARLSRUHE

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A Three-Dimensional Steady-State Multigroup Diffusion Theory Code in Rectangular Geometry, Based on Nodal Green's Function Method

Maria Manolova*

*delegated from the Institute for Nuclear Research and Nuclear Energy of Bulgarian Academy of Sciences in the framework of an IAEA-Fellowship

Kernforschungszentrum Karlsruhe GmbH, Karlsruhe

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Abstract

The Nodal Green's Function Method and the code, based on this method, developed bv R. Lawrence and J. Dorning for solving the 3-dimensional multigroup diffusion equations in rectangular geometry, are described. Some improvements of the code are presented. The method and code capabilities are demonstrated on the basis of a sample problem, namely the two-dimensional IAEA benchmark problem.

NGFM - ein Rechenprogramm zur Lösung der stationären Multigruppen-Diffusionsgleichung in Rechtecksgeometrie mit der Methode Nodaler Green'scher Funktionen

Zusammenfassung

Im vorliegenden Bericht werden die von R. Lawrence and J. Dorning entwickelte "Nodal Green's Function Method" zur Lösung der 3-dimensionalen Multigruppen-Neutronen Diffusionsgleichungen und das auf der Grundlage dieser Methode entwickelte Rechenprogramm beschrieben. Außerdem werden einige Verbesserungen dieses Verfahrens dargestellt. Die Methoden und die Möglichkeiten des Codes werden anhand von Musterbeispielen, insbesondere am 2-dimensionalen IAEA-Benchmark-Problem, vorgeführt.

CONTENTS

1. Introduction

The determination of the neutron flux and power distributions is of great importance for steady-state core performance or long-time burnup studies, as well as for safety analysis studies, i.e. analysis of short-time reactor transients.

In the past, low-order finite difference methods have been used normally for solving the neutron diffusion equation, especially for small Light Water reactors (LWRs) /4/. However these methods require an enormous amount of space meshpoints and consequently very much computer time. In addition, for large 3000 MW (thermal) power reactors one observes that the diameters, measured in diffusion lengths, are approximately 190 for an PWR and 36 for Liquid Metal cooled Fast Breeder Reactors (LMFBRs) /13/. This indicates the well known fact that the fuel subassemblies in large PWRs are neutronically very loosely coupled, so that the solution methods converge very slowly unless very efficient and partly sophisticated convergence acceleration techniques are applied. For three-dimensional problems with accuracy requirements for the average assembly powers in the range of one percent, the computer times for codes based on low-order difference methods became so excessive that such codes could hardly be used for routine calculations. For large fast breeder reactors the situation is somewhat relaxed with respect to the required number of spatial mesh points and also the neutronic coupling of the cores is not as loose as in the light water reactor cores. But, on the other hand, these reactors require more neutron energy groups for an adequate description.

For this reason much emphasis has been put on the development of more efficient numerical methods during the last decade. Especially the class of so-called nodal methods /2,5,6/ has been investigated to enable the calculation of flux and power distributions in modern nuclear reactor systems in the steady-state as well as in transient operation. The application of these methods involves the determination of equivalent homogenized group diffusion parameters /7,8/ representing relative large subregions of the reactor (so-called "nodes") such as entire fuel assemblies. Typical assembly sidelengths are: for LWR in rectangular geometry 20 cm (two-dimensional IAEA Benchmark Problem /14/) and for LMFBR in hexagonal geometry about 6.5 cm (three-dimensional INR Benchmark Problem /15/). Once these parameters are known, the global power distribution is obtained by solving the diffusion equation for

this "homogenized-assembly" reactor model, using simple approximations for the space dependence of the flux within the node. Therefore much attention has to be given to the coupling between the nodes. In this way the nodal methods can produce sufficiently accurate results when applied to a mesh corresponding to the dimensions of the homogenized fuel assemblies, i.e. by an appreciably smaller number of mesh points, than has been found to be necessary for finite-difference methods.

One of the modern nodal methods is the Nodal Green's Function Method /1,2,3/, described as a highly accurate and efficient tool for the analysis of LWRs as well as LMFBRs in multidimensional models. This method achieves a very high accuracy when it is applied on a mesh size, corresponding to the dimensions normally used for fuel assemblies for LWRs. Numerical comparisons show (Table I - reproduced from Table III in /3/) that the efficiency of NGFM can be higher by about three orders of magnitude than that of standard finite difference methods for three-dimensional LWR calculations. The use of such a nodal method should allow to perform three-dimensional reactor calculations with high accuracy much more on routine basis than it is presently possible using conventional finite difference methods.

The NGFM-computer code, developed by R. Lawrence /1/ is based on the solution of the three-dimensional multi-group steady-state diffusion equation in rectangular geometry by the Nodal Green's Function Method. The usual fixed source scheme (i.e. outer iteration formalism) is applied in this code. There are two possibilities to accelerate the outer iterations convergence: fission source extrapolation /10/ and coarse-mesh rebalancing technique /11/.

The present report contains a short description of the NGFM and the NGFM-Code, as well as the user's manual and a sample case for illustration.

In a forthcoming KfK-report related studies devoted especially to LMFBR-applications will be described.

2. Solution of the Multigroup Diffusion Equations by the Nodal Green's Function Method

The NGFM is based on the linear form of the nodal balance equation written in terms of the face-average partial currents across the surfaces of the node, namely

$$(1)\quad \sum_{u=x,y,z} \frac{1}{2\,a_u^k} \left[J_{gu}^{out,k}(a_u^k) - J_{gu}^{in,k}(a_u^k) + J_{gu}^{out,k}(-a_u^k) - J_{gu}^{in,k}(-a_u^k) \right]$$

$$+ \Sigma_g^{r,k}\, \bar{\phi}_g^k = \bar{Q}_g^k \qquad \begin{matrix} g=1,\ldots G. \\ k=1,\ldots K. \end{matrix}$$

where the average (or nodal) fluxes and group sources are defined by

$$(2a)\quad \bar{\phi}_g^k \equiv \frac{1}{V_k} \int_{V_k} \phi_g(r)\,dr^3$$

and

$$(2b)\quad \bar{Q}_g^k \equiv \frac{\chi_g}{\lambda} \sum_{g'=1}^{G} (\nu\Sigma_{g'}^{f,k} + \Sigma_{gg'}^{s,k})\, \bar{\phi}_{g'}^k$$

respectively.

Equation (1) is derived by integrating the steady-state diffusion equation in standard multigroup representation, assuming Cartesian geometry. The reactor configuration V is partitioned into an array of K homogeneous boxes (or nodes) V_k, k=1,...K, such that $V_k \cap V_1 = 0$, $k \neq 1$, and $\cup V_k = V$.

a_u^k, u = x,y,z denote the node halfwidths (Fig. 1), i.e.

$$V_k \equiv \left[-a_x^k, a_x^k \right] \times \left[-a_y^k, a_y^k \right] \times \left[-a_z^k, a_z^k \right]$$

All other quantities have their usual meaning.

The balance equation (1) is solved in two steps:

1) Coupling each node "k " with the adjacent nodes by the face-averaged partial currents across the surfaces of the node. The face-averaged outgoing and incoming partial currents across the node surfaces perpendicular to the u-direction are given by

$$(3a) \qquad J_{gu}^{out,k}(\pm a_u^k) \equiv \left[\frac{1}{4} \phi_{gu}^k(u) \mp \frac{1}{2} D_g^k \frac{\partial}{\partial_u} \phi_{gu}^k(u) \right]_{u=\pm a_u^k},$$

$$u = x,y,z$$

and

$$(3b) \qquad J_{gu}^{in,k}(\pm a_u^k) \equiv \left[\frac{1}{4} \phi_{gu}^k(u) \pm \frac{1}{2} D_g^k \frac{\partial \phi}{\partial_u}{}_{gu}^k(u) \right]_{u=\pm a_u^k},$$

$$u = x,y,z$$

where $\phi_{gu}^k(u)$ is a partially integrated flux, defined by

$$(4) \qquad \phi_{gu}^k(u) \equiv \frac{1}{4 a_w^k a_u^k} \int_{-a_w^k}^{a_w^k} dw \int_{-a_v^k}^{a_v^k} dv \quad \phi_g^k(u,v,w),$$

$$u = x,y,z, \qquad w \neq u \neq v$$

2) reduction of the solution of the three-dimensional problem to the solution of three successive one-dimensional problems, obtained by integrating the diffusion equations for each box over the two directions transverse to each coordinate direction. This reduction is made with the goal of receiving the additional relations between surface partial currents and the interior fluxes.

