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L. V. Boccaccini
Institut für Neutronenphysik und Reaktortechnik

Kernforschungszentrum Karlsruhe

KERNFORSCHUNGSZENTRUM KARLSRUHE

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# Time Averaging Procedure for Calculating the Mass and Energy Transfer Rates in Adiabatic Two Phase Flow

L.V. Boccaccini

Kernforschungszentrum Karlsruhe GmbH, Karlsruhe

Kernforschungszentrum Karlsruhe GmbH
Postfach 3640, 7500 Karlsruhe 1

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Abstract

To take advantages of the semi-implicit computer code models - to solve the two phase flow differential system - a proper averaging procedure is also needed for the source terms.

In fact, in some cases, the correlations normally used for the source terms - not time averaged - fail using the theoretical time step that arises from the linear stability analysis used on the right handside.

Such a time averaging procedure is developed with reference to the bubbly flow regime.

Moreover, the concept of mass that must be exchanged to reach equilibrium from a non-equilibrium state is introduced to limit the mass transfer during a time step.

Finally some practical calculations are performed to compare the different correlations for the average mass transfer rate developed in this work.

Ein Zeitmittelungsverfahren zur Berechnung von Massen- und Energietransportraten in adiabaten Zweiphasenströmungen


## Zusammenfassung

Damit man die Vorteile der halb-impliziten Rechenmodelle zur Lösung der Differentialgleichungssysteme in dem Bereich der Zweiphasenströmung nutzen kann, ist ein, auch für die Quellterme, geeignetes Mittelungsverfahren notwendig.
Die für die üblicherweise über die Zeit nicht gemittelten Quellterme geeigneten Beziehungen, können wohl manchmal deshalb versagen, weil sie einen theoretischen Zeitschritt verwenden, der aus den auf der rechten Seite der Gleichung benutzten Linearstabilitätsbetrachtungen resultiert.

Solch ein Zeitmittelungsverfahren wurde für das Blasenströmungsregime entwickelt.

Darüberhinaus wurde die Definition der Masse eingeführt, die zum Erreichen des Gleichgewichts von einem beliebigen Ungleichgewichtszustand aus ausgetauscht werden muß, um den Massenübergang während eines Zeitschrittes zu begrenzen.

Zum Schluß wurden einige Berechnungen durchgeführt, um die verschiedenen, in dieser Untersuchung bereitgestellten Beziehungen zur Ermittlung der mittleren Massenaustauschrate zu vergleichen.

Contents

## Nomenclature

Latin:

$a$ - thermal diffusivity

$a, b$ - parameters defined by (3.12)

$A_b$ - surface area of vapor bubble

$C_p$ - specific heat at constant pressure

$E_{ex}$ - energy per unit volume that must be exchanged from liquid to vapor to reach equilibrium, defined by (2.18).

$g$ - specific Gibbs free energy

$h$ - specific enthalpy

$h_{lv}$ - enthalpy of vaporization

$Ja$ - Jacob number, defined as

$$\frac{\rho_f C_{pf} \Delta T_s}{\rho_v h_{lv}}$$

$L$ - characteristic length, defined by (4.3)

$M_{ex}$ - mass per unit volume that must be exchanged from liquid to vapor to reach equilibrium, defined by (2.17)

$n_b$ - number of bubbles per unit volume

$p$ - pressure

$\dot{q}$ - heat exchange rate

$R$ - bubble radius

$R_g$ - reference radius for bubble growth

$R_o$ - initial bubble radius

$s$ - specific entropy

$t$ - time

$t_c$ - characteristic time of collapse

$t_g$ - reference time of bubble growth

$T$ - absolute temperature

$T_s$ - absolute temperature in saturated state at the pressure $p_f$

$u$ - specific internal energy

$v$ - specific volume

$V_{fb}$ - volume of associated liquid defined by (4.2)

$X$ - static quality

Greek:

$\alpha$ - void fraction

$\gamma_c$, $\gamma_g$ - volume change fraction defined by (3.6) and (3.1)

$\Delta t$ - time step

$\Delta t_c$ - mass exchange time defined as $t_2 - t_1$

$\Delta T$ - temperature difference

$\Delta T_s$ - temperature difference, defined as $|T_s - T_f|$

$\lambda$ - thermal conductivity

$\mu$ - mass exchange from liquid to vapor per unit volume

$\dot{\mu}$ - mass transfer rate per unit volume

$\bar{\dot{\mu}}$ - average mass transfer rate per unit volume defined by (1.8)

Subscript:

e - equilibrium property

f - liquid property

g - vapor property

i - non-equilibrium property

l - liquid property at saturation

m - mixture property

v - vapor property at saturation

## Introduction

The use of semi-implicit computer code models to solve the differential system that arises in the mathematical description of two phase flow, presents some advantages compared to explicit methods, to minimize the computational work needed for integration of the system /1/, /2/, /3/.

One of the features of this family of methods is the use of a relatively larger time step.

The equations numerically solved are time and volume averaged, but in common practice the source terms - like the mass and heat transfer rates between the phases - are not time and space averaged.

The result is - e.g. for a very intensive mass transfer rate - the semi-implicit methods fail to use the theoretical time step that arises from the linear stability analysis used on the right handside.

Therefore, we need a proper averaging procedure to take advantage of the relatively large time step, used in the semi-implicit integral technique.

The purpose of the present study is to develop a time-averaging procedure for the mass and energy exchange rates.

Moreover, to develop a methodology for practical evaluation of the parameters needed for this procedure, as well as a numerical procedure suitable for direct code use.
For calculating the mass and energy exchange transfer rates between the two phases in a non-equilibrium two phase flow model, we use a procedure based on the evaluation of the mass and the energy that must be exchanged to reach the equilibrium point, and on the evaluation of the time required for the transfer to take place.
At every time step and mesh point during the integration, therefore, the method must answer two questions.

First, if there are non-equilibrium conditions, what is the mass and energy that must be exchanged to reach equilibrium condition?
And second, what is the time required for this exchange of mass and energy?

From general balances (mass,energy and linear momentum) the general integration method has calculated the conditions of the liquid and the vapor. In general these are not equilibrium conditions.
Now one can calculate the equilibrium state with the hypothesis that there is no exchange of mass and heat with the outside for a fixed control volume.

One can so evaluate the mass ($M_{ex}$) and energy ($E_{ex}$) that must be exchanged, per unit volume, to reach equilibrium, only with general consideration of equilibrium.

The second question concerns the time that is required to reach equilibrium.

The evaluation of this time requires a physical model of the phenomena of heat and mass transfer. That is, it requires making an assumption on the kind of flow regime (bubbly, dispersed droplets, film, etc.) and proper micromodels to simulate the exchanges in the interface region.
In the course of this work, we will refer particularly to a special flow regime - bubbly flow - for analyzing some equations used in practice in the two phase flow computer codes. The procedure proposed can be generalized to the other flow regimes.

In Section 1 we will discuss the problems that come in the evaluation of the mass source term using a class of constitutive equations used in common practice. Moreover, we will consider the possibility to derive time-averaged expressions directly from these equations.

In Section 2 we will present the procedure for evaluating the parameters $M_{ex}$ and $E_{ex}$, and the use of $M_{ex}$ to develop a theoretical maximum in the mass exchange during a given time step.

In Section 3 and 4 we will develop new correlations to take into account the maximum mass transfer, discussed in the previous section.
For these equations two different physical models will be used to describe the growth and the collapse of a vapor bubble, one in a semi-infinite and one in a finite liquid medium.

Finally, in Section 5, results from some calculations performed with the expressions developed in this work will be shown, and comparisons are made among them.

## 1. Evaluation of the mass transfer rate

In the evaluation of the mass transfer rate from the liquid to the vapor phase, per unit volume, $\dot{\mu}(t)$, in the case of bubbly flow, one can derive $\dot{\mu}(t)$ from models that describe the growth and the collapse of a single vapor bubble.

The most common expressions that give the bubble radius as a function of time are based on the "thermal controlled growth or collapse theory" /4/.

Among these, we refer to a special class that can be expressed in this general form

$$\frac{R}{R_g} = \sqrt{\frac{t}{t_g}} \qquad \text{for the growth} \tag{1.1}$$

$$\frac{R}{R_o} = 1 - \sqrt{\frac{t}{t_c}} \qquad \text{for the collapse} \tag{1.2}$$

For example, according to the expression of Lobunzov /5/ for bubble growth,

$$t_g = \frac{\pi \cdot R_g^2}{12 \cdot J_a^2 \cdot \left[ 1 + \frac{1}{2} \left( \frac{\pi}{6 \cdot J_a} \right)^{2/3} + \frac{\pi}{6 \cdot J_a} \right] \cdot a_f} \tag{1.3}$$

where $R_g$ is a reference radius, used only to derive a dimensionless form.

Or, according to the Florschuetz and Chao "plane interface approximation" /6/ for bubble collapse,

$$t_c = \frac{\pi \cdot R_o^2}{4 \cdot J_a^2 \cdot a_f} \tag{1.4}$$

where $R_o$ is the initial radius. Note that, in contrast to $R_g$, $R_o$ has a physical meaning, and affects the expression of the radius versus time.

From eq.(1.1) and (1.2) one can derive expressions for $\dot{\mu}(t)$, defined as:

$$\dot{\mu}(t) = 4\pi\, n_b \rho_v \cdot R^2_{(t)}\, \dot{R}_{(t)} \tag{1.5}$$

These are, for bubble growth:

$$\dot{\mu}(t) = 2\pi \cdot n_b \cdot \rho_v \cdot \frac{R_g^{\,3}}{t_g^{\,3/2}} \sqrt{t} \qquad \text{for } t \geq 0 \tag{1.6}$$

and for bubble collapse:

$$\begin{cases} \dot{\mu}(t) = -2\pi \cdot n_b \cdot \rho_v \dfrac{R_o^3}{t_c^{\,3/2}} \dfrac{(\sqrt{t_c} - \sqrt{t})^2}{\sqrt{t}} & \text{for } 0 \leq t \leq t_c \\ \dot{\mu}(t) = 0 & \text{for } t > t_c \end{cases} \tag{1.7}$$

These functions of $\dot{\mu}$ versus time are shown in Fig.(1.1) and (1.2).

Usually in the semi-implicit methods one uses these equations for determining the value of the mass source term directly, without an averaging procedure.

From the evaluation of the actual bubble radius[1] one can obtain the starting time $t_1$, with the use of the eq.(1.1) or (1.2) respectively.

With the value of $t_1$, $\dot{\mu}(t_1)$ can be evaluated with the use of eq.(1.6) or (1.7). The source term during the time step $\Delta t$ is assumed to be $\dot{\mu}(t_1) \cdot \Delta t$.

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(1) The procedure to evaluate the actual radius of the bubble is based on the number of bubbles per unit volume, usually a user input. Then the actual radius can be evaluated with the equation:

$$R = \sqrt[3]{\frac{3 \cdot \alpha}{4\pi \cdot n_b}}$$

This kind of approach presents some problems. First, the variation of $\dot{\mu}(t_1)$ may be very great during a given time step. Then the use of a constant value of $\dot{\mu}(t_1)$ underestimates or overestimates the exchange of mass.

As one can see from Fig. 1.1 , the use of the value $\dot{\mu}(t_1)$ gives an exchange of mass during the time step $\Delta t$ - represented by the area ABCD - less than the value predicted - area AECD -.
This difference increases as $\Delta t$ increases,as well as $t_1$ approaches 0.

On the contrary (Fig. 1.2 ), for bubble collapse, there is an overestimation of the mass exchange, and this increases as $\Delta t$ increases as $t_1$ approaches 0.

For overcoming this problem we can use an average value to derive a correct value of mass exchange. This average value is then defined as:

$$\bar{\dot{\mu}}(t_1, \Delta t) \overset{\Delta}{=} \frac{1}{\Delta t} \int_{t_1}^{t_1+\Delta t} \dot{\mu}(t) \cdot dt \qquad (1.8)$$

Because we have, in this case, very simple analytical solutions, we can give the following expressions for the time-average mass transfer rate . For the growth:

$$\bar{\dot{\mu}}(t_1, \Delta t) = \frac{4\pi}{3} \frac{n_b \rho_v}{\Delta t} \frac{R_g^3}{t_g^{3/2}} \left[ (t_1 + \Delta t)^{3/2} - t_1^{3/2} \right] \qquad (1.9)$$

For the collapse:

$$\left[ \begin{array}{l} \bar{\dot{\mu}}(t_1, \Delta t) = \dfrac{-4\pi}{3} \dfrac{n_b \rho_v}{\Delta t} \dfrac{R_o^3}{t_c^{3/2}} \left| 3t_c\sqrt{t} + t^{3/2} - 3\sqrt{t_c} \cdot t \right|_{t_1}^{\hat{t}} \\ \\ \hat{t} = \min\ (t_1 + \Delta t,\ t_c) \end{array} \right. \qquad (1.10)$$

However, the use of the average values given by (1.9) and (1.10) does not overcome another problem.

The exchange of mass between the two phases is connected with non-equilibrium conditions of the mixture. This exchange will be complete when equilibrium is reached.

As noted in the introduction, it is possible to evaluate the mass that is connected with reaching the equilibrium condition ($M_{ex}$).
Then, for a given time step, we can use this value as the upper limit of the mass exchange.
In fact, if more mass is transferred, the system would arrive at another non-equilibrium point, but with the direction of mass transfer inverted.
If the situation repeats itself, in the next time steps we would observe oscillation around the equilibrium.
Using $M_{ex}$ as the upper limit of the mass transfer, this problem can be avoided.

In the next section we will discuss the calculation of the parameter $M_{ex}$, and the theoretical limitation of mass transfer that comes from it.

