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Abstract

Tritium handling implies tritium bearing waste streams. Part of this tritium is recovered in appropriate processing steps. An accountancy system is investigated for use in such a process. In particular, the chances of a safe and timely detection of anomalies are evaluated in quantitative terms.

Tritiumbilanzierung unter Beachtung typischer Betriebsabläufe

Zusammenfassung

Tritiumführende Systeme werden tritiumkontaminierte Abfallströme erzeugen; dieses Tritium kann über geeignete Prozeßschritte wiedergewonnen werden. Für eine solche Prozeßführung wird ein Bilanzierungssystem untersucht. Insbesondere werden Möglichkeiten einer sicheren und rechtzeitigen Aufdeckung von Anomalien in quantitativer Form ausgewertet.

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1. Introduction

Future nuclear fusion power plants which make use of the DT-reaction will require the nuclear fuel cycle to be complete. Careful tritium inventory taking will be necessary not only to cope with the issues of radiation protection, but to assist in the management of the nuclear fuel cycle as well. Obviously, inventory taking can be considered only as a further measure supplementing the already envisaged systems of containment and surveillance.

At present, only a few stations in the nuclear fuel cycle are being subjected to a more in-depth investigation. Above all on account of the physical and chemical properties of tritium, laboratories must be adequately equipped so that the investigations can be carried out. A laboratory of this type starts its operation on the premises of the Karlsruhe Nuclear Research Center (KfK). Thus, the study will benefit from an in situ test bed available. Although the investigation described here has been devised for the test bed at KfK, the methods supplied can be transferred to other laboratories, too. To guarantee this, simulation techniques are used. Normally, three simulation packages are necessary in this type of investigation, namely modeling of the underlying process, modeling of the measurements applied, and modeling of the accountancy system. This investigation deals with the third package. Information concerning the first two packages indicated is contained in references [1] and [2].

It should be underlined again in this context that the major task of inventory taking consists in the timely detection of any tritium anomaly and, if possible, in locating it. Similar criteria have been investigated for a rather long time and at great expenditure in the nuclear fission fuel cycle. Whereas in nuclear safeguards losses and thefts cannot be a priori excluded, we are presently not assuming a diversion for non-peaceful purposes in tritium bearing systems. However, it should be emphasized here that the highest court in Germany has decided to the effect that in Germany tritium shall be classified as a basically weapons grade material.

The simplified experimental setup is represented schematically in Figure 1. Here, P denotes a process in which wastes contaminated with tritium are produced. In the area of P measuring points are provided which allow - according to the state of the art - inventory taking to be made in an optimum manner. The waste streams are collected in tank B; their tritium content can be estimated rather inaccurately only. After a quite long period of time, the contents of tank B are returned into the process area of P where they are cleaned and are then available again for a relatively accurate inventory to be taken.

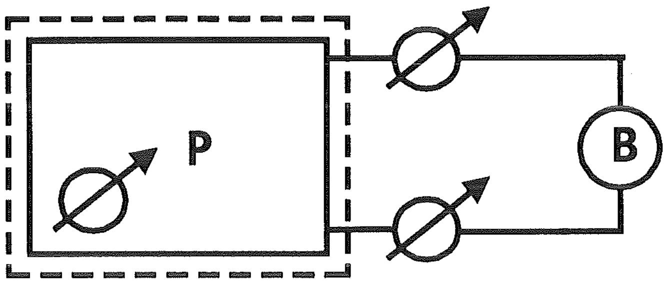

Fig. 1: Tritium bearing process P for waste streams contaminated with tritium (the Material Balance Area under consideration is marked by a dashed box).