- solving each one-dimensional problem by application of Green's functions for the one-dimensional in-group diffusion removal operator. The integral equations received represent an exact (local) solution to the coupled set of one-dimensional differential equations.

- spatial approximation of these one-dimensional integral equations, using a weighted residual procedure applied within each node /12/. The partially integrated group fluxes, group sources, and the transverse leakages are expanded in quadratic polynomials

$$\text{(5a)} \qquad \phi^k_{gu}(u) = \sum_{n=1}^{\bar{3}} \phi^k_{gun} P^k_{un}$$

$$\text{(5b)} \qquad Q^k_{gu}(u) = \sum_{n=1}^{\bar{3}} Q^k_{gun} P^k_{un}$$

$$\text{(5c)} \qquad L^k_{gu}(u) = \sum_{n=1}^{\bar{3}} L^k_{gun} P^k_{un}$$

$u = x,y,z,\ k = 1, \text{NET}$

NET - total number of nodes

$$P^k_{u1}(u) = 1$$

$$P^k_{u2}(u) = 2$$

and

$$P^k_{u3}(u) \equiv u^2 - \frac{1}{3}(a^k_u)^2$$

- solving the resulting matrix equation with expansion coefficients, which provides the necessary additional relationships between the interface partial currents and the flux within the node, in conjunction with the linear form of the nodal balance equation (1).

3. Basic Iteration Strategy and Acceleration Techniques

3.1 Iteration Scheme

The basic iteration strategy, applied in the NGFM-code, consists of the following steps:

1. Constructing the vector group source $\vec{Q}^{k(n)}_{gu}$, k=1,...K, u = x,y,z , and the average (nodal) group source $\bar{Q}^{k(n)}_{gu}$, k=1,...K, e.g.

$$(6a)\quad \bar{Q}^{k(n)}_{g} = \chi_g \frac{1}{\lambda^{(n-2)}} \bar{\Psi}^{k(n-1)} + \sum_{g'<g} \Sigma^{s,k}_{gg'} \bar{\phi}^{k(n)}_{g'},$$

where only down-scattering is allowed and the average (nodal) fission source $\bar{\Psi}^{k}$ is defined by

$$(6b)\quad \bar{\Psi}^{k(n)} \equiv \sum_{g'=1}^{G} \nu \Sigma^{f,k}_{g'} \bar{\phi}^{k(n)}_{g'},$$

2. Calculating the vector transverse leakage $\vec{L}^{k(n)}_{gx}$, k=1,...K, (K - total number of nodes) using the y-directed partial currents $J^{out,k(n-1)}_{gy}(\pm a^k_y)$, from the previous outer iteration, and then solving the matrix equations over each row of the computational mesh for the x-directed partial currents, namely:

$$(7a)\quad J^{out,k(n)}_{gx}(+a^k_x) = \left[\vec{G}^{\pm n}_{gx}\right]^T \{\vec{Q}^{k(n)}_{gx} - \vec{L}^{k(n)}_{gx}\} + R^k_{gx} J^{in,k(n)}_{gx}(a^k_x)$$

$$+ T^k_{gx} J^{in,k(n-1)}_{gx}(-a^k_x), \qquad n = 1,...K$$

$$g = 1,...G$$

(7b) $$J^{out,k(n)}_{gx}(-a^k_x) = \left[\vec{\bar{G}}^{+x}_{gx}\right]^T \{\vec{Q}^{k(n)}_{gx} - \vec{L}^{k(n)}_{gx}\} + R^k_{gx}\, J^{in,k(n-2)}_{gx}(-a^k_x)$$

$$+ T^k_{gx}\, J^{in,k(n)}_{gx}(a^k_x), \qquad k = 1,\ldots K \quad g = 1,\ldots G$$

where $\left[\vec{\bar{G}}^{+x}_{gx}\right]$ is a column vector, representing the convolution integral of the Green's functions and the expansion polynomials, and R^k_{gx} and T^k_{gz} are reflection and transmission coefficients, respectively, obtained from Green's functions, depending on width and material properties of the node /1/.

3. Calculating the vector transverse leakages $\vec{L}^{k(n)}_{gy}$, k=1,...K , using the x-directed partial currents just calculated and then solving equations (6) on each column of the computational mesh for the y-directed partial currents $J^{out,k(n)}_{gy(\pm a^k_y)}$

   Analogously, for three-dimensional problems calculating

   $\vec{L}^{k(n)}_{gz}$ using $J^{out,k(n)}_{gx}(\pm a^k_x)$ and $J^{out,k(n)}_{gy}(\pm a^k_y)$

   and then solving Eq. (7) on each one-dimensional block in z-direction for the z-directed partial currents $J^{out,k(n)}_{gz}(\pm a^k_z)$ *).

4. Calculating the vectors with the expansion coefficients $\vec{\phi}^{k(n)}_{gu}$ of the partially integrated fluxes, solving the one-dimensional matrix equations, obtained by the weighted residual procedure

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*) In order to avoid possible misunderstanding or misinterpretation it may be worthwhile to indicate that $\vec{L}^{k(u)}_{gz}$ is the leakage component <u>for</u> the one-dimensional calculation in the z-direction but it describes physically the leakage in the directions perpendicular to the z-direction.

$$(8)\qquad \vec{\phi}^{k(n)}_{gu} = \left[\vec{G}^{uu}_{gu}\right]\left\{\vec{Q}^{x(n)}_{gu} - \vec{L}^{k(n)}_{gu}\right\} + 2\left[\vec{G}^{u+}_{gu}\right] J^{in,k(n)}_{gu}(a^{k}_{u}) + 2\left[\vec{G}^{u-}_{gu}\right] J^{in,k(n)}_{gu}(-a^{k}_{u})$$

$$u = x,y,z,\ g = 1,\ldots G,\quad k = 1,\ldots K$$

where $\left[\vec{G}^{uu}_{gu}\right]$, $\left[\vec{G}^{u+}_{gu}\right]$ and $\left[\vec{G}^{u-}_{gu}\right]$ are 3×3 matrix and 3-entries vectors, respectively, defined in /1, p. 29/ by Green's functions $\vec{\phi}^{k(n)}_{gu}$, $\vec{Q}^{k(n)}_{gu}$ and $\vec{L}^{x(n)}_{gu}$ are column vectors with 3 entries, containing the expansion coefficients (Eq. (5)). $J^{in,k(n)}_{gu(\pm a^{k}_{u})}$ are the most recently calculated partial currents.

5. Calculating the eigenvalue:

$$\lambda^{(n)} = \lambda^{(n-1)}\ \frac{F^{(n)}}{F^{(n-1)}}$$

where

$$F^{(n)} = \sum_{k=1}^{\bar{k}} \bar{\Psi}^{k(n)}$$

Steps (1.) through (5.) constitute one fission source (or - outer) iteration; the inner loop, steps (1) through (4), is repeated for each energy group g,g=1,...G. The partial current equations (7) are solved iteratively using directed sweeps through the one-dimensional mesh /1/.

