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RESONANT AND NONRESONANT COULOMB BREAK UP OF ^{6}Li

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Abstract

The resonant and nonresonant cross section for break up of ^{6}Li in the Coulomb field of a heavy nucleus is theoretically studied on the basis of a DWBA approach and analysed in view of a possible experimental access to electromagnetic transition matrix elements between the ground state of the projectile and α+d continuum states at small relative energies. The calculation explicitly uses some simplifications appearing in the particular case of quadrupole transitions which dominate the considered case. Various sensitivities of the cross sections are discussed.

DER RESONANTE UND NICHTRESONANTE COULOMB-AUFBRUCH VON ^{6}Li

Der resonante und nichtresonante Wirkungsquerschnitt des Aufbruchs von ^{6}Li im Coulomb-Feld eines schweren Kerns wird theoretisch auf der Grundlage einer DWBA-Beschreibung untersucht und im Hinblick auf eine mögliche experimentelle Bestimmung von elektromagnetischen Matrixelementen für den Übergang vom Projektil-Grundzustand zu α+d Kontinuumszuständen analysiert. Die Rechnungen benutzen einige Vereinfachungen, die im Falle von Quadrupol-Übergängen auftreten, die den betrachteten Fall dominieren. Verschiedene Empfindlichkeiten des Wirkungsquerschnitts werden diskutiert.

1. Introduction

The dissociation of light-ion projectiles under the influence of the electromagnetic and nuclear fields, experienced while passing a target nucleus, is an important reaction mode in nucleus-nucleus interactions. (Baur et al. 1984, de Meijer and Kamermans 1985) The reaction mechanism may be virtually considered as a two-step process, first exciting the projectile in a continuum state with the relative momentum k of the fragments, then decaying into free fragment states. Two extreme situations do emerge, depending on the relation between the collision time and the lifetime of the excited projectile state. (Weidenmüller and Winther 1971) The so-called sequential break-up proceeds via an excitation of a particle-unstable resonance state with sufficiently long life time, so that it subsequently decays far-away from the excitation region. The (elastic) process can be fairly well separated into sequential steps

$$a + A \rightarrow a^* + A \rightarrow b + x + A$$

and the DWBA transition matrix element can be naturally written as (Rybicki and Austern 1972)

$$T_{fi} = \langle \chi^{(-)}_{\vec{Q}_f}(\vec{R})\ \phi^{(-)}_k(r)\,|V_{res}(\vec{R},\vec{r})|\,\chi^{(+)}_{\vec{Q}_i}(\vec{R})\ \phi_a(\vec{r})\rangle \tag{1.1}$$

where $\chi^{(+)}_{\vec{Q}_i}(\vec{R})$ and $\chi^{(-)}_{\vec{Q}_f}(\vec{R})$ denote the centre of mass motion of the initial and the final state with the momenta $\vec{Q}_i$ and $\vec{Q}_f$, respectively. The wave functions $\phi_a(\vec{r})$ and $\phi^{(-)}_k(\vec{r})$ describe the groundstate and the continuum states of the projectile. When the fragments b and x are observed with the momenta k_b and k_x, the momenta are given (in obvious notation) by

$$\vec{Q}_f = \vec{k}_b + \vec{k}_x \tag{1.2a}$$

$$\vec{k} = \frac{m_b}{m_a}\vec{k}_x - \frac{m_x}{m_a}\vec{k}_b \tag{1.2b}$$

In the case of a pure sequential process, ϕ_k is a resonance state, but the same matrix element may also describe nonresonant break up processes when adequate wave functions ϕ_k are introduced, describing the relative fragment motion with k far-away from a resonance. This is the break up model introduced by Rybicki and Austern (1972), while Baur et al. (1976, 1980, 1984) immediately replace $\chi_{Q_f}(\vec{R})\cdot\phi_k$ by $\chi_b(\vec{q}_b,\vec{R})\cdot\chi_x(\vec{q}_x,\vec{R})$, thus starting with the other extreme situation of a simultaneous direct break up.

In the present paper we consider some particular features of the matrix element (eq. 1.1) when used for describing the dissociation in the Coulomb field. Assuming point charge distribution for the constituent projectile cluster, the residual interaction is given (for R>r) by

$$V_{res} = Z_A e^2 \left(\frac{Z_b}{r_{bA}} + \frac{Z_x}{r_{xA}} - \frac{Z_a}{R}\right)$$

$$= 4\pi Z_A e^2 \sum_{L,M\geq 1} \left\{ Z_b\left(-\frac{m_x}{m_a}\right)^L + Z_x\left(\frac{m_b}{m_u}\right)^L \right\} \frac{r^L}{R^{L+1}} \qquad (1.3)$$

$$* \frac{1}{2L+1} \cdot Y^*_{LM}(\hat{R}) \cdot Y_{LM}(\hat{r})$$

In particular, we would like to call attention to the fact, that the matrix element

$$M^L_{\vec{Q}_i\vec{Q}_f} \propto \langle \chi^{(-)}_{\vec{Q}_f}(\vec{R}) \,|\, R^{-L-1}\, Y^*_{LM}(\hat{R}) \,|\, \chi^{(+)}_{\vec{Q}_i}(\vec{R}) \rangle \qquad (1.4)$$

for a quadrupole transition (L=2) can be factorized in a term independent from the momentum transfer $\vec{q} = \vec{Q}_i - \vec{Q}_f$ and a term which we call "contact term", being the product

