

# Weak localization in arrays of metallic quantum dots

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Combining scattering matrix formalism with non-linear  $\sigma$ -model technique we analyze weak localization effects in arrays of chaotic quantum dots connected via barriers with arbitrary distribution of channel transmissions. With the aid of our approach we evaluate magnetoconductance of two arbitrarily connected quantum dots as well as of  $N \times M$  arrays of identical quantum dots.

## I. INTRODUCTION

Quantum interference of electrons is fundamentally important for electron transport in disordered conductors<sup>1,2,3</sup>. Quantum coherent effects are mostly pronounced at low temperatures in which case certain interaction mechanisms are effectively “frozen out” and, hence, cannot anymore restrict the ability of electrons to interfere. At the same time, there exists at least one mechanism, electron-electron interactions, which remains important down to lowest temperatures and may destroy quantum interference of electrons down to  $T = 0$ . It is, therefore, highly desirable to formulate a general theoretical formalism which would allow to describe electron interference effects in the presence of disorder and electron-electron interactions at any temperature, including the limit  $T \rightarrow 0$ .

In a series of papers<sup>4</sup> we offered such an approach which extends Chakravarty-Schmid description<sup>3</sup> of weak localization (WL) and generalizes Feynman-Vernon path integral influence functional technique to fermionic systems with disorder and interactions. With the aid of this technique it turned out to be possible to quantitatively explain low temperature saturation of WL correction to conductance  $\delta G^{WL}(T)$  commonly observed in diffusive metallic wires<sup>5</sup>. It was demonstrated<sup>4</sup> that this saturation effect is caused by electron-electron interactions.

It is worth pointing out that low temperature saturation of WL correction and of the electron decoherence time  $\tau_\varphi$  (extracted from  $\delta G^{WL}(T)$  or by other means) has been repeatedly observed not only in metallic wires but also in virtually any type of disordered conductors ranging from individual quantum dots<sup>6</sup> to very strongly disordered 3d structures and granular metals<sup>7</sup>. It is quite likely that in all these systems we are dealing with *the same* fundamental effect of electron-electron interactions. In order to support (or discard) this conjecture it is necessary to develop a unified theoretical description which would cover essentially all types of disordered conductors. Although the approach<sup>4</sup> is formally an exact procedure treating electron dynamics in the presence of disorder and interactions, in some cases, e.g., for quantum dots and granular metals, it can be rather difficult to directly evaluate  $\delta G^{WL}(T)$  within this technique.

One of the problems in those cases is that the de-

scription in terms of quasiclassical electron trajectories may become insufficient, and electron scattering on disorder should be treated on more general footing. Another (though purely technical) point is averaging over disorder. In our approach<sup>4</sup> disorder averaging is (can be) postponed until the last stage of the calculation which is convenient in certain physical situations. In other cases – like ones studied below – it might be, in contrast, more appropriate to perform disorder averaging already in the beginning of the whole analysis. In addition, it is desirable to deal with the model which would embrace various types of conductors with well defined properties both in the long and short wavelength limits.

In this paper we make a first step towards this unified theory. Namely, we will describe a disordered conductor by means of an array of (metallic) quantum dots connected via junctions (scatterers) with an arbitrary distribution of transmissions of their conducting channels. This model will allow to easily crossover between the limits of granular metals and those with pointlike impurities and to treat spatially restricted and spatially extended conductors within the same theoretical framework. Electron scattering on each such scatterer will be treated within the most general scattering matrix formalism<sup>8,9</sup> adopted to include electron-electron interaction effects<sup>10,11,12,13,14</sup>. Averaging over disorder will be performed within the non-linear  $\sigma$ -model technique in Keldysh formulation as first proposed by Horbach and Schön<sup>15</sup> for non-interacting electrons. This method has certain advantages over the imaginary time approach<sup>16</sup> since it allows to treat both equilibrium and non-equilibrium problems and also enables one to include Coulomb interaction between electrons in a straightforward manner<sup>17</sup>. In this paper we will analyze WL corrections to conductance merely for non-interacting electrons and will include interaction effects only phenomenologically by introducing an effective electron dephasing time  $\tau_\varphi$  as a parameter of our theory. Systematic analysis of the effect of electron-electron interactions on weak localization within this formalism will be developed elsewhere.

The structure of the paper is as follows. In Sec. II we define the basic model of a 1d array of quantum dots and outline the key features of our formalism. In Sec. III we will introduce convenient parameterization of the non-linear  $\sigma$ -model which will then be used in Sec. IV

to derive WL correction to the system conductance for the model in question. This WL correction will be evaluated for various structures in Sec. V. Sec. VI contains direct generalization of our analysis and results to the case of 2d arrays of quantum dots and is followed by a brief summary in Sec. VII. Some technical details of our calculation are presented in Appendix.

## II. THE MODEL AND FORMALISM

Let us consider a 1d array of connected in series chaotic quantum dots (Fig. 1). Each quantum dot is characterized by its own mean level spacing  $\delta_n$ . Adjacent quantum dots are connected via barriers which can scatter electrons. Each such scatterer is described by a set of transmissions of its conducting channels  $T_k^{(n)}$  (here  $k$  labels the channels and  $n$  labels the scatterers). We will ignore spin-orbit scattering and, for the sake of definiteness and simplicity, we will first focus our attention on 1d arrays only. Generalization of our analysis to other situations can be performed in a straightforward manner, as it will be demonstrated in Sec. VI of the paper.

An effective action  $S[\check{Q}]$  of an array depicted in Fig. 1 depends on the fluctuating  $4 \times 4$  matrix fields<sup>18</sup>  $\check{Q}_n(t_1, t_2)$  defined for each of the dots ( $n = 1, \dots, N-1$ ). Each of these fields is a function of two times  $t_1$  and  $t_2$  and obeys the normalization condition

$$\check{Q}_n^2 = 1. \quad (1)$$

The action of an array can be represented as a sum of two terms

$$iS[\check{Q}] = iS_d[\check{Q}] + iS_t[\check{Q}]. \quad (2)$$

The first term,  $iS_d[\check{Q}]$ , describes the contribution of bulk parts of the dots. This term reads

$$iS_d[\check{Q}] = \sum_{n=1}^{N-1} \frac{\pi}{\delta_n} \text{Tr} \left[ \frac{\partial}{\partial t} \check{Q}_n - \alpha_n H^2 ([\check{A}, \check{Q}_n])^2 \right]. \quad (3)$$