3.2 Incorporation of Boundary Conditions

The boundary conditions, taken into account for solving Eq. (7), are the following ones:

$$(9)\qquad J^{in,1}_{gu}(a^{1}_{u}) = c\,J^{out,1}_{gu}(a^{1}_{u}),\qquad a^{1}_{u}\varepsilon^{S},$$

where S - reactor boundary;

ℓ - number of node, adjacent to S;

c - constant, which can assume the following values:

c	Boundary conditions, imposed on the solution of the diffusion equation
-1	zero flux
0	zero incoming partial current
1	zero net current (symmetry)

3.3 Acceleration Techniques

There are two possibilities to accelerate the convergence rate in the NGFM-Code:

- using the fission source extrapolation method when a corresponding criterium is satisfied /10/
- using (by option) the coarse-mesh rebalancing method /11/. For this procedure a coarse mesh grid different from the original one and also a different number of rebalancing iterations may be used.

4. Code Description

The NGFM-Code is written in FORTRAN-IV and is used at KfK on an IBM-360/168 and an IBM-3033 computer. The version described on 21 March 1983 has the following general features:

1. Only down-scattering is allowed. The maximal number of energy groups is 8.

2. The node numbering is performed rowwise from left to right and from bottom to top (in three dimensional cases). Jagged boundary is allowed on the right (in (x-y) plane). See Fig. 3.

3. The code can solve problems having three types of boundary conditions at external node sides: a) zero flux; b) zero incoming partial current; c) symmetry (mesh-edged or mesh-centered).
   The vacuum boundary conditions a) and b) can be imposed on each reactor boundary, the symmetry conditions - only on the left and on the lower boundaries (in (x,y) plane) and additionally on the bottom boundary (in z-direction).

4. The code admits full plane (360^{o}) and 90^{o} symmetry sector of the reactor core (in (x,y) plane) and full or half height (in z-direction).

5. The code requires ca. 1172 K Bytes virtual storage including LMAX = 140000 4-Bytes words storage locations.

The array dimensions, designated LEND, should not exceed LMAX:

LEND=1+NGRP*ND1*ND3+NGRP*ND2+ND1*ND4+ND1*ND5,

where the following code variable designations are used:

ND1=NODES - total number of nodes
ND1=NX*NY*NZ; NX, NY, NZ - number of nodes in x-, y-, z-direction, respectively.

ND2=2*IDIM*NODES; IDIM - number of dimensions ($2 \leq IDIM \leq 3$)
ND3=3*IDIM;

ND4=2*IDIM+1;

ND5=ND4

NGRP - number of energy groups (NGRP $\leq$ 8).

A block-diagram of the code is presented in Fig. 2. The code routines and their functions are described below. Additional information, e.g. on the meaning of various variables may be found in Chapter 5, describing the input data.

MAIN - sets limits for calculation of array dimensions. Calculates array dimensions and pointers. Calls subroutines: SETUP, OUTER, OUTERD, OUTPUT.

OUTERD - solves directly (i.e. without iterations) the partial current equation, for two-dimensional problems with node numbers less than 11 in x- and y-direction. If at least one of these numbers is greater than 11, subroutine OUTER is used. Calls PCMATR, INNERD.

PCMATR - constructs partial current matrix PCM.

INNERD - calculates nodal flux and flux expansion coefficients. Normalizes flux coefficients. Calls subroutines: QSOUR and JCALCD.

JCALCD - calculates directly partial currents in each dimension. Calls subroutine TVLEAK.

SETUP - checks for jagged outer boundary and locates surface nodes on this boundary. Sets up flat initial flux ($\bar{\phi}_k^o = 1$) and calculates the initial outgoing partial currents $J_k^{out,o} = 0.25$ (k=1,NET) on this basis. Calculates: node volumes and surface areas, number of nodes in each row (by jagged outer boundary), initial production rate. Normalizes the flux. Calls subroutines: MESH, PARAB, GREENF.

Some important designations:

X(NGRP,ND1,ND3) - flux expansion coefficients groupwise, nodewise and directionwise (Eq. (6a)).

XJ(NGRP,ND2) - partial currents (in- and outgoing) groupwise through each node surface, including boundary surfaces (Eq. (7)).

QOLD(ND1,ND5) - fission source expansion coefficients for each node, in x-, y-, z-direction, respectively (Eq. (5), (6)).

Creates arrays:

KEND(J,K), J=1,2;K=NPY,...1 - array with entries specifying the numbers of the first and the last nodes respectively in row K $K \leq 50$;

KEND(J,K) J=3,4;K=NPX,...1 - array with entries specifying the numbers of the first and the last nodes respectively in column K $K \leq 50$;

KROW(K1), K1=1,...NPY - number of nodes in row NPY-K1+1 (necessary for jagged boundary)

KROW(K1)=KEND(2,K1)-KEND(1,K1)+1

MESH - reads geometry model data, node material numbers, boundary conditions, iteration and rebalancing parameters. Sets up node mesh. Calculates:

NPX=NPU(1) - maximum number of nodes in x-direction;
NPY=NPU(2) - maximum number of nodes in y-direction;
NPZ=NPU(3) - maximum number of nodes in z-direction;
NPXY - number of nodes in (x,y) plane;
NET - total number of nodes.

Sets up:

NASX=NASU(1) - number of assemblies in x-direction;
NASY=NASU(2) - number of assemblies in y-direction;
NASZ=NASU(3) - number of assemblies in z-direction;
NASXY - number of assemblies in (x,y) plane;
NAST - total number of assemblies.

Creates arrays:

MATN(I), I=1,...20 - material number for each volume element (maximal 20 volume elements);

W(I,J), I=1,...20,J=1,2,3 - width of volume element "I" in direction "J";

NCOUP(K,M), K=1,...NET, M=1,...2*IDIM - array with pointers for incoming partial currents (for each node "K")

NCOUP(K,NRPT), K=1,...NET, NRPT=2*IDIM+1 - volume element type in node "K";

ISWP(3,9) - array with sweep parameters in x-, y-, z-direction, respectively.

Sets up coarse-mesh rebalancing parameters:

NCMX=NCMU(1) - number of nodes for coarse-mesh call in x-direction;

NCMY=NCMU(2) - number of nodes for coarse-mesh cell in y-direction;

NCMZ=NCMU(3) - number of nodes for coarse-mesh cell in z-direction;

NCMXY - number of nodes for coarse-mesh cell in (x,y) plane;

NCMR - total number of nodes for coarse-mesh cell, NCMR $\leq$ 256;

IDBLK, ID1D - block sizes for coarse-mesh inversion;

IDBLK=NCMX,ID1D=NCMY for two-dimensional problems;

IDBLK=NCMXY,ID1D=NCMZ for three-dimensional problems;

IREBAL = 0 - no rebalancing

1 - with rebalancing

KCMBY(J,1), J=1,...2*IDIM	- highest numbered node on outer surface J;
KCMBY(IDIR,2) IDIR=1,...IDIM	- increment for coarse-mesh cell numbering in direction IDIR;
NCMC(ICM,IDIR) ICM=1,...NCMU (IDIR),IDIR=1, ...,IDIM	- array with entries specifying the number of nodes per coarse-mesh cell "ICM" in IDIR-direction; ICM $\leq$ 12.
FR(I),I=1,...NCMR	- rebalancing factors
RHO	- eigenvalue, calculated by coarse-mesh rebalancing
RHO=1.	- initial guess for eigenvalue

Prints input data, as well as iteration and rebalancing parameters. Calls subroutine NUCDAT.

NUCDAT - reads and prints nuclear data. Calls subroutine SIGMIN, if parameter NFILE > 0.

SIGMIN - transfers macroscopic cross sections of a SIGMN-block into NGFM-own storage areas.

GREENF - calculates 4 types of Green's function matrices for volume element type NR in each direction IDIR(IDIR=1,...IDIM) and in each energy group NG.

