# MEASURING STATIC AND SLOWLY CHANGING LOADS USING PIEZOELECTRIC SENSORS

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#### Abstract

Due to their high dynamics, piezoceramic materials are widely used for sensing and actuating. Recently there have been developments to use piezoelectric transducers in an adaptronic sense, as self-sensing actuators, combining these two functions. Thus, they are very interesting for the use in machine tools, e.g. for vibration damping or correction of positions via force control. However, force controlled correction of static or quasistatic deflections resulting from production tolerances, installation faults and process loads is rather problematic since the discharging resistance of the ceramic material strongly reduces the measuring signal, making measuring of low-frequency signals ( $\ll 1$  Hz) difficult and measuring of static signals impossible.

This problem can be reduced using the functional principle of a scale with a vibrating string. Observing the change of eigenfrequency of the vibrating string using frequency counters or phase-locked loops, the load on the string and thus the load on the piezoelectric transducer can be determined.

Implemented in a strut of a machine tool the described system is used to develop an adaptronic component for compensation of geometric errors.

Keywords: adaptronics, piezoceramics, frequency counters, phase-locked loops

## Introduction

Geometric faults in parts of machine tools with parallel kinematics, such as, for example, errors in the lines of the guiding skids, lead to stresses in the structure and deflections of the tool center point (TCP), reducing the quality of the workpiece. Fur-

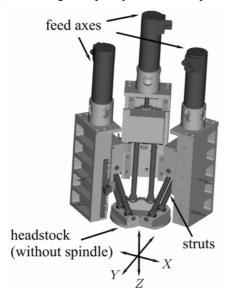
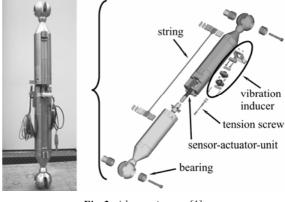


Fig. 1: Parallel kinematic machine tool with three degrees of freedom [1].

thermore, a machine tool with ideally three degrees of freedom, as shown exemplarily in Fig.1, exhibits a full three-dimensional behavior in six axes. Translational deflections can be compensated by additionally moving the guiding skids, the compensation of rotational deflections, however, is restricted. To reduce this drawback, an adaptronic strut was developed as shown in Fig.2. The strut, similar in shape to a general strut in machine tools, comprises a lower and an upper half, and a sensor-actuator unit implemented in-between. An edge-knife bearing and a tension screw connect the two halves with each other. Along the strut, prestressing the piezoelectric element, a thin metal sheet is mounted which during the operation is excited by a vibration inducer. This metal sheet, in the following due to a comparable behavior also referred to as string, is required for



# Fig. 2: Adaptronic strut [1].

measuring static and slowly changing ( $\ll$ 1Hz) loads on the strut. Due to the discharging resistance of the ceramic material it is, if even possible, very

cumbersome to use a piezosensor for measuring such slowly changing loads [2]. Via a lever mechanism a string vibrating in resonance induces vibrations onto the piezoelectric sensor. The measured signal then comprises the sought load of the static or slowly changing process, respectively, as well as the superposed dynamic load of the string vibration and therefore it is easy to acquire with the piezosensor.

According to the working principle of a scale with a vibrating string the signal of the piezosensor can be processed further by analyzing the frequency behavior using e.g. frequency counters or phase-locked loops. Thus, the load on the strut can easily be determined.

Based on this working principle the mode of operation of the adaptronic strut and a simple mechanical model are introduced. Different methods for frequency measurement are presented and the functionality of the adaptronic strut is shown.

#### Model

Corresponding to the design of the adaptronic strut as depicted in Fig.2 a simple model can be built, shown in Fig.3. Via a lever of length b the vibrating string induces longitudinal forces onto the piezoelectric transducer. The string itself is excited in resonance by a solenoid force F(t).

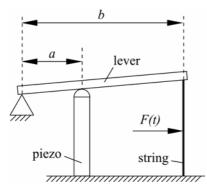


Fig. 3: Model of adaptation [3].