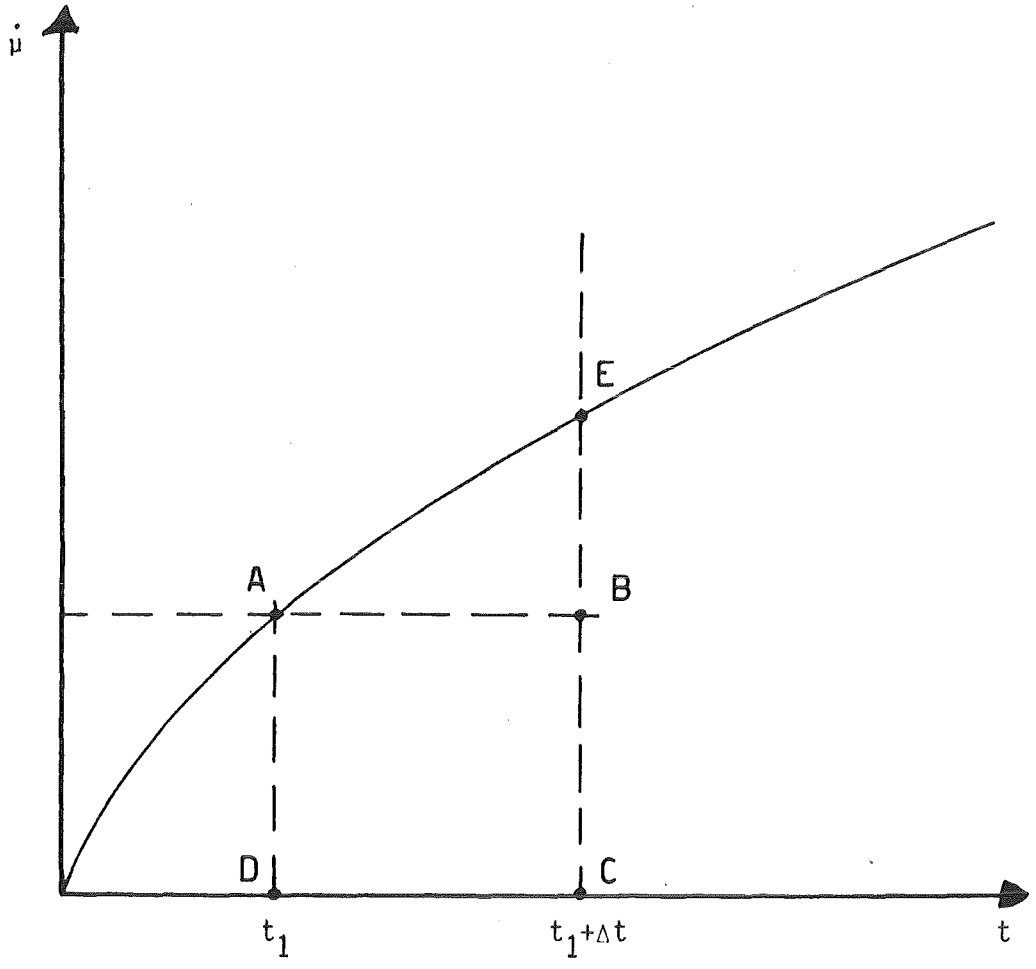



Fig. 1.1 Mass transfer rate for bubble growth

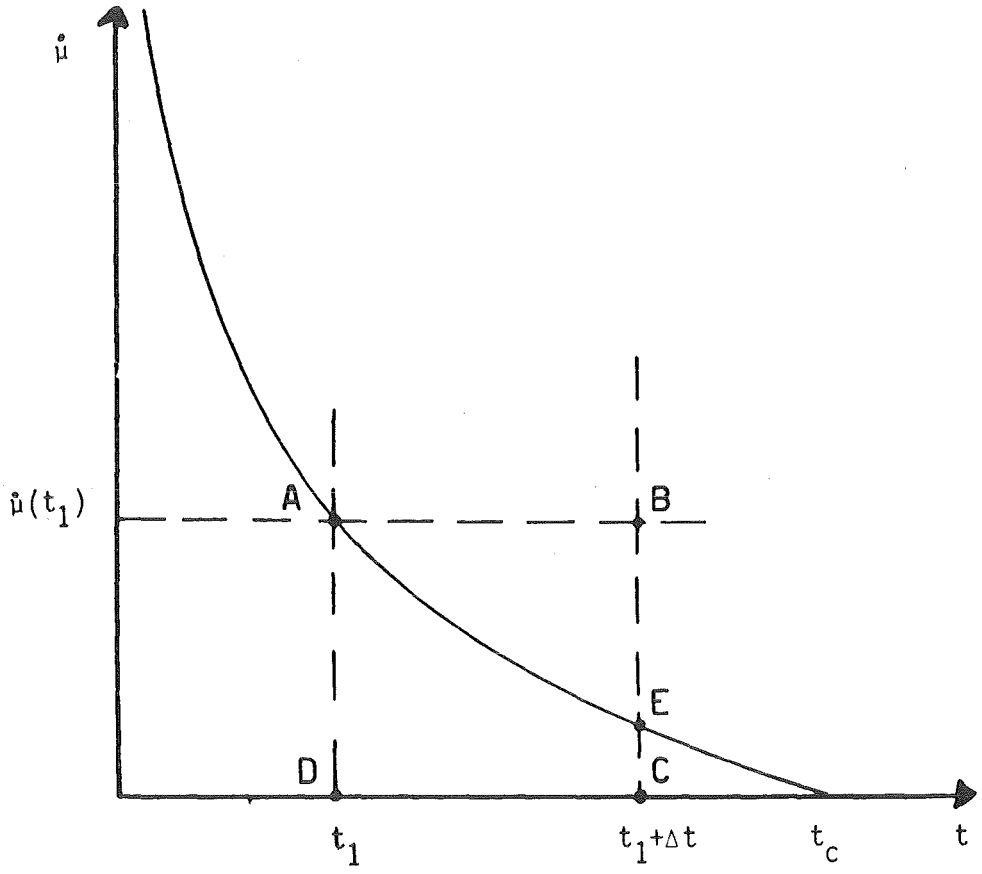



Fig. 1.2 Mass transfer rate for bubble collapse

## 2. Evaluation of the mass and energy that must be exchanged to reach equilibrium conditions

For evaluating the mass and energy that must be exchanged between the phases to reach equilibrium from a given non-equilibrium condition of the mixture, we suppose that in a control volume (Fig. 2.1) the transformation from "i" to "e" is without exchange of mass and energy between the control volume and the outside.
This means that during this transformation the specific internal energy and the specific volume of the mixture must be constant.

$$u_m = u_{gi}X_i + u_{fi}\cdot(1-X_i) = u_{ge}X_e + u_{fe}\cdot(1-X_e) \tag{2.1}$$

$$v_m = v_{gi}X_i + v_{fi}\cdot(1-X_i) = v_{ge}X_e + v_{fe}\,(1-X_e) \tag{2.2}$$

The following conditions characterize the equilibrium point of the mixture.

$$p_{ge} = p_{fe} \qquad \text{mechanical equilibrium} \tag{2.3}$$

$$T_{ge} = T_{fe} \qquad \text{thermal equilibrium} \tag{2.4}$$

$$g_{ge} = g_{fe} \qquad \text{chemical equilibrium} \tag{2.5}$$

From these equations three different cases (for a water pressure range from 700 Pa and $210\cdot10^5$ Pa) arise:

A - mixture in saturated state

$$u_{ve}X_e + u_{le}\cdot(1-X_e) = u_m \tag{2.6}$$

$$v_{ve}X_e + v_{le}\cdot(1-X_e) = v_m \tag{2.7}$$

B - only subcooled liquid

$$u_{fe} = u_m \tag{2.8}$$

$$v_{fe} = v_m \tag{2.9}$$

C - only superheated vapor

$$u_{ge} = u_m \tag{2.10}$$

$$v_{ge} = v_m \tag{2.11}$$

The methodology used is based on attempting to solve the first set of equations (A); if it is impossible to do this, then we select one of the others.

We can use an iterative procedure. From a trial value of p, it is possible to calculate two values of X from the equations (2.6) and (2.7).

$$X_u = \frac{u_m - u_l(p)}{u_v(p) - u_l(p)} \tag{2.12}$$

$$X_v = \frac{v_m - v_l(p)}{v_v(p) - v_l(p)} \tag{2.13}$$

If p is the equilibrium pressure of the mixture then $X_u$ and $X_v$ must be equal:

$$p = p_e \longrightarrow \quad X_u = X_v \tag{2.14}$$

If these two values are different, we will evaluate a new pressure until the condition (2.14) is satisfied.
For evaluating this new value for p, we may use a linear interpolation based on the values of p, $X_u$ and $X_v$ in the actual step and in the previous iteration. Using the subscript 1 for the previous values and 2 for the actual values, we can give an estimation $p_a$ of $p_e$ (Fig.2.2):

$$p_a = p_1 + \frac{(X_{v1} - X_{u1})\,(p_2 - p_1)}{(X_{u2} - X_{u1} - X_{v2} + X_{v1})} \tag{2.15}$$

In this way we arrive at a value for X.
If X is between 0 and 1, we are in the A condition, if $X < 0$ in B, if $X > 1$ in C.
The value of $X_e$ is then:

$$\begin{cases} X_e = X & \text{if} \quad 0 \leq X \leq 1 \\ X_e = 0 & \text{if} \quad X < 0 \\ X_e = 1 & \text{if} \quad X > 1 \end{cases} \tag{2.16}$$

Then, the mass that must be exchanged from the liquid to the vapor phase, per unit volume, ($M_{ex}$) is given by :

$$M_{ex} = \frac{-X_i + X_e}{v_m} \tag{2.17}$$

The energy that must be exchanged from the liquid to the vapor phase, per unit volume, is given by:

$$E_{ex} = \frac{-u_{gi}X_i + u_{ge}X_e}{v_m} \tag{2.18}$$

In Appendix A is a listing of the subroutine EQUIL that performs this evaluation.

Figs. 2.3 - 2.8 show some results from the calculations made with the subroutine EQUIL. In Figs. 2.3 - 2.6, we report the exchange parameters $M_{ex}$ and $E_{ex}$ as functions of the non-equilibrium void fraction for different liquid and vapor conditions. In Figs. 2.7 and 2.8, the equilibrium void fraction as function of non-equilibrium void fraction is shown.

One practical consequence of this kind of approach is the presence of a limiting curve for $\bar{\dot{\mu}}$ . In fact, if we suppose that entire mass $M_{ex}$ is exchanged at time t = 0, the expression for the average mass transfer rate is given by

$$\bar{\dot{\mu}}(\Delta t) = \frac{M_{ex}}{\Delta t} \tag{2.19}$$

This curve represents the maximum value that $\bar{\dot{\mu}}$ can reach for a given time step $\Delta t$. Note that (2.19) is based only on equilibrium considerations.

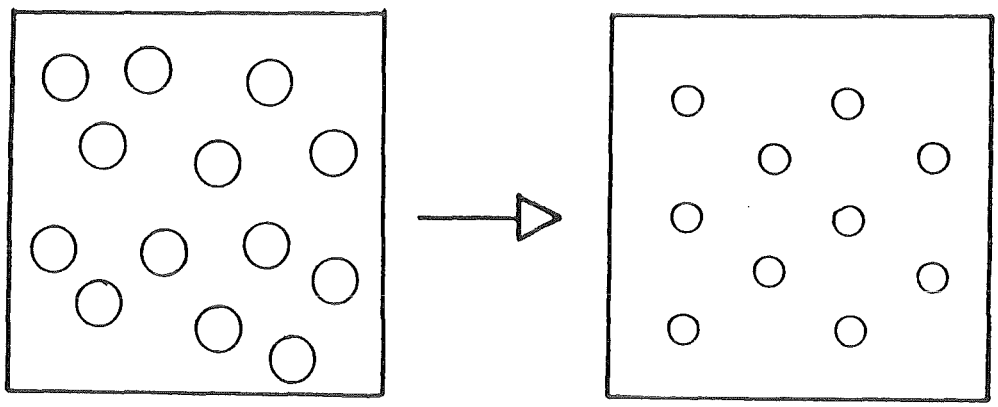
If we suppose that the exchanging process is completed at the time $\Delta t_c$ - this is evaluated by a proper model of mass exchange - the value of $\bar{\dot{\mu}}(\Delta t)$ lies on the limiting curve (2.19) for $\Delta t \geq \Delta t_c$. For $\Delta t < \Delta t_c$ it lies below this curve (Fig. 2.9).

Therefore, the curve (2.19) can be used as criterion for judging of the correctness of the previous results.

As we will show in the last section with a practical calculation, the expressions (3.6), (3.7), (3.9) and (3.10) fail in some cases, because they predict a mass exchange during the time step greater than the value of $M_{ex}$.

To try to overcome this problem one might introduce a limitation in the calculated value of $\bar{\mu}$, based on eq.(2.19), or introduce this limitation inside the expression of $\dot{\mu}(t)$, before integrating, to obtain the average value.