Some important designations:

IFLAG - print parameter

$$\text{IFLAG} = \begin{cases} 0 - \text{no print} \\ \neq 0 - \text{print all matrices} \end{cases}$$

NAPROX = 3 - degree of polynomial approximation

IPF=(IDIR-1)*NAPROX - counter for matrix element $\left[G^{uu}_{gu}\right]_{mn}$, (see /1/, p. 144), n=1,...NAPROX,m=IPF+1,...IPF+NAPROX; u=x,y,z (resp. IDIR=1,2,3),g=1,...NGG; NGG - total number of energy groups;

IPF=2*(IDIR-1)+1 - node surface number in "-" direction

$$IPJ = \begin{cases} 1 - \text{for surface } (-x) \\ 3 - \text{for surface } (-y) \\ 5 - \text{for surface } (-z) \end{cases}$$

NRNG=NGG*(NR-1)+NG - groupwise counter, sequentially for each volume element NR; NRNG $\leq$ 60.

The following 4 types of matrices GVV, GVS, GSV and GSS are calculated in GREENF (for readers more interested in the details see /1/, pp. 144 - 147):

1) volume-volume

$$GVV(NRNG,IPF+1,J)=\left(A^{-1}\cdot\left[G^{uu}_{ug}\right]\right)_{IJ},g\equiv NG,$$

NG=1,NGG;

I,J=1,NAPROX;

IPF+I $\leq$ 9; J $\leq$ 3;

u=x,y,z;

2) volume-surface

$$GVS(NRNG,IPF+I,IPY)=2\cdot\left(A^{-1}\cdot\left[\vec{G}^{u-}_{gu}\right]\right)_{I},I=1,...NAPROX$$

$$GVS(NRNG,IPF+I,IPJ+1)=2\cdot\left(A^{-1}\cdot\left[\vec{G}^{u+}_{gu}\right]\right)_{I},I=1,...NAPROX,IPJ\leq 6$$

3) surface-volume

$$GSV(NRNG,IPJ,IPF+I)=\left[\vec{G}^{-u}_{gu}\right]_{I/2},\ I=1,\ldots NAPROX$$

$$GSV(NRNG,IPJ+1,IPF+I)=\left[\vec{G}^{+u}_{gu}\right]_{I/2},\ I=1,\ldots NAPROX$$

4) surface-surface

GSS(NRNG,IPJ,IPJ)=REFL - reflection coefficient for surface IPJ=1,3,5

GSS(NRNG,IPJ+1,IPJ+1)=REFL - reflection coefficient for surface IPJ+1=2,4,6

GSS(NRNG,IPJ,IPJ+1)=TRAN - transmission coefficient from surface IPJ+1 to surface IPJ

GSS(NRNG,IPJ+1,IPJ)=TRAN - transmission coefficient from surface IPJ to surface IPJ+1

OUTER - contains the outer iteration loop. Calculates the eigenvalue, if there is no rebalance acceleration. Checks whether the convergence criteria are fulfilled. The outer iterations terminate, if these criteria are satisfied, or/and the maximum number of outer iterations NMAX(1) is reached.

Pointwise (i.e. nodewise) fission source convergence criterion:

$$(10)\qquad ERRQ = \max_{k=1,..NET} \left| \frac{\bar{\psi}^{k}(n) - \bar{\psi}^{k}(n-1)}{\bar{\psi}^{k}(n)} \right| \leq ERR\ (1),$$

where $\bar{\psi}^{k}(n)$ - average fission source in node K for outer iteration "n" (Eq. (6b))

ERR(1) - input value

Flux convergence criterion*):

$$(11) \qquad ERRM = \max_{x,g} \left| \frac{\bar{\phi}_g^{k(n)} - \phi_g^{k(n-1)}}{\bar{\phi}_g^{k(n)}} \right| \leq ERR\ (2),$$

where ERR(2) is an input value.

The outer iterations are terminated, if criteria (10) and (11) are satisfied, or when the maximum number of iterations MMAX(1) is reached.

Prints iteration, rebalancing and fission source extrapolation parameters for each outer iteration. Calls subroutines INNER and REBAL (if IREBAL > 0).

INNER - calculates group flux expansion coefficients on the basis of Eq. (8) starting with the top plane (in each plane (x,y) from top to bottom, from the right to the left). Calculates the nodal flux using nodal balance equation (1). Normalizes flux coefficients.

Computes maximum flux error nodewise and groupwise**):

$$ERRF = \max_{\substack{k=1,\ldots K \\ g=1,\ldots G}} \left| \frac{\bar{\phi}_g^{k(n)} - \bar{\phi}_g^{k(n-1)}}{\bar{\phi}_g^{k(n)}} \right|$$

Computes maximum deviation from 1 for flux normalization coefficients FACT(NG,K)***), i.e.

$$ERRM = \max_{NG,K} \left| 1 - FACT(NG,K) \right|$$

Calls QSOUR and JCALC.

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*) The implementation and application of the flux criterion is a new feature compared to the original code as described in /1/.

) and *) are new options in comparison with the original NGFM-code /1/.

QSOUR - constructs group source. Computes pointwise (i.e. nodewise) the fission source error

(12) $$ERRQ = \max_{k=1,\ NET} \left| \frac{\bar{\psi}^{k(n)} - \bar{\psi}^{(k(n-1)}}{\bar{\psi}^{k(n)}} \right|$$

where $\bar{\psi}^{k(n)}$ - average fission source in node "K", for outer iteration "n"

Computes fission source extrapolation (FSE) parameter

(13) $$OMEG = \frac{\sqrt{\sum_{k=1}^{NET} (\bar{\psi}^{k(n)} - \bar{\psi}^{k(n-1)})^2}}{1 - \sqrt{\sum_{k=1}^{NET} (\bar{\psi}^{k(n)} - \bar{\psi}^{k(n-1)})^2}} = \omega^{(n)}$$

Checks for asymptotic convergence

(14) $$\left| \frac{\omega^{(n)} - \omega^{(n-2)}}{\omega^{(n)}} \right| \leq \varepsilon',$$

where ε' is an input value. Usually $\varepsilon' = 1.10^{-1}$ when the source extrapolation criterion (14) is satisfied, the fission source term and the partial currents are extrapolated with the parameter.

Some important designations:

QOLD(ND1),ND5) - array with entries specifying the fission source expansion coefficients nodewise (for outer iteration "n")

X(NG,K,N) - fission and down-scatter source expansion coefficients groupwise and nodewise for fixed outer iteration "n".
NG=1,NGG
K=1,NET
N=1,NAPROX

JCALC - calculates partial currents in each direction, solving Eq. (7). Calls subroutines TVLK1 and TVLK2.

TVLEAK - calculates one-dimensional transverse leakage expansion coefficients for fixed one-dimensional blocks on the basis of Lawrence's approximation (/1/, p. 32 - 33, p. 148).

REBAL - scales (with rebalance factors) nodal fluxes, flux expansion coefficients, outgoing currents and incoming currents on outer surfaces. Calculates new Wielandt estimate

```
ERR(4) = 1.0+ABS(RHO-1.0)
```

where RHO is the previously calculated eigenvalue. Calls subroutines: EXTEND, CMM2D, CMM3D and DIRECT.

EXTEND - extends partial currents on (jagged) outer boundary to square outer boundary for rebalancing.

CMM2D - calculates coarse mesh rebalance matrix H(80,80,12) for two-dimensional problems.

CMM3D - calculates coarse mesh rebalance matrix H(80,80,12) for three-dimensional problems.

DIRECT - calls the Harwell subroutine MB01B for the inversion of the matrix H /23/. [In Lawrence's version is used subroutine INVERS, but when applying it, some LMFBR Benchmark Problems could not be solved on an IBM computer even with double word precision because of the occurrence of divide checks.] Performs NCMI coarse mesh iterations, where NCMI is an input value. Solves for rebalancing factors F. Computes new estimate for eigenvalue - RHO. Computes F-maximum deviations from 1, i.e. the value ERRCM.

