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Abstract

A formalism developed by Benoist, to calculate the neutron leakage in a voided fast reactor lattice, was modified and simplified using analytic chord length distributions. The new method (computer routine ARIADNE) allows for a rapid calculation of buckling-dependent anisotropic cell diffusion coefficients for voided and unvoided periodic lattices with adequate accuracy. Results are compared with Monte Carlo calculations carried out at Argonne National Laboratory, and with results from other authors.

Zusammenfassung

Eine neue vereinfachte Methode zur Berechnung der Streaming-Reaktivität in Stabgittern schneller Reaktoren

Ein Formalismus, der von Benoist zur Berechnung der Neutronen-Leckage in Natrium-entleerten Stabgittern schneller Reaktoren entwickelt wurde, wurde modifiziert und unter Benutzung analytisch angenäherter Sehnenlängenverteilungen vereinfacht. Die neue Methode (Rechenprogramm ARIADNE) ermöglicht eine schnelle Berechnung von Buckling-abhängigen anisotropen Diffusionskoeffizienten für Zellen mit und ohne Natrium mit ausreichender Genauigkeit. Die Ergebnisse werden mit Monte-Carlo-Rechnungen verglichen, die im Argonne National Laboratory durchgeführt wurden. Weitere Vergleiche mit publizierten Ergebnissen werden vorgestellt.

Contents

1. Introduction

If the pin lattice of a Fast Reactor has voided coolant channels, neutron leakage is enhanced as compared to the case of a homogeneous material distribution in the unit cell. It is important to calculate this enhanced leakage ("streaming effect") for a gas cooled fast reactor in its operating state, and for a sodium cooled reactor under conditions of hypothetical core discruptive accidents, when the sodium has boiled out. E.g., in an unprotected loss-of-flow accident, pin failure into a voided channel leads to a quasi-homogeneous material distribution, and therefore to an increase in reactivity because the leakage is reduced.

For lattices which involve two-dimensional plane void gaps, the directional cell diffusion coefficients depend strongly on the bucklings B^2 of the global flux distribution. On the other hand, the standard KfK cell code KAPER[1] does not have the capability to calculate B^2-dependent diffusion coefficients. The method included in KAPER underestimates the leakage in voided pin lattices. This is unsatisfactory because the streaming effect is a safety-relevant parameter, and it is important to calculate it with reasonable accuracy.

Therefore, a computer model ARIADNE was developed at KfK, which calculates B^2-dependent diffusion coefficients in a 2-region approximation (fuel pin and coolant channel region). It is based to some extent on a formalism proposed by Benoist[2]. In the present paper, ARIADNE is described. For verification of the method, results for cell diffusion coefficients are compared with data available in the literature.

2. Methods to Calculate Cell Diffusion Coefficients

The standard tool for homogenisation calculations at KfK is the cell code KAPER[1]. Direction-dependent diffusion coefficients are calculated using the collision probability method by Benoist[3], which leads to the simple equation

$$D_k = \frac{\sum_i V_i \Phi_i \sum_j P_{ij,k} / \Sigma_j}{3 \sum_i V_i \Phi_i} \qquad (1)$$

where the $P_{ij,k}$ are the directional (axial or radial) collision probabilities introduced by Benoist. The cell is taken as cylindrical, with white boundary conditions; i.e. neutrons returning from the boundary have isotropic angular distribution. This is adequate for a sodium-filled hexagonal lattice of a fast reactor. However, a voided lattice contains two dimensional plane gaps (see Fig. 1) if it is wide enough; i.e. if $p/d > 2/\sqrt{3}$, which is usually the case for fast reactor lattices. Then, the "cylindical cell" method involves an artificial cut of the plane gaps by introducing a fictitious cell boundary, which is located in vaccum. Therefore, the long neutron paths which are present in such a voided lattice are not adequately treated, and the leakage (or the cell diffusion coefficient) is underestimated. A slightly different method, which is also available in KAPER, is the use of Dancoff factors for the coolant channel region. This alternative leads to a similar underestimate of the leakage in voided cells as the cylindrical cell method.

The principle of cell homogenisation leads to different definitions of the cell-averaged cross sections, and the cell diffusion coefficients, depending on how different terms of the neutron balance are grouped together. This was discussed e.g. by Hughes[4]. However, for fast reactors, the pin cells are usually optically thin, and the differences between the different definitions are usually negligibly small. It is, therefore, completely adequate to use the simplest definition, which is termed "Benoist uncorrected"[4]. On the other hand, as mentioned above, it is important to calculate B^2-dependent diffusion coefficients for lattices with 2-dimensional void planes. This is still not standard in most cell codes. In the following, the different methods are briefly discussed.

Köhler and Ligou[5] were the first ones to calculate B^2-dependent diffusion coefficients for gas cooled fast reactor lattices. These authors used a numerical method to solve the basic equations in their code DIFFAX. This method does not involve

any modelling approximations and is, therefore, accurate and useful as a reference method. However, streaming corrections are quoted in their paper only for the axial direction, and only for voided lattices. For a sodium cooled reactor, it is desirable to have a method which is valid for both voided and unvoided lattices, and for both directions.

Eisemann[6] developed a method to calculate streaming in the hexagonal lattice of a gas cooled reactor. This method is also restricted to voided lattices. Besides, the B^2-dependence was suppressed, and it is not clear from the documentation how convergence of the numerical integration was achieved. Therefore, this method was not considered suitable for further development.

Gelbard and Lell[7] and Lell[8] derived a perturbation equation to calculate the change Δk due to streaming for given B^2, and solved it by a Monte Carlo method. The published results quoted in Table I were produced by a one-group calculation; therefore, ΔD's can be derived from them. A direct use of the quoted $\Delta k/k$ to the real reactor is, however, questionable, for once because of the one-group approximation, and also because blankets are ignored in these zero-dimensional calculations.

Benoist extended his original theory[3] to the case of lattices with voided coolant channels[2]. His approach follows the principle to retain the B^2-dependence only to the lowest order necessary. This is consistent with using the diffusion approximation in the whole core calculation.

For a periodic 2-region lattice (fuel pin and coolant channel), the important quantity is the reduced collision probability p_{cck} (c = coolant channel). It is related to the collision provbability P_{cck} (probability that a neutron born in the coolant channel makes its first collision in the coolant channel) through the equation

$$p_{cck} = \frac{2 P_{cck}}{l_c \Sigma_c}$$

Note that p_{cck} (unlike P_{cck}) remains finite in the case of a voided channel, i.e. if $\Sigma_c = 0$.

This quantity is split into contributions Q_k and Q'_k, where Q_k is due to neutron paths which never leave the coolant channel region. It is calculated by a numerical procedure, adding the contributions of each ring of fuel pins separately. The second term Q'_k, accounts for neutron paths which traverse at least one fuel pin. The assumption is made that neutrons entering a pin have isotropic angular distribution. This assumption leads to an overestimate of the diffusion coefficients,

which is obvious by looking at the results published by Benoist and Duracz[9]. Note that for a typical fast reactor lattice, Q'_k is a lot larger than Q_k.

Gho[10], in his dissertation, developed a comprehensive computer model TRIFFAX. It is based in part on DIFFAX, but extensions include e.g. the effect of the wrapper tubes of a hexagonal lattice. The results of this code for an undisturbed periodic lattice are very close to the DIFFAX results.

3. The KfK Method to Calculate B^2-Dependent Cell Diffusion Coefficients for Pin Lattices

The following method is valid for both voided and for sodium filled periodic pin lattices. It can therefore be used to calculate the streaming effect associated with the removal of sodium in a consistent manner. The method is, in its present form, restricted to two region cells, which consist of the fuel pin and the coolant channel.

3.1 Approximate Projected Chord Length Distribution

The method is based in part on a formalism developed by Benoist[2]. An important quantity which is needed in Benoist's equations is the distribution of the chord lengths g (ρ) in the coolant region, projected on a horizontal plane. Thus, the projected chord length ρ is given by

$$\rho = l \sin \theta$$

where l is the true chord length, and θ is the angle of the flight path with the z-axis.
Assume a pin lattice, with "a" the pin radius, and "s" the lattice pitch. If $s > 4a/\sqrt{3}$ (which is usually the case in fast reactor lattices) then the lattice contains plane void spaces of the thickness

$$\varepsilon = \frac{\sqrt{3}}{2}\left(s - \frac{4a}{\sqrt{3}}\right) \tag{2}$$

For a lattice where $\varepsilon < 0$, the following method is irrelevant, and the simple KAPER method should be used instead. However, if $\varepsilon > 0$, the distribution of projected chord lengths g (ρ) contains an asymptotic term, which behaves, for large values of ρ, as

$$g_{as}(\rho) = \frac{A}{\rho^3} \tag{3}$$

where

$$A = \frac{3\varepsilon^2 s}{\pi a} \tag{4}$$

It is due to this asymptotic term that a formalism which is independent of B^2 would give infinitely large cell diffusion coefficients for voided lattices. This is connected with the fact that the mean square of the chord length is infinity

$$\overline{\rho^2} = \int_0^\infty g_{as}(\rho)\,\rho^2 d\rho \to \infty$$

The complete distribution of the projected chord lengths,

$$g(\rho) = g_1(\rho) + g_2(\rho) \tag{5}$$

where g_1 is the asymptotic term, is clearly a rather complex function of ρ for a hexagonal lattice. In an early paper by Sauer[11], it was suggested to approximate complex chord length distributions by simple analytic functions, which lead again to simple, but rather accurate analytic expressions for the collision probabilities. Sauer uses the following ansatz for the (three dimensional) chord length distribution in coolant channel regions of pin lattices

$$f(l) = \begin{cases} 0 & l < \bar{l}\,\tau \\ Ce^{-l/\bar{l}} & l > \bar{l}\,\tau \end{cases} \tag{6}$$

where $\bar{l}$ is the mean chord length, C is a normalization constant, and τ is a dimensionless free parameter. Clearly, very small chord lengths do not occur. Therefore, $f(l) = 0$ for small l. Sauer obtained the best fit if he required that the logarithmic moment of the distribution, $\overline{\ln l/\bar{l}}$, was adjusted to values obtained by numerical (Monte Carlo) calculations.