Here  $H$  is an external magnetic field,  $\alpha_n = b_n(e^2/h^2c^2)v_F d_n^2 \min\{l_e, d_n\}$ ,  $b_n$  is a geometry dependent numerical prefactor<sup>9,19</sup>,  $d_n$  is the size of  $n$ -th dot,  $l_e$  is the elastic mean free path in the dot, and  $\check{A}$  is  $4 \times 4$  matrix:

$$\check{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (4)$$

The second term in Eq. (2),  $iS_t[\check{Q}]$ , describes electron transfer between quantum dots. It has the form<sup>20</sup>

$$iS_t[\check{Q}] = \frac{1}{2} \sum_{n=1}^N \sum_k \text{Tr} \ln \left[ 1 + \frac{T_k^{(n)}}{4} (\{\check{Q}_{n-1}, \check{Q}_n\} - 2) \right]. \quad (5)$$

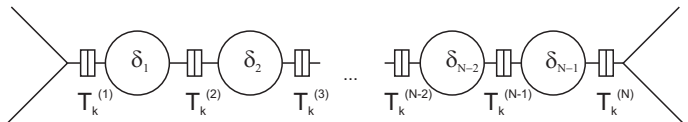


FIG. 1: 1d array of  $N-1$  quantum dots coupled by  $N$  barriers. Each quantum dot is characterized by mean level spacing  $\delta_n$ . Each barrier is characterized by a set of transmissions of its conducting channels  $T_k^{(n)}$ .

A similar expression was also considered within the imaginary time technique<sup>19,21</sup>.

Note that here the magnetic field  $H$  is included only in the term (3) describing the quantum dots while it is ignored in the term (5). Usually this approximation remains applicable at not too low magnetic fields. We will return to this point in Sec. VI.

An equilibrium saddle point configuration  $\check{\Lambda}(t_1 - t_2)$  of the matrix field  $\check{Q}(t_1, t_2)$  depends only on the time difference and has the form

$$\check{\Lambda}(t) = \int \frac{dE}{2\pi} e^{-iEt} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ g^K(E) & 0 & 1 & 0 \\ 0 & -g^K(E) & 0 & -1 \end{pmatrix}, \quad (6)$$

where  $g^K(E) = 2[1 - 2f_F(E)] = 2 \tanh(E/2T)$ . This choice of the saddle point corresponds to the following structure of the  $4 \times 4$  matrix Green function  $\check{G}$ :

$$\check{G} = \begin{pmatrix} G^A & 0 & 0 & 0 \\ 0 & \mathcal{T}G^{A*\mathcal{T}} & 0 & 0 \\ -G^K & 0 & G^R & 0 \\ 0 & \mathcal{T}G^{K*\mathcal{T}} & 0 & \mathcal{T}G^{R*\mathcal{T}} \end{pmatrix}. \quad (7)$$

Here we defined the time inversion operator  $\mathcal{T}$ :

$$\mathcal{T}f(t) = f(t_f - t), \quad (8)$$

where  $t_f$  will be specified later. Note that the function  $\check{G}$  in Eq. (7), defined for a given disorder configuration, should be contrasted from the Green function

$$\check{G}_Q = \left[ i \frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \frac{i}{2\tau_e} \check{Q} \right]^{-1} \quad (9)$$

defined for a given realization of the matrix field  $\check{Q}$ . In Eq. (9) we also introduced the electron elastic mean free time  $\tau_e$ .

## III. GAUSSIAN APPROXIMATION

In order to evaluate the WL correction to conductance we will account for quadratic (Gaussian) fluctuations of the matrix field  $\check{Q}_n$ . This approximation is always sufficient provided the conductance of the whole sample exceeds  $e^2/h$ , in certain situations somewhat softer applicability conditions can be formulated. Expanding in powers

of such fluctuations we introduce the following parameterization

$$\begin{aligned}\check{Q}_n &= e^{i\check{W}_n} \check{\Lambda} e^{-i\check{W}_n} \\ &= \check{\Lambda} + i[\check{W}_n, \check{\Lambda}] + \check{W}_n \check{\Lambda} \check{W}_n - \frac{1}{2} \{\check{W}_n^2, \check{\Lambda}\} + \mathcal{O}(W^3).\end{aligned}\quad (10)$$

It follows from the normalization condition (1) that only 8 out of 16 matrix elements of  $\check{W}$  are independent parameters. This observation provides certain freedom to choose an explicit form of this matrix. A convenient parameterization to be used below is

$$\check{W}_n = \begin{pmatrix} 0 & u_{1n} & b_{1n} & 0 \\ u_{2n} & 0 & 0 & b_{2n} \\ a_{1n} + b_{1n} & 0 & 0 & v_{1n} \\ 0 & a_{2n} + b_{2n} & v_{2n} & 0 \end{pmatrix}. \quad (11)$$

With this choice the quadratic part of the action takes the form

$$iS^{(2)} = iS_{ab}^{(2)}[a, b] + iS_{uv}^{(2)}[u, v], \quad (12)$$

where  $iS_{ab}^{(2)}[a, b]$  does not depend on  $H$  and describes diffuson modes, while  $iS_{uv}^{(2)}[u, v]$  is sensitive to the magnetic field and is responsible for the Cooperons. The diffuson part of the action  $iS_{ab}^{(2)}[a, b]$  was already analyzed in Ref. 14 and will be omitted here. Below we will focus our attention on the Cooperon contribution which reads

$$\begin{aligned}iS_{uv}^{(2)}[u, v] &= \sum_{n=1}^{N-1} \frac{2\pi}{\delta_n} \text{Tr} \left[ \frac{\partial}{\partial t} [u_{1n}, u_{2n}] - 16\alpha_n H^2 u_1 u_2 \right] \\ &+ \sum_{n=1}^{N-1} \frac{2\pi}{\delta_n} \text{Tr} \left[ \frac{\partial}{\partial t} [v_{2n}, v_{1n}] - 16\alpha_n H^2 v_1 v_2 \right] \\ &- \sum_{n=1}^N \frac{g_n}{2} \text{Tr} \left[ (u_{1n} - u_{1,n-1})(u_{2n} - u_{2,n-1}) \right. \\ &\left. + (v_{1n} - v_{1,n-1})(v_{2n} - v_{2,n-1}) \right], \quad (13)\end{aligned}$$

where  $g_n = 2 \sum_k T_k^{(n)} = 2\pi\hbar/e^2 R_n$  is the dimensionless conductance of  $n$ -th barrier. With the aid of the action (13) we can derive the pair correlators of the fields  $u_{1,2}$  and  $v_{1,2}$ :

$$\begin{aligned}\langle u_{1n}(t_1, t_2) u_{2m}(t', t'') \rangle &= \langle v_{1n}(t', t'') v_{2m}(t_1, t_2) \rangle \\ &= \frac{\delta_m}{2\pi} \delta(t_1 - t_2 + t' - t'') C_{nm}(t'' - t_1), \quad (14)\end{aligned}$$

where we defined a discrete version of the Cooperon  $C_{nm}(t)$  obeying the equation

$$\begin{aligned}\left( \frac{\partial}{\partial t} + \frac{1}{\tau_{Hn}} + \frac{1}{\tau_{\varphi n}} \right) C_{nm} + \frac{\delta_n}{4\pi} [(g_n + g_{n+1}) C_{nm} \\ - g_n C_{n-1,m} - g_{n+1} C_{n+1,m}] = \delta_{nm} \delta(t).\end{aligned}\quad (15)$$

This equation should be supplemented by the boundary condition  $C_{nm}(t) = 0$  which applies whenever one

of the indices  $n$  or  $m$  belongs to the lead electrode. Here  $\tau_{Hn} = 1/16\alpha_n H^2$  is the electron dephasing time due to the magnetic field. In Eq. (15) we also introduced an additional electron decoherence time in  $n$ -th quantum dot  $\tau_{\varphi n}$  which can remain finite in the presence of interactions. In this paper we are not aiming to further specify the interaction mechanisms and only account for them phenomenologically by keeping the parameter  $\tau_{\varphi n}$  in the equation for the Cooperon.