Considering the metal sheet as Euler-Bernoulli beam by using Hamilton's Principle

$$\delta \int_{t_0}^{t_1} (T - V) dt + \int_{t_0}^{t_1} \delta W dt = 0 , \qquad (1)$$

its equations of motion can be derived to

$$-\mu u_{tt} + EA\left(u_x + \frac{1}{2}w_x^2\right)_x = 0$$

$$-EIw_{xxxx} - \mu w_{tt} + EA\left(u_x w_x\right)_x = -F_{mag}\left(t\right).$$
(2)

Thus, the corresponding longitudinal force acting on the clamping on the lever can be determined. Via the lever, the force applied on the piezosensor and thus, the electric voltage induced in the piezosensor can be determined using the constitutive equations of the piezoelectric material according to [4]:

$$T_{p} = c_{pq}^{E} S_{q} - e_{kp} E_{k}$$

$$D_{i} = e_{iq} S_{q} + \varepsilon_{ik}^{S} E_{k} .$$
(3)

# Analogy to functional principle of a scale with vibrating string

The sensor measures a dynamic force, the static part of the applied load due to geometric errors superposed by the dynamic part from the string. By measuring the frequency of the signal the current state of stress can be determined according to the principle of a scale with a vibrating string. Starting from a reference frequency, the eigenfrequency of the vibrating string,

$$f_0 = \frac{1}{2l_0} \sqrt{\frac{\sigma}{\rho}},\tag{4}$$

the frequency changes with changing stress  $\sigma = E \Delta l/l$  and corresponding changing length  $\Delta l$  of the string. By measuring the new frequency

$$f_0^* = f_0 + \Delta f = \frac{1}{2(l_0 + \Delta l)} \sqrt{\frac{E}{\rho} \frac{\Delta l}{l_0}}$$
(5)

the elongation of the string and, thus, the external load can be determined. However, it must be kept in mind, that this frequency change can only be measured during a small period of time, instantaneously after the load on the strut changed, since the whole system is excited by a specified forced excitation frequency. To continuously excite the string in current resonance, the excitation frequency must instantly be adjusted during the process, which requires fast frequency measurement methods.

#### **Frequency measurement**

In the following two different methods for measuring the frequency of a signal are discussed. In general, an oscillating signal with time variant phase can be described in the form

$$x(t) = \hat{x} \sin[\phi^*(t)]. \tag{6}$$

If the signal mainly consists of a constant basic angular frequency  $\omega_0$ , equation (6) can be rewritten as

$$x(t) = \hat{x} \sin[\omega_0 t + \phi(t)] \tag{7}$$

with a time variant phase  $\phi(t)$ . A sudden change of frequency, for example in shape of a step function from  $\omega_0$  to  $\omega_1$ , can then easily be described by

$$\phi(t) = \Delta \omega \ t \ \sigma(t - t_0). \tag{8}$$

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The first measuring principle bases upon the determination and inversion of the period of the signal, since f = 1/T. All instants of time when the signal crosses a specific line (e.g. zero) with a falling slope (or rising slope, respectively), as shown in Fig.4, are detected and the current period of the signal can be

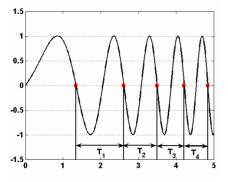


Fig. 4: Detecting zero crossings to measure period.

determined. In the following this will be referred to as the period method (PM). Instead of using the instants of time of zero crossings directly, a reference signal in combination with a counter can be used. The reference signal, e.g. a Walsh function, has a specified frequency. Every time there is a rising flank, the counter increases the number. Counting is started at the first zero crossing of the measuring signal and finished and restarted at the next. The counted number is exported and the counter is reset to zero. The product of the period of the reference signal with the output number leads to the period of the measuring signal. Again, by inversion, the frequency of the measuring signal is obtained. In the following, this method will be referred to as the impulse method (IM).

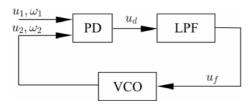


Fig. 5: Linear phase-locked loop.

The second measuring principle is based upon the behavior of phase-locked loops (PLL) as usually used in radio transmission. This method is based on the correlation of angular frequency and phase of a signal. According to [5], this relationship is given by

$$\omega(t) = \frac{d\phi^*(t)}{dt},\tag{9}$$

the angular frequency  $\omega$  being the first time derivative of the phase. A linear phase-locked loop, as shown in Fig.5, usually consists of a voltage controlled oscillator (VCO), a low-pass filter (LPF), usually a simple multiplier. The PLL tracks the phase and therefore the frequency  $\omega_1$  of the input signal. The VCO oscillates at an angular frequency  $\omega_2$ , determined by the output signal  $u_f$  of the LPF by