Sauer's argument leads, for a hexagonal lattice, to the following approximate expression for the free parameter τ

$$\tau = \frac{0.9523\sqrt{1 + \dfrac{V_c}{V_u}} - 1}{V_c / V_u} - 0.12 \tag{7}$$

where V_c / V_u is the volume ratio "coolant/fuel pin". This approximation (Fig. 2) is used in the KfK cell code KAPER, and gives good results for sodium-filled lattices.

Note that, according to Sauer[11], the logarithmic moment of the chord length distribution is connected with τ by the equation

$$\overline{\ln l/\bar{l}} = \ln \tau + \exp\left(\frac{\tau}{1-\tau}\right) E_1\left(\frac{\tau}{1-\tau}\right) \tag{8}$$

where E_1 is the exponential integral.

In the present work, an approximation is desired for the projected chord length distribution rather than for the three-dimensional chord length distribution as used by Sauer. It should include the asymptotic term, so that it can be used for voided lattices, too. The following approximate distribution function is used

$$g(\rho) = \begin{cases} 0 & \rho < t \\ \dfrac{A}{\rho^3} + \left(1 - \dfrac{A}{2t^2}\right) \dfrac{1}{\lambda} e^{-(\rho - t)/\lambda} & \rho > t \end{cases} \tag{9}$$

The distribution (9) is normalized; λ follows from the condition that the mean ρ has the correct value

$$\bar{\rho} = \frac{\pi}{4} \bar{l}$$

where $\bar{l}$ is the mean chord length in the coolant channel. This condition can be written

$$\frac{\lambda}{\bar{l}} = \frac{\dfrac{\pi}{4} - \dfrac{A}{\bar{l}\, t}}{1 - \dfrac{A}{2t^2}} - \frac{t}{\bar{l}} \tag{10}$$

The method described by Sauer is then used to determine another dimensionless parameter $\tilde{\tau} = t/\bar{l}$, which is characteristic of the projected chord length distribution. One finds easily that the logarithmic moment is connected with the parameter $\tilde{\tau}$ through the relation

$$\overline{\ln l/\bar{l}} = \ln \tilde{\tau} + \left(1 - \frac{A}{2\tilde{\tau}^2 \bar{l}^2}\right) \exp\left(\frac{\tilde{\tau}\, \bar{l}}{\lambda}\right) E_1\left(\frac{\tilde{\tau}\, \bar{l}}{\lambda}\right) + \frac{A}{4\tilde{\tau}^2 \bar{l}^2} + \ln 2 - \frac{1}{2} \tag{11}$$

Then, assuming that Sauer's parameter τ is given as a function of the volume ratio V_c/V_u by Sauer's approximation (7), one obtains the logartihmic moment from eq. (8), and the parameter $\tilde{\tau}$ from eq. (11). This procedure gives $\tilde{\tau}$, as shown by the solid

line in Fig. 3. For easy use in the code, this function is approximated by the following analytic fit

$$\tilde{\tau} = \frac{t}{\bar{l}} = \frac{0.9523\sqrt{1 + \frac{V_c}{2V_u}} - 1}{\frac{V_c}{2V_u}} - \frac{0.03}{0.2 + \frac{V_c}{V_u}} \qquad (12)$$

The circles in Fig. 3 show the fit by this equation. The KfK method (computer routine ARIADNE) consists of using the approximate distribution (9) to simplify Benoist's formalism. This will be outlined in the following Section. Note that t in eq. (9) is given by eq. (12).

3.2 Outline of the Basic Benoist Approach[2]

The equations for cell homogenisation are generally derived from the neutron transport equation, which reads in the case of one energy group, with isotropic scattering

$$(\vec{\Omega}\nabla + \Sigma_t) F(\vec{r}, \vec{\Omega}) = \frac{\Sigma_s}{4\pi} \int F(\vec{r}, \vec{\Omega}) d\vec{\Omega} + \frac{1}{4\pi} S \qquad (13)$$

where F is the angular flux, and S is an external source.

Then, one assumes that the flux can be factored as follows

$$F = f \exp(i\vec{B}\vec{r}) \qquad 14$$

where f has the periodicity of the lattice, and the global flux is described by the exponential term. One obtains a system of two coupled equations

$$\begin{aligned} (\vec{\Omega}\nabla + \Sigma_t) g - (\vec{\Omega}\vec{B}) h &= \frac{\Sigma_s}{4\pi} \int g d\vec{\Omega} + \frac{1}{4\pi} S \\ (\vec{\Omega}\nabla + \Sigma_t) h + (\vec{\Omega}\vec{B}) g &= \frac{\Sigma_s}{4\pi} \int h d\vec{\Omega} \end{aligned} \qquad (15)$$

where g and h are the real and imaginary part of the periodic cell flux f.

Starting from these equations, one can work out different definitions for the cell diffusion coefficients. Benoist[2] uses the simplest definition, which is usually termed "Benoist uncorrected" in the literature[2,4]. It is given by

$$\sum_k D_k B_k^2 = - \frac{\int dV \int d\vec{\Omega} h \sum_k (\Omega_k B_k)}{\int dV \int d\vec{\Omega} g} \tag{16}$$

where the volume integral is over the unit cell, and the integral $d\Omega$ over the full solid angle.

Benoist evaluates eq. (16) in an approximation where the B^2-dependence is retained only to the lowest order necessary, and the small "angular correlation" terms are neglected[2]. This way, he finds that the expression (1) for the cell diffusion coefficient can be generalized to include the buckling dependence in first approximation, by including non-leakage factors in the directional collision probabilities; i.e. replacing P_{ijk} by the expressions

$$P_{ijk}(B) = \int_{V_j} dV \int_{V_i} dV' P_k(\vec{r}', \vec{r}) \cos(\vec{B}\,\vec{R}) \tag{17}$$

where $\vec{R} = \vec{r}' - \vec{r}$. For a two-region cell, the diffusion coefficients are then

$$D_k = \frac{1}{3\Sigma_u} \left\{ 1 + \frac{V_c}{V_u + V_c} \left(1 - \frac{\Sigma_c}{\Sigma_u} \right) \left[1 + \frac{l_c \Sigma_u}{2} \left(1 - \frac{\Sigma_c}{\Sigma_u} \right) \left(Q_k + Q'_k \right) \right] \right\} \tag{18}$$

where Q_k and Q'_k are related to the reduced collision probabilities

$$Q_k + Q'_k = 2 P_{cck} / l_c \Sigma_c \tag{19}$$

P_{cc} is the probability of a neutron born in the coolant channel region to make its next collision in the coolant channel. The P_{cck} are the corresponding directional collision probabilities. Q_k refers to neutrons which never leave the coolant channel region, while Q'_k accounts for those neutrons which traverse at least one fuel pin. Note that the reduced collision probabilities (i.e. also Q_k and Q'_k) are non-zero even in the void case where P_{cc} is zero.

3.3 Evaluation of the Quantities Q_z and Q_r

According to Benoist, Q_z and Q_r are defined by the following four-fold integrals

$$Q_z(\Sigma_c, B_z, B_r) = \frac{24}{\pi l_c^2} \int_0^{\pi/2} d\theta \cos^2\theta \int_0^{\infty} d\rho\, \rho^2 g(\rho) \int_0^1 \eta\, d\eta \int_0^1 d\chi \exp$$

$$\left| -\chi\eta\left(\Sigma_c + iB_z\cos\theta\right)\rho/\sin\theta \right| J_0(\chi\eta B_r \rho) \qquad (20)$$

and

$$Q_r(\Sigma_c, B_z, B_r) = \frac{12}{\pi l_c^2} \int_0^{\pi/2} d\theta \sin^2\theta \int_0^{\infty} d\rho\, \rho^2 g(\rho) \int_0^1 \eta\, d\eta \int_0^1 d\chi \exp$$

$$\left| -\chi\eta\left(\Sigma_c + iB_z\cos\theta\right)\rho/\sin\theta \right| \left(J_0(\chi\eta B_r \rho) - J_2(\chi\eta B_r \rho)\right) \qquad (21)$$

where J_0 and J_2 are Bessel functions.

We first evaluate the contributions due to the first (asymptotic) term of the distribution function (9). These contributions are termed $Q_z^{(1)}$, $Q_r^{(1)}$. The integrals over ρ have the form

$$W_n^{(1)} = \int_t^{\infty} \frac{d\rho}{\rho} \exp\left| -\chi\eta\left(\frac{\Sigma_c}{\sin\theta} + iB_z \operatorname{ctg}\theta\right)\rho \right| J_n(\chi\eta B_r \rho) \qquad (22)$$

where $n = 0$ or 2. Introducing the variables $x = \chi\eta\Sigma_c\rho$, $x_0 = \chi\eta\Sigma_c t$, $a_r = B_r/\Sigma_c$, $a_z = B_z/\Sigma_c$, $\Gamma = 1/\sin\theta + ia_z \operatorname{ctg}\theta$, one has

$$W_0^{(1)} = \int_{x_0}^{\infty} \frac{dx}{x} e^{-\Gamma x} J_o(a_r x)$$

This can be written

$$W_0^{(1)} = \int_0^{\infty} \frac{dx}{x} e^{-\Gamma x}\left(J_0(a_r x) - e^{-a_r a x}\right) + E_1((\Gamma + a a_r)x_o) - \int_0^{x_0} \frac{dx}{x} e^{-\Gamma x}\left(J_0(a_r x) - e^{-a_r a x}\right) \qquad (23)$$

where α is a free parameter. The first integral on the right side of eq. (23), which we call $W_{01}^{(1)}$, can be evaluated in closed form

$$W_{01}^{(1)} = \ln\left[\frac{2(\Gamma + \alpha a_r)}{\sqrt{\Gamma^2 + a_r^2} + \Gamma}\right] \tag{24}$$

We notice that the exponential integral E_1 has the expansion

$$E_1(z) = -\ln z - \gamma + \Delta E_1(z) \tag{25}$$

where γ is Euler's constant. Thus

$$W_0^{(1)} = \ln\frac{2}{\sqrt{\Gamma^2 + a_r^2} + \Gamma} - \ln x_0 - \gamma + \Delta E_1\left(\frac{x_0}{\sin\theta}\right) \tag{26}$$

where ΔE_1 is a small correction of order x_0; therefore B was set equal to zero in this term.