$$|\chi^{(+)}_{\vec{Q}_i}(0) \cdot \chi^{(-)*}_{\vec{Q}_f}(0)|.$$

This speciality of L = 2 transitions has been already noticed by Mullin and Guth (1951) and may lead to considerable simplifications in actual cases. Such a case is given by the ^{6}Li $\rightarrow$ α+d break up (Hansteen and Wittern 1965), where dipole transitions are largely suppressed ($Z_x/m_x = Z_b/m_b$), and the sequential break up proceeding through L = 2 transitions to the 3^+_1 state in ^{6}Li has been found to be dominant (Scholz et al. 1977, Gemmeke et al. 1978). Referring to the case of 156 MeV-^{6}Li break up, we consider the interplay of resonant and nonresonant Coulomb dissociation processes, study various sensitivities and present calculated triple-differential cross sections. In particular, we try to estimate the value of the nonresonant dissociation cross section at a center-of-mass energy of the α-d-system $E_{CM} \lesssim 0.7$ MeV. In this energy region the direct capture reaction, which is in some sense the inverse process, is of astrophysical interest (Robertson et al. 1981) in view of the question of ^{6}Li production in the big bang, but a direct experimental access to the capture cross sections at low energies is very difficult. It has been recently proposed (Rebel 1985, Baur 1985) to study the inverse process, the photodisintegration by the virtual photon field, seen by the projectile when passing the Coulomb field of a heavy nucleus. This implies an enhancement of the cross section and experimental advantages.

2. Evaluation of the orbital matrix element.

In the plane wave Born approximation the matrix element $M^L_{\vec{Q}_i\vec{Q}_f}$ (e.g. 1.4) is written (with z-axis along $\vec{q}$)

$$\begin{aligned} M^L_{\vec{Q}_i\vec{Q}_f}\ (\mathrm{PWBA}) &= Z_A e \sqrt{\frac{4\pi}{2L+1}} \int \frac{e^{i\vec{q}\cdot\vec{R}}}{R^{L+1}}\ Y_{LM}\ (\hat{R})\ d\vec{R} \\ &= 4\pi\cdot Z_A e\ i^L \int \frac{j_L(qR)}{R^{L+1}}\ R^2 dR \\ &= 4\pi\cdot Z_A e\ i^L\ q^{L-2} \lim \left[\frac{j_{L-1}(qR)}{(q\cdot R)^{L-1}}\right]_{R\to 0} \\ &= 4\pi\cdot Z_A e\ i^L\ \frac{q^{L-2}}{(2L-1)!!} \end{aligned} \qquad (2.1)$$

This matrix element has already the endearing property of being independent of $\vec{q}$, $\vec{Q}_i$ and $\vec{Q}_f$ for L = 2. Introducing the Fourier transforms of the scattering wave functions*

* This procedure was suggested by R. Serber as quoted by Mullin and Guth (1951)

$$\chi_{\vec{Q}_i}(\vec{R}) = \int \tilde{\chi}_i(\vec{k}_i) e^{i \vec{k}_i \cdot \vec{R}} d\vec{k}_i \tag{2.2a}$$

$$\chi_{\vec{Q}_f}(\vec{R}) = \int \tilde{\chi}_f(\vec{k}_f) e^{i \vec{k}_f \cdot \vec{R}} d\vec{k}_f \tag{2.2b}$$

eq. 1.4 is transformed in

$$M^L_{\vec{Q}_i \vec{Q}_f} = \int d\vec{k}_i d\vec{k}_f \; M^L_{\vec{k}_i \vec{k}_f}(\text{PWBA}) \; \tilde{\chi}_i(k_i) \; \tilde{\chi}_f(k_f) \tag{2.3}$$

and for L = 2

$$M^{L=2}_{\vec{Q}_i \vec{Q}_f} = M^{L=2}_{\vec{Q}_i \vec{Q}_f}(\text{PWBA}) \cdot \chi^{(+)}_{\vec{Q}_i}(R=0) \chi^{(-)}_{Q_f}(R=0) \tag{2.4}$$

$$= M^{L=2}_{\vec{Q}_i \vec{Q}_f}(\text{PWBA}) \; \left[\frac{u^i_0(Q_i R)}{Q_i R}\right]^*_{R=0} \; \left[\frac{u^f_0(Q_f R)}{Q_f R}\right]_{R=0}$$

where u^i_0 and u^f_0 are s-partial waves of the scattering states, evaluated at the origin. Obviously the use of correct distorted waves modifies the PWBA cross section just by a factor. In the case of pure Coulomb scattering from a point nucleus, this factor

$$C^2 = [2\pi\eta_i/(e^{2\pi\eta_i}-1)] \; [2\pi\eta_f/(e^{2\pi\eta_f}-1)] \tag{2.5}$$

with the Coulomb parameter

$$\eta_{i,f} = Z_p \cdot Z_A \; e^2 / \hbar \; v_{i,f}$$

and represents a measure of the penetration of the particle into the Coulomb barrier. It is quite clear that this factor changes when the attractive nuclear field is taken into account. Therefore, the "contact term" $C^2 = |\chi^{(+)}_{\vec{Q}_i}(R=0) \cdot \chi^{(-)*}_{\vec{Q}_f}(R=0)|^2$ has to be evaluated with distorted waves, derived from a Coulomb potential of realistic charge distribution and from adequate nuclear potentials, thus introducing absorption effects. The corresponding wave functions can be easily provided from any optical model scattering code.

3. Studies of various sensitivities and theoretical cross sections

We consider the example of the elastic break of 156 MeV ^{6}Li ions in the Coulomb field of ^{208}Pb(η_i = 7.6), where fragment-α-particles and deuterons are emitted at various angle pairs (Θ_α,Θ_d). The relative angle ΔΘ defines the minimum value of k, observable in a particular kinematical situation. Fig. 1 displays the variation of k with the energy of the α-particle fragment, which is coincidently observed with the fragment deuteron, while the target nucleus remains in the ground state (elastic break up).