Thus, the problems associated with balancing accountancy concerning the Material Balance Area defined by the process P can be represented in qualitative terms as follows: At time t_0 the inventory I_0 is measured in the process; the tank B is empty. After a rather long period $[t_0, t_n]$ the inventory I^P_n present in process P is measured and the tank B is again empty. During the time intervals $[t_{j-1}, t_j]$, $j = 1$ ..., n-1 the "small tritium amounts" $A^P_{j-1,j}$ are discharged from the process into the tank and estimated. Moreover, at the times t_1 until t_{n-1} the process inventories I^P_j, $j = 1$, ... n-1 can be measured. During the period $[t_{n-1}, t_n]$ the "large amount" $E^P_{n-1,n}$ is discharged from the tank into the process. Let the variances associated with the random measurement errors of all inventories introduced so far be:

$$\mathrm{var}(I_j) = \sigma_I^2, \mathrm{var}(A^P_{j-1,j}) = \sigma_A^2 \text{ for } j = 0, \ldots, n-1 \text{ and } \mathrm{var}(E^P_{n-1,n}) = \sigma_E^2 . \quad (1)$$

We assume that an inspecting authority will check the balance regularly at time t_n and that, in addition, it will select at random within the reference period $[t_0, t_n]$ one of the points in time t_j, $j = 1, \ldots, n-1$ and check the balance for that period $[t_0, t_j]$. The idea inherent in this approach is that at the end of the reference period - e.g. at the end of the year - a relatively accurate balance can be made up, i.e. any anomalies can be detected with a high probability. The interim balance selected at random can then serve for a timely detection of anomalies.

2. Material Balance Test for the Reference Period $[t_0, t_n]$

We define for the period $[t_0, t_n]$ the balance statistics $X^P{}_{0,n}$ as follows:

$$X^P_{0,n} = I^P_0 - I^P_n . \qquad (2)$$

If there were no measuring errors and anomalies, $X^P{}_{0,n}$ should equal zero; should the anomaly $\mu > 0$ occur, we should find $X^P{}_{0,n}$ equals μ. However, as measuring errors cannot be avoided, a statistical test must be performed which will help decide whether a non-vanishing value of $X^P{}_{0,n}$ can be explained or not solely by measuring errors.

Under the null hypothesis H_0, we assume that the expected value of $X^P{}_{0,n}$ equals zero; under the alternative hypothesis H_1, we assume the expected value of $X^P{}_{0,n}$ to be positive:

$$\begin{aligned} H_0 &: E_0 (X^P_{0,n}) = 0 \\ H_1 &: E_1 (X^P_{0,n}) = \mu > 0 . \end{aligned} \qquad (3)$$

In order to be able to make the choice between these two hypotheses, the significance threshold s_n is defined. The inspector rejects H_0, if the observed value $X^P{}_{0,n}$ of $X^P{}_{0,n}$ exceeds the threshold s_n:

$$X^P_{0,n} > s_n : \ H_0 \text{ is rejected.} \qquad (4)$$

The value of s_n can be calculated if the probability of error of the first kind α_n (also called false alarm probability) is given; it is defined as the probability that $X^P{}_{0,n}$ exceeds s_n if in fact the null hypothesis H_0 is true:

$$\alpha_n := \text{prob} (X^P_{0,n} > s_n \mid H_0) . \qquad (5)$$

For the special case of Gaussian or normally distributed measurement errors, the following explicit relation between α_n, s_n and σ holds:

$$1 - \alpha_n = \phi \left(\frac{s_n}{\sigma} \right), \quad (6)$$

where σ^2 denotes the variance of $X^P_{0,n}$:

$$\sigma^2 = \mathrm{var}\,(X^P_{0,n}) = \mathrm{var}\,(I_0) + \mathrm{var}\,(I_n) = 2\ \sigma_I^2 , \quad (7)$$

and where $\phi\,(\cdot)$ represents the Gaussian distribution:

$$\phi\,(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2} \right) dt . \quad (8)$$

The efficiency of the test procedure is described by the probability of error of the second kind β_n (also called no-detection probability). It is defined as the probability that $X^P_{0,n}$ does not exceed s_n if in fact the alternative hypothesis H_1 is true:

$$\beta_n := \mathrm{prob}\,(X^P_{0,n} < s_n \mid H_1) . \quad (9)$$

For our case of normally distributed measurement errors, we get

$$1 - \beta_n = \phi \left(\frac{\mu - s_n}{\sigma} \right) . \quad (10)$$

Using (6) we can eliminate s_n and get

$$1 - \beta_n = \phi \left(\frac{\mu}{\sigma_I \cdot \sqrt{2}} - \phi^{-1}\,(1 - \alpha_n) \right), \quad (11)$$

where $\phi^{-1}\,(\cdot)$ denotes the inverse function of $\phi\,(\cdot)$.