Inserting into (20), and carrying out the integrals over χ and η, one obtains

$$Q_z^{(1)}(\Sigma_c, B_z, B_r) = \frac{12A}{\pi l_c^2}\left\{\frac{\pi}{4}(1-\gamma) - \int_0^{\pi/2} d\theta \cos^2\theta \ln\left|t\left|\Sigma_c + iB_z\cos\theta + \sqrt{(\Sigma_c + iB_z\cos\theta)^2 + B_r^2\sin^2\theta}\right|\right| + 2\int_0^1 \eta d\eta \int_0^1 d\xi \int_0^{\pi/2} d\theta \cos^2\theta\, \Delta E_1\left(\frac{x_0}{\sin\theta}\right)\right\} \tag{27}$$

To evaluate the last term (the integral over the function ΔE_1) we observe that the integral over the E_1 function can be written as follows

$$\int_0^{\pi/2} d\theta \cos^2\theta\, E_1\left(\frac{x_0}{\sin\theta}\right) = \int_{x_0}^{\infty} \frac{du}{u}\left(Ki_1(u) - Ki_3(u)\right) \tag{28}$$

where the Ki_n are the Bickley functions, or Bessel function integrals[12], of index n.

In a fast reactor, the coolant channels are always optically thin; thus, $x_0 \ll 1$. Therefore, we want to find a series expansion in powers of x_0. Using the series expansions of the Bessel function integrals Ki_n, we obtain

$$\int_{x_0}^{\infty} \frac{du}{u}\left(Ki_1(u) - Ki_3(u)\right) = Con_1 + \frac{\pi}{4}\ln\frac{1}{x_0} + \left(\psi(2) + \ln\frac{2}{x_0}\right)x_0$$

$$+ \frac{\pi}{8}x_0^2 - \left(\psi(4) + \frac{1}{2} + \frac{1}{3} + \ln\frac{2}{x_0}\right)\frac{x_0^3}{36} \qquad (29)$$

where the constant Con_1 is

$$Con_1 = \int_1^{\infty} \frac{du}{u}\left(Ki_1(u) - Ki_3(u)\right) + \int_0^1 \frac{du}{u}\left(Ki_1(u) - Ki_3(u) - \frac{\pi}{4}\right) \qquad (30)$$

The constant can be evaluated by a very simple argument. The integral over ΔE_1 is given, according to eq. (25), by

$$\int_0^{\pi/2} d\theta \cos^2\theta\, \Delta E_1\left(\frac{x_0}{\sin\theta}\right) = \int_0^{\pi/2} d\theta \cos^2\theta\, E_1\left(\frac{x_0}{\sin\theta}\right) + \frac{\pi}{4}\ln x_0 + \frac{\pi}{4}\left(\gamma + \ln 2 + \frac{1}{2}\right) \qquad (31)$$

As the left hand side is of order of x_0, its expansion has a constant term which is zero. Therefore by comparison

$$Con_1 = -\frac{\pi}{4}\left(\gamma + \ln 2 + \frac{1}{2}\right) \qquad (32)$$

The integrals over η and ξ in the last term of eq. (27) are then trivial, leading to the following contribution to $Q_z^{(1)}$

$$Q_z^{(1)}(\text{contribution of } \Delta E_1) = \frac{24A}{\pi l_c^2}\left[\left(\psi(4) + \ln\frac{2}{p}\right)\frac{p}{6} + \frac{\pi}{8}\frac{p^2}{12}\right.$$

$$\left. - \left(\psi(6) + \frac{1}{2} + \frac{1}{3}\ln\frac{2}{p}\right)\frac{p^3}{720}\right] \qquad (33)$$

where $p = \Sigma_c t$.

The equations (27) and (33) represent $Q_z^{(1)}$ in terms of a single integral over θ, plus a fast-converging series expansion. Eq. (27) shows clearly that the buckling dependence becomes important when $\Sigma_c \lesssim B_z$, and/or $\Sigma_c \lesssim B_r$. Besides, it is clear that the dependence on the two bucklings, B_r and B_z, has a different form.

To evaluate the corresponding radial quantitiy $Q_r^{(1)}$, as defined by the eq. (21), we need, in addition to $W_0^{(1)}$, the quantity $W_2^{(1)}$, which according to eq. (22) is given by

$$W_2^{(1)} = \int_{x_0}^{\infty} \frac{dx}{x} e^{-\Gamma x} J_2(a_r x)$$

$$= \int_0^{\infty} \frac{dx}{x} e^{-\Gamma x} J_2(a_r x) - \int_0^{x_0} \frac{dx}{x} e^{-\Gamma x} J_2(a_r x) \qquad (34)$$

In this equation, the second term on the right side is of higher order in B_r, and can be neglected.

The first integral, from zero to infinity, can be evaluated in closed form

$$W_2^{(1)} = \frac{\left(\sqrt{\Gamma^2 + a_r^2} - \Gamma\right)^2}{2 a_r^2} \qquad (35)$$

Then, inserting the expressions (23) and (35) into eq. (21), and carrying out the integrals over η and ξ, one obtains for the radial term $Q_r^{(1)}$

$$Q_r^{(1)}(\Sigma_c, B_z, B_r) = \frac{6A}{\pi l_c^2} \left\{ \frac{\pi}{4}(2-\gamma) - \int_0^{\pi/2} d\theta \sin^2\theta \ln\left| t\left|\Sigma_c + iB_z\cos\theta + \sqrt{(\Sigma_c + iB_z\cos\theta)^2 + B_r^2\sin^2\theta}\right|\right|\right.$$

$$- \frac{B_r^2}{2} \mathrm{Re} \int_0^{\pi/2} \frac{d\theta \sin^4\theta}{2(\Sigma_c + iB_z\cos\theta)^2 + B_r^2\sin^2\theta + 2(\Sigma_c + iB_z\cos\theta)\sqrt{(\Sigma_c + iB_z\cos\theta)^2 + B_r^2\sin^2\theta}}$$

$$\left. + 2\int_0^1 \eta\, d\eta \int_0^1 d\xi \int_0^{\pi/2} d\theta \sin^2\theta\, \Delta E_1\left(\frac{x_0}{\sin\theta}\right) \right\} \qquad (36)$$

The last term in the integral, which involves ΔE_1, can be evaluated in a similar way as for the axial case. One has

$$\int_0^{\pi/2} d\theta \sin^2\theta\, E_1\left(\frac{x_0}{\sin\theta}\right) = \int_{x_0}^{\infty} \frac{du}{u} Ki_3(u) \qquad (37)$$

and, proceeding to a series expansion

$$\int_{x_0}^{\infty} \frac{du}{u} \mathrm{Ki}_3(u) = \frac{\pi}{4} \ln \frac{1}{x_0} + \mathrm{Con}_2 + x_0 - \frac{\pi}{8} x_0^2 + \left(\psi(4) + \frac{1}{3} + \ln \frac{2}{x_0} \right) \frac{x_0^3}{18} \qquad (38)$$

where the constant Con_2 is

$$\mathrm{Con}_2 = \int_1^{\infty} \frac{du}{u} \mathrm{Ki}_3(u) + \int_0^1 \frac{du}{u} \left(\mathrm{Ki}_3(u) - \frac{\pi}{4} \right) \qquad (39)$$

Again, the constant can be found from the requirement that the integral over ΔE_1 is of order x_0; then

$$\mathrm{Con}_2 = -\frac{\pi}{4} \left(\ln 2 + \gamma - \frac{1}{2} \right) \qquad (40)$$

The contribution of the last term in eq. (36) is then

$$Q_r^{(1)}(\text{contribution of } \Delta E_1) = \frac{12A}{\pi l_c^2} \left[\frac{p}{6} - \frac{\pi}{8} \frac{p^2}{12} + \left(\psi(6) + \frac{1}{3} + \ln \frac{2}{p} \right) \frac{p^3}{360} \right] \qquad (41)$$

This completes the evaluation of the terms $Q_z^{(1)}$ and $Q_r^{(1)}$, which are now represented by single integrals over θ, plus fast-converging series expansions in the parameter $p = \Sigma_c t$. The principle to keep the B^2 dependence only in the lowest order necessary was followed, but no other approximations were made. Note that the approximate splitting in two terms, $Q(\Sigma_c, B) = Q(\Sigma_c, B=0) + \delta Q(\Sigma_c=0, B)$, as suggested by Benoist[2], was not used. A few remarks on further simplication will be made later. In the following, the contributions of the second (exponential) term in the distribution of the projected chord lengths, eq. (9), will be evaluated. It is sufficient to evaluate these terms for zero buckling because they are finite in this case.

Inserting the exponential term of the distribution function g (ρ) into eqs. (20) and (21), one finds after some manipulations

$$Q_z^{(2)}(\Sigma_c, 0) = \frac{24\, e^q}{\pi l_c^2 \Sigma_c^3 \lambda} \left(1 - \frac{A}{2t^2} \right) \int_p^{\infty} du\, e^{-u/\Sigma_c \lambda} \left[\mathrm{Ki}_3(u) - \mathrm{Ki}_5(u) - \frac{\pi}{16} + \frac{u}{3} \right] \qquad (42)$$

and

$$Q_r^{(2)}(\Sigma_c,0) = \frac{12\,e^q}{\pi\, l_c^2 \Sigma_c^3 \lambda}\left(1 - \frac{A}{2\,t^2}\right)\int_p^\infty du\, e^{-u/\Sigma_c\lambda}\left[Ki_5(u) - \frac{3\,\pi}{16} + \frac{2}{3}\,u\right] \qquad (43)$$

where $q = t/\lambda$.