OUTPUT - normalizes node power densities in such a way that the total power density is 1. Prints primary results. By option prints node power densities and normalized average assembly fluxes.

5. Input and Output Description

The input consists of following cards:

Card Type	Format	Item	Comment
1	10A4	(TITLE(I),I=1,20)	Identification heading: two strings of 40 characters
Geometry model data (read in subroutine MESH)			
2	14I5	IDIM	Dimension of the problem: 2 - two-dimensional problem 3 - three-dimensional problem
		NGG	Number of energy groups (NGG $\leq$ 8)
		NMAT	Number of materials (NMAT $\leq$ 11)
		NDE	Number of different (or unique) volume elements (which are simply computational mesh cells or "nodes") as determined by material composition and mesh spacings in each coordinate direction (NDE $\leq$ 20)
		NREG	Number of different overlays used in setting up the mesh. A total of NREG cards are read in subroutine MESH in order to specify the mesh.
		IPRT1	Printing option for input data: 0 - only axial buckling is printed; 1 - all nuclear data as well as geometry model are printed.

Card Type	Format	Item	Comment
		IPRT2	Printing option for output data: 0 - print primary results and average power distribution 1 - 0 as well as power distributions in each layer in z-direction 2 - 1 as well as normalized fluxes group - and nodewise
3	14I5	NBDY(I), I=1,2*IDIM	Boundary conditions: -1 - vacuum (ingoing current is 0); 0 - vacuum (flux is 0); 1 - edge-centered symmetry; 2 - mesh-centered symmetry imposed on the boundaries: -x, +x, -y, +y and -z, +z (for 3-dimensional problems)
4	14I5	NPTS	Number of subdivisions (in x- and y-directions) of the basic computational mesh. This mesh consists of "assemblies" and thus a total of NPTS*NPTS nodes per assembly (in x-y plane) are obtained. The power density edits are performed on the assembly-size mesh. When NPTS=1, each node corresponds to an assembly.
		NPTSZ	Number of subdivisions (in z-direction) of the basic computational mesh (i.e. number of nodes per assembly)
		NASU(I), I=1,IDIM	Number of assemblies in x-, y-, z-direction, respectively.

Card Type	Format	Item	Comment
5	14I5	MMAX(2)	Number of rebalance iterations per outer iteration. If MMAX(2)=0 - no rebalancing is performed.
		NCMPX	Such that NCMPX*NCMPX assemblies are combined to form one coarse mesh rebalance region.
		NCMPZ	Such that NCMZ z-planes (in the basic computational mesh) are combined into one coarse-mesh rebalance region.
6	I5,5E10.0	MMAX(1)	Maximum number of outer iterations
		ERR(1)	Outer iteration convergence criterion
		ERR(2)	Flux convergence criterion
		ERR(5)	Fission source extrapolation criterion
For each region NR=1, NREG is given the following card (7):			
7	2I5,2I5, E10.0, 2I5,E10.0, 2I5,E10.0	NTYPE	Volume element type (NTYP $\leq$ 20) NTYPE=0, for nodes outside the jagged boundary.
		NM	Material number for this volume element.
		(NU1(I),NU2(I), WU(I),I=1,IDIM)	
		NU1(I)	Number of the first assembly in I-direction, containing the volume element NTYPE
		NU2(I)	Number of the last assembly in I-direction, containing NTYPE
		WU(I)	Dimensions of these assemblies containing NTYPE in the corresponding direction "I".

Card Type	Format	Item	Comment
Nuclear data description (reading in subroutine NUCDAT)			
8	(5E14.0)	(CHI(I),I=1,NGG)	Fission spectrum (the same for all material regions).
		BSQ	Axial buckling (for the two-dimensional problem only).
9	1DI5	NFILE	Read option parameter: 0 - read nuclear data >0 - no reading of nuclear data; they are transferred by subroutine SIGMIN.
For each material NM are given consistently NGG cards with cross sections:			
10	5E14.0	DIFCO(NM,NG)	Diffusion coefficient for material NM in energy group NG, i.e. D_{NM}^{g} (assumed to be isotropic).
		SIGR(NM,NG)	Removal cross section for material NM in energy group NG, i.e. $\Sigma_{NM}^{R,g}$
		VSIGF(NM,NG)	Production cross section for material NM in energy group NG, i.e. $\nu\Sigma_{NM}^{f,g}$
11	5E14.0	(SCAT(NM,NG,NG1), NG1=1,NG-1)	Down-scattering cross-section, i.e. $\Sigma_{NM}^{S,g\to g'}$, only for NG > 1.

At the end of each outer iteration the following values are printed:
(not all of these variables were included in the original version of the code - see footnotes in chapter 4 - subroutines INNER and OUTER)

NOUT - outer iteration number
XFKEFF - recent estimate for eigenvalue
ERRQ - maximum relative fission source error
ERRF - maximum relative flux error
NUMG - energy group to which ERRF is attributed
NUMF - node to which ERRF is attributed
NR5 - energy group, for which the flux normalization factor has maximum deviation from 1
NRN - node, for which this deviation is maximal
ERRM - maximum deviation of flux normalization factor from 1
ERRCM - maximal deviation of rebalancing factors from 1
OMEG - fission source extrapolation factor (see Eq. (13))
ISE - FSE parameter: 0 - no FSE;
1 - FSE criterion is satisfied and FSE is performed.

6. Sample Problem

In the following the input data and output edit for the two-dimensional IAEA Benchmark Problem /14/ are given to illustrate some NGFM-Code capabilities.

```
2-D IAEA BENCHMARK PROBLEM
JAGGED OUTER BOUNDARY(COR.DIF.COEF)
   2    2    4    4   24    1    1    0
   2   -1    2   -1
   1    1    9    9
   2    2    2
 100   1.0E-04   1.0E-03   1.0E-01
   4    4    1    9      180.0    1    9      180.0
   3    3    1    1       20.0    1    1       20.0
   2    2    2    4       60.0    1    4       80.0
   3    3    5    5       20.0    1    1       20.0
   2    2    6    6       20.0    1    4       80.0
   2    2    7    7       20.0    1    2       40.0
   1    1    8    8       20.0    1    3       60.0
   2    2    1    1       20.0    2    4       60.0
   2    2    5    5       20.0    2    4       60.0
   1    1    7    7       20.0    3    5       60.0
   3    3    1    1       20.0    5    5       20.0
   2    2    2    2       20.0    5    7       60.0
   2    2    3    4       40.0    5    6       40.0
   3    3    5    5       20.0    5    5       20.0
   1    1    6    6       20.0    5    6       40.0
   2    2    1    1       20.0    6    7       40.0
   1    1    3    3       20.0    7    8       40.0
   1    1    4    4       20.0    7    7       20.0
   1    1    5    5       20.0    6    7       40.0
   1    1    1    2       40.0    8    8       20.0
   0    4    5    9      100.0    9    9       20.0
   0    4    7    9       60.0    8    8       20.0
   0    4    8    9       40.0    7    7       20.0
   0    4    9    9       20.0    5    6       40.0
   0
    1.000E+00      0.000E+00      0.000E+00
          1.5            0.03            0.0
          0.4            0.08          0.135
         0.02
          1.5            0.03            0.0
          0.4           0.085          0.135
         0.02
          1.0            0.03            0.0
          0.4            0.13          0.135
         0.02
          2.0            0.04            0.0
          0.3            0.01          0.000
         0.04
          2.0            0.04            0.0
          0.3           0.055            0.0
         0.04
C===== (5) INPUT FOR TWO-DIMENSIONAL IAEA BENCHMARK PROBLEM.
C=====     REFERENCE: ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK,
C=====     ANL-7416, SUPPLEMENT 2, ARGONNE NATIONAL LAB. (1977).
```