In the following, we call $1-\beta_n$ the accountancy effectiveness since we use this quantity as the yardstick for the quality of the accountancy system. It increases monotonically with increasing loss μ, and increasing probability of error first kind, as well as with decreasing variance σ^2, as we would expect.

3. Material Balance Test for Time Intervals $[t_0, t_j]$ within the Reference Period $[t_0, t_n]$

We now assume that, in addition to the test for the material balance for the whole reference period $[t_0, t_n]$, the inspector wishes to check the balance at any point in time t_j, $j=1, ..., n-1$, for the period $[t_0, t_j]$ in order to detect an eventual anomaly earlier than only at time t_n. Thus, the related balance statistics read:

$$X^P_{0,j} = I^P_0 - \sum_{i=1}^{j} A^P_{i-1,i} - I^P_j, \qquad j=1, ..., n-1. \tag{12}$$

For the period $[t_0, t_n]$ there are two possibilities for establishing the material balance test statistics: If we remember that during $[t_{n-1}, t_n]$ the large amount $E^P_{n-1,n}$ is discharged from the tank into the process then it is given by

$$I^P_0 - \sum_{i=1}^{n-1} A^P_{i-1,i} + E^P_{n-1,n} - I^P_n. \tag{13}$$

On the other hand, we assume - under H_0 - that all material discharged from the process during $[t_0, t_{n-1}]$ is given back to the process during $[t_{n-1}, t_n]$. This means that we should have at t_n the same process inventory as at t_0[1)] and thus the material balance test statistics is given by

$$I^P_0 - I^P_n. \tag{14}$$

In fact, we used this already according to (2), since its variance is smaller than the former one.

1) We assume that all inventory data are corrected for radioactive decay.

Specifying the two hypotheses H_0 and H_1 by

$$H_0: \quad E_0(X^P_{0,j}) = 0 \tag{15}$$

$$H_1: \quad E_1(X^P_{0,j}) = \mu_j > 0 \tag{16}$$

and the significance levels s_j, we obtain relations corresponding to Eqs. (6), (7) and (11), in particular

$$1 - \alpha_j = \Phi\left(\frac{s_j}{\sigma_j}\right), \tag{17}$$

where σ_j^2 is given by

$$\sigma_j^2 = \begin{cases} 2\,\sigma_I^2 + j\sigma_A^2 & \text{for } j = 1, \ldots, n\text{-}1 \\ 2\,\sigma_I^2 & \text{for } j = n, \end{cases} \tag{18}$$

and furthermore

$$1 - \beta_j = \Phi\left(\frac{\mu_j}{\sigma_j} - \Phi^{-1}(1 - \alpha_j)\right), \qquad j = 1, \ldots, n. \tag{19}$$

At this point of the process the false alarm probabilities α_j for the test performed at the time t_j, $j = 1, \ldots, n\text{-}1$, which is selected at random, and α_n for the test which takes place at any rate at time t_n must be fixed. For reasons of pragmatism we set $\alpha_j = \alpha$ for $j = 1, \ldots, n$, but would like to mention that both test procedures are not independent because of the commonly used measurements I_0 and $A^P_{i-1,\,i}$ for $i = 1 \ldots, j$ and therefore the total false alarm probability α_{tot} for both tests is given by

$$1 - \alpha_{tot} = B\left(\Phi^{-1}(1 - \alpha),\ \Phi^{-1}(1 - \alpha);\ \rho_j\right), \tag{20}$$