If p is small, which is always the case for a fast reactor pin lattice, the following power series expansions are useful

$$Q_z^{(2)}(\Sigma_c,0) = \frac{24\,\lambda^2}{\pi\, l_c^2}\left(1 - \frac{A}{2\,t^2}\right)\left[\frac{\pi}{8}\,\phi_2(q) - \left(\psi(4) - 1 - \ln\frac{p}{2q}\right)\phi_3(q)\,\frac{p}{6\,q}\right.$$

$$+ A_3(q)\,\frac{p}{6q} - \frac{\pi}{48}\,\phi_4(q)\,\frac{p^2}{q^2} + \left(\psi(6) + \frac{1}{2} - \ln\frac{p}{2q}\right)\phi_5(q)\,\frac{p^3}{240\,q^3}$$

$$\left. - A_5(q)\,\frac{p^3}{240\,q^3}\right] \qquad (44)$$

$$Q_r^{(2)}(\Sigma_c,0) = \frac{12\,\lambda^2}{\pi\, l_c^2}\left(1 - \frac{A}{2\,t^2}\right)\left[\frac{\pi}{8}\,\phi_2(q) - \phi_3(q)\,\frac{p}{6\,q} + \frac{\pi}{48}\,\phi_4(q)\,\frac{p^2}{q^2}\right.$$

$$\left. - \left(\psi(6) - \ln\frac{p}{2q}\right)\phi_5(q)\,\frac{p^3}{120\,q^3} + A_5(q)\,\frac{p^3}{120\,q^3}\right] \qquad (45)$$

where

$$\phi_n(q) = q^n + nq^{n-1} + n(n-1)\,q^{n-2} + \ldots + n(n-1)\ldots 3\,q + n\,(n-1)\ldots 2.1$$

and

$$A_3(q) = \phi_3(q)\ln q + (q^2 + 5\,q + 11) + 6\,e^q E_1(q)$$

$$A_5(q) = \phi_5(q)\ln q + (q^4 + 9\,q^3 + 47\,q^2 + 154\,q + 274) + 120\,e^q E_1(q)$$

These expansions are used in the computer routine ARIADNE.

We now discuss further simplifications of the terms $Q_z^{(1)}$ and $Q_r^{(1)}$, and quote the results for the important case where the coolant channels are voided.

3.4 Limiting Cases

The expression for $Q_z^{(1)}$ can be further simplified if $B_r = 0$; i.e. if the (small) influence of the radial buckling on the axial leakage is neglected. In that case, the remaining θ integral in eq. (27) can be carried out, and one has

$$Q_z^{(1)}(\Sigma_c, B_z, B_r = 0) = \frac{24\,A}{\pi l_c^2}\left\{\frac{\pi}{8}\left(\frac{1}{2} - \gamma + \ln\frac{1}{p} - \ln\left(1 + \sqrt{1+\alpha_z^2}\right) - \frac{\sqrt{1+\alpha_z^2} - 1}{\alpha_z^2}\right)\right.$$

$$\left. + \left(\psi(4) + \ln\frac{2}{p}\right)\frac{p}{6} + \frac{\pi}{8}\frac{p^2}{12} - \left(\psi(6) + \frac{1}{2} + \frac{1}{3} + \ln\frac{2}{p}\right)\frac{p^3}{720}\right\} \qquad (46)$$

This expression holds for any value of the ratio $\alpha_z = B_z / \Sigma_c$. Especially, it has a finite value in the void case, $\Sigma_c = 0$. In the latter case, the total Q_z reads

$$Q_z(\Sigma_c = 0, B_z, B_r = 0) = Q_z^{(1)} + Q_z^{(2)}$$

$$= \frac{3\,A}{l_c^2}\left|\ln\frac{1}{B_z t} + \frac{1}{2} - \gamma\right| + \frac{3\,\lambda^2}{l_c^2}\left(1 - \frac{A}{2t^3}\right)(q^2 + 2q + 2) \qquad (47)$$

The asymptotic part (in the first bracket) is nearly the same as the one given by Köhler and Ligou, except that ε (Köhler and Ligou) is replaced by t.

The similar case, where $Q_r^{(1)}$ is to be calculated neglecting the small influence of the axial buckling, i.e. setting $B_z = 0$; is not as simple. The two integrals over θ, which appear in eq. (36), read in this case

$$\int_0^{\pi/2} d\theta \sin^2\theta \left|\ln p + \ln\left(1 + \sqrt{\alpha_r^2 \sin^2\theta}\right)\right| \qquad (48)$$

and

$$\frac{\alpha_r^2}{2}\int_0^{\pi/2} \frac{d\theta \sin^4\theta}{2 + \alpha_r^2\sin^2\theta + 2\sqrt{1+\alpha_r^2\sin\theta}} = \frac{1}{2\alpha_r^2}\int_0^{\pi/2} d\theta\left(\sqrt{1+\alpha_r^2\sin^2\theta} - 1\right)^2 \qquad (49)$$

Both integrals can only be evaluated in closed form in the limits $\alpha_r \to \infty$ (void case) and $\alpha_r = 0$ (buckling dependence negligible).
In the first case (void case) the integrals are, respectively

$$\frac{\pi}{4}\left(\frac{1}{2} + \ln\frac{\alpha_r}{2}\right) \quad \text{and} \quad \frac{\pi}{8}$$

Thus, Q_r is in the void case

$$Q_r(\Sigma_c = 0, B_z = 0, B_r) = \frac{3A}{2l_c^2}\left(\ln\frac{2}{B_r t} + 1 - \gamma\right) + \frac{3\lambda^2}{2l_c^2}\left(1 - \frac{A}{2t^2}\right)(q^2 + 2q + 2) \qquad (50)$$

In the opposite case, $\alpha_r = 0$, the two integrals are

$$\frac{\pi}{4}\ln(2p) \quad \text{and} \quad 0$$

respectively. To arrive at a "working approximation", the following procedure is used.

First, find $Q_r^{(1)}$ for $B_r = 0$, assuming $\Sigma_c \neq 0$ (though it may be very small). Then, the correction for $B_r \neq 0$, as obtained in the limit of small Σ_c, is proportional to

$$\ln\frac{4\Sigma_c}{B_r} - 1 \qquad (51)$$

As $Q_r^{(1)}$ can only decrease if B_r is considered, this correction is applied only if it is negative, i.e. if

$$\frac{\Sigma_c}{B_r} < \frac{e}{4}$$

Eq. (46) for $Q_z^{(1)}$, and the "working approximation" for $Q_r^{(1)}$ are used in the standard ARIADNE routine. They give results which are very close to those of the equations (27) and (33).

3.5 Calculation of Q'_k

The quantity Q'_k is the reduced collision probability for neutrons which traverse one or more fuel pins, and then collide in a coolant channel. A simple way to calculate Q'_k is to represent the total collision probability as a product of single-zone collision probability, and probabilities to traverse a zone. Then

$$Q' = (1 - P_{cc}) \sum_{n=1}^{\infty} (1 - \Gamma_u)^n (1 - \Gamma_c)^{n-1} \frac{2\Gamma_c}{l_c \Sigma_c} \qquad (52)$$

where the index n accounts for neutrons which traverse n fuel pins. Γ_u and Γ_c are the probabilities that a neutron entering medium u or c makes a collision in this medium; $1 - P_{cc}$ is the escape probability from the coolant.

Eq. (52) holds for the isotropic case. It involves the assumption that the neutrons entering a medium (u or c) have an isotropic angular distribution. To calculate the anisotropic Q'_z, one has to assume that the source neutrons have an angular distribution proportional to $\cos^2\theta$. Then, in eq. (52), also the Γ_u and Γ_c must be calculated for neutrons entering with an angular distribution $\sim \cos^2\theta$. This is, however, not a good approximation because neutrons with small θ decay much faster, after traversing a few pins, than neutrons with $\theta = \pi/2$ (i.e. in the horizontal plane). Then, the angular distribution is no longer $\sim \cos^2\theta$.

A significantly improved method was proposed by Benoist in the Appendix 1 of his paper[2]. One assumes isotropy of the incident neutrons only for the azimuthal angle Φ (i.e. in the x-y plane), whereas the integration over the elevation angle θ is carried out explicitly. With these assumptions, one has

$$Q'_k = \frac{3\pi}{2} \int_0^{\pi/2} d\theta\, \Omega_k^2 \, \frac{(1 - \omega^*_{cc})^2 \omega^{(u)}_{ss}}{1 - \left[1 - \frac{\pi \Sigma_c l_c}{4 \sin\theta} (1 - \omega^*_{cc})\right] \omega^{(u)}_{ss}} \tag{53}$$

$\omega^{(u)}_{ss}$ is the transmission probability through a fuel pin in the x-y plane, averaged over the azimuth angle Φ. It is a function of the quantity $z = 2a\,\Sigma_u/\sin\theta$, and is defined by[2]

$$\omega^{(u)}_{ss}(\sin\theta) = \int_0^{\pi/2} d\phi \cos\phi \exp\left(-\frac{2a\,\Sigma_u \cos\phi}{\sin\theta}\right) \tag{54}$$

The collision probability in the coolant channel in the x-y plane, for a uniform and isotropic source is

$$\omega^*_{cc}(v) = \frac{4A}{\pi v l_c t}\left[-\frac{1}{2} + v + E_3(v)\right]$$

$$+ \frac{4t}{\pi l_c v}\left(1 - \frac{A}{2t^2}\right)\left[\frac{v}{q}(1+q) - 1 + \frac{e^{-v/q}}{1 + v/q}\right] \tag{55}$$

where the variable v is defined as

$$v = \frac{\Sigma_c t}{\sin\theta} = \frac{p}{\sin\theta}$$

Note that ω^*_{cc} is independent of the buckling, because for calculating Q'_k the buckling dependence is neglected.