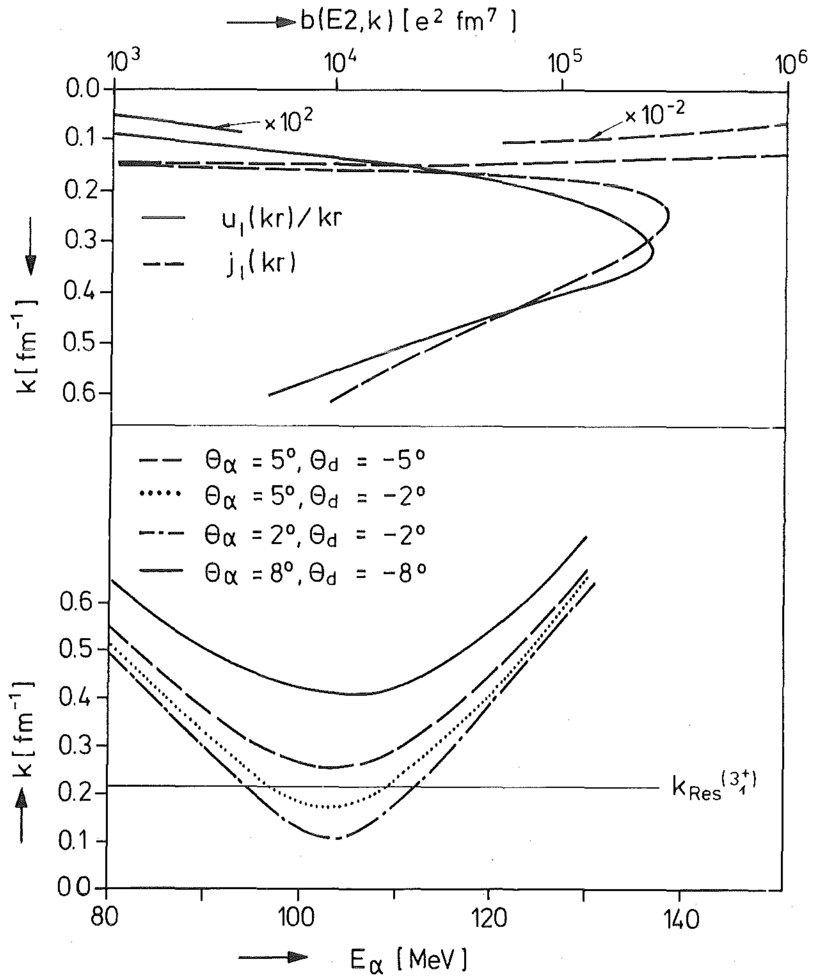

Fig. 1 Variation of the relative α-d momentum k with the observed α-particle energy E_α^{Lab} in the elastic break up of 156 MeV-^{6}Li-ions, and the nonresonant b(E2,k) distribution

The evaluation of the full matrix element (1.1) requires a specification of the wave functions $\phi_{Li}(\vec{r})$ and ϕ_k. The ground state wave function is generated by a bound-state potential of Saxon-Woods form reproducing the binding energy while the nonresonant continuum states were generated in the α-d potential given by McIntyre and Haeberli 1967 (see also Robertson et al. 1981).

The internal part of the matrix elements (1.1) can be written

$$M(E2,M) = \sqrt{\frac{4\pi}{5}} \{Z_\alpha e (\frac{m_d}{m_{Li}})^2 + Z_d e (\frac{m_\alpha}{m_{Li}})^2\} * \langle\phi_k | r^2 \; Y_{LM} | \phi_{Li}\rangle \tag{3.1}$$

which is related to the reduced transition probability (De Shalit and Talmi 1963)

$$b(EL,k,J_i \to J_f) = \{Z_\alpha e (\frac{m_d}{m_{Li}})^2 + Z_d \; e (\frac{m_\alpha}{m_{Li}})^2\}^2 * \frac{(2J_f+1)(2l_i+1)(2l_f+1)(2L+1)}{4\pi} * \begin{pmatrix} l_i & l_f & L \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} l_f & l_i & L \\ J_i & J_f & s_f \end{Bmatrix}^2 |\langle R^L\rangle|^2 \tag{3.2}$$

We denote

$$\langle R^L\rangle = 4\pi \; i^L \sqrt{\frac{2L+1}{4\pi}} \int_0^\infty \frac{u_{l_f}(kr)}{kr} \; r^L \; \Phi_{Li}(r) r^2 \; dr \tag{3.3}$$

with Φ_{Li} being the radial part of Φ_{Li}, $u_{l_f}(kr)/kr$ the same of the continuum states.

For the case of the $1^+_{gr} \to 3^+_1$ transition in ^6Li ($l_i=0$, $l_f=2$, $s_f=1$, $L=2$) we get

$$b(E2,k,\; 1^+ \to 3^+) = \{Z_\alpha e (\frac{m_d}{m_{Li}})^2 + Z_d e (\frac{m_\alpha}{m_{Li}})^2\}^2 \; \frac{7}{12\pi} \; |\langle R^2\rangle|^2 \tag{3.4}$$

The quantity b(E2,k) represents a transition density [in units of e^2 fm^7] and is displayed in Fig. 1. For comparison, the b(E2,k) distribution is additionally calculated by using simply plane waves for describing the continuum states. It is obvious that at very low k-values considerable differences occur as compared to the use of more correct scattering states.