This second method is used in the next section, where the equations (1.1) and (1.2) are used only to estimate the characteristic time of the mass exchange while the amount of mass exchanged is based on $M_{ex}$.



i- non-equilibrium e- equilibrium

Fig. 2.1 Transformation of the fluid inside the control volume

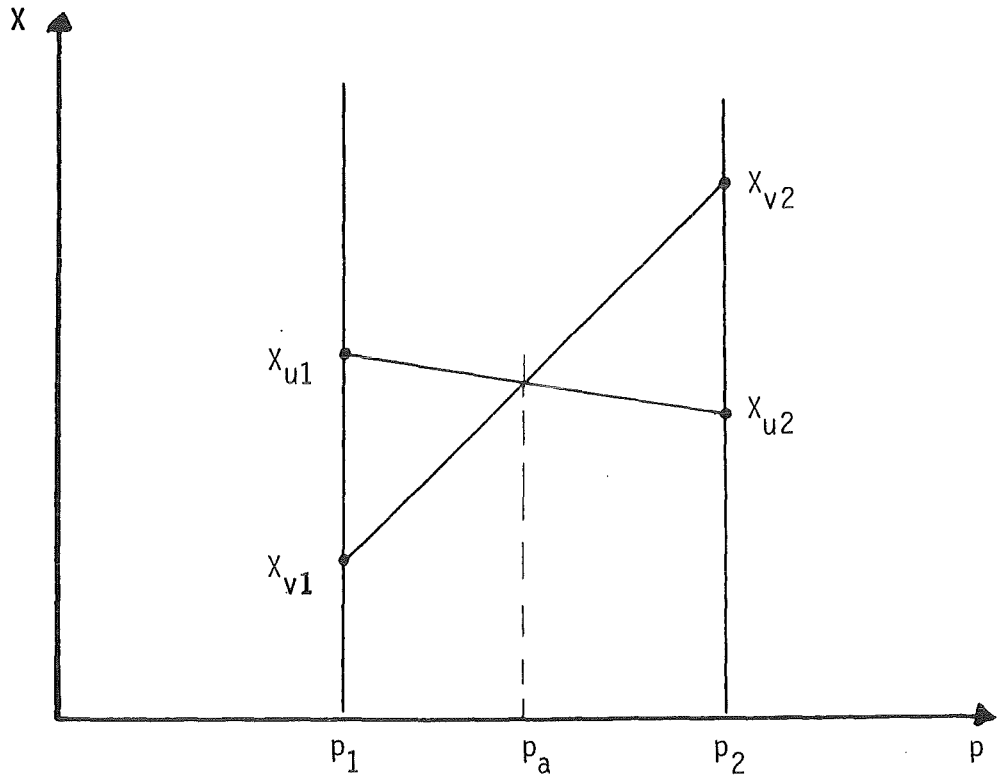



Fig. 2.2 Interpolation methodology to evaluate an approximate value $p_a$ of $p_e$.

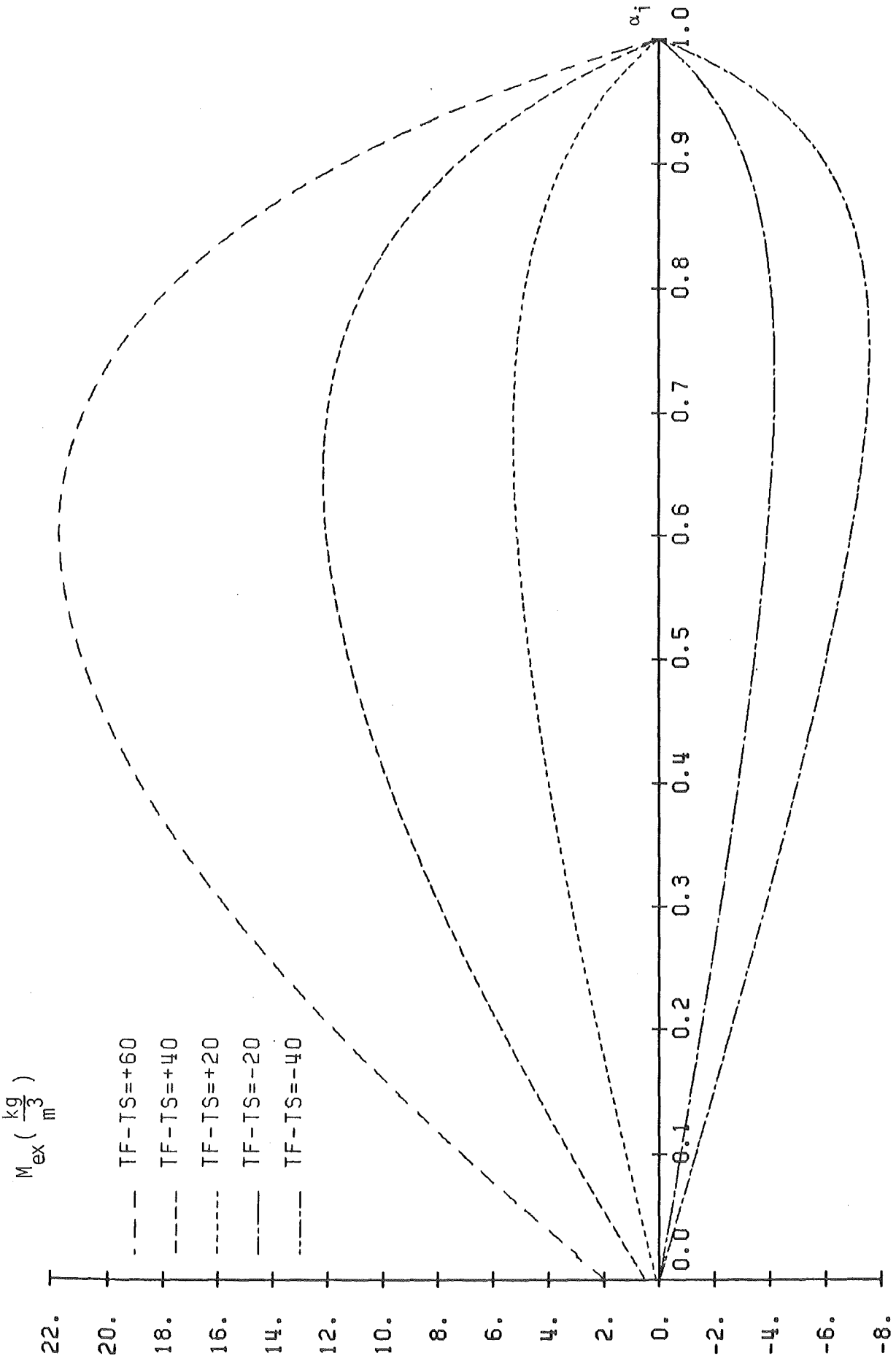



Fig. 2.3 Mass exchange at 5.E6 PA ($T_g - T_s = 0$)

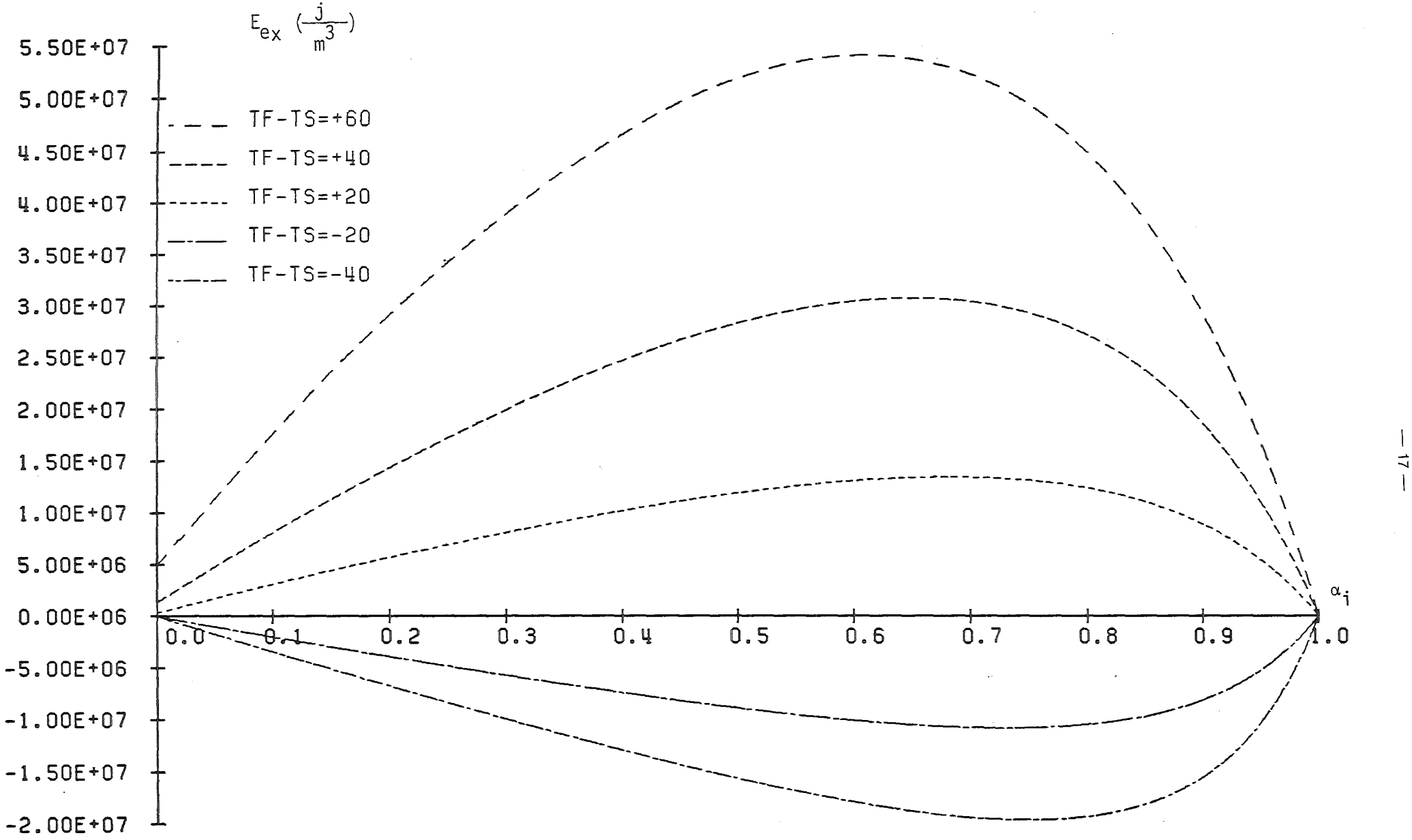



Fig. 2.4 Energy exchange at 5.E6 PA ($T_g - T_s = 0$)

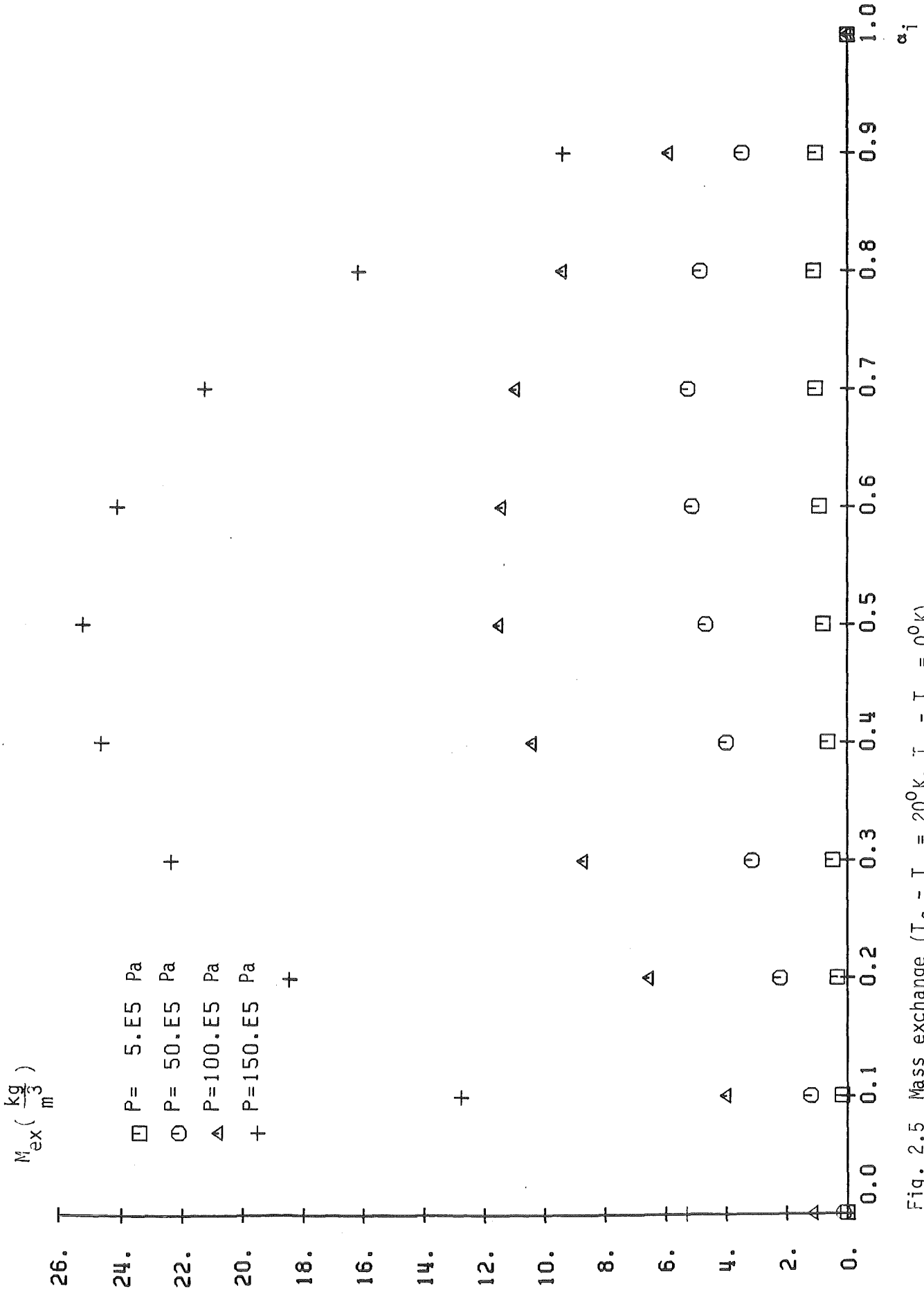



Fig. 2.5 Mass exchange ($T_f - T_s = 20^oK$, $T_g - T_s = 0^oK$).