2-D IAEA BENCHMARK PROBLEM
JAGGED OUTER BOUNDARY(COR.DIF.COEF)

NODAL GREENS FUNCTION METHOD:

NUMBER OF SPACE DIMENSIONS	=	2
NUMBER OF GROUPS	=	2
NUMBER OF MATERIAL COMPOSITIONS	=	4
NUMBER OF DIFFERENT VOLUME ELEMENTS	=	4
NUMBER OF REGIONS	=	24

GEOMETRY:

NODES PER ASSEMBLY (X-Y PLANE)	=	1 X 1
AXIAL NODES PER AXIAL MESH DIVISION	=	1
NODES IN X-DIRECTION	=	9
NODES IN Y-DIRECTION	=	9
NODES IN Z-DIRECTION	=	1
TOTAL NUMBER OF NODES	=	81

MESH CODE:

ELEMENT TYPE	DELTA X	DELTA Y	DELTA Z	MATERIAL NUMBER
1	20.0000	20.0000	0.0	1
2	20.0000	20.0000	0.0	2
3	20.0000	20.0000	0.0	3
4	20.0000	20.0000	0.0	4

MESH LAYOUT:

AXIAL PLANE NO. 1

```
4 4 4 4 0 0 0 0 0
1 1 1 4 4 4 0 0 0
2 2 1 1 1 4 4 0 0
2 2 2 2 1 1 4 4 0
```

```
3 2 2 2 3 1 1 4 0
2 2 2 2 2 2 1 4 4
2 2 2 2 2 2 1 1 4
2 2 2 2 2 2 2 1 4
3 2 2 2 3 2 2 1 4
```

BOUNDARY CONDITIONS:

MINUS X: 2	-1 = VACUUM (ZERO J-IN)
PLUS X:-1	0 = VACUUM (ZERO FLUX)
MINUS Y: 2	1 = EDGE-CENTERED SYMMETRY
PLUS Y:-1	2 = MESH-CENTERED SYMMETRY
MINUS Z: 3	3 = NOT USED IN 2-D PROBLEM
PLUS Z: 3	

NUCLEAR DATA

MATERIAL NUMBER 1

GROUP	DIFFUSION	REMOVAL	VUFISS	FISSION	CHI
1	1.5000000E+00	3.0000001E-02	0.0	0.0	1.0000000E+00
2	3.9999998E-01	7.9999983E-02	1.3499999E-01	1.3499999E-01	0.0

SCATTERING MATRIX

GROUP	1	2
1	0.0	0.0
2	2.0000000E-02	0.0

MATERIAL NUMBER 2

GROUP	DIFFUSION	REMOVAL	VUFISS	FISSION	CHI
1	1.5000000E+00	3.0000001E-02	0.0	0.0	1.0000000E+00
2	3.9999998E-01	8.4999979E-02	1.3499999E-01	1.3499999E-01	0.0

SCATTERING MATRIX

GROUP	1	2
1	0.0	0.0
2	2.0000000E-02	0.0

MATERIAL NUMBER 3

GROUP	DIFFUSION	REMOVAL	VUFISS	FISSION	CHI
1	1.0000000E+00	3.0000001E-02	0.0	0.0	1.0000000E+00
2	3.9999998E-01	1.3000000E-01	1.3499999E-01	1.3499999E-01	0.0

SCATTERING MATRIX

GROUP	1	2
1	0.0	0.0
2	2.0000000E-02	0.0

MATERIAL NUMBER 4

GROUP	DIFFUSION	REMOVAL	VUFISS	FISSION	CHI
1	2.0000000E+00	3.9999999E-02	0.0	0.0	1.0000000E+00
2	3.0000001E-01	1.0000002E-02	0.0	0.0	0.0

SCATTERING MATRIX

GROUP	1	2
1	0.0	0.0
2	3.9999999E-02	0.0

AXIAL BUCKLING = 0.0

ITERATION PARAMETERS:

REBALANCING (0/1 = NONE/DIRECT)	=	1
REBALANCE ITERATIONS PER OUTER ITERATION	=	2
NUMBER OF NODES PER COARSE MESH CELL (X-Y)	=	2 X 2
NUMBER OF NODES PER COARSE MESH CELL (Z)	=	1
MAXIMUM NUMBER OF OUTER ITERATIONS	=	100
OUTER ITERATION CONVERGENCE CRITERION	=	1.0E-04
FLUX CONVERGENCE CRITERION	=	1.0E-03
SOURCE EXTRAPOLATION CRITERION	=	1.0E-01

COARSE-MESH LAYOUT:

COARSE-MESH DIVISION	NODES PER COARSE-MESH DIVISION X-DIRECTION	Y-DIRECTION	Z-DIRECTION
1	1	1	
2	2	2	
3	2	2	
4	2	2	
5	2	2	

TOTAL NUMBER OF COARSE-MESH CELLS = 5 X 5 X 1 = 25

OUTER ITERATIONS:

NO.	EIGENVALUE	POINTWISE CONVERGENCE	POINT-GROUPWISE CONVERGENCE	GROUP AND NODE WITH MAX. FLUX-ERR		GROUP AND NODE WITH MAX(FAC-1.0)		MAX(F-1.0)	OMEGA	SOURCE EXTRAP.?
1	1.00496292	1.00E+00	1.00E+00	2	1	1	69	3.294E-01	6.23E-01	1.6E-01	0
2	1.03922081	3.12E+00	1.18E+00	2	1	2	44	1.450E-01	4.85E-01	4.1E-01	0
3	1.03045177	4.32E-01	5.35E-01	2	61	2	53	1.213E-01	1.00E-01	1.5E+00	0
4	1.03135490	1.17E-01	1.57E-01	2	76	2	53	1.055E-01	5.15E-02	4.4E-01	0
5	1.03230095	8.65E-02	9.78E-02	2	53	2	53	1.009E-01	4.64E-02	2.5E+00	0
6	1.03282166	7.61E-02	8.13E-02	1	61	2	53	8.858E-02	3.92E-02	8.8E+00	0
7	1.03322601	6.41E-02	6.55E-02	1	61	2	53	7.528E-02	3.35E-02	5.2E+00	0
8	1.03352928	5.46E-02	5.51E-02	1	61	2	53	6.316E-02	2.88E-02	5.8E+00	0
9	1.03524590	4.65E-02	2.29E+00	2	53	2	53	4.352E-01	1.83E-02	6.3E+00	1
10	1.03554344	4.20E-02	4.63E-01	2	53	2	61	2.920E-01	2.54E-02	7.8E-01	0
11	1.03519344	5.39E-02	3.03E-01	2	61	2	69	1.439E-01	8.64E-03	7.9E+00	0
12	1.03503227	1.97E-02	6.00E-02	2	61	2	69	9.263E-02	6.11E-03	6.9E-01	0
13	1.03497219	1.33E-02	3.20E-02	2	69	2	69	5.760E-02	4.32E-03	2.9E+00	0
14	1.03493500	9.46E-03	1.91E-02	2	69	2	69	3.501E-02	3.23E-03	3.5E+00	0
15	1.03491020	7.05E-03	1.18E-02	2	69	2	69	2.077E-02	2.50E-03	4.8E+00	0
16	1.03489304	5.46E-03	7.27E-03	2	69	2	69	1.201E-02	1.97E-03	5.0E+00	0
17	1.03482914	4.31E-03	4.69E-02	2	61	2	36	5.491E-03	1.30E-03	5.1E+00	1
18	1.03481483	3.60E-03	8.81E-03	2	52	2	61	9.013E-03	1.58E-03	1.5E+00	0
19	1.03483009	3.61E-03	1.66E-02	2	61	2	69	3.491E-03	4.66E-04	2.5E+00	0
20	1.03483677	1.09E-03	2.75E-03	2	69	2	69	2.112E-03	3.14E-04	8.0E-01	0
21	1.03483963	6.96E-04	1.53E-03	2	69	2	69	1.218E-03	2.34E-04	1.9E+00	0
22	1.03484058	5.06E-04	9.25E-04	2	69	2	76	8.533E-04	1.84E-04	2.7E+00	0
23	1.03484154	3.84E-04	5.88E-04	2	69	2	76	6.255E-04	1.45E-04	4.0E+00	0
24	1.03484344	2.98E-04	3.86E-04	2	69	2	76	4.594E-04	1.17E-04	4.2E+00	0
25	1.03484821	2.35E-04	2.44E-03	2	61	2	76	5.627E-04	7.34E-05	4.4E+00	1
26	1.03484821	1.73E-04	5.13E-04	2	52	2	61	5.705E-04	6.96E-05	7.7E-01	0
27	1.03484726	1.66E-04	8.01E-04	2	61	2	69	3.175E-04	9.54E-06	-1.1E+02	0
28	1.03484726	2.44E-05	9.58E-05	2	69	2	69	2.081E-04	4.77E-06	6.8E-01	0