4. Comparison of Results

Table I shows a comparison of ARIADNE and Monte Carlo[8] one-group calculations for the Clinch River Breeder Reactor and for a Gas Cooled Fast Reactor. The results for the axial leakage are in very good agreement, whereas the ARIADNE method underestimates somewhat the radial leakage; the largest deviation is of the order of two standard deviations. The overall results can be considered satisfactory. Note that ARIADNE should not be applied to high-leakage cores (e.g. EBR-II, where $k_\infty = 2.009$) because the B^2-dependence is retained only to a low order.

To compare ARIADNE and the cylindrical cell approximation in KAPER with MOBIDIC, calculations for a Gas Breeder cell, but with variable Σ_c were performed, and are shown in Tables II and III. The code MOBIDIC is based on Benoist's theory[2]. The results quoted in Tables II and III are taken from a publication by Benoist and Duracz[9]. The geometry data are a = 0.37 cm, s = 1.08 cm, and the bucklings $B_z^2 = 1 \times 10^{-4}$ cm^{-2}, $B_r^2 = 2 \times 10^{-4}$ cm^{-2}. For large coolant cross sections, the streaming corrections are small, and the results of KAPER and ARIADNE are comparable. In the void case, there is a fairly large underestimate by KAPER. It is interesting to see that MOBIDIC gives larger streaming corrections than ARIADNE. This is probably due to an overestimte of MOBIDIC in the cases where the fuel cross section is small, as is typical for fast reactors. This point will be discussed below, see comments on Table IV. The B^2 dependence is shown in the Figures 4 and 5 for the case $\Sigma_u = 0.59657$ cm^{-1}. In the void case, the streaming corrections increase with decreasing bucklings.

Fig. 6 shows a comparison with some results available in the literature. The axial streaming correction for a gas cooled fast reactor lattice is shown. The lattice is voided, the variable homogeneous diffusion coefficients on the abscissa correspond to variable fuel cross sections. DIFFAX can be considered as a reference because it uses numerical integration. The ARIADNE method is rather close to DIFFAX except for small diffusion coefficients, where ARIADNE underestimates the effect. This range corresponds to the large fuel cross sections at thermal energies and is not relevant for applications to fast reactors. It is obvious that the "cylindrical cell" method, which is used in KAPER, strongly underestimates the streaming correction. The overestimate by MOBIDIC, as published by Benoist and Duracz[9], may be due to an inaccurate method to calculate Q'_z (see above).

This latter statement can be understood better when looking at Table IV, which shows the streaming correction for different fuel cross sections. For the large Σ_u (thermal neutrons), very few neutrons traverse a pin. Thus, Q'_k is small compared to Q_k. MOBIDIC agrees well with DIFFAX, but the simple ARIADNE method underestimates the streaming correction (D/D_{hom} - 1) by about 10 %. This must be due to an underestimate of Q_k, because Q'_k is small and has little influence on the results. For the smaller Σ_u, which are the cases of interest for a fast reactor, the streaming correction is smaller. In these cases, Q_k remains the same, but Q'_k becomes important. It is interesting to see that ARIADNE compares well with DIFFAX. On the other hand, MOBIDIC overestimates the effect, obviously because it calculates Q'_k too high.

5. Calculation of the Streaming Reactivity in the SNR-300

The streaming reactivity in the voided lattice of SNR-300 was calculated by D. Thiem, using a procedure for neutronics calculations which includes cross section preparation, cell homogenization, and whole-core diffusion calculations. The cell diffusion coefficients were obtained with ARIADNE. The procedure and the results are described and discussed in some detail in Ref.[13].

Appendix: Extension of ARIADNE to the Case of a Cell with Three Regions

The original computer program ARIADNE uses two regions, namely the fuel pin and the coolant channel. Thus, the fuel and the clad must be smeared over the pin volume. This approximation is adequate for many cases because the clad is usually thin, and the transport cross sections of oxide fuel and the clad are not too much different. It is, however, desirable to have the capability for a separate treatment of the fuel and the clad. Therefore, the program ARIADNE3Z for 3 zones (u = uranium, cl = clad, c = coolant) was developed. The extension was carried out along the lines suggested by Benoist in the Appendix of his papers[2].

For this extension, the assumption is made that neutrons which cross the pin boundary, either entering the coolant channel or the pin, have isotropic angular distribution in the x-y plane. The distribution in the azimuth angle θ is calculated explicitly. This is more accurate than the usual assumption of complete isotropy in 3 dimensions (as used e.g. in KAPER), and is an important improvement when directional collision probabilities (and diffusion coefficients) are required. Note that no assumption about the angular distribution at the u-cl surface is made.

To follow up this idea, one introduces 2-dimensional collision probabilities ω_{ij} in the x-y-plane by the definition

$$P_{ij,k} = \int 3\Omega_k^2 \, d\theta \sin\theta \, \omega_{ij}(\theta) \qquad \text{(A 1)}$$

and

$$\omega_{ij} = \frac{\Sigma_j'}{V_i} \int_{V_i} dr' \int_{V_j} dr \, \frac{\exp(-\Sigma'\rho)}{2\pi\rho} \qquad \text{(A 2)}$$

where ρ is the distance in the x-y plane, and

$$\Sigma_j' = \frac{\Sigma_j}{\sin\theta} \qquad \text{(A 3)}$$

Inserting (A 2) into (A 1) leads immediately to the usual definition of $P_{ij,k}$. Note that the $P_{ij,k}$, or at least some of them, depend on the bucklings.

They are the quantities needed in Benoist's expression for the diffusion coefficients[2, 3]

$$D_k = \frac{\sum_i V_i \sum_j P_{ij,k}/\Sigma_j}{3\,V_t} \qquad \text{(A 4)}$$

For the ω_{ij}, both the reciprocity relations

$$V_i \Sigma_i' \omega_{ij} = V_j \Sigma_j' \omega_{ji} \qquad \text{(A 5)}$$

and the completeness relations

$$\sum_j \omega_{ij} = 1$$

hold.

For a 3-region cell, the collision probabilities for neutrons starting in the coolant channel will be given first. With the above assumption, the ω_{cc} for the lattice can be approximated as follows

$$\omega_{cc} = \omega^*_{cc} + \omega_{cs}\,\omega^{(p)}_{ss}\,\tilde{\omega}_{sc} \qquad \text{(A 6)}$$

where

- ω^*_{cc} probability for a neutron to collide in c without leaving the region
- $\omega_{cs} = 1 - \omega^*_{cc}$ probability for a neutron to leave region c (s = surface)
- $\omega^{(p)}_{ss}$ probability for a neutron which enters the pin to traverse it without a collision
- $\tilde{\omega}_{sc}$ probability for a neutron which enters c to eventually collide in c (perhaps after traversing one or more pins)

The quantity $\tilde{\omega}_{sc}$ is determined by the relation

$$\tilde{\omega}_{sc} = \omega_{sc} + \omega^{(c)}_{ss}\,\omega^{(p)}_{ss}\,\tilde{\omega}_{sc} \qquad \text{(A 7)}$$

where

- ω_{sc} probability for a neutron which enters c to collide without leaving the region
- $\omega^{(c)}_{ss}$ probability for a neutron which enters c to traverse it without a collision

In the ARIADNE method, ω^*_{cc} is given by eq. (55). Note that the buckling dependence is neglected. For small values of the argument, the expression (55) is replaced by a series expansion.

In additon, we need the following probabilities

$\omega^{(p)}_{su}$ probability for a neutron which enters a pin to collide in u in the same pin

$\omega^{(p)}_{scl}$ probability for a neutron which enters a pin to collide in cl in the same pin.

Clearly

$$\omega^{(p)}_{su} + \omega^{(p)}_{scl} + \omega^{(p)}_{ss} = 1 \qquad \text{(A 8)}$$

These probabilities can be expressed by the integrals

$$\omega^{(p)}_{su} = \int_0^{\operatorname{atan} R_u/R} d\phi \cos\phi \exp(-\Sigma'_{cl}\rho_{cl})(1 - \exp(-\Sigma'_u \rho_u))$$

$$= \frac{1}{R}\int_0^{R_u} dh \exp(-\Sigma'_{cl}\rho_{cl})(1 - \exp(-\Sigma'_u \rho_u)) \qquad \text{(A 9)}$$

where R pin radius

R_u radius of region u

$$\rho_u = 2\sqrt{R_u^2 - h^2}$$

$$\rho_{cl} = \sqrt{R^2 - h^2} - \sqrt{R_u^2 - h^2}$$

and

$$\omega^{(p)}_{ss} = \frac{1}{R}\left[\int_0^{R_u} dh \exp\left(-2\Sigma'_{cl}\rho_{cl} - \Sigma'_u \rho_u\right) + \int_{R_u}^{R} dh \exp\left(-2\Sigma'_{cl}\sqrt{R^2 - h^2}\right)\right] \qquad \text{(A 10)}$$

The lattice collision probabilities for neutrons born in c are then

$$\omega_{cc} = \omega^{*}_{cc} + \frac{\omega_{cs}\,\omega^{(p)}_{ss}\,\omega_{sc}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 11}$$

$$\omega_{cu} = \frac{\omega_{cs}\,\omega^{(p)}_{su}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 12}$$

$$\omega_{c,cl} = \frac{\omega_{cs}\,\omega^{(p)}_{scl}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 13}$$