The triple differential cross section

$$\frac{d^3\sigma}{d\Omega_\alpha d\Omega_d dE_\alpha} = \frac{2\pi\ \mu}{\hbar^2 Q_i} \frac{1}{2J_i+1} \sum_{M_i M} |T_{fi}|^2 \cdot \rho \qquad (3.5)$$

with ρ being the three body phase space factor (see Baur and Trautmann 1976a and Ohlsen 1965) can be concisely written as

$$\frac{d^3\sigma}{d\Omega_\alpha d\Omega_d E_\alpha} = \frac{(4\pi)^4}{90\hbar} \frac{Z_A^2\ e^2}{(2J_f+1)} \sqrt{\frac{m_{Li}}{E_{Li}^{Lab}}}\ C^2\ b(E_\alpha,E)\ \rho \qquad (3.6)$$

(with E being the relative energy of the fragments)

as

$$\frac{1}{2J_i+1} \cdot \sum_{M_i M} |T_{if}|^2 = [\frac{4\pi}{3} Z_A e]^2 \cdot \frac{1}{2J_f+1} (\frac{4\pi}{5})\ b(E2,E) \qquad (3.7)$$

Figs. 2 - 4 show the nonresonant triple differential cross section for some pairs of emission angles. The results in Fig. 2 differ by the values of the penetration factor C^2. The value for a point charge is (at E_α = 104 MeV)

C^2 (point charge) $= 0.36 \cdot 10^{-38}$

while the realistic values

C^2 (homog. charge) $= 0.5482$

C^2 (homog. charge + nucl.) $= 0.80 \cdot 10^{-5}$

are calculated with a spherical homogeneous charge distribution ($R_c = 1.3\ A_{Target}^{1/3}$) and using optical model parameters of a Saxon-Woods form as given by Cook et al. 1982 and Neumann et al. 1982.

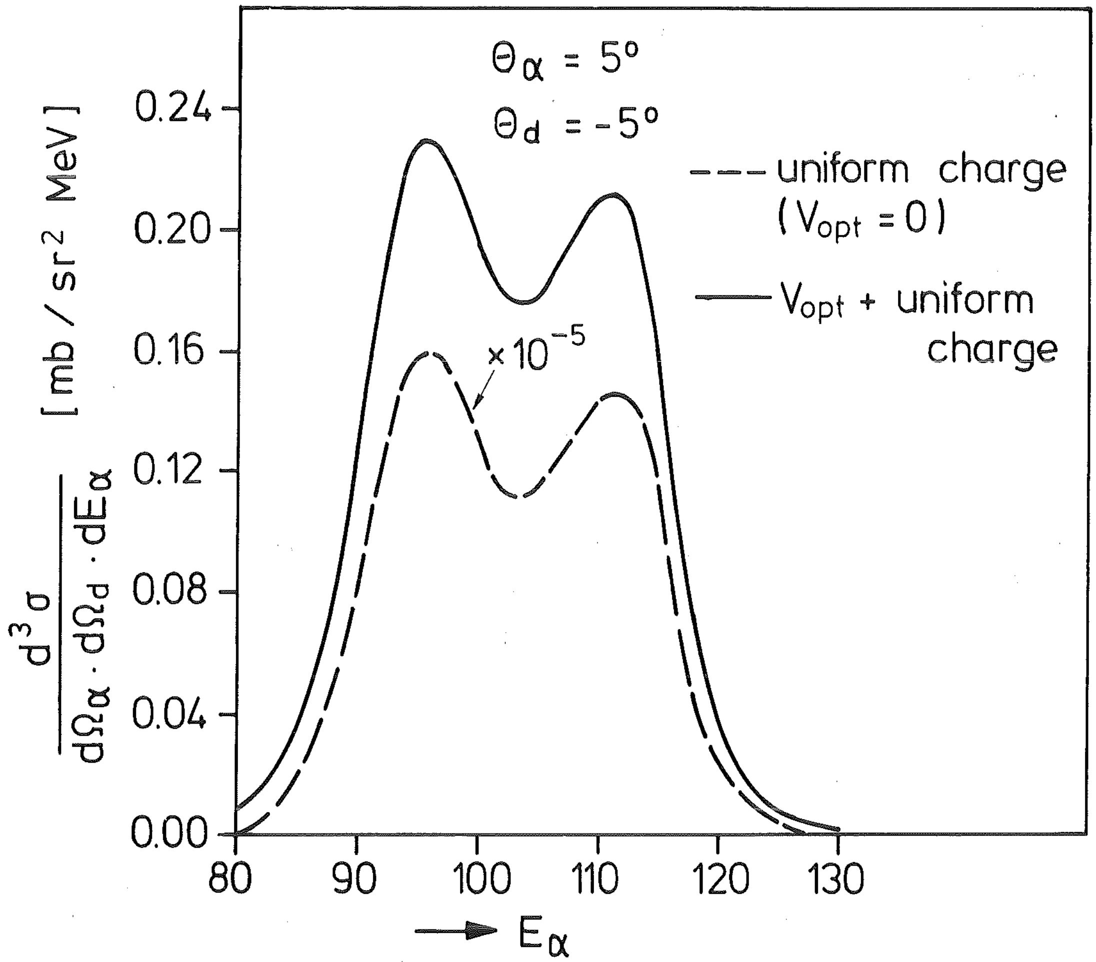

Fig. 2 Triple differential cross section and shape of the correlated α-particle spectrum in the nonresonant ^{208}Pb(^{6}Li, αd) ^{208}Pb break up reaction at E_{Li} = 156 MeV.