#### IV. WL CORRECTIONS

Let us now derive an expression for WL correction to the conductance in terms of the fluctuating fields  $u$  and  $v$ . In what follows we will explicitly account for the discrete nature of our model and specify the WL correction for a single barrier in-between two adjacent quantum dots in the array.

We start, however, from the bulk limit, in which case the Kubo formula for the conductivity tensor  $\sigma_{\alpha\beta}$  reads

$$\begin{aligned}\sigma_{\alpha\beta}(\mathbf{r}, \mathbf{r}') &= -i \int_{-\infty}^t dt' (t - t') \\ &\times \langle j_\beta(t', \mathbf{r}') j_\alpha(t, \mathbf{r}) - j_\alpha(t, \mathbf{r}) j_\beta(t', \mathbf{r}') \rangle.\end{aligned}\quad (16)$$

Following the standard procedure<sup>1,2,3</sup>, approximating the Fermi function as  $-\partial f_F(E)/\partial E \approx \delta(E)$  (which effectively implies taking the low temperature limit) and using a phenomenological description of interactions as mediated by external (classical) fluctuating fields, from Eq. (16) one can derive the WL correction in the form:

$$\begin{aligned}\delta\sigma_{\alpha\beta}^{WL}(\mathbf{r}, \mathbf{r}') &= -\frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \\ &\times (\nabla_{\mathbf{r}_1}^\alpha - \nabla_{\mathbf{r}_2}^\alpha)_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}} (\nabla_{\mathbf{r}'_1}^\beta - \nabla_{\mathbf{r}'_2}^\beta)_{\mathbf{r}'_1=\mathbf{r}'_2=\mathbf{r}'} \\ &\times \langle G^R(t, \mathbf{r}_1; t'', \mathbf{r}'_2) G^A(t', \mathbf{r}'_1; t, \mathbf{r}_2) \rangle_{\text{dis, max cross}}, \quad (17)\end{aligned}$$

which implies summation over all maximally crossed diagrams<sup>1,2,3</sup>, as indicated in the subscript. At the same time, averaging over fluctuations of  $\check{Q}$  within Gaussian approximation is equivalent to summing over all ladder diagrams. Since we are not going to go beyond the above approximation, we need to convert maximally crossed diagrams in Eq. (17) into the ladder ones. Technically this conversion can be accomplished by an effective time reversal procedure for the advanced Green function which can be illustrated as follows.

Consider, e. g., the second order correction to  $G^A$  in the disorder potential  $U_{\text{dis}}(\mathbf{x})$

$$\begin{aligned}\delta^{(2)} G^A(t', \mathbf{r}'_1; t, \mathbf{r}_2) &= -i \int_{t'}^t d\tau_2 \int_{t'}^{\tau_2} d\tau_1 \int d^3 \mathbf{x}_2 d^3 \mathbf{x}_1 \\ &\times G^A(t', \mathbf{r}'_1; \tau_1, \mathbf{x}_1) U_{\text{dis}}(\mathbf{x}_1) G^A(\tau_1, \mathbf{x}_1; \tau_2, \mathbf{x}_2) \\ &\times U_{\text{dis}}(\mathbf{x}_2) G^A(\tau_2, \mathbf{x}_2; t, \mathbf{r}_2).\end{aligned}\quad (18)$$

Making use of the property  $G^A(X_1, X_2) = G^{R*}(X_2, X_1)$ , we get

$$\begin{aligned} \delta^{(2)}G^A(t', \mathbf{r}'_1; t, \mathbf{r}_2) &= -i \int_{t'}^t d\tau_2 \int_{t'}^{\tau_2} d\tau_1 \int d^3\mathbf{x}_2 d^3\mathbf{x}_1 \\ &\times G^{R*}(t, \mathbf{r}_2; \tau_2, \mathbf{x}_2) U_{\text{dis}}(\mathbf{x}_2) G^{R*}(\tau_2, \mathbf{x}_2; \tau_1, \mathbf{x}_1) \\ &\times U_{\text{dis}}(\mathbf{x}_1) G^{R*}(\tau_1, \mathbf{x}_1; t', \mathbf{r}'_1). \end{aligned} \quad (19)$$

Setting  $t_f = t + t'$ , we rewrite this expression as follows

$$\begin{aligned} \delta^{(2)}G^A(t', \mathbf{r}'_1; t, \mathbf{r}_2) &= -i \int_{t_f-t}^{t_f-t'} d\tau_2 \int_{t_f-t}^{\tau_2} d\tau_1 \\ &\times \int d^3\mathbf{x}_2 d^3\mathbf{x}_1 G^{R*}(t_f - t', \mathbf{r}_2; \tau_2, \mathbf{x}_2) \\ &\times U_{\text{dis}}(\mathbf{x}_2) G^{R*}(\tau_2, \mathbf{x}_2; \tau_1, \mathbf{x}_1) \\ &\times U_{\text{dis}}(\mathbf{x}_1) G^{R*}(\tau_1, \mathbf{x}_1; t_f - t, \mathbf{r}'_1). \end{aligned} \quad (20)$$

Close inspection of the right hand side of Eq. (20) allows to establish the following relation

$$\delta^{(2)}G^A(t', \mathbf{r}'_1; t, \mathbf{r}_2) = \mathcal{T} \delta^{(2)}G^{R*}(t', \mathbf{r}_2; t, \mathbf{r}'_1) \mathcal{T}, \quad (21)$$

which turns out to hold in all orders of the perturbation theory in  $U_{\text{dis}}$ . As before, the time inversion operator  $\mathcal{T}$  is defined in Eq. (8) with  $t_f = t + t'$ .