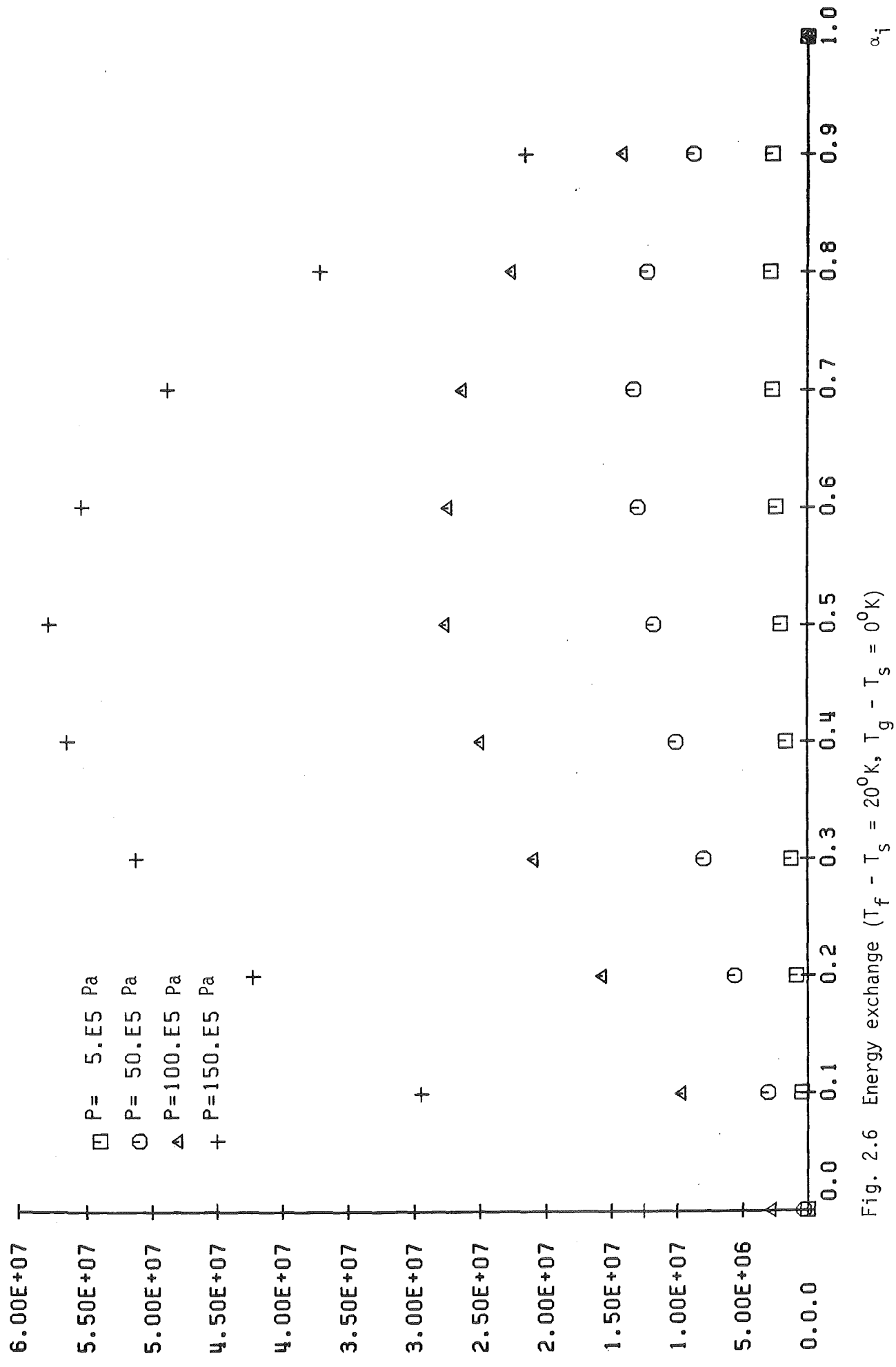



Fig. 2.6 Energy exchange ($T_f - T_s = 20^oK$, $T_g - T_s = 0^oK$)

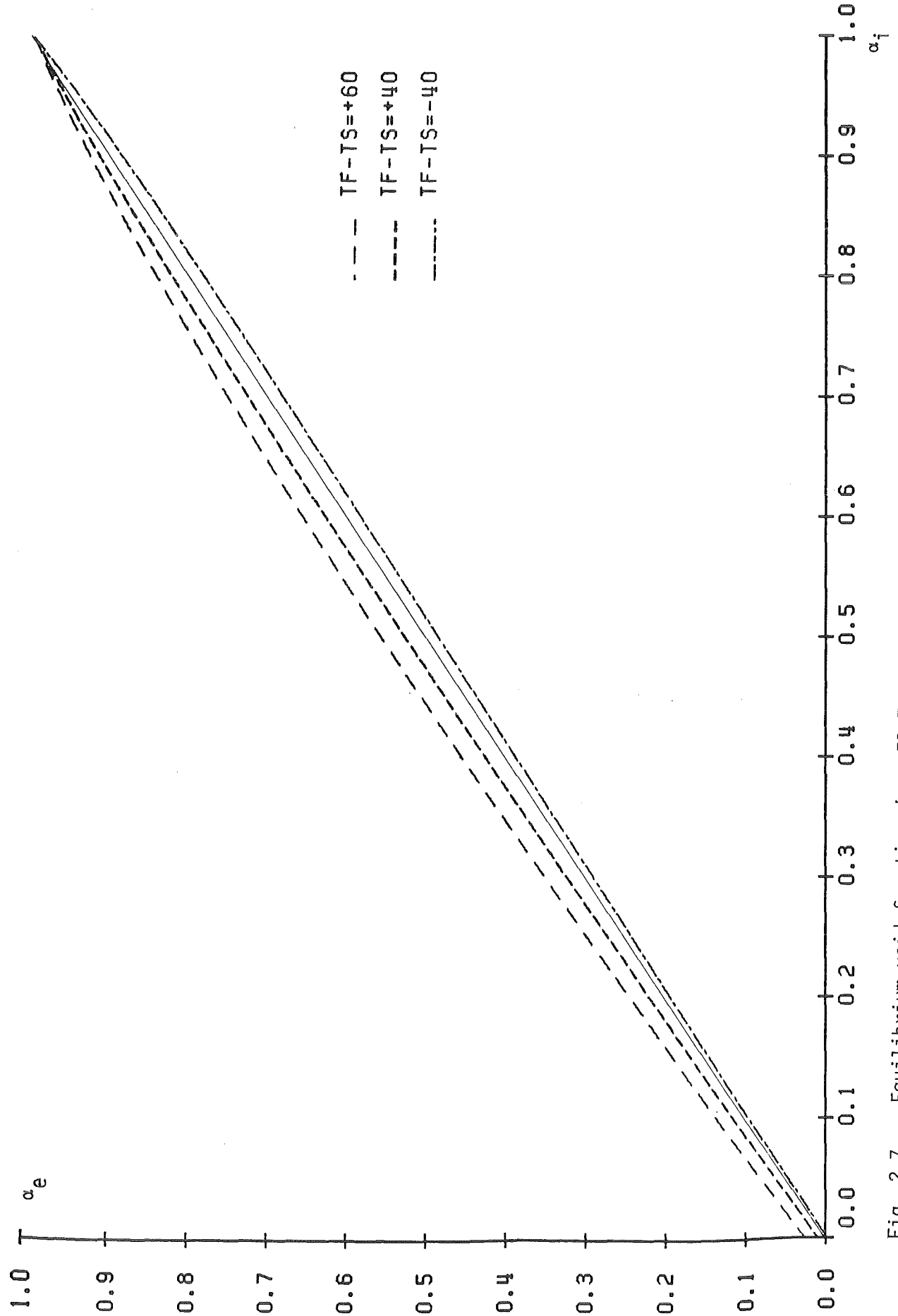



Fig. 2.7 Equilibrium void fraction ($p = 50.E5$ Pa, $T_g - T_s = 0$)

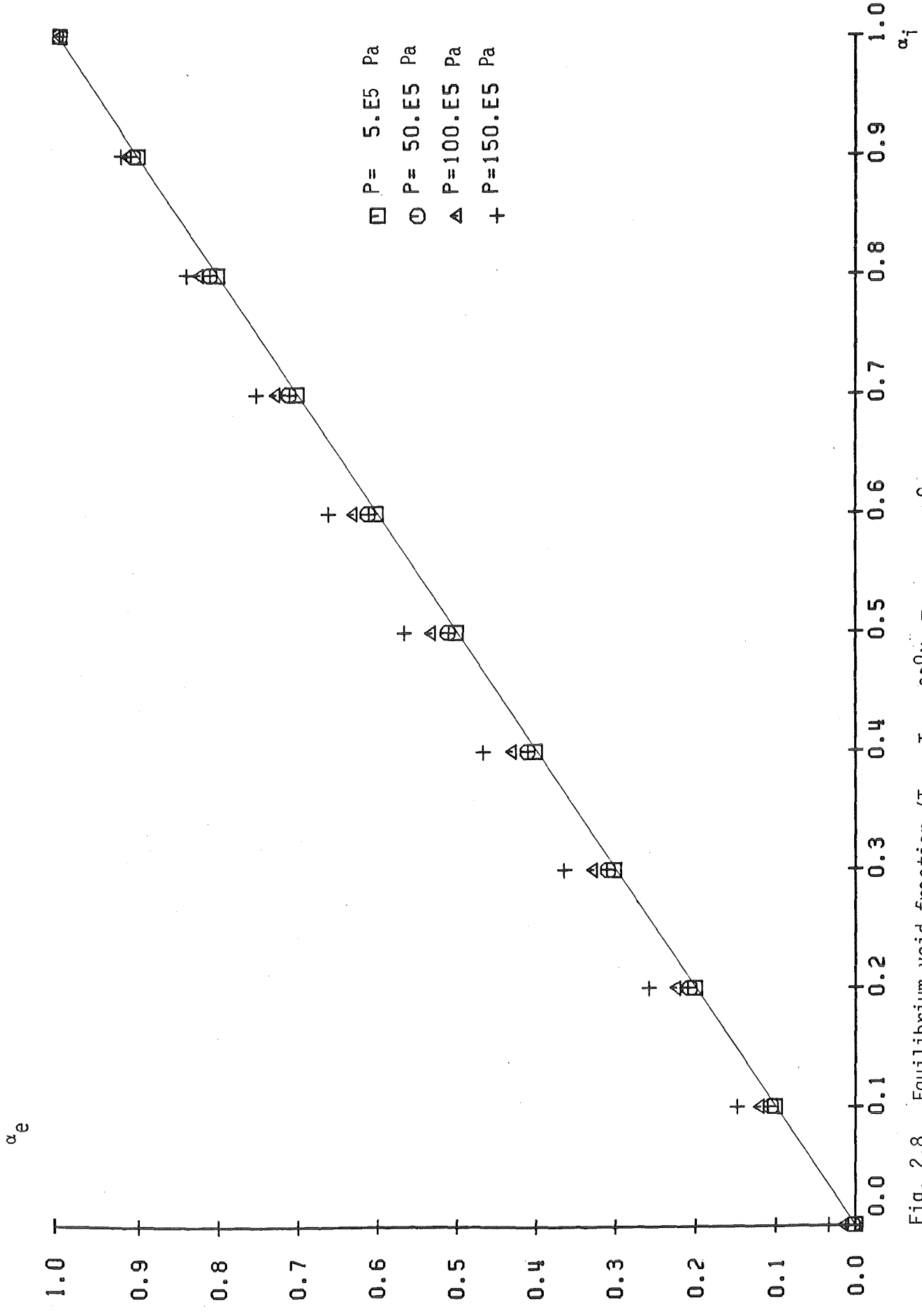



Fig. 2.8 Equilibrium void fraction ($T_f - T_s = 20^oK$, $T_g - T_s = 0^oK$)

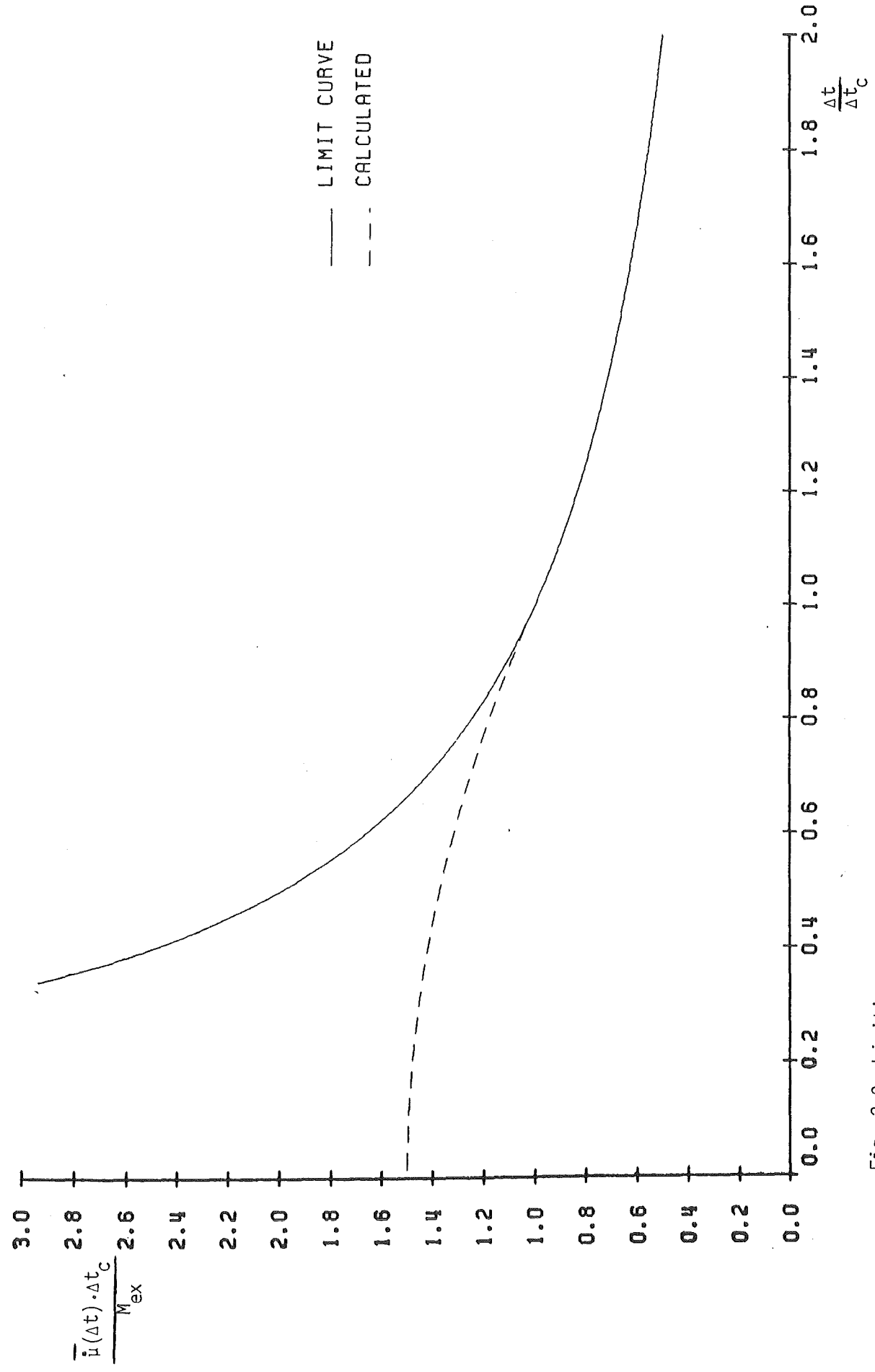



Fig. 2.9 Limiting curve

## 3. Development of equations based on the limitation of mass exchange during the time step

In Section 2 the evaluation of the void fraction in the corresponding equilibrium state, $\alpha_e$, from the non-equilibrium value $\alpha_i$ has been shown. From these values it is possible to calculate the radius $R_1$, connected with $\alpha_i$, and the radius $R_2$, connected with $\alpha_e$, using the equation presented in Note 1 of Section 1.