$$\omega_2(t) = \omega_0 + K_0 u_f(t), \qquad (10)$$

with  $K_0$  being the gain and  $\omega_0$  being the center angular frequency of the VCO. The PD compares the phases of the input signal  $u_1$  and the output signal  $u_2$ of the VCO. Within a bounded range the output signal  $u_d$  of the PD can be linearized to

$$u_d(t) \approx K_d \Delta \phi(t),$$
 (11)

with  $K_d$  being the gain of the PD. The LPF finally filters the signal. The tracking attitude of the PLL can easily be seen. As long as the two signals  $u_1$  and  $u_2$  are equal in phase and therewith in frequency, no phase error is detected in the PD and the signals  $u_d$ and  $u_f$  equal zero. When there is a change, the VCO is caused to change its frequency  $\omega_2$  such that the phase error vanishes [5].

If the frequency range of the measuring signal is known, which is likely to be the case in this application due to the idea of resonant excitation of the vibrating string, the angular center frequency  $\omega_0$  as well as the appropriate pull-in, lock and hold range of the PLL can be specified.

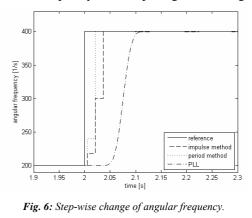
#### Results

In the following the results of two examples for frequency detection using the described methods are presented.

First, the angular frequency  $\omega_m$  of a sinusoidal measuring signal can ideally be described as a step function by

$$\omega_{\rm m} = 200 \, [1 + \sigma \, (t-2)].$$
 (12)

In Fig.6 the measured angular frequency using IM, PM and PLL is shown. It can easily be seen that there is a time delay in all three measuring signals. Since at least one period of the signal is required to determine two consecutive zero crossings using IM or PM, this delay in unavoidable and solely depending on the frequency of the input signal. It is slightly



**ACTUATOR 2006**, J. Wauer, C. Rudolf, J. Fleischer, C. Munzinger Measuring static and slowly changing loads using piezoelectric sensors. different for the PLL. The delay, larger than for IM and PM, results mainly from the properties of the low-pass filter. It can be reduced by modifying the time constant of the filter. However, this is not arbitrarily viable since the properties of the filter are exceedingly influencing the overall behavior of the PLL concerning hold, lock and pull-in range. The second major observation is the shape of the signal. While both signals of the IM and the PM are digitalized functions, the shape of the PLL signal is continuous. These digital signals result from the process of detecting the zero crossing line of the measuring signal, not occurring with the linear PLL.

A similar result can be observed in Fig. 7. The sinusoidal angular frequency function

$$\omega_{\rm m} = 300 + 10\{1 - \cos[10(t-1)]\} \quad (13)$$

of the measuring signal is detected by the IM, PM and the PLL. Similar observations with respect to continuity can be done. For the current settings, the time delays found are approximately 10ms for the PM, 40ms for the IM and 70ms for the PLL.

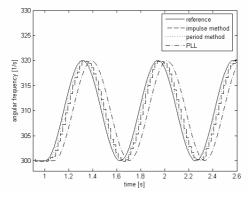
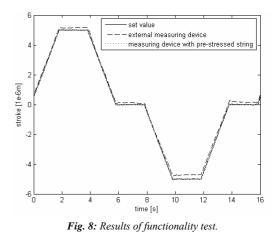


Fig. 7: Sinusoidal change of angular frequency.

Additionally to the presented simulation results the functionality of the adaptronic strut including the frequency detection using the impulse counting method was tested in a test-rig. As shown in equation (5), the change of the frequency also indicates a change of length of the string. Therefore, during the test the stroke of the actuator was to follow a specified function of time, elongating the strut and its tip moving correspondingly. The tip motion was detected by two independent measuring devices, an external one, measuring the change of length of the total strut, and the depicted internal one, using the change of frequency of the vibrating string by use of the impulse method. The results are shown in Fig.8. As easily can be seen there are only small deviations and negligible time delays when the prestressed string is used for measuring and the method proves to be promising for further application as selfsensing actuator.



### Conclusion

In the current test-rig the frequency analysis of the signal is done digitally using the impulse method. However, if the speed of the PLL can be increased, it might prove interesting to remove the digital analysis and its required measuring hardware, substituting it by a PLL element. Due to the analogue signal of the PLL the total information of the signal could be kept compared to the deficient digitalized signal. Additionally, the overall size of the PLL might be an advantage when it must be fitted into a narrow construction.

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