As a result, the expression for  $\delta\sigma_{\alpha\beta}^{WL}$  takes the form:

$$\begin{aligned} \delta\sigma_{\alpha\beta}^{WL}(\mathbf{r}, \mathbf{r}') &= -\frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \\ &\times (\nabla_{\mathbf{r}_1}^\alpha - \nabla_{\mathbf{r}_2}^\alpha)_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}} (\nabla_{\mathbf{r}'_1}^\beta - \nabla_{\mathbf{r}'_2}^\beta)_{\mathbf{r}'_1=\mathbf{r}'_2=\mathbf{r}'} \\ &\times \langle G^R(t, \mathbf{r}_1; t'', \mathbf{r}'_2) \mathcal{T} G^{R*}(t', \mathbf{r}_2; t, \mathbf{r}'_1) \mathcal{T} \rangle_{\text{dis, ladder}} \end{aligned} \quad (22)$$

Rewriting Eq. (22) in terms of the matrix elements of the Green function (7), we obtain

$$\begin{aligned} \delta\sigma_{\alpha\beta}^{WL}(\mathbf{r}, \mathbf{r}') &= -\frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \\ &\times (\nabla_{\mathbf{r}_1}^\alpha - \nabla_{\mathbf{r}_2}^\alpha)_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}} (\nabla_{\mathbf{r}'_1}^\beta - \nabla_{\mathbf{r}'_2}^\beta)_{\mathbf{r}'_1=\mathbf{r}'_2=\mathbf{r}'} \\ &\times \langle G_{33}(t, \mathbf{r}_1; t'', \mathbf{r}'_2) G_{44}(t', \mathbf{r}_2; t, \mathbf{r}'_1) \rangle_{\text{dis, ladder}} \end{aligned} \quad (23)$$

Our next step amounts to expressing WL correction via the Green function  $\tilde{G}_Q$  (9). For that purpose we will use the following rule of averaging

$$\begin{aligned} &\langle G_{33}(t, \mathbf{r}_1; t'', \mathbf{r}'_2) G_{44}(t', \mathbf{r}_2; t, \mathbf{r}'_1) \rangle_{\text{dis}} \\ &= \langle G_{33;Q}(t, \mathbf{r}_1; t'', \mathbf{r}'_2) G_{44;Q}(t', \mathbf{r}_2; t, \mathbf{r}'_1) \rangle_Q \\ &- \langle G_{34;Q}(t, \mathbf{r}_1; t, \mathbf{r}'_1) G_{43;Q}(t', \mathbf{r}_2; t'', \mathbf{r}'_2) \rangle_Q. \end{aligned} \quad (24)$$

One can check that within our Gaussian approximation in  $u$  and  $v$  the first term in the right hand side of Eq. (24) does not give any contribution. Hence, we find

$$\begin{aligned} \delta\sigma_{\alpha\beta}^{WL}(\mathbf{r}, \mathbf{r}') &= \frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \\ &\times (\nabla_{\mathbf{r}_1}^\alpha - \nabla_{\mathbf{r}_2}^\alpha)_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}} (\nabla_{\mathbf{r}'_1}^\beta - \nabla_{\mathbf{r}'_2}^\beta)_{\mathbf{r}'_1=\mathbf{r}'_2=\mathbf{r}'} \\ &\times \langle G_{34;Q}(t, \mathbf{r}_1; t, \mathbf{r}'_1) G_{43;Q}(t', \mathbf{r}_2; t'', \mathbf{r}'_2) \rangle_Q. \end{aligned} \quad (25)$$

Let us now turn to our model of Fig. 1 in which case the voltage drops occur only across barriers. In this case Eq. (25), which only applies to bulk metals, should be generalized accordingly. Consider the conductance of an individual barrier determined by the following Kubo formula

$$\begin{aligned} G &= -i \int_{-\infty}^t dt' (t-t') \langle I(t', x') I(t, x) \\ &- I(t, x) I(t', x') \rangle. \end{aligned} \quad (26)$$

Here  $I(t, x)$  is the operator of the total current flowing in the lead (or dot) and  $x$  is a longitudinal coordinate chosen to be in a close vicinity of the barrier. Due to the current conservation the conductance  $G$  should not explicitly depend on  $x$  and  $x'$ . Comparing Eqs. (26) and (16), and making use of Eq. (25) and the relation  $I(t, x) = \int d^2\mathbf{z} j_x(t, x, \mathbf{z})$ , where  $j_x$  is the current density in the  $x$ -direction and  $\mathbf{z}$  is the vector in the transversal direction, we conclude that WL correction to the conductance of a barrier between the left and right dots should read

$$\begin{aligned} \delta G_{LR}^{WL} &= \frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \int d^2\mathbf{z} d^2\mathbf{z}' \\ &\times (\nabla_{x_1} - \nabla_{x_2})_{x_1=x_2=x} (\nabla_{x'_1} - \nabla_{x'_2})_{x'_1=x'_2=x'} \\ &\times \langle G_{34;Q}(t, x_1, \mathbf{z}; t, x'_1, \mathbf{z}') G_{43;Q}(t', x_2, \mathbf{z}; t'', x'_2, \mathbf{z}') \rangle_Q. \end{aligned} \quad (27)$$

In what follows we will assume that both coordinates  $x$  and  $x'$  are on the left side from and very close to the corresponding barrier. Let us express the Green function in the vicinity of the barrier in the form

$$\begin{aligned} \tilde{G}_Q(t, x, \mathbf{z}; t', x', \mathbf{z}') &= \sum_{nm} \{ e^{ip_n x_1 - ip_m x'_1} \tilde{G}_{mn}^{++}(t, t', x, x') \\ &+ e^{-ip_n x + ip_m x'} \tilde{G}_{mn}^{--}(t, t', x, x') \\ &+ e^{ip_n x + ip_m x'} \tilde{G}_{mn}^{+-}(t, t', x, x') \\ &+ e^{-ip_n x - ip_m x'} \tilde{G}_{mn}^{-+}(t, t', x, x') \} \Phi_n(\mathbf{z}) \Phi_m^*(\mathbf{z}'), \end{aligned} \quad (28)$$

where  $\Phi_n(\mathbf{z})$  are the transverse quantization modes which define conducting channels,  $p_n$  is projection of the Fermi momentum perpendicular to the surface of the barrier, and the semiclassical Green function  $\mathcal{G}_{mn}^{\alpha\beta}$  slowly varies in space. Eq. (27) then becomes

$$\begin{aligned} \delta G_{LR}^{WL} &= \frac{e^2}{4\pi m^2} \int_{-\infty}^t dt' \int dt'' \\ &\times \sum_{mnl} \sum_{\alpha\beta\gamma\delta=\pm 1} (\alpha p_n - \gamma p_k) (\beta p_m - \delta p_l) \\ &\times \langle \mathcal{G}_{mn;34}^{\alpha\beta}(t, t, x, x') \mathcal{G}_{kl;43}^{\gamma\delta}(t', t'', x, x') \rangle_Q \\ &\times e^{i\alpha p_n x_1 - i\beta p_m x'_1 + i\gamma p_k x_2 - i\delta p_l x'_2} \Big|_{x_1=x_2=x; x'_1=x'_2=x'}. \end{aligned} \quad (29)$$