For the bubble growth process, we can use eq.(1.1) to obtain this expression of the volume change fraction, $\gamma_g$, defined in the following equation:

$$\begin{cases} \gamma_g \triangleq \dfrac{V - V_1}{V_2 - V_1} = \dfrac{t^{3/2} - t_1^{3/2}}{t_2^{3/2} - t_1^{3/2}} & \text{for } t_1 \le t \le t_2 \\ \gamma_g = 1 & \text{for } t > t_2 \end{cases} \tag{3.1}$$

where

$$\begin{cases} t_1 = t_g \left(\dfrac{R_1}{R_g}\right)^2 \\ t_2 = t_g \left(\dfrac{R_2}{R_g}\right)^2 \end{cases} \tag{3.2}$$

Eq.(3.1) describes the change of the volume of the bubble as function of time.

Because the properties of the vapor do not change during the time of bubble growth based on the "thermal controlled theory", one can use the same expression to describe the exchange of mass in the interval $[\, t_1, t_2 \,]$.

$$\frac{\mu(t) - \mu(t_1)}{\mu(t_2) - \mu(t_1)} \tilde{=} \gamma_g(t) \tag{3.3}$$

where $\mu(t_1) = 0$, at the starting point $t_1$.

Then, to determine the expression of the net mass transfer $\mu(t)$, it remains only to evaluate $\mu(t_2)$.

Following the proposed methodology for the total mass exchanged in the time interval $\Delta t_c = t_2 - t_1$, we will use the value $M_{ex}$ instead of the integral of (1.6) in the same time interval.

In this way we have the following expression for the net mass transfer from the liquid to the vapor phase per unit volume.

$$\begin{cases} \mu(t) = M_{ex} \dfrac{t^{3/2} - t_1^{3/2}}{t_2^{3/2} - t_1^{3/2}} & \text{for } t_1 \leq t \leq t_2 \\ \mu(t) = M_{ex} & \text{for } t > t_2 \end{cases} \tag{3.4}$$

From this expression one can derive - from the eq.(1.8) - the expression for the average mass transfer rate:

$$\begin{cases} \bar{\mu}(\Delta t) = \dfrac{M_{ex}}{\Delta t} \dfrac{(t_1 + \Delta t)^{3/2} - t_1^{3/2}}{t_2^{3/2} - t_1^{3/2}} & \text{for } \Delta t \leq t_2 - t_1 \\ \bar{\mu}(\Delta t) = \dfrac{M_{ex}}{\Delta t} & \text{for } \Delta t > t_2 - t_1 \end{cases} \tag{3.5}$$

The same procedure can be applied to also get an expression in the case of bubble collapse. The volume change fraction $\gamma_c$ is defined as:

$$\gamma_c \triangleq \frac{V_1 - V}{V_1 - V_2} = \frac{\left[1 - \sqrt{\frac{t_1}{t_c}}\right]^3 - \left[1 - \sqrt{\frac{t}{t_c}}\right]^3}{\left[1 - \sqrt{\frac{t_1}{t_c}}\right]^3 - \left[1 - \sqrt{\frac{t_2}{t_c}}\right]^3} \quad \text{for } t_1 \leq t \leq t_2$$

$$\gamma_c = 1 \quad \text{for } t > t_2 \tag{3.6}$$

with

$$t_1 = \frac{t_c}{R_o^2}\left[R_o - R_1\right]^2$$

$$t_2 = \frac{t_c}{R_o^2}\left[R_o - R_2\right]^2 \tag{3.7}$$

As seen in the growth of the bubble, one can use the same expression for $\gamma_c$ to get the net mass transfer:

$$\dot{\mu}(t) = M_{ex} \cdot \frac{\left[1 - \sqrt{\frac{t_1}{t_c}}\right]^3 - \left[1 - \sqrt{\frac{t}{t_c}}\right]^3}{\left[1 - \sqrt{\frac{t_1}{t_c}}\right]^3 - \left[1 - \sqrt{\frac{t_2}{t_c}}\right]^3} \quad \text{for } t_1 \leq t \leq t_2$$

$$\dot{\mu}(t) = M_{ex} \quad \text{for } t > t_2 \tag{3.8}$$

And from this equation, the average mass transfer rate:

$$\bar{\dot{\mu}}(\Delta t) = \frac{M_{ex}}{\Delta t} \frac{\left[\sqrt{t_c} - \sqrt{t_1}\right]^3 - \left[\sqrt{t_c} - \sqrt{\Delta t + t_1}\right]^3}{\left[\sqrt{t_c} - \sqrt{t_1}\right]^3 - \left[\sqrt{t_c} - \sqrt{t_2}\right]^3} \quad \text{for } \Delta t \leq t_2 - t_1$$

$$\bar{\dot{\mu}}(\Delta t) = \frac{M_{ex}}{\Delta t} \quad \text{for } \Delta t > t_2 - t_1 \tag{3.9}$$

We rewrite eqs.(3.5) and (3.9), with the introduction of some parameters, so that comparison with the limiting curve (2.9) is more meaningful.

For the growth:

$$\begin{cases} \bar{\dot{\mu}}(\Delta t) = \dfrac{M_{ex}}{\Delta t} \dfrac{\left[\dfrac{\Delta t}{t_2 - t_1} \cdot (1-a) + a\right]^{3/2} - a^{3/2}}{1 - a^{3/2}} & \text{for } \Delta t \leq t_2 - t_1 \\ \bar{\dot{\mu}}(\Delta t) = \dfrac{M_{ex}}{\Delta t} & \text{for } \Delta t > t_2 - t_1 \end{cases} \tag{3.10}$$

For the collapse:

$$\begin{cases} \bar{\dot{\mu}}(\Delta t) = \dfrac{M_{ex}}{\Delta t} \dfrac{\left[1 - \sqrt{ab}\right]^3 - \left[1 - \sqrt{ab + b(1-a)\dfrac{\Delta t}{t_2 - t_1}}\right]^3}{\left[1 - \sqrt{ab}\right]^3 - \left[1 - \sqrt{b}\right]^3} & \text{for } \Delta t \leq t_2 - t_1 \\ \bar{\dot{\mu}}(\Delta t) = \dfrac{M_{ex}}{\Delta t} & \text{for } \Delta t > t_2 - t_1 \end{cases} \tag{3.11}$$

where

$$\begin{cases} a = t_1/t_2 & 0 \leq a \leq 1 \\ b = t_2/t_c & 0 \leq b \leq 1 \end{cases} \tag{3.12}$$

In Fig. 3.1, eqs.(3.10) and (3.11) - in a dimensionless form - are compared with the limiting curve (2.9).

The use of this type of correlation to describe the growth and the collapse of the bubbles is based on a initial hypothesis: the conditions of the liquid (subcooled or superheated) surrounding the bubble remain constant during the heat and mass transfer process, i.e. the driving force - the difference of temperature between vapor and liquid - remains constant.

However, the driving force becomes smaller and smaller as the system approaches equilibrium. This means that the previous hypothesis underestimates the time necessary for the transport of mass to be accomplished.

In the next section we discuss a very simple model that takes the reduction of the driving force into account.

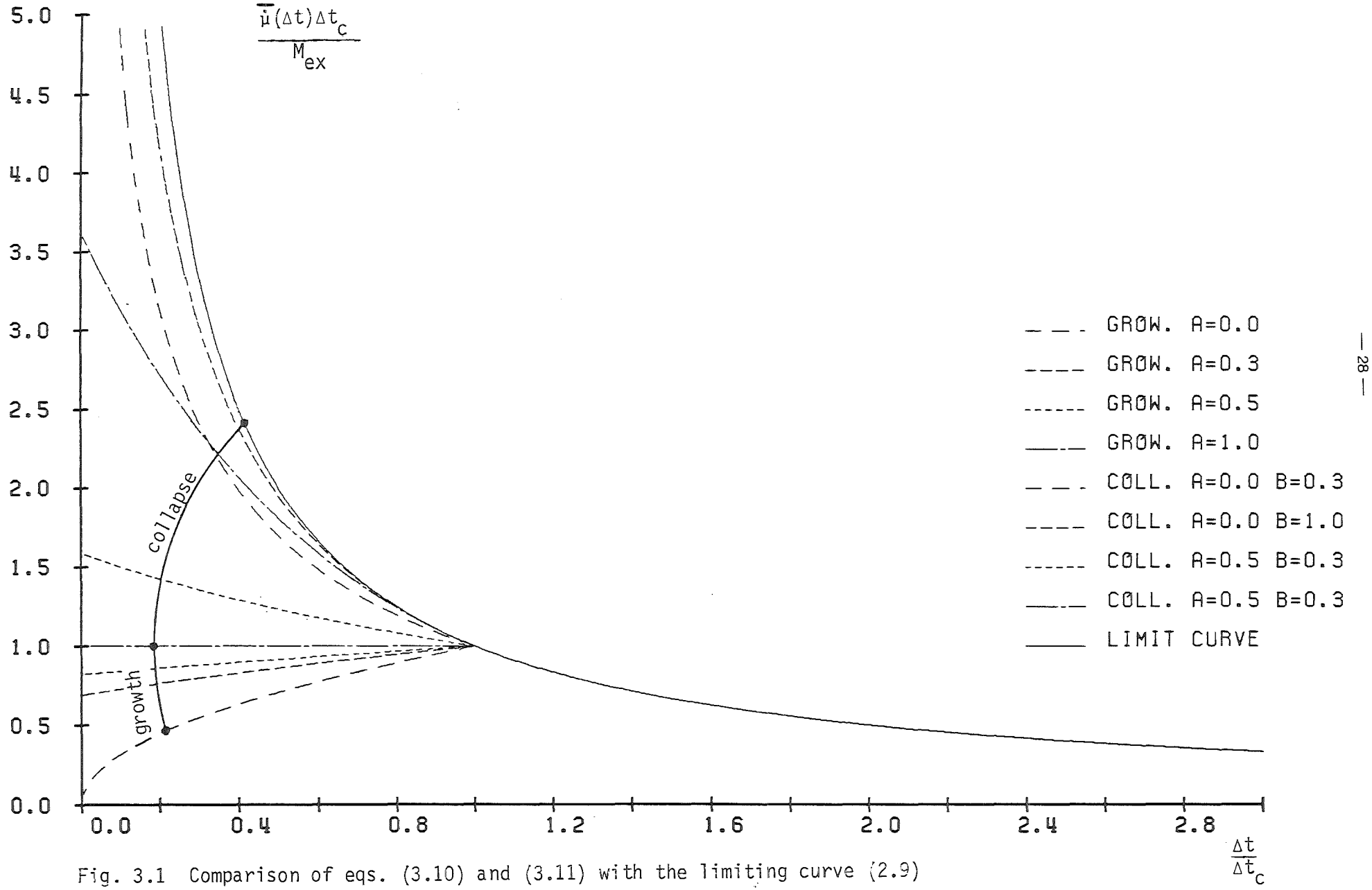



Fig. 3.1 Comparison of eqs. (3.10) and (3.11) with the limiting curve (2.9)

## 4. Driving force reduction model

In this model, we keep the previous hypothesis that the vapor is in a saturated state at the same liquid pressure. In contrast to the model in Section 3, the liquid changes its state continuously to reach the equilibrium point.
We suppose that the equilibrium conditions are very close to the non-equilibrium conditions, so that we can assume that the transformation occurs at constant pressure and the change of the void fraction is small.

Fig. 4.1 is a P-T diagram of the transformation model, for the evaporation, "a", and condensation, "b", of the vapor.
We suppose that for each bubble the same quantity of liquid that must be heated (in case of condensation) or cooled (in case of evaporation) is available. The exchanging of heat is possible only from the bubble to its associated volume of liquid (Fig. 4.2).

If we use a plane interface model for the transient heat conduction problem, we can represent the bubble model as shown in Fig. 4.3.

The area of the exchanging surface for a single bubble is:

$$A_b = 4\pi \left( \frac{3\alpha}{4\pi n_b} \right)^{2/3} \tag{4.1}$$

The volume of the "associated" liquid is:

$$V_{fb} = \frac{1 - \alpha}{n_b} \tag{4.2}$$

The parameter L, the characteristic length, is then:

$$L = \frac{V_{fb}}{A_b} = \frac{1 - \alpha}{4\pi n_b \left( \frac{3\alpha}{4\pi n_b} \right)^{2/3}} \tag{4.3}$$

Now one can solve the transient conduction problem. Note that from the initial hypothesis, L remains constant during the heat transfer.