Similarly, one has for neutrons born in u

$$\omega_{uu} = \omega^{*}_{uu} + \frac{\omega_{us}\,\omega^{(c)}_{ss}\,\omega^{(p)}_{su}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 14}$$

$$\omega_{u,cl} = \omega^{*}_{u,cl} + \frac{\omega_{us}\,\omega^{(c)}_{ss}\,\omega^{(p)}_{scl}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 15}$$

$$\omega_{uc} = \frac{\omega_{us}\,\omega_{sc}}{1 - \omega^{(p)}_{ss}\,\omega^{(c)}_{ss}} \tag{A 16}$$

where

$$\omega^{*}_{uu} = 1 - \frac{2}{V_u \Sigma'_u} \int_0^{R_u} dh\,(1 - \exp(-\Sigma'_u \rho_u)) \tag{A 17}$$

$$\omega^{*}_{u,cl} = \frac{2}{V_u \Sigma'_u} \int_0^{R_u} dh\,(1 - \exp(-\Sigma'_u \rho_u))(1 - \exp(-\Sigma'_{cl} \rho_{cl})) \tag{A 18}$$

$$\omega_{us} = \frac{2}{V_u \Sigma'_u} \int_0^{R_u} dh\,(1 - \exp(-\Sigma'_u \rho_u)) \exp(-\Sigma'_{cl} \rho_{cl}) \tag{A 19}$$

The eq. (A 4) for the diffusion coefficient reads for a 3-region cell (V_t = cell volume)

$$3\,V_t\,D_k = V_1(P_{11k}/\Sigma_1 + P_{12k}/\Sigma_2) + 2\,V_2\,P_{31k}/\Sigma_1 + V_2(P_{21k}/\Sigma_1 + P_{22k}/\Sigma_2) + 2\,V_3\,P_{32k}/\Sigma_2 + V_3\,P_{33k}/\Sigma_3 \qquad \text{(A 20)}$$

As region 3 is the coolant channel, one has

$$P_{33k} = \frac{1}{2}\,l_c\,\Sigma_c\,(Q_k + Q'_k) \qquad \text{(A 21)}$$

where Q_k depends on the buckling; Q'_k is given by eq. (53), where the transmission probability $\omega^{(u)}_{ss}$ has to be replaced by $\omega^{(p)}_{ss}$. In the present model, the completeness relations hold approximately for the P_{ij}. Therefore, one has in a approximation which is consistent with the 2-region case

$$P_{31} = 1 - P_{32} - P_{33}$$

$$P_{11} = 1 - P_{12} - \frac{V_3\,\Sigma_3}{V_1\,\Sigma_1}\,P_{31}$$

$$P_{21} = \frac{V_1\,\Sigma_1}{V_2\,\Sigma_2}\,P_{12}$$

$$P_{22} = 1 - P_{21} - \frac{V_3\,\Sigma_3}{V_2\,\Sigma_2}\,P_{32}$$

Thus, the three collision probabilities which must be calculated explicitly are P_{33}, P_{32}, P_{12}.

For comparison between a 2-region and a 3-region model, calculations were carried out for the Gas Breeder cell with variable Σ_c, see Tables II and III. The fuel radius is 0.33 cm, the pin radius 0.37 cm. To obtain a significant difference, it was assumed arbitrarily that the transport cross section of the fuel is more than twice the one of the clad. The results are shown in Table V and Table VI. The diffusion coefficients calculated by the 3-region model are up to 2 % larger than the ones of the 2-region model. Note, however, that this is due to the unphysically large difference between the cross sections, chosen to demonstrate the method.

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Table I: Streaming-Reactivity (in % $\Delta k/k$) Calculated with Monte Carlo[7] and ARIADNE (1-Group, Zero-Dimension)

		CRBR	GCFR
B^2_{ax} (cm^{-2})		4.69422×10^{-4}	4.69422×10^{-4}
B^2_{rad} (cm^{-2})		4.9×10^{-4}	3.88607×10^{-4}
$1 + L^2_{hom}B^2$		1.2520	1.4425
$(\Delta k/k)_{ax}$	MC	0.326 ± 0.062	1.162 ± 0.113
	AR	0.280	1.119
$(\Delta k/k)_{rad}$	MC	0.217 ± 0.062	0.676 ± 0.083
	AR	0.138	0.500

Table II: Diffusion Coefficients for a Gas Breeder Lattice with Variable Σ_c. Comparisons KAPER-MOBIDIC-ARIADNE. $\Sigma_u = 0.59657$

			D_z / D_{hom}			D_r / D_{hom}		
	D_{hom}	KAPER[a]	MOBIDIC	ARIADNE	KAPER[a]	MOBIDIC	ARIADNE
$\Sigma_c =$ .4	.6891	1.0048		1.0050	1.0026		1.0028
.3	.7820	1.0126		1.0135	1.0067		1.0074
.2	.9037	1.0269		1.0299	1.0140		1.0163
.1	1.0703	1.0524	1.0798	1.0630	1.0265	1.0376	1.0338
.05	1.1790	1.0723		1.0952	1.0357		1.0504
.02	1.2555	1.0879		1.1303	1.0426		1.0685
.01	1.2832	1.0942		1.1490	1.0454		1.0798
.005	1.2976	1.1040		1.1602	1.0442		1.0822
0	1.3122	1.1070	1.2343	1.1697	1.0454	1.1089	1.0844

a) Cylindrical cell approximation

Table III: Diffusion Coefficients for a Gas Breeder Lattice with Variable Σ_c. Comparisons KAPER-MOBIDIC-ARIADNE. $\Sigma_u = 0.29828$

		D_z / D_{hom}			D_r / D_{hom}		
	D_{hom}	KAPER[a]	MOBIDIC	ARIADNE	KAPER[a]	MOBIDIC	ARIADNE
$\Sigma_c =$ .2	1.3783	1.0025		1.0027	1.0013		1.0015
.1	1.8074	1.0136	1.0218	1.0160	1.0069	1.0099	1.0088
.05	2.1407	1.0259		1.0332	1.0129		1.0182
.02	2.4070	1.0374		1.0540	1.0184		1.0296
.01	2.5111	1.0424		1.0655	1.0207		1.0366
.005	2.5666	1.0488		1.0723	1.0204		1.0387
0	2.6245	1.0516	1.1100	1.0789	1.0216	1.0469	1.0409

a) Cylindrical cell approximation

Table IV: Streaming correction with different codes (Gas Breeder cell, $\Sigma_c = 0$)

Σ_u	2.9828		0.59657		0.29828	
D_{hom}	0.26246		1.3123		2.6246	
	Axial	Radial	Axial	Radial	Axial	Radial
D/D_{hom} ARIADNE	2.070	1.507	1.170	1.084	1.079	1.091
MOBIDIC (Benoist and Duracz)	2.176	1.551	1.234	1.109	1.110	1.047
DIFFAX (Köhler and Ligou)	2.152		1.187		1.082	
TRIFFAX (Gho)	2.157	1.600				
Q_k (AR)	3.608	1.940	3.608	1.940	3.608	1.940
Q'_k (AR)	0.238	0.360	3.254	3.751	7.622	8.246

For large Σ_u (thermal neutrons), the term Q_k dominates. ARIADNE slightly underestimates the streaming (~ 10 %).
For small Σ_u (fast neutrons), both Q_k and Q'_k are important. ARIADNE agrees very well with DIFFAX.

Table V: Diffusion Coefficients for a Gas Breeder Lattice with Variable Σ_c. Two-region model ($\Sigma_u = \Sigma_{cl} = 0.59657$) and three-region model ($\Sigma_u = 0.67284$, $\Sigma_{cl} = 0.3$)

	D_{hom}	D_z/D_{hom}		D_r/D_{hom}	
		2 region	3 region	2 region	3 region
$\Sigma_c = 0.3$	0.7820	1.0135	1.0191	1.0074	1.0103
0.2	0.9037	1.0299	1.0379	1.0163	1.0205
0.1	1.0703	1.0630	1.0747	1.0338	1.0397
0.05	1.1790	1.0952	1.1098	1.0504	1.0575
0.02	1.2555	1.1303	1.1472	1.0685	1.0765
0.01	1.2832	1.1490	1.1670	1.0798	1.0882
0.005	1.2976	1.1602	1.1787	1.0822	1.0907
0.002	1.3064	1.1687	1.1853	1.0856	1.0921
0	1.3122	1.1697	1.1889	1.0844	1.0932

Table VI: Diffusion Coefficients for a Gas Breeder Lattice with Variable Σ_c. Two-region model ($\Sigma_u = \Sigma_{cl} = 0.29828$) and three-region model ($\Sigma_u = 0.36818$, $\Sigma_{cl} = 0.15$)

	D_{hom}	D_z/D_{hom}		D_r/D_{hom}	
		2 region	3 region	2 region	3 region
$\Sigma_c = 0.2$	1.3783	1.0027	1.0067	1.0015	1.0035
0.1	1.8074	1.0160	1.0244	1.0088	1.0132
0.05	2.1407	1.0332	1.0452	1.0182	1.0243
0.02	2.4070	1.0540	1.0691	1.0296	1.0371
0.01	2.5111	1.0655	1.0820	1.0366	1.0448
0.005	2.5666	1.0723	1.0897	1.0387	1.0471
0	2.6245	1.0789	1.0971	1.0409	1.0495