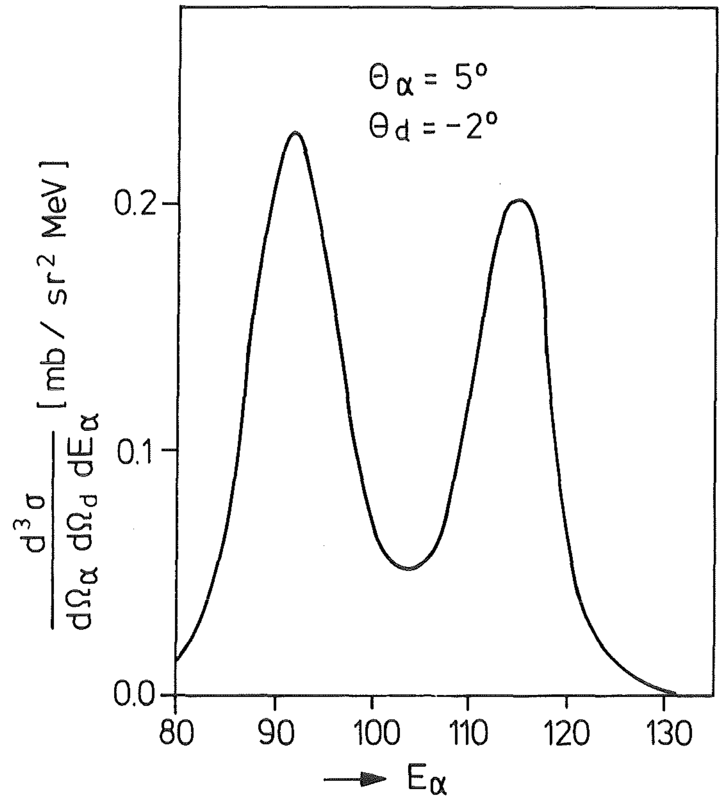

Fig. 3 Triple differential cross section of the nonresonant $^{208}Pb(^{6}Li,\alpha d)^{208}Pb$ reaction

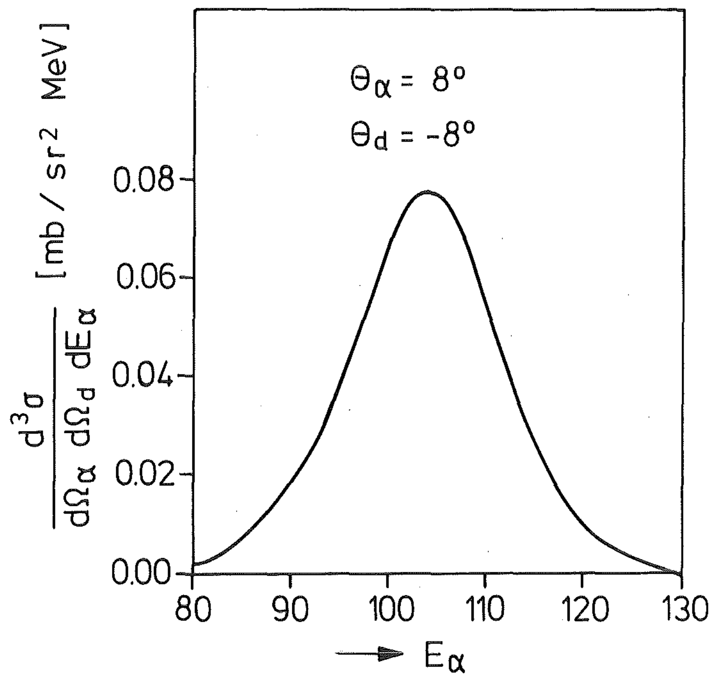

Fig. 4 Triple differential cross section of the nonresonant $^{208}Pb(^{6}Li,\alpha d)^{208}Pb$ reaction

Fig. 5 displays the variation of various factors with the laboratory energy of the α-particle observed in a particular kinematical arrangement of the detectors.

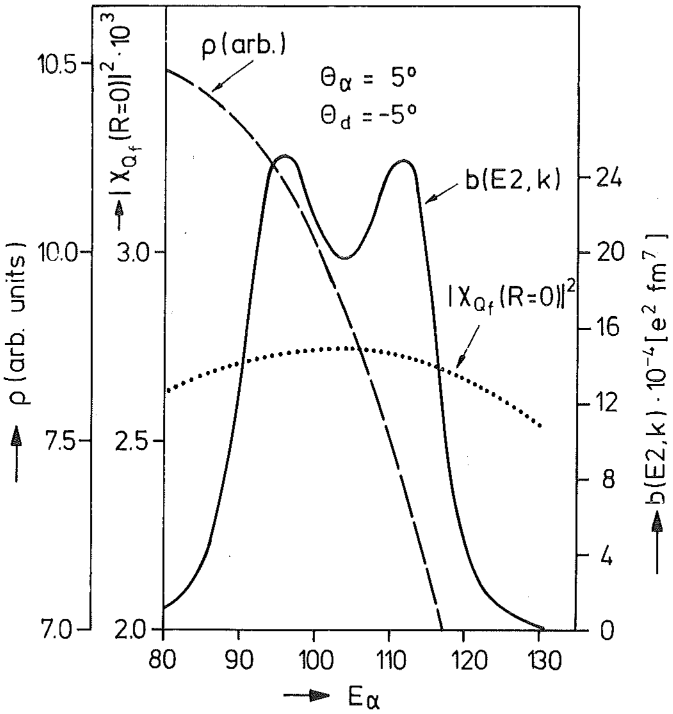

Fig. 5 The variation of the phase space factor ρ, of the contact term ($\propto |\chi_{Q_f}(R=0)|^2$) and of the reduced transition probability with the α-particle energy

In Fig. 6 the resonance excitation of the 3_1^+ state in $^6Li(k_{res} = 0.21\ fm^{-1})$ is included with a width Γ (=26 keV)* corresponding to $B(E2;\ 1^+ - 3_1^+) = 45\ e^2\ fm^4$ (Endt 1979) by writing

$$B(EL, J_i \to J_f) = \frac{1}{(2\pi)^3} \int b(EL, k, J_i \to J_f)\ \ |f|^2 \cdot k^2\ dk \qquad (3.8)$$

Here

$$|f|^2 = \left| \frac{i\ \Gamma/_2}{(E - E_{res}) + i\Gamma/_2} \right|^2 \qquad (3.9)$$

is a Breit-Wigner resonance factor with $E = \hbar^2 k^2/2\ \mu_{\alpha d}$ and $E_{res} = \hbar^2\ k_{res}^2/2\ \mu_{\alpha d}$. For the very narrow 3_1^+ resonance, we replaced