The differential equation is:

$$\frac{\partial T}{\partial t} = a_f \cdot \frac{\partial^2 T}{\partial x^2} \qquad | \; 0 \leq x \leq L \; | \tag{4.4}$$

with the boundary conditions:

$$\begin{cases} T\,(x,o) = T_i & | \; 0 < x \leq L \; | \\ T\,(L,t) = T_e & | \; t \geq 0 \; | \\ \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \end{cases} \tag{4.5}$$

Where $T_i$ is the initial liquid temperature and $T_e$ is the vapor temperature - at saturation - constant during entire transient.

The solution is /7/:

$$T(x,t) = T_e + 2(T_i - T_e) \sum_{n=0}^{\infty} \left\{ \exp\left[ \frac{-(n+\frac{1}{2})^2 \pi^2 a_f}{L^2} t \right] \cdot \right. \tag{4.6}$$

$$\left. \cdot \frac{(-1)^n}{(n+\frac{1}{2}).\pi} \cdot \cos\left[ (n+\frac{1}{2}) \frac{\pi x}{L} \right] \right\} \qquad | \; n = 1,2,3,\ldots |$$

The expression of the heat transfer throughout the surface of the bubble (x=L) can be easily derived:

$$\frac{\dot{q}}{A_b} \triangleq \lambda_f \left.\frac{\partial T}{\partial x}\right|_{x=L} = \lambda_f \frac{2(T_i - T_e)}{L} \sum_{n=0}^{\infty} \exp(-t/t_n) \tag{4.7}$$

where

$$\begin{cases} t_n = \dfrac{\bar{\tau}}{(2n+1)^2} & | \; n = 0,1,2\ldots | \\ \bar{\tau} = \dfrac{4L^2}{\pi^2 a_f} \end{cases} \tag{4.8}$$

Because the properties of the vapor do not change during the exchange process and all the heat is used to evaporate or condense vapor, one can use equation (4.7) to describe the mass transfer process. As noted in Section 3, we introduce the same limit for mass exchange, using this expression (4.7) only to evaluate the characteristic time of the process, while the amount of mass exchange is based on $M_{ex}$.

Then, for the mass transfer rate $\dot{\mu}(t)$ we can derive the following expression:

$$\dot{\mu}(t) = \frac{8}{\pi^2} \frac{M_{ex}}{\bar{\tau}} \sum_{n=0}^{\infty} \exp(-t/t_n) \tag{4.9}$$

To show the importance of each exponential term, we can rewrite eq.(4.9):

$$\left[\begin{array}{l} \dot{\mu}(t) = \sum_{n=0}^{\infty} \dot{\mu}_n(t) \\ \\ \dot{\mu}_n(t) = \frac{8}{\pi^2} \frac{M_{ex}}{\bar{\tau}} \exp(-t/t_n) \end{array}\right. \tag{4.10}$$

Fig. 4.4 shows - in dimensionless form - the behaviour of the terms $\dot{\mu}_n(t)$.

One can derive the average mass transfer rate

$$\left[\begin{array}{l} \bar{\dot{\mu}}(\Delta t) = \sum_{n=0}^{\infty} \bar{\dot{\mu}}_n(\Delta t) \\ \\ \bar{\dot{\mu}}_n(\Delta t) = \frac{8}{\pi^2(2n+1)^2} \frac{M_{ex}}{\Delta t} \left[ 1 - \exp(-\frac{\Delta t}{t_n}) \right] \end{array}\right. \tag{4.11}$$

In Fig. 4.5 the single terms $\bar{\dot{\mu}}_n(\Delta t)$ are given in dimensionless form, while in Fig. 4.6 the function $\bar{\dot{\mu}}(\Delta t)$ is given.

The model described in this section seems more consistent with the model of mass exchange developed in Section 2, because it takes the reduction of the "driving force" into account.

Moreover, in condensation, every consideration about the initial radius of collapse $R_o$ is avoided.

Nevertheless, one can make the following remarks:

1. the plane interface approximation might not be a good hypothesis;
2. the hypothesis of a constant L is - as previously stated - proper only for initial conditions not far from the equilibrium state;
3. in this model all historical effects are neglected, because one supposes that the temperature gradient at the bubble boundary is infinite at the beginning of each time step.

A consequence of the first point is that there is no difference in the equations describing the phenomena of growth or collapse of the bubble. The difference in results is only due to the different value of $M_{ex}$. Extension to spherical geometry would require a more complex mathematical solution.

Similarly the second point is justified by the simpler mathematical model.

In contrast, the third consideration brings up a problem that is common for every correlation that uses a dynamical model. To take historical effects into account one might store some parameters (the boundary velocity of the bubble, e.g.) to be used in the next time step. Additionally, the form of eq.(4.6) must be modified to take the influence of the previous time step on the initial condition in the new time step into account.

In the next section we discuss some results of the values calculated with the different expression obtained for the mass source term, and make a comparison among them.

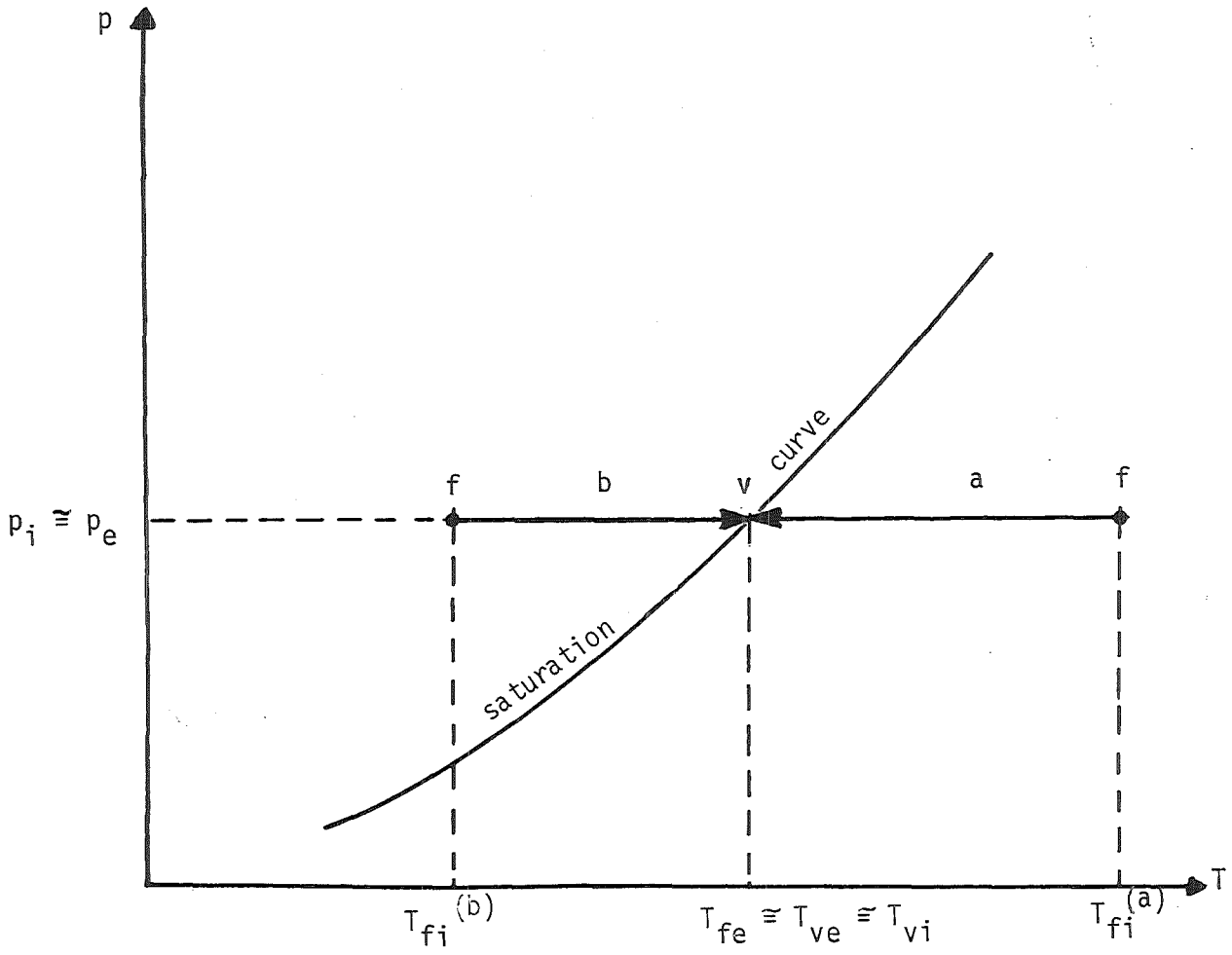



Fig. 4.1 Models for evaporation (a) and condensation (b)