Next we require  $\delta G_{LR}^{WL}$  to be independent on  $x$  and  $x'$ , i.e. in Eq. (29) we omit those terms, which contain

quickly oscillating functions of these coordinates. This requirement implies that  $\alpha p_n + \gamma p_k = 0$  and  $\beta p_m + \delta p_l = 0$ . These constraints in turn yield  $\gamma = -\alpha$ ,  $\delta = -\beta$ ,  $k = n$  and  $l = m$ . Thus, we get

$$\delta G_{LR}^{WL} = \frac{e^2}{\pi m^2} \sum_{mn} \sum_{\alpha\beta=\pm 1} \int_{-\infty}^t dt' \int dt'' \alpha\beta p_n p_m \times \left\langle \mathcal{G}_{mn;34}^{\alpha\beta}(t, t, x, x') \mathcal{G}_{nm;43}^{-\alpha, -\beta}(t', t'', x, x') \right\rangle_Q. \quad (30)$$

Let us choose the basis in which transmission and reflection matrices  $\hat{t}$  and  $\hat{r}$  are diagonal. In this basis the semiclassical Green function is diagonal as well,  $\mathcal{G}_{mn} \propto \mathcal{G}_{nn} \delta_{nm}$ , and Eq. (30) takes the form

$$\delta G_{LR}^{WL} = \frac{e^2}{\pi} \sum_n \frac{p_n^2}{m^2} \int_{-\infty}^t dt' \int dt'' \times \left\langle \mathcal{G}_{L,nn;34}^{++}(t, t) \mathcal{G}_{L,nn;43}^{--}(t', t'') \right. \\ \left. + \mathcal{G}_{L,nn;34}^{--}(t, t) \mathcal{G}_{L,nn;43}^{++}(t', t'') \right. \\ \left. - \mathcal{G}_{L,nn;34}^{+-}(t, t) \mathcal{G}_{L,nn;43}^{-+}(t', t'') \right. \\ \left. - \mathcal{G}_{L,nn;34}^{-+}(t, t) \mathcal{G}_{L,nn;43}^{+-}(t', t'') \right\rangle_Q. \quad (31)$$

What remains is to express WL correction in terms of the field  $\hat{Q}$  only. This goal is achieved by establishing an explicit relation between the Green function  $\hat{\mathcal{G}}$  and the field  $\hat{Q}$ . A derivation of this relation is presented in Appendix A. Here we only display the final result expressed via the fluctuating fields  $v_1$  and  $v_2$ . We obtain

$$\delta G_{LR}^{WL} = -\frac{e^2}{\pi} \sum_n \int_{-\infty}^t dt' \int dt'' \times \left\langle T_n [v_{1L}(t, t) v_{2R}(t', t'') + v_{1R}(t, t) v_{2L}(t', t'')] \right. \\ \left. + T_n^2 [v_{1L}(t, t) - v_{1R}(t, t)] [v_{2L}(t', t'') - v_{2R}(t', t'')] \right\rangle. \quad (32)$$

Note that the contribution linear in  $T_n$ , which contains the product of the fluctuating fields on two different sides of the barrier, vanishes identically provided fluctuations on one side tend to zero, e.g. if the barrier is directly attached to a large metallic lead. In contrast, the contribution  $\propto T_n^2$  in Eq. (32) survives even in this case.

Finally, applying the contraction rule (14) we get

$$\delta G_{LR}^{WL} = -\frac{e^2 g}{4\pi^2} \int_0^\infty dt \left\{ \beta [\delta_R C_{LR}(t) + \delta_L C_{RL}(t)] \right. \\ \left. + (1 - \beta) [\delta_R C_{RR}(t) + \delta_L C_{LL}(t)] \right\}. \quad (33)$$

Here  $\delta_{L,R}$  is the mean level spacing in the left/right quantum dot,  $g = 2 \sum_k T_k$  is the dimensionless conductance of the barrier and  $\beta = \sum_k T_k (1 - T_k) / \sum_k T_k$  is the corresponding Fano factor.

Likewise, the WL correction to the  $n$ -th barrier conductance in 1d array of  $N - 1$  quantum dots with mean level spacings  $\delta_n$  connected by  $N$  barriers with dimensionless conductances  $g_n$  and Fano factors  $\beta_n$  reads

$$\delta G_n^{WL} = -\frac{e^2 g_n}{4\pi^2} \int_0^\infty dt \left\{ \beta_n [\delta_n C_{n-1,n}(t)] \right.$$

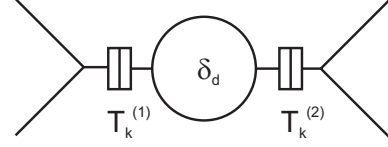


FIG. 2: Single quantum dot connected to the leads via two barriers.

$$+ \delta_{n-1} C_{n,n-1}(t) + (1 - \beta_n) [\delta_n C_{nn}(t) \\ + \delta_{n-1} C_{n-1,n-1}(t)] \}. \quad (34)$$

So far we discussed the local properties, namely WL corrections to the conductivity tensor,  $\delta\sigma_{\alpha,\beta}^{WL}(\mathbf{r}, \mathbf{r}')$ , and to the conductance of a single barrier,  $\delta G_{LR}^{WL}$ . Our main goal is, however, to evaluate the WL correction to the conductance of the whole system. For bulk metals one finds that at large scales the WL correction (17) is local,  $\delta\sigma_{\alpha,\beta}^{WL}(\mathbf{r}, \mathbf{r}') \propto \delta(\mathbf{r} - \mathbf{r}')$ . In general though, there can exist other, non-local, contributions to the conductivity tensor<sup>22</sup>. Without going into details here, we only point out that, even if these non-local terms are present, one can still apply the standard Ohm's law arguments in order to obtain the conductance of the whole sample. Specifically, in the case of 1d arrays one finds (cf.<sup>23</sup>):

$$\delta G^{WL} = \frac{1}{\sum_{n=1}^N (G_n + \delta G_n^{WL})^{-1}} - \frac{1}{\sum_{n=1}^N G_n^{-1}} \\ = \frac{\sum_{n=1}^N \delta G_n^{WL} / g_n^2}{\left( \sum_{n=1}^N 1/g_n \right)^2} + \text{higher order terms}. \quad (35)$$

Eqs. (33), (34) and (35) will be used to evaluate WL corrections for different configurations of quantum dots considered below.