where the correlation ρ_j between $XP_{0,j}$ and $XP_{0,n}$ is given by

$$\rho_j = \left(4 + 2j \cdot \frac{\sigma_A^2}{\sigma_I^2} \right)^{-1/2} \tag{21}$$

and where B (h, k; ρ) is the bivariate standard normal distribution:

$$B(h, k; \rho) = \frac{1}{\sqrt{1-\rho^2}} \int_{-\infty}^{h} dt_1 \int_{-\infty}^{k} dt_2 \exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}(t_1^2 - 2\rho t_1 t_2 + t_2^2) \right). \tag{22}$$

Evidently, the procedure adopted may also consist in defining in advance α_{tot}. Should α_j and α_n be selected to be identical for the individual tests, they are defined by Eq. (20). However, they may be selected also to be different from each other in case the inspector deems more important one or the other test information.

It should be added that for $4+2j\ \sigma^2_A/\sigma^2_I >> 1$, using Eq. (19), $\rho_j \approx 0$ follows and, hence, in general terms, using Eq. (20) and Eq. (22)

$$1 - \alpha_{tot} = (1 - \alpha)^2 ;$$

moreover, it follows with $j \cdot \sigma^2_A/\sigma^2_I >> 1$ from Eq. (21) that $\rho = \frac{1}{2}$, actually independent of j.

4. Timely Detection of Anomalies

As the interim balance check by the inspector serves the timely detection of anomalies, as has already been mentioned, it suggests itself to ask which time will elapse between the occurrence of the anomaly and its detection. In the following studies the assumed anomaly will be so large that it will be detected with certainty in the test performed at time t_n.

Two cases are investigated, namely

- an anomaly gradually developing from the beginning of the reference period (protracted anomaly);
- a unique anomaly occurring at time t_{j0} (abrupt anomaly).

4.1 Protracted Anomaly

If the interim balance is checked at time t_j, the (conditional) expected time until detection (expressed by the number of periods for which inventories have been taken) is $j \cdot (1 - \beta_j) + n \cdot \beta_j$.

Should the interim balance for any point in time be established with the same probability 1/(n-1), the (unconditional) expected time until detection is

$$E_1(T) = \sum_{j-1}^{n-1} \left[j \cdot (1 - \beta_j) + n \cdot \beta_j \right] \cdot \frac{1}{n-1} = \frac{n}{2} + \sum_{j=1}^{n-1} \frac{n-j}{n-1} \cdot \beta_j \,. \tag{23}$$

With a uniform distribution of the anomaly $\mu_j = \mu/n$ for $j = 1, \ldots, n$, the probability β_j of no detection is

$$\beta_j = \Phi \left(\Phi^{-1}(1-\alpha) - \frac{j}{n} \cdot \frac{\mu}{\sqrt{2\,\sigma_I^2 + j\sigma_A^2}} \right), \qquad j = 1, \ldots, n-1 \,. \tag{24}$$

Two special cases of (23) are of interest, since they illustrate this formula and are a valuable check for numerical calculations. For $\mu = 0$ we get from (19) $\beta_j = 1 - \alpha$ for $j = 1, \ldots, n$ and therefore from (23)

$$E_0(t) = \frac{n}{2} + \frac{1-\alpha}{n-1} \sum_{j=1}^{n-1} (n-j) = n \left(1 - \frac{\alpha}{2} \right); \tag{25}$$

this is the expected time until the occurrence of an error first kind (false alarm).

For $\mu >> \sigma_j$, $j = 1, \ldots, n$, we get from (19) $\beta_j \simeq 0$ and therefore from (23)

$$E_1(t) = \frac{n}{2};$$

if the inspection authority detects the anomaly with certainty if he checks the balance, and if all points $t_1, \ldots, t_{n-1}$ are chosen with equal probability, then the expected detection time is just in the middle of the interval $[t_1, t_{n-1}]$.