* Due to the "magnifying glass" effect of the three body kinematics Γ corresponds to $\approx$ 260 keV on the E_α axis in Fig. 5

$$B(EL, J_i \rightarrow J_f)_{Res} = \frac{\mu_{\alpha d}\, k_{res}}{(2\pi)^3\, \hbar^2}\, b(EL, k_{res}) \cdot \pi \cdot \frac{\Gamma}{2}$$

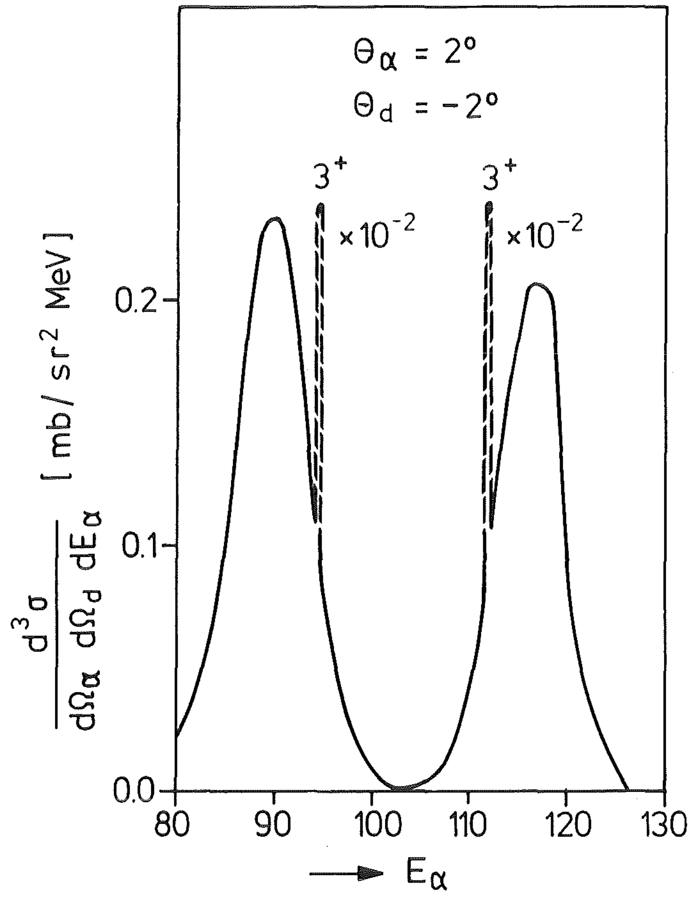

Fig. 6 Resonant and nonresonant excitation of the α+d continuum in ^{6}Li by projectile break up in the Coulomb field of ^{208}Pb at E_{Li} = 156 MeV

The result shown in Fig. 6 demonstrates the dominance of the "sequential break up" via the 3_1^+ resonance. However, the resonance peak disappears in other kinematical arrangements (see Fig. 1).

4. Discussion

In view of the experimental difficulties in measuring radiative capture reaction cross sections at low relative energies, being of considerable interest for nuclear astrophysics, it has been proposed (Rebel 1985) to study the inverse reactions: the electromagnetically induced decay of a nucleus into two fragments, which subsequently emerge from the reaction with low relative energies E, however on a pedestal of sufficiently high laboratory energies, which are convenient for the experimental detection. Such a process is the break up of a complex nuclear projectile while moving through the Coulomb field of a nucleus, which only acts as a catalyst for the electromagnetically induced dissociation. In addition to obvious advantages, partly arising from the flexibility of the three body kinematics, there is an enhancement (Rebel 1986, Baur 1985) of the dissociation cross section as compared to the photo dissociation cross section $\sigma_\gamma(a+\gamma\rightarrow b+x)$. The latter is directly related via detailed balance to the capture cross section $\sigma_{cap}(b+x\rightarrow a+\gamma)$

$$\sigma_\gamma(E_\gamma=E+Q) = \frac{(2j_b+1)(2j_x+1)}{(2j_a+1)\cdot 2} \frac{k^2}{k_\gamma{}^2} \sigma_{capt}(E) \tag{4.1}$$

The enhancement is due to the virtual photon flux and can be estimated by the Weizsäcker-Williams method (see Jackson 1975), writing (Hoffmann and Baur 1984)

$$d\sigma^L_{Diss}(E) = N^L_\gamma(E_\gamma = E+Q) \cdot \sigma_\gamma(E_\gamma) \tag{4.2}$$

with $N^L_\gamma(E_\gamma,b)$ the virtual photon spectrum (multipolarity L), experienced by the projectile, when scattered with the impact parameter b. The quantity N_γ is of the order of 10^3 MeV^{-1} for forward scattering of light ion projectiles at 30 MeV/amu.

In the present paper we investigated the Coulomb break up of light-ion projectiles on the basis of a DWBA approach. We adopted the reaction model of Rybicki and Austern (1972), assuming the process as evolving from a two-step-mechanism: excitation to a resonant or nonresonant continuum state of the fragment-system, whose center of mass motion in the field of the catalyst-nucleus is described by a distorted wave $\chi_{\vec{Q}}(\vec{R})$. This picture differs from the DWBA approach of Baur et al. (1976, 1980, 1984), ignoring any final state interaction between the fragments and describing their motion separately by distorted waves $\chi_{\vec{q}_x}(\vec{r}_x)\cdot\chi_{\vec{q}_b}(\vec{r}_b)$. Although it is not clear, how these two pictures are mutually related (see Srivastava and Rebel 1985) and to which extent they really differ, we feel that the quasi-sequential approach is more adequate, when studying final states with small relative energies of the fragments.