## V. EXAMPLES

### A. Single quantum dot

We start from the simplest case of a single quantum dot depicted in Fig. 2. In this case the solution of Eq. (15) reads

$$C_{11}(t) = \exp \left[ -\frac{t}{\tau_D} - \frac{t}{\tau_H} - \frac{t}{\tau_\varphi} \right], \quad (36)$$

where  $\tau_D = 4\pi/(g_1 + g_2)\delta_d$  is the dwell time, and  $\delta_d$  is the mean level spacing in the quantum dot. All other components of the Cooperon are equal to zero. From Eq. (33) we get

$$\delta G_1^{WL} = -\frac{e^2 g_1 (1 - \beta_1) \delta_d}{4\pi^2} \frac{1}{1/\tau_D + 1/\tau_H + 1/\tau_\varphi}, \\ \delta G_2^{WL} = -\frac{e^2 g_2 (1 - \beta_2) \delta_d}{4\pi^2} \frac{1}{1/\tau_D + 1/\tau_H + 1/\tau_\varphi}. \quad (37)$$

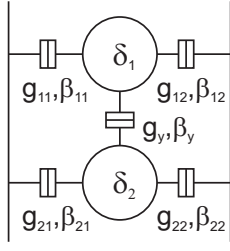


FIG. 3: Most general system with two quantum dots

According to Eq. (35) the total WL correction becomes

$$\delta G^{WL} = -\frac{e^2 \delta}{4\pi^2} \frac{g_1 g_2^2 (1 - \beta_1) + g_1^2 g_2 (1 - \beta_2)}{(g_1 + g_2)^2 (1/\tau_D + 1/\tau_\varphi + 1/\tau_H)}. \quad (38)$$

Since  $1/\tau_H \propto H^2$ , the magnetoconductance has the Lorentzian shape<sup>9</sup>. In the limit  $H = 0$  and in the absence of interactions ( $\tau_\varphi \rightarrow \infty$ ) Eq. (38) reduces to<sup>24</sup>

$$\delta G^{WL} = -\frac{e^2}{\pi} \frac{g_1 g_2^2 (1 - \beta_1) + g_1^2 g_2 (1 - \beta_2)}{(g_1 + g_2)^3}. \quad (39)$$

## B. Two quantum dots

Next we consider the most general setup composed of two quantum dots with the corresponding conductances and Fano factors defined as in Fig. 3. The Cooperon is represented as a  $2 \times 2$  matrix which zero frequency component satisfies the following equation

$$\begin{pmatrix} g_{11} + g_{12} + g_y + \gamma_1 & -g_y \\ -g_y & g_{21} + g_{22} + g_y + \gamma_2 \end{pmatrix} \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} 4\pi/\delta_1 & 0 \\ 0 & 4\pi/\delta_2 \end{pmatrix}, \quad (40)$$

where

$$\gamma_{1,2} = \frac{4\pi}{\delta_{1,2}} \left( \frac{1}{\tau_{H1,2}} + \frac{1}{\tau_{\varphi 1,2}} \right). \quad (41)$$

Defining  $\Delta = (g_{11} + g_{12} + g_y + \gamma_1)(g_{21} + g_{22} + g_y + \gamma_2) - g_y^2$ , we get

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \frac{4\pi}{\Delta} \begin{pmatrix} (g_{21} + g_{22} + g_y + \gamma_2)/\delta_1 & g_y/\delta_2 \\ g_y/\delta_1 & (g_{11} + g_{12} + g_y + \gamma_1)/\delta_2 \end{pmatrix}.$$

With the aid of Eq. (33) we derive WL corrections for all five barriers in our setup:

$$\begin{aligned} \delta G_{11}^{WL} &= -\frac{e^2 g_{11} \delta_1 (1 - \beta_{11})}{4\pi^2} C_{11} \\ &= -\frac{e^2 g_{11} (g_{21} + g_{22} + g_y + \gamma_2) (1 - \beta_{11})}{\pi \Delta}, \\ \delta G_{12}^{WL} &= -\frac{e^2 g_{12} \delta_1 (1 - \beta_{12})}{4\pi^2} C_{11} \end{aligned}$$

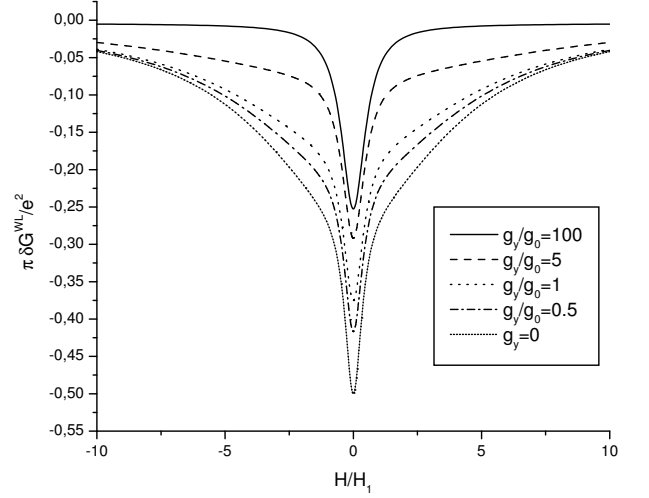


FIG. 4: The magnetoconductance of two dots of Fig. 3 for  $d_1, d_2 \gg l_e$ ,  $d_1/d_2 = 5$ ,  $g_{ij} = g_0$ ,  $\beta_{ij} = 0$ ,  $\beta_y = 0$ ,  $\tau_{\varphi 1} = \tau_{\varphi 2} = \infty$ . Here  $H_1 = 1/4\sqrt{\alpha_1 \tau_{D1}}$  is the field at which weak localization is effectively suppressed in the first dot. For  $g_y = 0$  the magnetoconductance is given by superposition of two Lorentzians with different widths (decoupled dots), while for large  $g_y$  only one Lorentzian survives corresponding to the contribution of a one “composite dot”.

$$\begin{aligned} \delta G_{21}^{WL} &= -\frac{e^2 g_{21} \delta_1 (1 - \beta_{21})}{4\pi^2} C_{22} \\ &= -\frac{e^2 g_{21} (g_{11} + g_{12} + g_y + \gamma_1) (1 - \beta_{21})}{\pi \Delta}, \\ \delta G_{22}^{WL} &= -\frac{e^2 g_{22} \delta_1 (1 - \beta_{22})}{4\pi^2} C_{22} \\ &= -\frac{e^2 g_{22} (g_{11} + g_{12} + g_y + \gamma_1) (1 - \beta_{22})}{\pi \Delta}, \\ \delta G_y^{WL} &= -\frac{e^2 g_y}{4\pi^2} [\beta_y (\delta_1 C_{21} + \delta_2 C_{12}) \\ &\quad + (1 - \beta_y) (\delta_1 C_{11} + \delta_2 C_{22})] \\ &= -\frac{e^2 g_y}{\pi \Delta} [2g_y \beta_y + (1 - \beta_y) (g_{11} + g_{12} + g_{21} \\ &\quad + g_{22} + 2g_y + \gamma_1 + \gamma_2)]. \quad (42) \end{aligned}$$