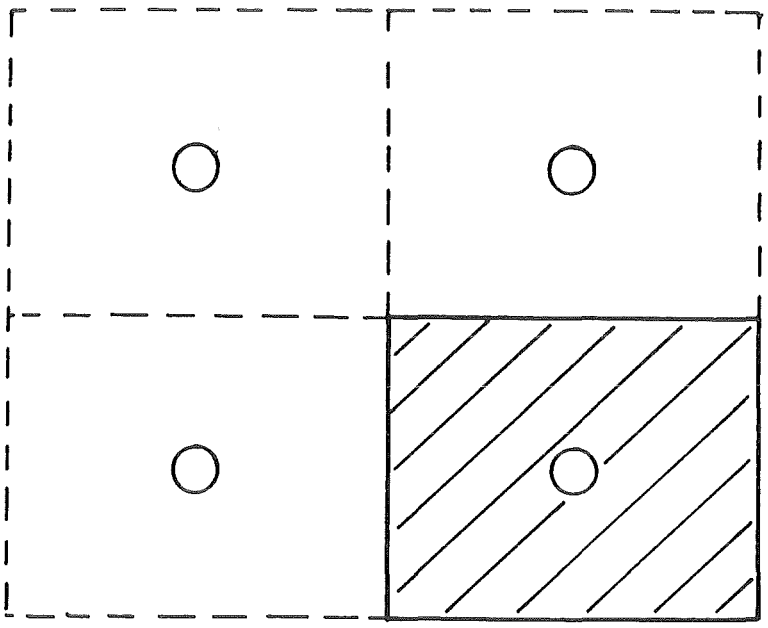


Fig. 4.2 "Associated " volume

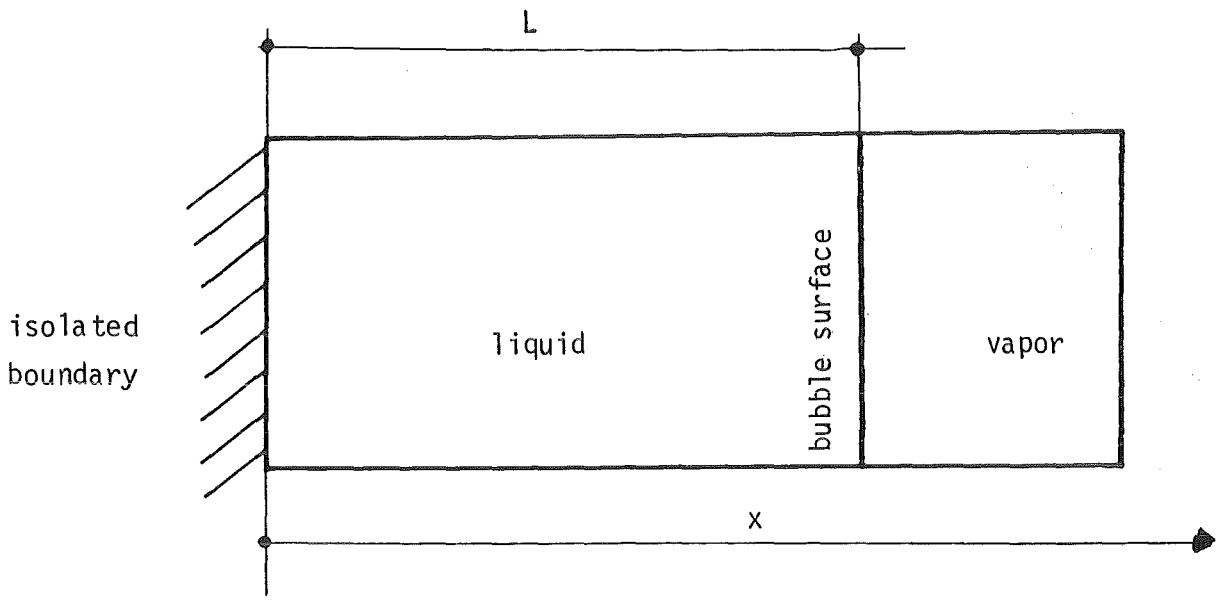



Fig. 4.3 Bubble model

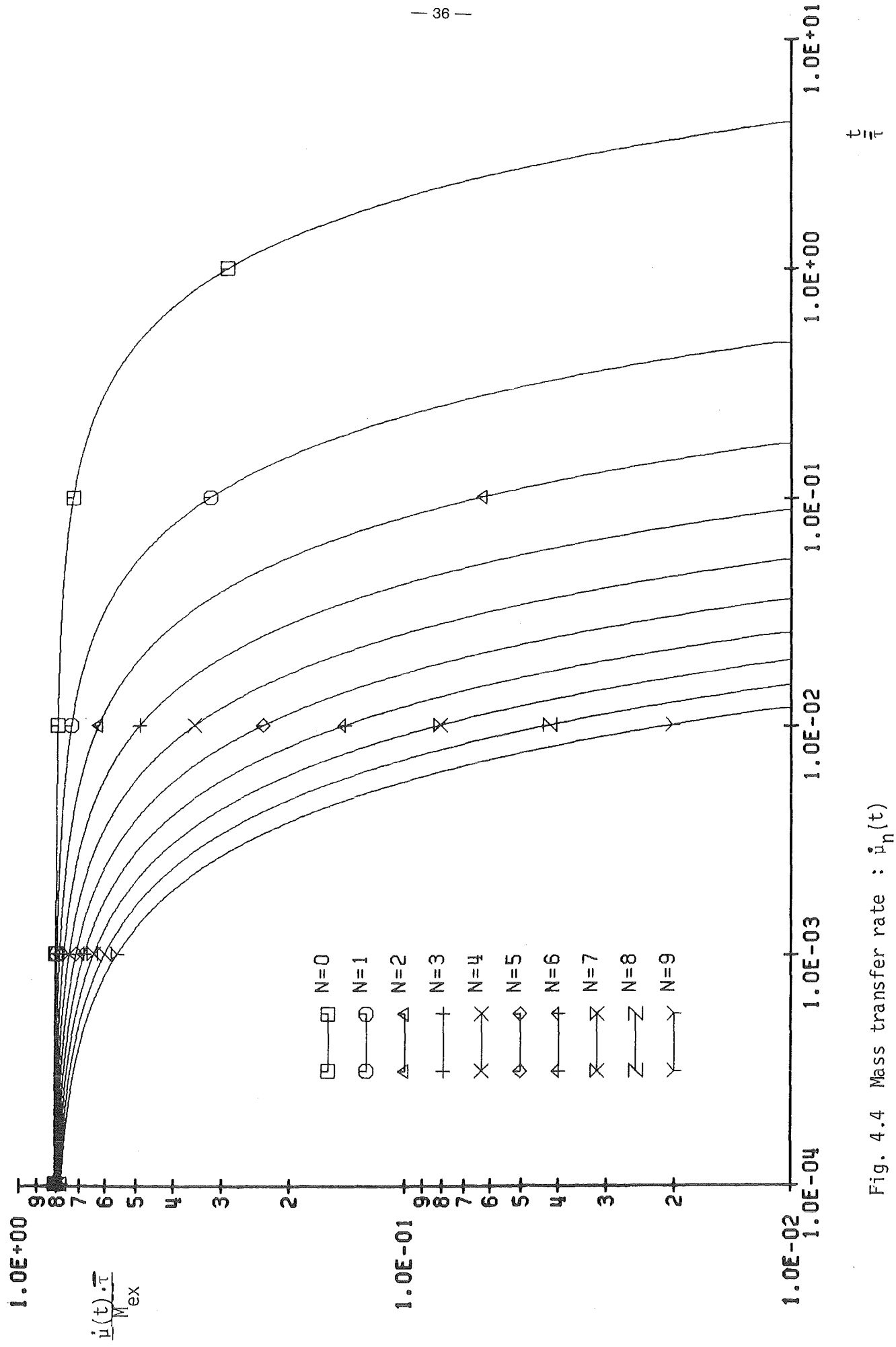



Fig. 4.4 Mass transfer rate : $\dot{\mu}_n(t)$

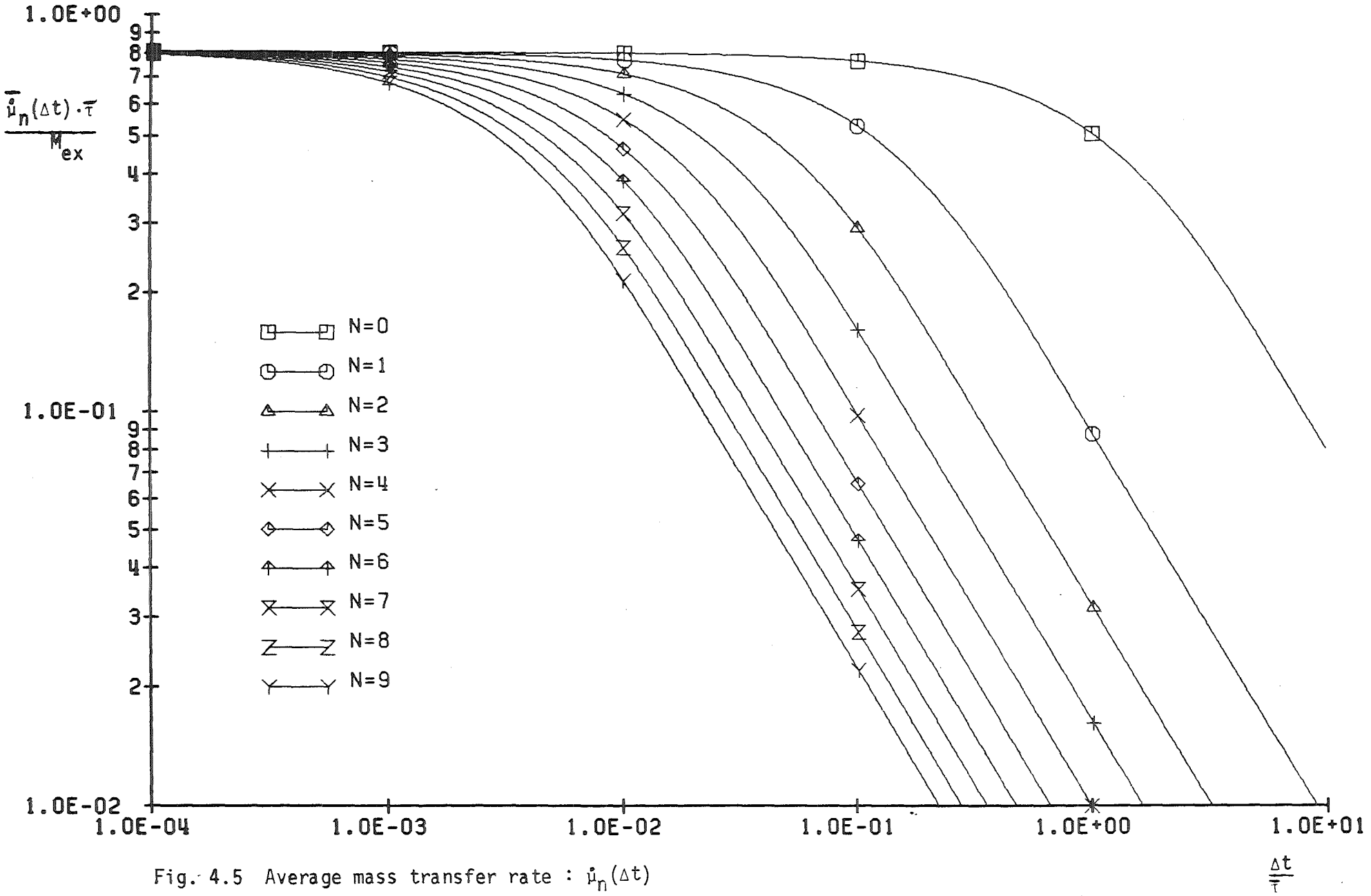



Fig. 4.5 Average mass transfer rate : $\dot{\mu}_n(\Delta t)$

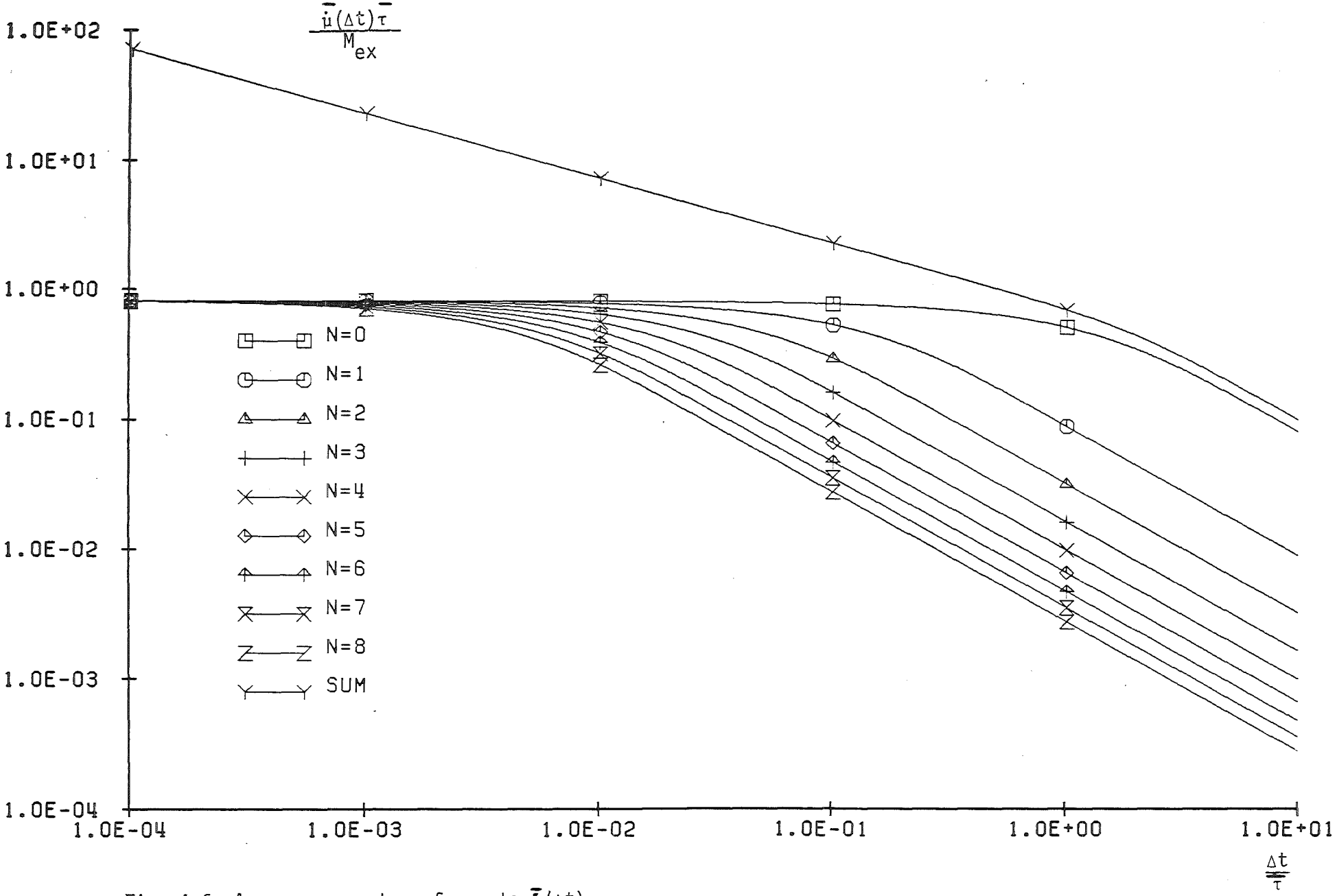



Fig. 4.6 Average mass transfer rate $\bar{\dot{\mu}}(\Delta t)$ .

## 5. Numerical values and conclusions

We have performed some calculations with the previous correlations for practical cases. We have taken into account six conditions, whose initial parameters are shown in Table 5.1.

For each case $M_{ex}$ and $\alpha_e$ have been calculated with the subroutine EQUIL. In Table 5.2 these calculated values are shown.

In Table 5.3 and 5.4 expressions for the mass source term developed in this work are summarized, for evaporation and condensation in bubbly flow respectively.

In Tables 5.5 - 5.10 the results of the calculations are shown for each case.

Analysis of these tables points out the different behaviour of the first two correlations, with respect to the last two, when compared to the limiting value (Case 3). Delimited by heavier lines are the results that exceed the limiting value for a given time step.
This occurs for the largest value of $\Delta t$ in each table shown.

Otherwise the results of Cases 4 and 5 lie considerably under the limiting curve (Fig. 2.9) for the smallest values of $\Delta t$ and approach this curve for the largest values.
Case 5 displays, in general, the smallest value with respect to Case 4 over a wide range of practical interest of $\Delta t$. In fact, the "driving force reduction model" delays the complete transfer of mass, with respect to the models based on equations (1.1) and (1.2).

Nevertheless, for the smallest values of $\Delta t$, the values in Curve 5 becomes greater than those in curve 4, because

$$\lim_{\Delta t \to 0} \dot{\bar{\mu}}_5(\Delta t) = \pm \infty$$

while the limit of the curve 4 in $\Delta t \to 0$ is finite.

We can observe the effects of the strong dependence of the parameter $R_o$ in the collapse equation (1.7) on the equations developed for $\mu(\Delta t)$.
The value of $R_o$ has been changed from 1.2 to 2.

The increase in $R_o$ with respect to $R_1$ produces large reduction of the mass source term. Of course this has no effect on expressions 3 and 5, because these equations are not based on equation (1.7).

The analysis performed on the bubbly flow regime can be considered satisfactory for the range of $\Delta t$ generally used in the computer codes.

Nevertheless, further theoretical considerations might be done in the range of the smallest value of $\Delta t$, where the theory developed can give values of $\mu(\Delta t)$ that are infinite or too large. In fact, the "thermal controlled theory" can fail in some cases, neglecting the inertial resistance of the liquid, and especially at the starting point of the bubble collapse.

The present theory can be completed by developing analogous equations for $\overline{\dot{\mu}}(\Delta t)$ based on the "inertial controlled theory". This can set some limitations on the maximum values of $\overline{\dot{\mu}}(\Delta t)$ especially in the range of the smallest values of $\Delta t$.

Another set of equations can be developed using the same methodology for the other flow regimes, or to take into account other phenomena, such as the presence of a non-condensible gas in the gas field.
In this way one can complete the package of equations describing the transfer of mass (and energy, using a theory based on $E_{ex}$) between the phases in non-equilibrium.
At this point, the next problem to consider is to generalize the different equations developed into only one mathematical form. This generalization would be very suitable for practical computation and to perform a linear stability analysis that also takes the source terms into account.