We considered the particular case of Coulomb break up of ^{6}Li. Here the dominant contributions (resonant and nonresonant) are of multipolarity L = 2. In this specific case the influence of the catalyst-nucleus on the orbital motion is absorbed by a "contact-term" with the role of a penetration factor. The absolute magnitude of the break up cross section depends sensitively on the assumptions about this factor, which is defined by the values of the wavefunctions $\chi_{\vec{Q}}(\vec{R})$ of the orbital motion at the origin R=0. Here the question of the use of correct distorting and absorbing potentials enters. In fact, when considering the break up in the strong nuclear field, the Rybicki and Austern-approach has been found to be less sucessful and affected by coupling effects, requiring unusual optical potentials in a formal DWBA description (Austern 1984). However, in the case of Coulomb break up it can be reasonably expected that coupling effects are of minor importance.

The quantity which relates the break up cross section to the radiative capture cross section is the reduced transition probability b(EL,E) $\delta(E-E_f)$. It enters into the capture cross section by

$$\sigma_{capt}(EL,E) = \frac{e^2}{\hbar v}\;\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\;k_\gamma^{2L+1}\cdot b_{capt}(EL,E) \qquad (4.3)$$

The b(E2,E), calculated with α+d continuum states for the considered ^{6}Li case are consistent with values theoretically extrapolated by an analysis of $^2H(\alpha,\gamma)^6Li$ capture reaction data measured at $E \geq 1$ MeV (Robertson et al. 1981). Therefore, the calculated cross sections represent realistic estimates within the adopted description of the reaction mechanism. Measurements of cross sections of this order of magnitude seem to be feasible (Jelitto et al. 1985)

They would sensitively check the theoretical apparatus, which, in turn, mediates the access to electromagnetic transition matrix element at rather small relative energies of the interacting nuclear particles.

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Appendix A

The multipole expansion of residual Coulomb potential for a uniform charge distribution of radius Rc.

The differential Coulomb potential experienced by the projectile 'a' in the field of the target A is given

$$V_{res} = V^C_{bA}(\vec{r}_{bA}) + V^C_{xA}(\vec{r}_{xA}) - V^C_{aA}(r_{aA}) \qquad \text{A.1}$$

where

$$V^C_{iA}(r) = \frac{Z_A \cdot Z_i \;\; e^2}{r} \qquad \text{if } r > R_c$$

and

$$= \frac{Z_A \cdot Z_i \;\; e^2}{2\;R_c} \quad (3 - \frac{r^2}{R_c{}^2}) \quad \text{if } r \leq R_c. \qquad \text{A.2}$$

For very large values of $\vec{R} = \vec{r}_{aA}$, the multipole expansion is given by

$$V_{res} = 4\pi Z_A e^2 \sum_{LM} \frac{1}{(2L+1)} \{Z_b(-\frac{m_x}{m_a})^L + Z_x(\frac{m_b}{m_a})^L\} \cdot \frac{r^L}{R^{L+1}} \cdot Y^*_{LM}(\hat{R}) \cdot Y_{LM}(\hat{r}). \qquad \text{A.3}$$

A more general expression for the multipole expansion can be derived by noting that (see appendix B)

$$V^C_{iA}(r) = \int \tilde{V}_i(\vec{q}) \cdot e^{-i\,\vec{q}\cdot\vec{r}} \cdot d\vec{q} \qquad \text{A.4}$$

where

$$\tilde{V}_i(\vec{q}) = \frac{1}{(2\pi)^3} \cdot \int V^C_{iA}(r)\; e^{i\,\vec{q}\cdot\vec{r}} \cdot d\vec{r} \qquad \text{A.5}$$

$$= \frac{1}{2\pi^2} \; \frac{3\; Z_A\; Z_i\; e^2}{q^2} \cdot [\frac{j_1(qR_c)}{q\; R_c}] \qquad \text{A.6}$$

If $R_c = 0$, the quantity inside the square brackets reduces to unity. Putting the expressions for $\vec{r}_{bA}$ and $\vec{r}_{xA}$ in terms of $\vec{R}_{aA}(=\vec{R})$ and $\vec{r}_{bx}(=\vec{r})$ in (A.4) we get

$$V_{res}(\vec{R},\vec{r}) = (4\pi)^2 \cdot \sum_{LM} [(-)^L \int \tilde{V}_b(q)\; j_L(\frac{m_x}{m_a} \cdot qr)\; j_L(qR) q^2 \cdot dq$$

$$+ \int \tilde{V}_x(q)\; j_L(\frac{m_b}{m_a} \cdot qr) j_L(qR) q^2\; dq]$$

$$Y^*_{LM}(R) \cdot Y_{LM}(\hat{r})$$

$$- \quad V_{aA}(R) \tag{A.7}$$

Now the Coulomb form-factor can be written as

$$F_c(\vec{k},\vec{R}) = \langle \phi^{(-)}(\vec{k},\vec{r}) \mid V_{res}(\vec{R},\vec{r}) \mid \phi_a(\vec{r}) \rangle \tag{A.8}$$

Taking the ground state wave function as

$$\phi_a(\vec{r}) = \phi(r) \tag{A.9}$$

and

$$\phi^{(+)}(\vec{k},\vec{r}) = 4\pi \sum_{LM} i^L \; \phi_L(k,r)\; Y_{LM}(\hat{k})\; Y^*_{LM}(\hat{r}) \tag{A.10}$$

we get, using the condition of their orthogonality,

$$F_c(\vec{k},\vec{R}) = (4\pi)^3 \sum i^{-L} \quad Y_{LM}(\hat{k}) \cdot Y^*_{LM}(\hat{R}) \quad F_L(k,R) \tag{A.11}$$

where

$$F_L(k,R) = \int_0^\infty q^2 dq \; [(-1)^L \; \tilde{V}_b(q) \cdot I^L_b + \tilde{V}_x(q) \cdot I^L_x] \; j_L(qR) \tag{A.12}$$

with

$$I^L_b(k,q) = \int r^2 dr \;\; \phi_L(k,r) \;\; j_L(\frac{m_x}{m_a} qr) \cdot \phi(r) \tag{A.13}$$

and

$$I^L_x(k,q) = \int r^2 dr \;\; \phi_L(k,r) \;\; j_L(\frac{m_b}{m_a} qr) \cdot \phi(r) \tag{A.14}$$