WL correction to the conductance of the whole structure  $\delta G^{WL}$  is obtained from the general expression for the conductance determined by Ohm’s law:

$$\begin{aligned} G &= [G_{11} G_{12} (G_{21} + G_{22}) + G_{21} G_{22} (G_{11} + G_{12}) \\ &\quad + G_y (G_{12} + G_{22}) (G_{11} + G_{21})] \\ &\quad / [(G_{11} + G_{12}) (G_{21} + G_{22}) \\ &\quad + G_y (G_{11} + G_{12} + G_{21} + G_{22})]. \quad (43) \end{aligned}$$

Substituting  $G_{ij} \rightarrow G_{ij} + \delta G_{ij}^{WL}$  into this formula and

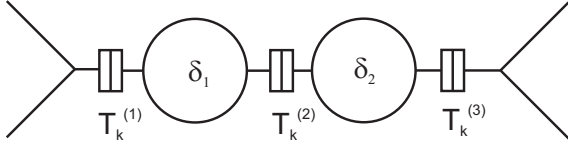


FIG. 5: Two quantum dots in series.

expanding the result to the first order in  $\delta G_{ij}^{WL}$ , we get

$$\delta G^{WL} = \sum_{i,j=1,2} \frac{\partial G}{\partial G_{ij}} \delta G_{ij}^{WL} + \frac{\partial G}{\partial G_y} \delta G_y^{WL}. \quad (44)$$

Combining Eqs. (42)-(44) we arrive at the final result for the WL correction to the conductance of the whole structure. This general result is rather cumbersome. It is illustrated in Fig. 4 for a particular choice of the system parameters. Below we will specifically consider two important limits.

First we analyze the system of two quantum dots connected in series, as shown in Fig. 5, i.e. in the general structure of Fig. 3 we set  $G_{12} = G_{21} = 0$ ,  $G_{11} = G_1$ ,  $G_y = G_2$ ,  $G_{22} = G_3$ ,  $\beta_{11} = \beta_1$ ,  $\beta_y = \beta_2$  and  $\beta_{22} = \beta_3$ . We also assume  $H = 0$  and  $\tau_\varphi = \infty$ . WL corrections to the barrier conductances then take the form

$$\begin{aligned} \delta G_1^{WL} &= -\frac{e^2 g_1(g_2 + g_3)(1 - \beta_1)}{\pi g_1 g_2 + g_2 g_3 + g_1 g_3}, \\ \delta G_2^{WL} &= -\frac{e^2 g_2(g_1 + g_3)(1 - \beta_2) + 2g_2^2}{\pi g_1 g_2 + g_2 g_3 + g_1 g_3}, \\ \delta G_3^{WL} &= -\frac{e^2 g_3(g_1 + g_2)(1 - \beta_3)}{\pi g_1 g_2 + g_2 g_3 + g_1 g_3}, \end{aligned} \quad (45)$$

while Eq. (43) reduces to

$$G = \frac{G_1 G_2 G_3}{G_1 G_2 + G_1 G_3 + G_2 G_3}. \quad (46)$$

WL correction for the whole system then reads

$$\begin{aligned} \delta G^{WL} &= -\frac{e^2 g_1 g_2^2 g_3^2 (g_2 + g_3)(1 - \beta_1)}{\pi (g_1 g_2 + g_2 g_3 + g_1 g_3)^3} \\ &\quad - \frac{e^2 g_1^2 g_2 g_3^2 (g_1 + g_3)(1 - \beta_2)}{\pi (g_1 g_2 + g_2 g_3 + g_1 g_3)^3} \\ &\quad - \frac{e^2 g_1^2 g_2^2 g_3 (g_1 + g_2)(1 - \beta_3)}{\pi (g_1 g_2 + g_2 g_3 + g_1 g_3)^3} \\ &\quad - \frac{2e^2 g_1^2 g_2^2 g_3^2}{\pi (g_1 g_2 + g_2 g_3 + g_1 g_3)^3}. \end{aligned} \quad (47)$$

In the limit of open quantum dots, i.e.  $\beta_{1,2,3} = 0$ , we reproduce the result of Ref. 23. It is easy to see that provided the conductance of one of the barriers strongly exceeds two others, Eq. (47) reduces to Eq. (39). If all three barriers are tunnel junctions,  $\beta_{1,2,3} \rightarrow 1$ , the first three contributions in Eq. (47) vanish, and only the last contribution – independent of the Fano factors – survives in this limit. If, on top of that, one of the tunnel

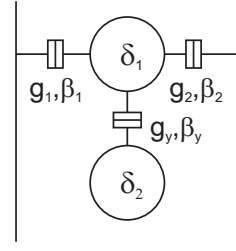


FIG. 6: The system of two connected quantum dots only one of which is in turn connected to the leads.

junctions, e.g. the central one, is less transparent than two others,  $g_2 \ll g_1, g_3$ , the result acquires a particularly simple (non-Lorentzian) form

$$\delta G^{WL} = -\frac{2e^2 g_2^2}{\pi (g_1 + \gamma_1)(g_3 + \gamma_2)}, \quad (48)$$

with  $\gamma_{1,2}$  defined in Eq. (41). Note that  $\delta G^{WL} \propto g_2^2$ , i.e. this result is dominated by the second order tunneling processes across the second barrier.

Our second example is the system depicted in Fig. 6 which corresponds to the following choice of parameters in Fig. 3:  $G_{11} = G_1$ ,  $G_{12} = G_2$ ,  $G_{21} = G_{22} = 0$ ,  $\beta_{11} = \beta_1$  and  $\beta_{12} = \beta_2$ . In addition, we assume that electrons are subject to dephasing only in the second quantum dot, i.e.  $\tau_{\varphi 1} = \infty$  while  $\tau_{\varphi 2}$  is finite. This setup allows one to analyze the so-called dephasing by voltage probes<sup>25,26</sup>. We obtain

$$C_{11} = \frac{4\pi}{\delta_1} \frac{g_y + 4\pi/\delta_2 \tau_{\varphi 2}}{(g_1 + g_2)g_y + 4\pi(g_1 + g_2 + g_y)/\delta_2 \tau_{\varphi 2}}. \quad (49)$$

In the limit  $\tau_{\varphi 2} \rightarrow \infty$  this result reduces to

$$C_{11} = \frac{4\pi}{\delta_1} \frac{1}{g_1 + g_2} \quad (50)$$

and we again arrive at Eq. (39), i.e. the second quantum dot attached to the first one does not affect the expression for WL correction. In the opposite limit of short decoherence times,  $\tau_{\varphi 2} \rightarrow 0$ , we find

$$C_{11} = \frac{4\pi}{\delta_1} \frac{1}{g_1 + g_2 + g_y} \quad (51)$$

and arrive at the WL correction<sup>26</sup>

$$\delta G^{WL} = -\frac{e^2 g_1 g_2^2 (1 - \beta_1) + g_1^2 g_2 (1 - \beta_2)}{\pi (g_1 + g_2)^3 (1 + \tau_D/\tau_\varphi^{\text{eff}})}, \quad (52)$$

where

$$\frac{1}{\tau_\varphi^{\text{eff}}} = \frac{g_y}{g_1 + g_2} \frac{1}{\tau_D} \quad (53)$$

is the electron decoherence rate induced in the first quantum dot due to coupling to the second one acting as an effective voltage probe.