4.2 Abrupt Anomaly

In this case we deem the time until detection to be the interval from the point in time t_{j0} of occurrence of the anomaly until its detection. Thus, we obtain instead of Eq. (23)

$$E_1(T; j0) = \begin{cases} \sum_{j=j0+1}^{n-1} \left[(j-j0)(1-\beta_j) + (n-j0) \cdot \beta_j \right] \frac{1}{n-1} & \text{for } 0 \leq j0 \leq n-2; \\ 1 & \text{for } j0 = n-1 \end{cases}$$

or after some transformations

$$E_1(T; j0) = \begin{cases} \frac{(n-j0)(n-j0-1)}{2(n-1)} + \sum_{j=j0+1}^{n-1} \frac{n-j}{n-1} \beta_j & \text{for } 0 \leq j0 \leq n-2 \\ 1 & \text{for } j0 = n-1; \end{cases} \tag{26}$$

where, in case of an anomaly μ occurring at j0,

$$\beta_j = \Phi\left(\Phi^{-1}(1-\alpha) - \frac{\mu}{\sqrt{2\sigma_I^2 + j\sigma_A^2}} \right) \quad \text{for } j > j0 . \tag{27}$$

Again we consider the same special cases as before.

For $\mu=0$ we get $\beta_j = 1-\alpha$, $j = 1, \ldots, n-1$, and therefore from (26)

$$E_0\,(T;\ j0) = \frac{(n-j0)\,(n-j0-1)}{2\,(n-1)} + \frac{1-\alpha}{n-1} \sum_{j=j0+1}^{n-1} (n-j)$$

$$= \frac{(n-j0)\,(n-j0-1)}{2\,(n-1)} + \frac{1-\alpha}{n-1}\,\frac{(n-j0)\,(n-j0-1)}{2}$$

$$= \frac{(n-j0)\,(n-j0-1)}{(n-1)} \left(1 - \frac{\alpha}{2}\right) \quad \text{for } 0 \leq j0 \leq n-2,$$

which again is the expected time until the occurrence of an error first kind (false alarm).

For $\mu >> \sigma_j$ for $j = 1, \ldots, n$ we get from (19) $\beta_j \approx 0$ for $j \geq j0$ and therefore from (23)

$$E_1\,(T;\ j0) = \frac{(n-j0)\,(n-j0-1)}{2\,(n-1)} \quad \text{for } 0 \leq j0 \leq n-2\,.$$

5. Numerical Results and Conclusions

Some results of numerical calculations will be presented below; the input data were chosen according to the present knowledge of the operational strategies and according to the state of the art. Based on these input data were obtained in $\sigma^2_I = 0.0001$ [g^2 H^3], $\sigma^2_A = 0.002$ [g^2 H^3]; furthermore, the reference time is one year including 10 inventory periods.