For large values of R, the multipole expansion of the residual potential (A.3) can be used to get

$$F_L(k,R) = \frac{(4\pi)^2}{(2L+1)} \cdot Z_A \cdot e^2 \quad \left\{ Z_b\left(-\frac{m_x}{m_a}\right)^L + Z_x\left(\frac{m_b}{m_a}\right)^L \right\} / R^{L+1}$$

$$\int_0^\infty r^2 dr \; \phi_L(k,r) \; r^L \cdot \phi(r) \qquad \text{A.15}$$

Appendix B

Fourier transform of the Coulomb potential for a nucleus having a uniform charge distribution.

The Coulomb potential experienced by a point projectile with the charge Z_p in the field of a nucleus of the charge Z_T with a uniform charge distribution of radius R_c is given by,

$$V_c(r) = \frac{Z_p \cdot Z_T \cdot e^2}{2\ R_c} \cdot [3 - \frac{r^2}{R_c{}^2}] \qquad \text{for } r < R_c \qquad \text{B.1}$$

and

$$V_c(r) = \frac{Z_p \cdot Z_T \cdot e^2}{r} \qquad \text{for } r > R_c \qquad \text{B.2}$$

The Fourier transform of the distribution is given by

$$\tilde{V}_c(q) = \frac{1}{(2\pi)^3} \cdot \int V_c(r) \cdot e^{i\,\vec{q}\cdot\vec{r}}\, d\vec{r} \qquad \text{B.3}$$

$$= \frac{Z_p \cdot Z_T \cdot e^2}{2\pi^2} \cdot \left[\int_0^{R_c} \frac{1}{2R_c} [3 - \frac{r^2}{R_c{}^2}] \cdot r^2 \cdot \frac{\sin qr}{qr} \cdot dr \right.$$

$$\left. + \int_{R_c}^{\infty} \frac{1}{r} \cdot \frac{\sin qr}{qr} \cdot r^2\, dr \right]$$

In this above, the $[0,R_c]$ integral is evaluated easily, and the $[R_c,\infty]$ part is evaluated by inserting a convergence factor $e^{-\varepsilon r}$, integrating and then taking the limit $\varepsilon\to 0$. This yields

$$\tilde{V}_c(q) = \frac{1}{2\pi^2} \cdot Z_p \cdot Z_T \cdot e^2 \cdot \frac{3}{q^2} \cdot \left[\frac{j_1(q\ R_c)}{q\ R_c} \right] \qquad \text{B.4}$$

The validity of the expression B.4 can be checked by writing,

$$V_c(r) = \int \tilde{V}_c(q) \cdot e^{-i\,\vec{q}\cdot\vec{r}}\, d\vec{q} \qquad \text{B.5}$$

$$= 4\pi \int \tilde{V}_c(q) \; \frac{\sin qr}{qr} \cdot q^2\, dq$$

$$= \frac{3\cdot Z_p \cdot Z_T \cdot e^2}{R_c\sqrt{r\cdot R_c}} \cdot \int_0^\infty \frac{1}{q^2} \; J_{3/2}(q\,R_c)\; J_{1/2}(qr)\; dq \qquad \text{B.6}$$

If $r > R_c$

$$K = \int_0^\infty \frac{1}{q^2} \cdot J_{3/2}(q\,R_c) \cdot J_{1/2}(qr)\; dq = \frac{R_c^{\,3/2}}{3\sqrt{r}} \qquad \text{B.7}$$

and thus

$$V_c(r) = \frac{Z_p \cdot Z_T \cdot e^2}{r}$$

For $r < R_c$

$$K = \frac{3\; r^{1/2}}{2\; R_c^{\,-1/2}} \cdot \{3 - \frac{r^2}{R_c^{\,2}}\} \qquad \text{B.8}$$

and thus

$$V_c(r) = \frac{Z_p \cdot Z_T \cdot e^2}{2\; R_c} \cdot \{3 - \frac{r^2}{R_c^{\,2}}\}$$

which are the original expressions (B.1, B.2) for the potential.

Appendix C

Predicted non-resonant break up triple differential cross-section for $\Theta_\alpha^L = 2°$ and $\Theta_\alpha^L = -2°$

E_α^L [MeV]	$\frac{d^3\sigma}{d\Omega_\alpha^L \, d\Omega_d^L \, dE_\alpha^L}$ [mb/sr^2 · MeV]
80.	0.0210
82.	0.0382
84.	0.0755
86.	0.1452
88.	0.2263
90.	0.2506
92.	0.2022
94.	0.1282
96.	0.0651
98.	0.0260
100.	0.0085
102.	0.0030
104.	0.0024
106.	0.0054
108.	0.0173
110.	0.0483
112.	0.1043
114.	0.1748
116.	0.2196
118.	0.1825
120.	0.0968
122.	0.0414
124.	0.0185
126.	0.0096

Predictions for the triple differential cross-section for the break up of ^{6}Li via the 3_1^+-state, over the lower energy sequential peak for $\Theta_\alpha^L = 2°$ and $\Theta_\alpha^L = -2°$.

E_α^L(MeV)	$d^3\sigma/d\Omega_\alpha^L\ d\Omega_d^L\ dE_\alpha^L$(MeV)
94.0	96.8
94.1	146.4
94.2	248.2
94.3	480.0
94.4	1170.5
94.5	2385.6
94.6	1680.0
94.7	544.3
94.8	306.0
94.9	192.1
95.0	111.2