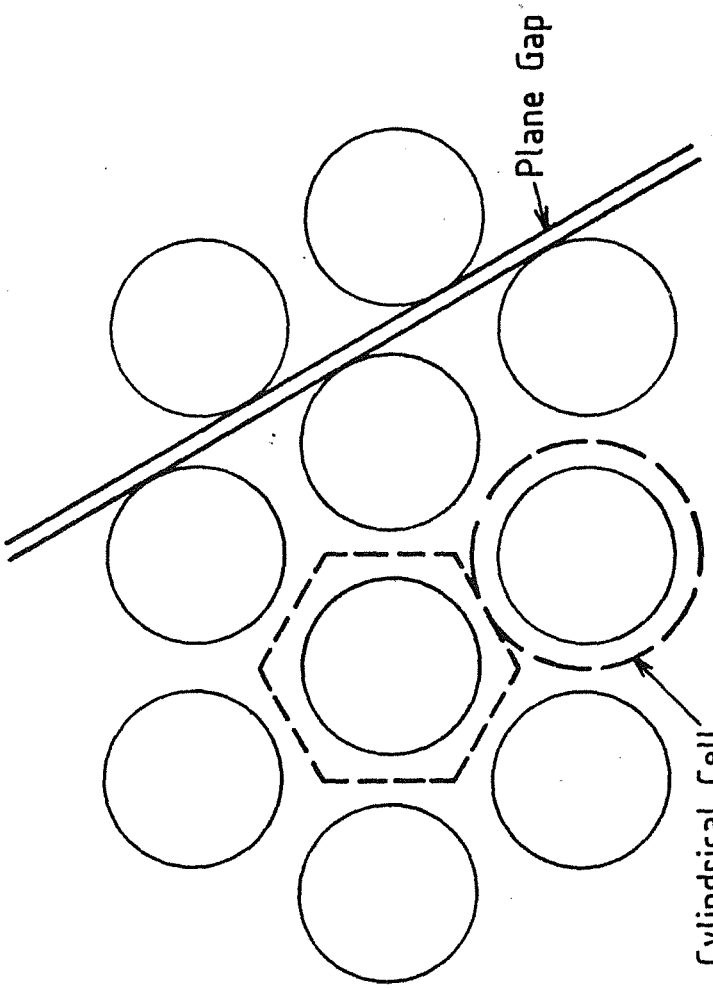

Fig. 1: Hexagonal Lattice

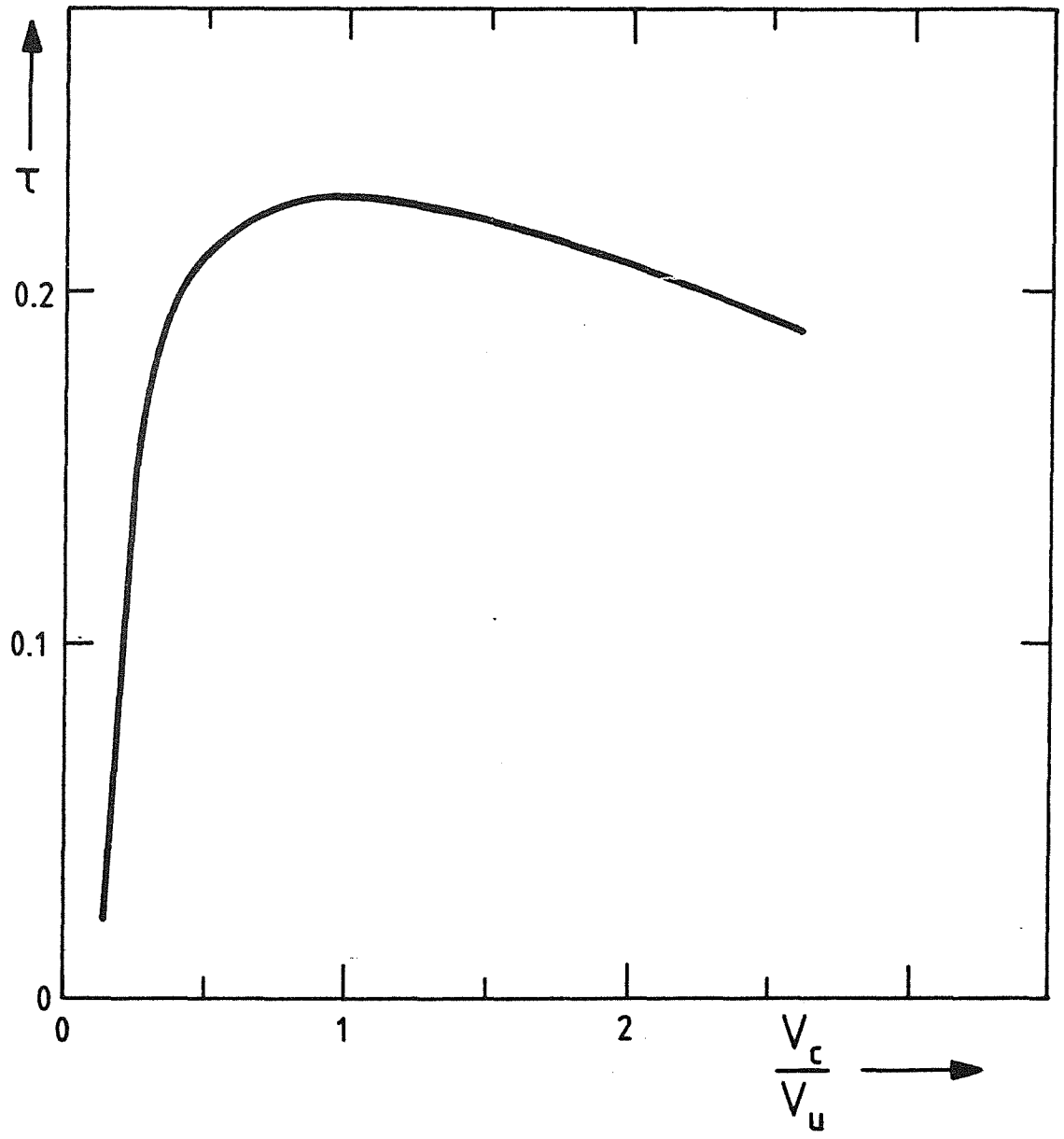

Fig. 2 Geometric Index τ vs. the volume ratio V_c/V_u for a hexagonal Latice (after Sauer)