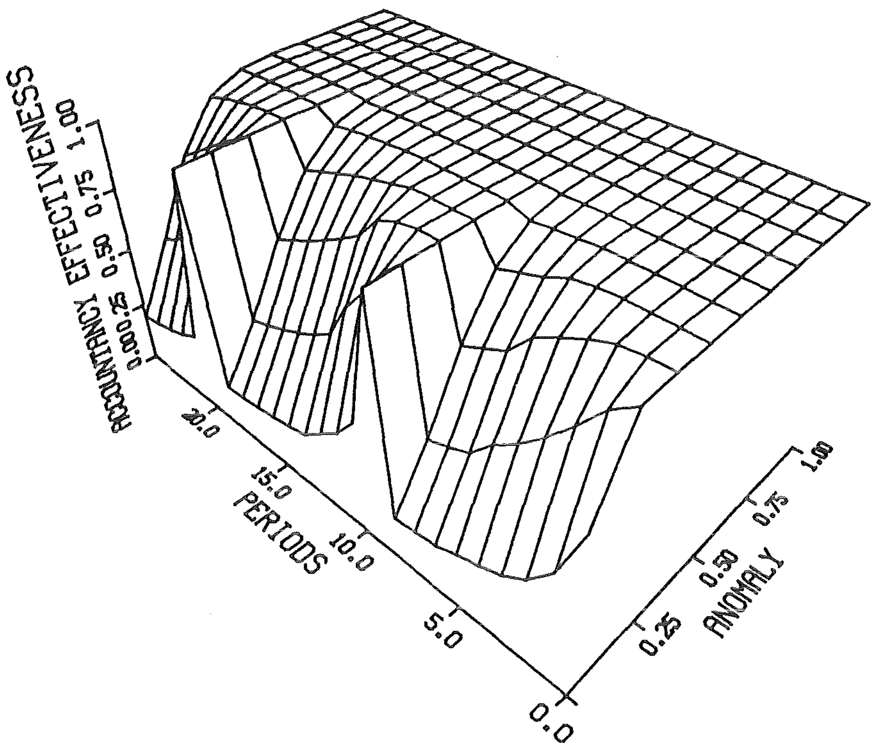

Fig. 2: Accountancy effectiveness according to (19) for $\alpha_j = \alpha = 0.05$, $\mu_j = \mu/10$, $j = 1, \dots 9$, as function of time (expressed as inventory periods) and extent of presumed anomaly.

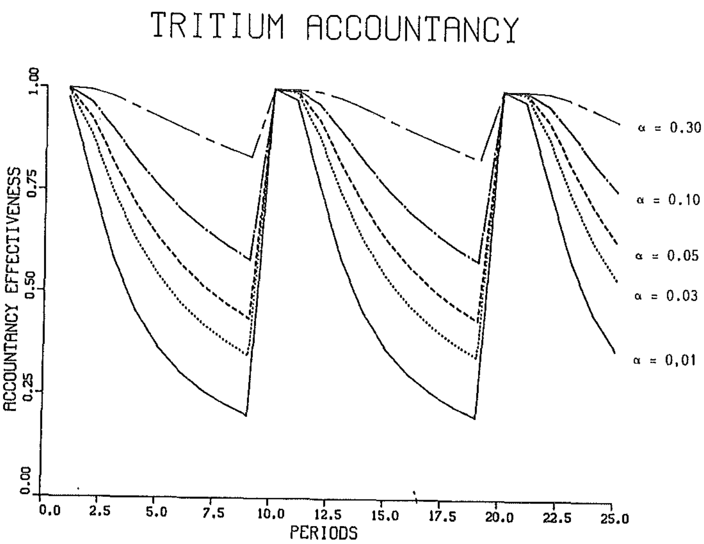

Fig. 3: Accountancy effectiveness according to (19) as function of time (expressed as inventory periods) with same single false alarm probability α for all tests as parameter.

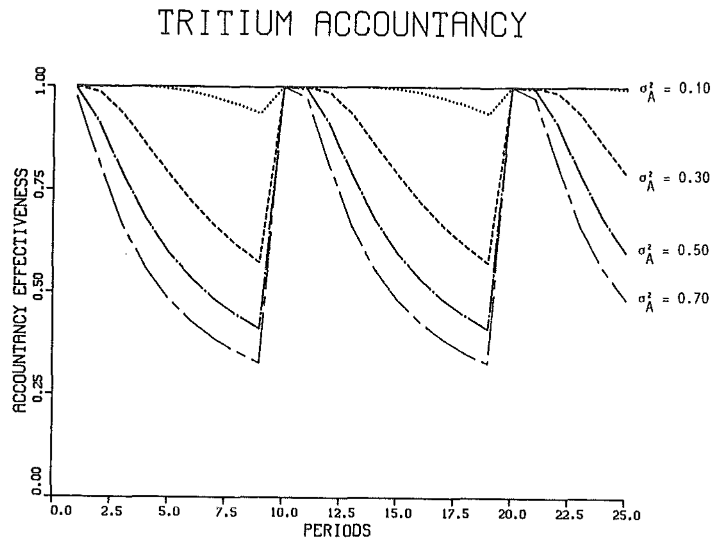

Fig. 4: Accountancy effeciveness according to (19) as function of time (expressed as inventory periods) with variance $\sigma^2{}_A$ of waste measurements as parameter.