The kind of analysis performed in this work constitutes a first approach to the problem of developing a well based time-averaging procedure for the source term. The next step in this work will concern testing the correlation developed by comparison between calculated and experimental data.

Table 5.1 : Initial conditions for each of the calculations performed

| Calculation | Pressure (Pa) | Void fraction | $T_f - T_s$ ($^oK$) | $T_g - T_s$ ($^oK$) | $\frac{R_o}{R_1}$ (for collapse) | $n_b (m^{-3})$ |
|---|---|---|---|---|---|---|
| a1 | $5 \cdot 10^5$ | 0.3 | + 20 | 0 | - | $10^9$ |
| a2 | $5 \cdot 10^5$ | 0.3 | - 20 | 0 | 1.2 | $10^9$ |
| a3 | $5 \cdot 10^5$ | 0.3 | - 20 | 0 | 2. | $10^9$ |
| b1 | $5 \cdot 10^6$ | 0.3 | + 20 | 0 | - | $10^9$ |
| b2 | $5 \cdot 10^6$ | 0.3 | - 20 | 0 | 1.2 | $10^9$ |
| b3 | $5 \cdot 10^6$ | 0.3 | - 20 | 0 | 2. | $10^9$ |

Table 5.2 : Calculated values of $M_{ex}$ and $\alpha_i$ for each cases.

| | Calculations performed | | | | | |
|---|---|---|---|---|---|---|
| Calculated variables | a1 | a2 | a3 | b1 | b2 | b3 |
| $M_{ex}(\frac{kg}{m^3})$ | 0.486 | -0.326 | -0.326 | 3.164 | -2.229 | -2.229 |
| $\alpha_i$ | 0.3009 | 0.2994 | 0.2994 | 0.3089 | 0.2947 | 0.2947 |

Table 5.3. : Average mass transfer rate ($\bar{\dot{\mu}}$) for bubble growth.

| | | |
|---|---|---|
| 1 | $2\pi \cdot n_b \cdot \rho_v \cdot \frac{R_g^3}{t_g^{3/2}} \sqrt{t_1}$ | (1.6) |
| 2 | $\frac{4\pi}{3} \cdot \frac{n_b \rho_v}{\Delta t} \cdot \frac{R_g^3}{t_g^{3/2}} \left[ (t_1 + \Delta t)^{3/2} - t_1^{3/2} \right]$ | (1.9) |
| 3 | $\frac{M_{ev}}{\Delta t}$ | (2.19) |
| 4 | $\begin{cases} \frac{M_{ex}}{\Delta t} \cdot \frac{(t_1 + \Delta t)^{3/2} - t_1^{3/2}}{t_2^{3/2} - t_1^{3/2}} & \text{for } \Delta t \leq t_2 - t_1 \\ \frac{M_{ex}}{\Delta t} & \text{for } \Delta t > t_2 - t_1 \end{cases}$ | (3.5) |
| 5 | $\sum_{n=0}^{\infty} \frac{8}{\pi^2 (2n+1)^2} \cdot \frac{M_{ex}}{\Delta t} \cdot \left[ 1 - \exp(-\frac{\Delta t}{t_n}) \right]$ | (4.11) |

Table 5.4 : Average mass transfer rate ($\dot{\mu}$) for bubble collapse

| | | |
|---|---|---|
| 1 | $-2\pi \cdot n_b \cdot \rho_v \dfrac{R_o^3}{t_c^{3/2}} \cdot \dfrac{(\sqrt{t_c} - \sqrt{t_1})^2}{\sqrt{t_c}}$ \| for $0 \leq t_1 \leq t_c$ \|<br>$0$ \| for $t_1 > t_c$ \| | (1.7) |
| 2 | $\left[ \dfrac{-4\pi}{3} \cdot \dfrac{n_b \rho_v}{\Delta t} \cdot \dfrac{R_o^3}{t_c^{3/2}} \right] 3t_c\sqrt{t} + t^{3/2} - 3\sqrt{t_c} \cdot t \Big\|_{t_1}^{\hat{t}}$<br>with $\hat{t} = \min(t_1 + \Delta t, t_c)$ | (1.10) |
| 3 | $\dfrac{M_{ex}}{\Delta t}$ | (2.19) |
| 4 | $\dfrac{M_{ex}}{\Delta t} \cdot \dfrac{(\sqrt{t_c} - \sqrt{t_1})^3 - (\sqrt{t_c} - \sqrt{\Delta t + t_1})^3}{(\sqrt{t_c} - \sqrt{t_1})^3 - (\sqrt{t_c} - \sqrt{t_2})^3}$ \| for $\Delta t \leq t_2 - t_1$ \|<br>$\dfrac{M_{ex}}{\Delta t}$ \| for $\Delta t > t_2 - t_1$ \| | (3.9) |
| 5 | $\sum_{n=0}^{\infty} \dfrac{8}{\pi^2(2n+1)^2} \cdot \dfrac{M_{ex}}{\Delta t} \cdot \left[ 1 - \exp(-\dfrac{\Delta t}{t_n}) \right]$ | (4.11) |

Table 5.5 : Results of the a1 calculation : $\bar{\dot{\mu}}$ $\left(\frac{kg}{m^3 sec}\right)$

| eqs. (Tab.5.3) \ $\Delta t$(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | 1065. | 1065. | 1065. | 1065. | 1065. | 1065. | 1065. |
| 2 | 1065. | 1068. | 1089. | 1275. | 2403. | 6797. | 2120. |
| 3 | 486000. | 48600. | 4860. | 486.0 | 48.60 | 4.860 | 0.4860 |
| 4 | 207300. | 48600. | 4860. | 486.0 | 48.60 | 4.860 | 0.4860 |
| 5 | 1142. | 227.3 | 70.80 | 22.39 | 7.081 | 2.238 | 0.4796 |

Table 5.6 : Results of a2 calculation : $\overline{\dot{\mu}}$ ( $\frac{kg}{m^3 sec}$ )

| eqs. (Tab.5.4) \ Δt(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | - 1423. | - 1423. | - 1423. | - 1423. | - 1423. | - 1423. | - 1423. |
| 2 | - 1420. | - 1494. | - 1193. | - 555.5 | - 79.98 | - 7.998 | - 0.7998 |
| 3 | - 325900. | - 32590. | - 3259. | - 325.9 | - 32.59 | - 3.259 | - 0.3259 |
| 4 | - 300800. | - 32590. | - 3259. | - 325.9 | - 32.59 | - 3.259 | - 0.3259 |
| 5 | - 765.1 | - 151.9 | - 47.29 | - 14.96 | - 4.729 | - 1.495 | - 0.3214 |

Table 5.7 : Results of a3 calculation : $\dot{\bar{\mu}}$ $\left(\frac{kg}{m^3 sec}\right)$

| eqs. (Tab.5.4) \ Δt(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | - 284.6 | - 284.6 | - 284.6 | - 284.6 | - 284.6 | - 284.6 | - 284.6 |
| 2 | - 284.5 | - 284.1 | - 279.6 | - 240.3 | - 79.63 | - 7.998 | - 0.7998 |
| 3 | - 325900. | - 32590. | - 3259. | - 325.9 | - 32.59 | - 3.259 | - 0.3259 |
| 4 | - 60270. | - 32590. | - 3259. | - 325.9 | - 32.59 | - 3.259 | - 0.3259 |
| 5 | - 765.1 | - 151.9 | - 47.29 | - 14.96 | - 4.729 | - 1.495 | - 0.3214 |

Table 5.8 : Results of b1 calculation : $\overline{\dot{\mu}}$ ( $\frac{kg}{m^3 sec}$ )

| eqs. (Tab.5.3) \ Δt(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | 218.9 | 218.9 | 218.9 | 218.9 | 218.9 | 218.9 | 218.9 |
| 2 | 218.9 | 218.9 | 219.0 | 219.9 | 229.0 | 303.1 | 681.7 |
| 3 | 3164000. | 316400. | 31640. | 3164. | 316.4 | 31.64 | 3.164 |
| 4 | 3076. | 3076. | 3078. | 3091. | 316.4 | 31.64 | 3.164 |
| 5 | 7243. | 1345. | 441.2 | 131.0 | 41.43 | 13.10 | 3.071 |

Table 5.9 : Results of the b2 calculation : $\bar{\dot{\mu}}$ ( $\frac{kg}{m^3 sec}$ )

| eqs. (Tab.5.4) \ Δt(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | - 207.5 | - 2.07.5 | - 207.5 | - 207.5 | - 207.5 | - 207.5 | - 207.5 |
| 2 | - 207.5 | - 207.4 | - 206.8 | - 201.2 | - 161.3 | - 62.42 | - 7.614 |
| 3 | - 2223000. | - 222300 . | - 22230 . | - 2223. | - 222.3 | - 22.23 | - 2.223 |
| 4 | - 3411. | - 3411. | - 3401. | - 2223. | - 222.3 | - 22.23 | - 2.223 |
| 5 | - 5171. | - 1003. | - 311.3 | - 98.44 | - 31.13 | - 9.842 | - 2.182 |

Table 5.10 : Results of the b3 calculation : $\bar{\dot{\mu}}$ ( $\frac{kg}{m^3 sec}$ )

| eqs. (Tab.5.4) \ $\Delta t$(sec) | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 |
|---|---|---|---|---|---|---|---|
| 1 | - 41.50 | - 41.50 | - 41.50 | - 41.50 | - 41.50 | - 41.50 | - 41.50 |
| 2 | - 41.50 | - 41.50 | - 41.49 | - 41.39 | - 40.39 | - 32.23 | - 7.614 |
| 3 | - 2223000. | - 222300. | - 22230. | - 2223. | - 222.3 | - 22.23 | - 2.223 |
| 4 | - 682.3 | - 682.3 | - 682.1 | - 680.5 | - 222.3 | - 22.23 | - 2.223 |
| 5 | - 5171. | - 1003. | - 311.3 | - 98.44 | - 31.13 | - 9.842 | - 2.182 |

APPENDIX A : Listing of the subroutine EQUIL

```
      SUBROUTINE EQUIL(P,TF,TG,AL,RHF,RHG,HF,HG,MEX,EEX,ALEQ,XEQ,RERU,
     *                 RERV,ERROR)
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LW,LD,MEX
      LOGICAL FLAG,ERROR
C
      ERROR=.FALSE.
C
C     STATIC QUALITY CALCULATION
C
      XG=AL*RHG/(AL*RHG+(1.-AL)*RHF)
C
C     SPEC. VOLUME AND INTERNAL ENERGY CALCULATION
C
      VF=1./RHF
      VG=1./RHG
      UG=HG-P*VG
      UF=HF-P*VF
C
      VM=(1.-XG)*VF+XG*VG
      UM=(1.-XG)*UF+XG*UG
C
C     ITERATIONS
C
      N=0
      FLAG=.FALSE.
      P1=P
      PX=P1+10.E5
      IF(P1.GT.100.E5) PX=P1-10.E5
C
    1 CALL SAET01(TE,P1,VW,VD,HW,HD,HWD,SW,SD,
     *              TSP,CPW,CPD,EW,ED,LW,LD,SIGMA,1)
C
      UW=HW-P1*VW
      UD=HD-P1*VD
C
      UWD=UD-UW
      VWD=VD-VW
C
      XGU1=(UM-UW)/UWD
      XGV1=(VM-VW)/VWD
C
      IF(.NOT.FLAG) GO TO 100
C
C     ERROR CONTROLL
C
      XEQ=XGV1
      IF(XEQ.LE.0.) XEQ=0.
      IF(XEQ.GE.1.) XEQ=1.
C
```

```
      UME=XEQ*UD+(1.-XEQ)*UW
      VME=XEQ*VD+(1.-XEQ)*VW
      ERU=UME-UM
      ERV=VME-VM
      RERU=ERU/UM*100.
      RERV=ERV/VM*100.
C
      SCR1=DABS(RERU)
      SCR2=DABS(RERV)
      IF(SCR1.LT.0.1.AND.SCR2.LT.0.1) GO TO 200
C
      PX=P1+(XGV1-XGU1)*(P2-P1)/(XGU2-XGU1-XGV2+XGV1)
C
      IF(PX.LE.700.0) PX=700.0
      IF(PX.GE.21.E6) PX=21.E6
C
  100 N=N+1
      IF(N.GE.100) GO TO 300
C
      P2=P1
      P1=PX
      XGV2=XGV1
      XGU2=XGU1
C
      FLAG=.TRUE.
      GO TO 1
C
  200 MEX=(XEQ-XG)/VM
      EEX=(UD*XEQ-UG*XG)/VM
      ALEQ=XEQ/(VW*(XEQ/VW+(1.-XEQ)/VD))
C
      RETURN
C
  300 ERROR=.TRUE.
      RETURN
      END
```

Literature

/1/ Liles D., et al.,
TRAC-FD2, An Advanced-Best-Estimate Computer Program for pressurized Water Reactor Loss-of-Coolant Accident Analysis, NUREG/CR-2054, LA-8709 MS (April 1981)

/2/ TRAC-PF1: An Advanced Best-Estimate Computer Program for pressurized Water Reactor Analysis, NUREG/CR-3567, LA-9844-MS, (February 1984)

/3/ Kolev N.I.,
Transiente Dreiphasen-Dreikomponenten-Strömung.
Teil 3 : 3D-Dreifluid-Diffusionsmodell, KfK-4080, Karlsruhe (April 1986)

/4/ Zuber N.,
The dynamics of vapor bubbles in nonuniform temperature fields,
Int. J. Heat Mass Transfer, Vol.2, pp.83-98, 1961

/5/ Kolev N.I.,
Transiente Zweiphasenströmung, Springer-Verlag 1986
(ISBN 3-540-15907-x)

/6/ Florschuetz L.W., Chao B.T.,
On the mechanics of vapor bubble collapse, Transactions of the ASME (Sec.C), J. of Heat Transfer, Vol. 87, pp. 209-220, 1965

/7/ Kreith F.,
Principles of Heat Transfer, Intext Educational Publisher, New York, Third Edition 1973