SUMMARY OF CALCULATION: 2-D IAEA BENCHMARK PROBLEM JAGGED OUTER BOUNDARY(COR.DIF.COEF)

EIGENVALUE	=	1.03484726
NO. OF OUTER ITERATIONS	=	28
NO. OF REBALANCE ITERATIONS	=	56
MAXIMUM FLUX ERROR	=	2.44E-05
CPU TIME FOR EIGENVALUE CALC.	=	0.0
TOTAL CPU TIME	=	0.0

AVERAGE ASSEMBLY POWERS: AXIAL AVERAGE:

	1	2	3	4	5	6	7	8	9
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.7525	0.7326	0.6831	0.0	0.0	0.0	0.0	0.0	0.0
7	0.9320	0.9405	0.9667	0.8410	0.5984	0.0	0.0	0.0	0.0
6	0.9328	1.0296	1.0658	0.9087	0.6932	0.5730	0.0	0.0	0.0
5	0.5843	1.0786	1.1932	0.9807	0.4521	0.6932	0.5984	0.0	0.0
4	1.2288	1.3198	1.3443	1.1987	0.9807	0.9087	0.8410	0.0	0.0
3	1.4766	1.4812	1.4583	1.3443	1.1933	1.0658	0.9667	0.6831	0.0
2	1.3261	1.4406	1.4813	1.3198	1.0786	1.0297	0.9405	0.7327	0.0
1	0.7171	1.3261	1.4766	1.2288	0.5843	0.9328	0.9320	0.7525	0.0
	1	2	3	4	5	6	7	8	9

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TABLE I

Comparison of Results for the Three-Dimensional IAEA Benchmark Problem
(reproduced from Table III /3/)

Method[a]	Reference	ε_{max} (%)	Execution Time (s)	Computer	Symmetry	Pointwise Convergence
Analytic method (QUANDRY)	17,22	0.7	29	IBM 370/168	One-eighth core	10^{-4}
Finite difference method[b] (VENTURE)	20	2.1	21 600	IBM 360/195	One-quarter core	10^{-5}
Finite element method[c] (FEM3D)	20	4.0	82 800	B-6700	One-eighth core	10^{-3}
Nodal expansion method	16,21	0.9	50	CDC 6600	One-eighth core	10^{-5}
NGFM	1,19	0.4	62	CYBER 175	One-quarter core	10^{-5}
NODLEG method	18	1.0	672	CDC 6500	One-quarter core	10^{-5}

[a] Except where noted, all calculations were done using a 20-cm radial mesh.

[b] 1.67-cm radial mesh.

[c] 5.0-cm radial mesh.