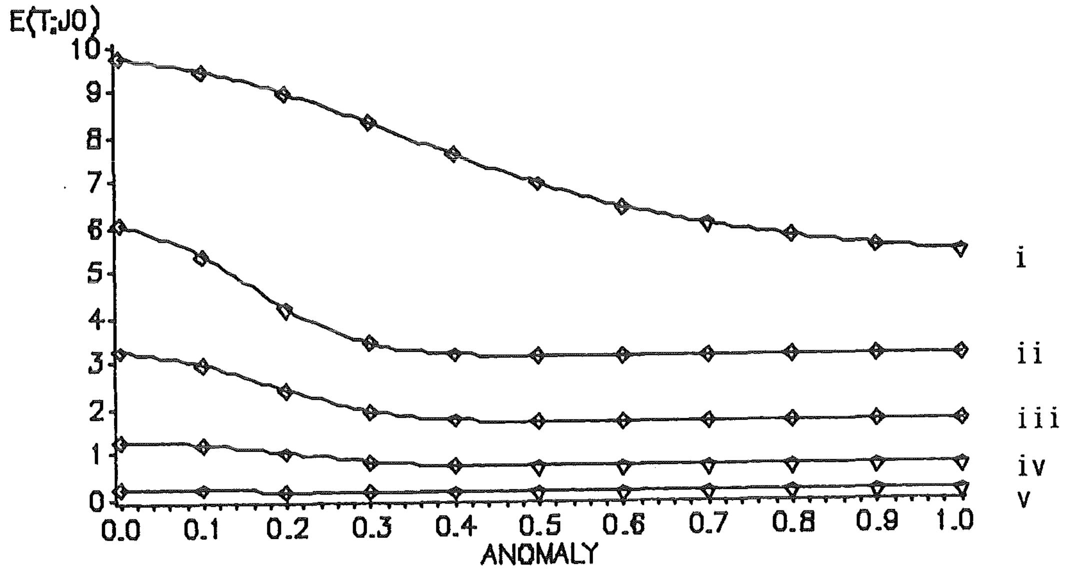

Fig. 5: Expected detection time as a function of the extent of the presumed anomaly with $\alpha = 0.05$; (I) is the protracted case according to (23); (II) through (V) are the abrupt cases according to (26) for $j0 = 2, 4, 6$, and 8, respectively.

The accountancy effectiveness as presented in Figures 2, 3 and 4 may be interpreted as providing the most complete information available to the operator about all material balances in the reference time interval.

Figure 2 shows that with increasing anomaly the accountancy effectiveness improves and furthermore that after the material has been collected again in the main accountancy area of the process at the end of the reference period, the accountancy effectiveness also sharply improves. Figure 3 shows the influence of the false alarm probability α; the wide range of the accountancy effectiveness indicates that both the inspector and the operator should carefully consider their purposes for accounting before they select the value of the false alarm probability. Finally, Figure 4 demonstrates the effect of the variance σ^2_A of the waste measurement on the accountancy effectiveness; looking at cost benefit considerations this should be known if improvements of the measurement equipment are envisaged.

The expected detection time as given in Figure 5 is of interest exclusively to the inspection authority. In fact, as mentioned before the random inspection between t_0 and t_n serves the only purpose of shortening the detection time for the inspector. Figure 3 also shows that the protracted case may be considered to be the most pessimistic assumption. The abrupt case implies a shorter detection time, the later the anomaly occurs: Even though the chances of inspection during j0 and t_n get smaller with increasing j0 so that the expected detection time gets longer, the time between the occurrence of the anomaloy and its safe detection at t_n gets shorter and thus, consequently, reduces the expected detection time.

6. References

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[2] G. Spannagel et al., Near Real Time Accountancy for Tritium Handling Systems, to be publ. (1992).

Acknowledgement

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