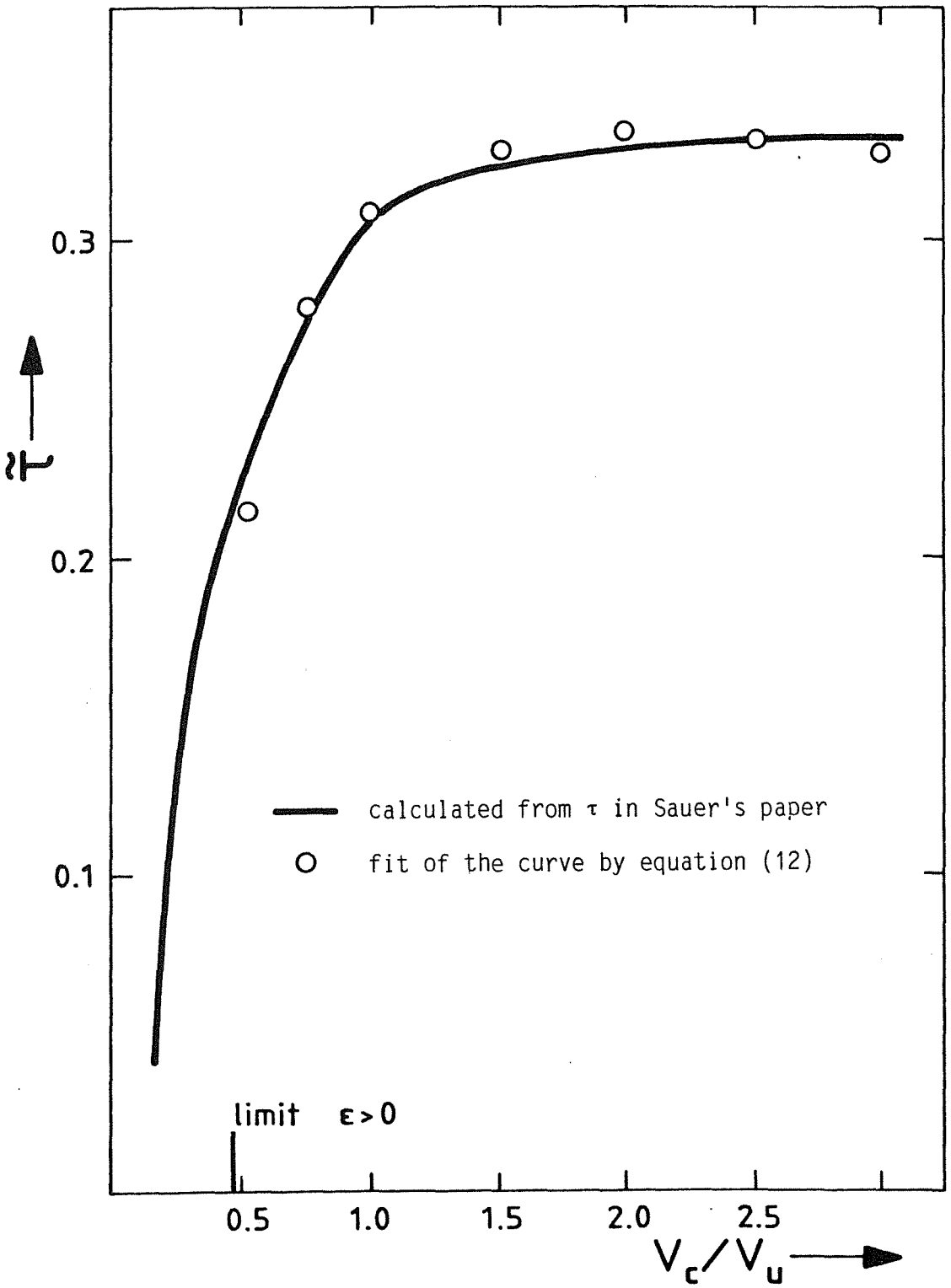

Fig. 3: Parameter $\tilde{\tau}$ Versus the Volume Ratio V_c/V_u

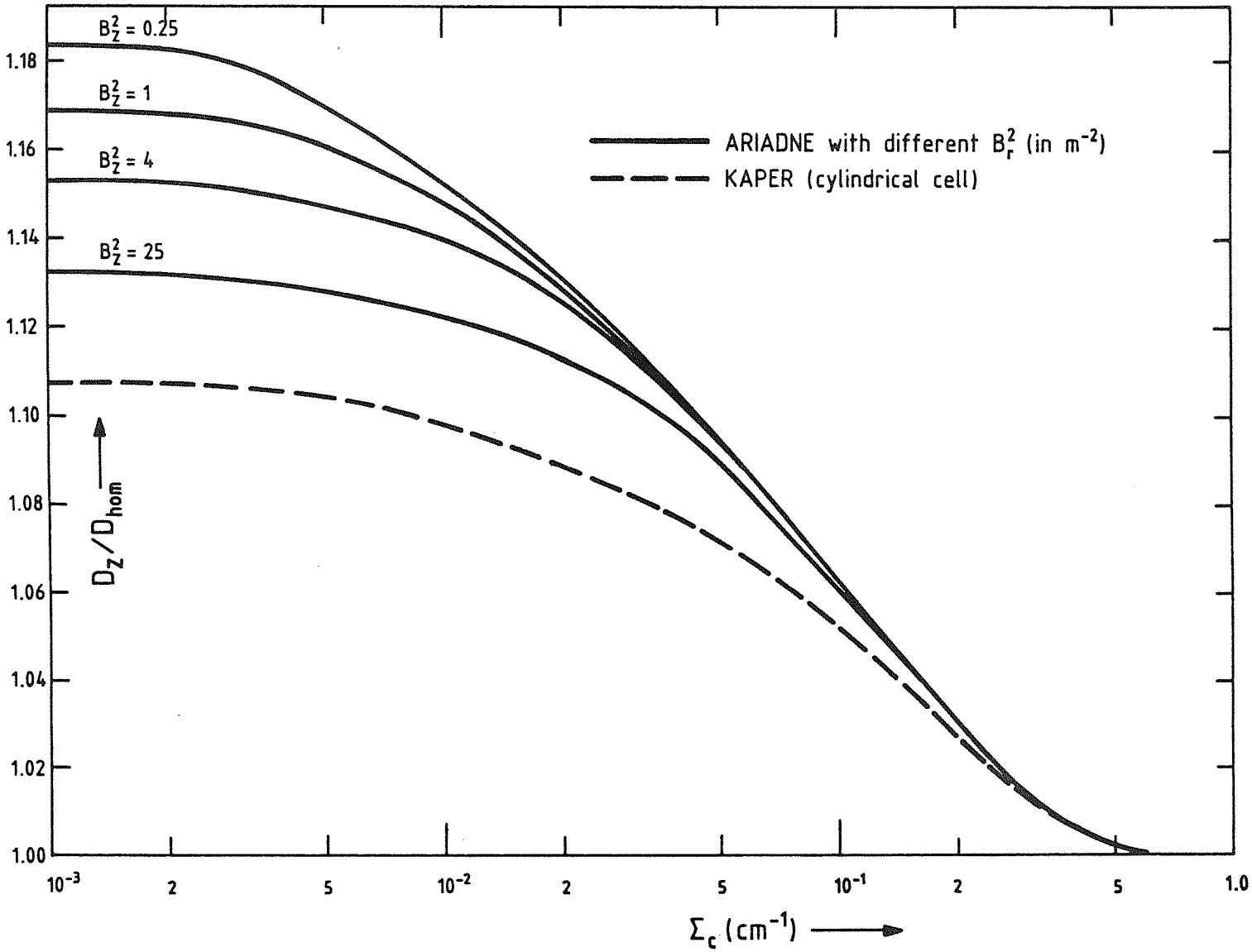

Fig. 4 Normalized Axial Cell Diffusion Coefficient vs. Coolant Cross Section for Different Bucklings

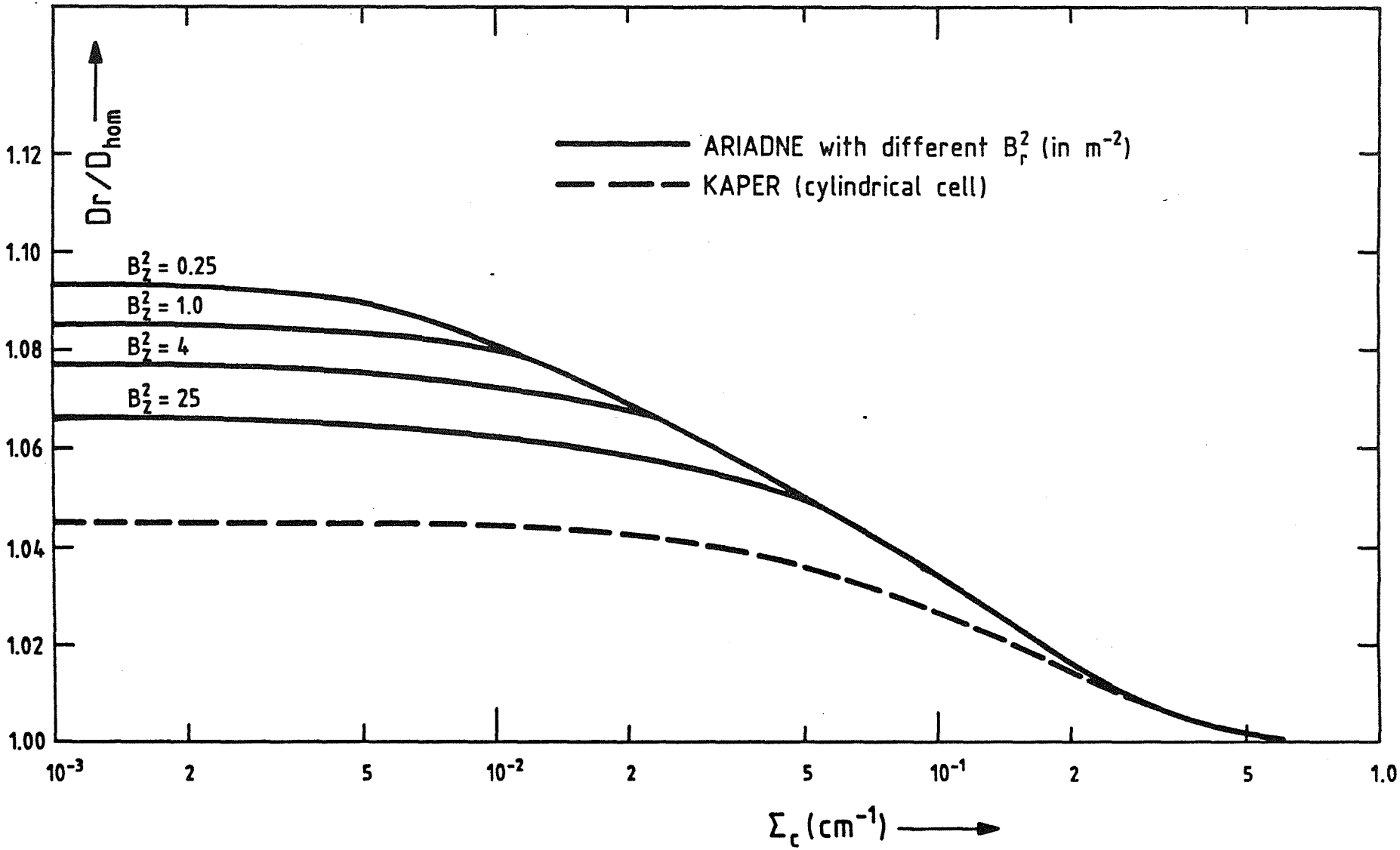

Fig. 5 Normalized Radial Cell Diffusion Coefficient vs. Coolant Cross Section for Different Bucklings

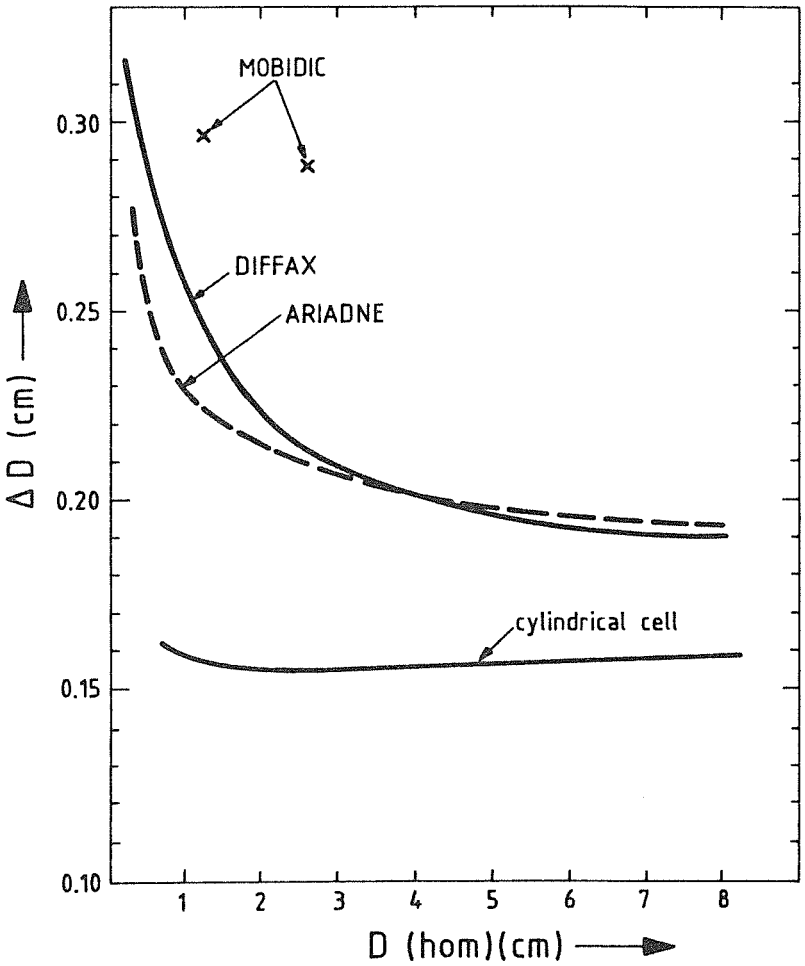

Fig. 6: Axial Streaming Correction for a Gas Cooled Fast Reactor Lattice