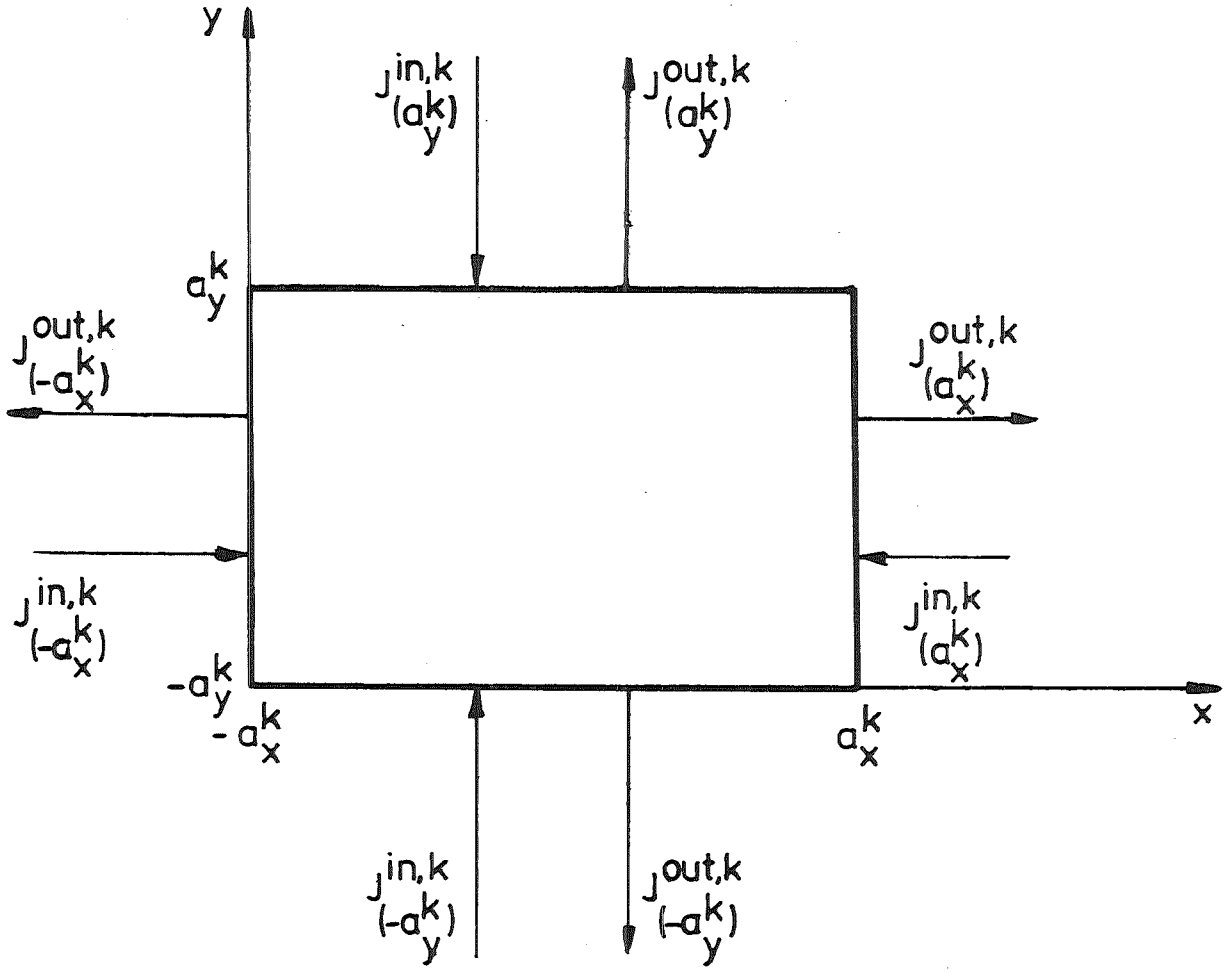

Fig 1: Scheme of the node "k" in the two-dimensional geometry model

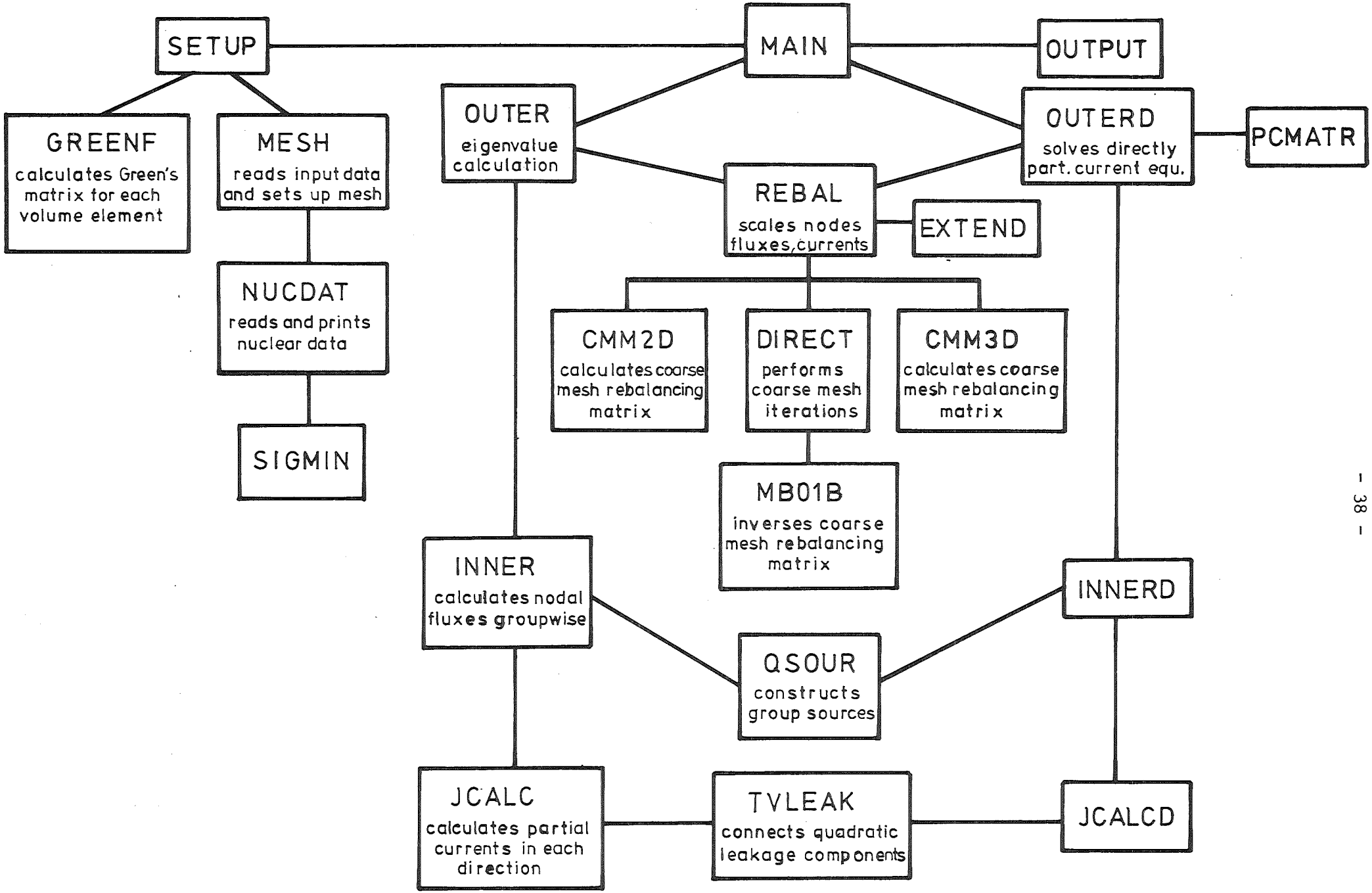

Fig. 2: Block - diagram of the NGFM - code

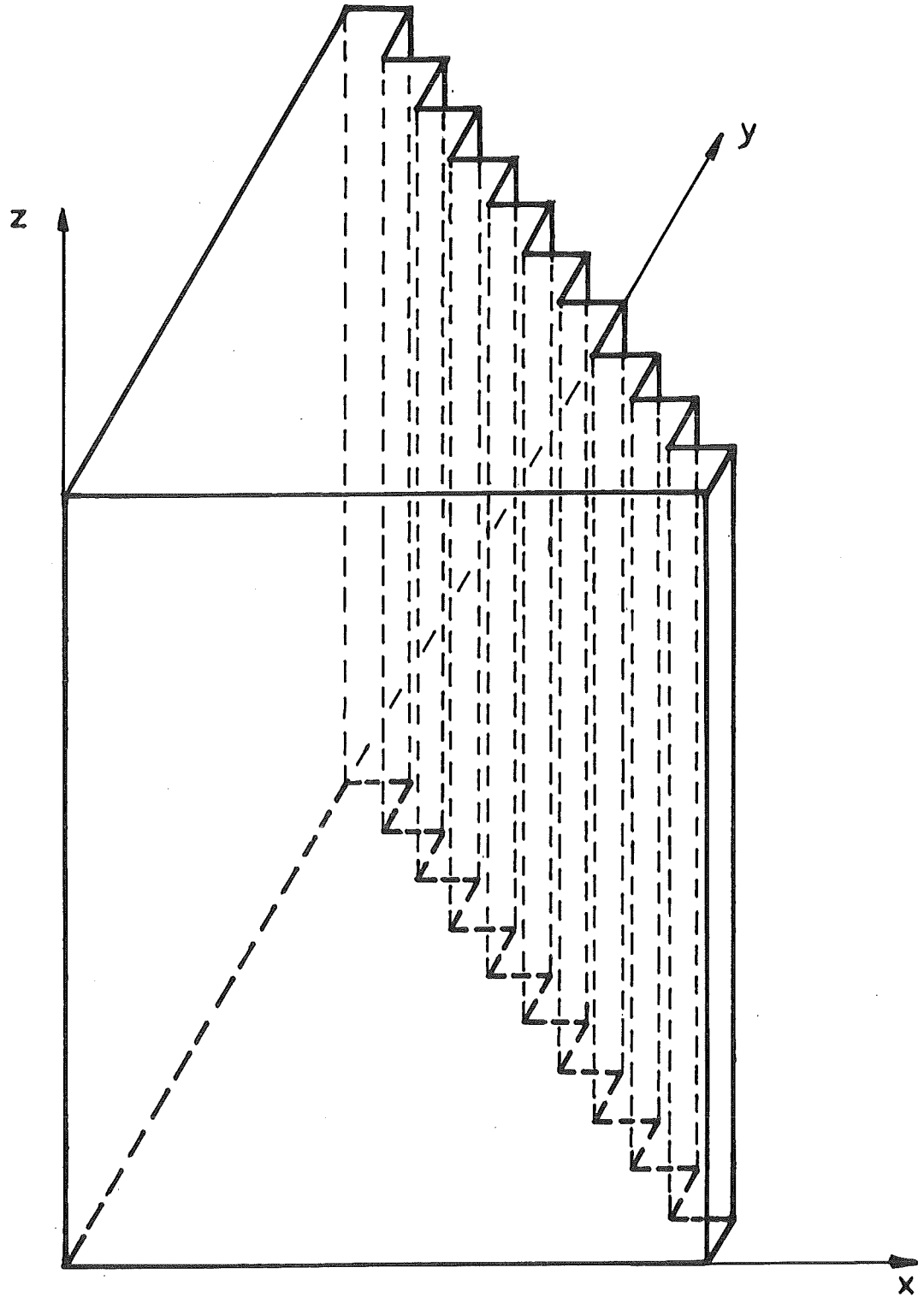

Fig. 3: NGFM-geometry calculational-model
(jagged boundary)

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