### C. 1D array of identical quantum dots

Let us now turn to 1d arrays of quantum dots depicted in Fig. 1. For simplicity, we will assume that our array consists of  $N - 1$  identical quantum dots with the same level spacing  $\delta_n \equiv \delta_d$  and of  $N$  identical barriers with the same dimensionless conductance  $g_n \equiv g$  and the same Fano factor  $\beta_n \equiv \beta$ . We will also assume that the quantum dots have the same shape and size so that  $\tau_{Hn} \equiv \tau_H$  and  $\tau_{\varphi n} \equiv \tau_{\varphi}$ . For this system the Cooperon can also be found exactly. The result reads

$$C_{nm}(\omega) = \frac{2}{N} \sum_{q=1}^{N-1} \frac{\sin \frac{\pi q n}{N} \sin \frac{\pi q m}{N}}{-i\omega + \frac{1}{\tau_H} + \frac{1}{\tau_{\varphi}} + \frac{1 - \cos \frac{\pi q}{N}}{\tau_D}}. \quad (54)$$

Here  $\tau_D = 2\pi/g\delta_d$  and  $\tau_H = 1/16\alpha H^2$ .

The WL correction then takes the form

$$\delta G^{WL} = -\frac{e^2 g \delta_d}{2\pi^2 N^2} \sum_{q=1}^{N-1} \frac{\beta \cos \frac{\pi q}{N} + 1 - \beta}{\frac{1}{\tau_H} + \frac{1}{\tau_{\varphi}} + \frac{1 - \cos \frac{\pi q}{N}}{\tau_D}}. \quad (55)$$

The sum over  $q$  can be handled exactly and yields

$$\delta G^{WL} = -\frac{e^2}{\pi N^2} \left[ \left( N \frac{1 + u^{2N}}{1 - u^{2N}} - \frac{1 + u^2}{1 - u^2} \right) \times \frac{\beta(1 + u^2) + 2(1 - \beta)u}{1 - u^2} - (N - 1)\beta \right], \quad (56)$$

where

$$u = 1 + \frac{\tau_D}{\tau_H} + \frac{\tau_D}{\tau_{\varphi}} - \sqrt{\left( 1 + \frac{\tau_D}{\tau_H} + \frac{\tau_D}{\tau_{\varphi}} \right)^2 - 1}. \quad (57)$$

In the tunneling limit  $\beta = 1$  and for  $\tau_{\varphi} \rightarrow \infty$  our result defined in Eqs. (56)-(57) becomes similar – though not exactly identical – to the corresponding result<sup>27</sup>.

If  $\tau_{\varphi}$  is long enough, namely  $1/\tau_{\varphi} \lesssim E_{\text{Th}}$ , where  $E_{\text{Th}} = \pi^2/2N^2\tau_D$  is the Thouless energy of the whole array, in Eqs. (55)-(56) it is sufficient to set  $\tau_{\varphi} = \infty$ . In this case the magnetic field  $H$  significantly suppresses WL correction provided  $1/\tau_H \gtrsim E_{\text{Th}}$  or, equivalently, if

$$H \gtrsim H_N, \quad H_N = \frac{1}{8N} \sqrt{\frac{\pi g \delta_d}{\alpha}}. \quad (58)$$

In the opposite limit  $1/\tau_{\varphi} \gtrsim E_{\text{Th}}$  we find

$$\delta G^{WL} = -\frac{e^2}{\pi N} \left[ \frac{\beta \left( 1 + \frac{\tau_D}{\tau_H} + \frac{\tau_D}{\tau_{\varphi}} \right) + 1 - \beta}{\sqrt{\left( 1 + \frac{\tau_D}{\tau_H} + \frac{\tau_D}{\tau_{\varphi}} \right)^2 - 1}} - \beta \right]. \quad (59)$$

In particular, in the diffusive limit  $\tau_H, \tau_{\varphi} \gg \tau_D$  we get

$$\delta G^{WL} = -\frac{e^2}{\pi N d} \sqrt{\frac{D \tau_H \tau_{\varphi}}{\tau_H + \tau_{\varphi}}}, \quad (60)$$

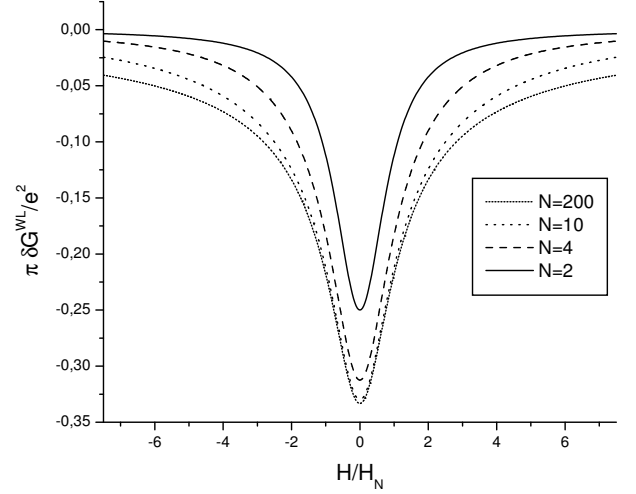


FIG. 7: Magnetoconductance of a 1d array of  $N - 1$  identical open ( $\beta = 0$ ) quantum dots in the absence of interactions ( $\tau_{\varphi} \rightarrow \infty$ ). The field  $H_N$  is defined in Eq. (58).

where we introduced the diffusion coefficient

$$D = d^2/2\tau_D. \quad (61)$$

Eq. (60) coincides with the standard result for quasi-1d diffusive metallic wire. Note, however, that the values of  $\tau_H$  within our model may differ from those for a metallic wire. The ratio of the former to the latter is  $\tau_H^{\text{qd}}/\tau_H^{\text{met}} \sim \tau_H/\tau_D$ , where  $\tau_H \sim d/v_F$  is the flight time through the quantum dot. Since typically  $\tau_H < \tau_D$  we conclude that for the same value of  $D$  the magnetic field dephases electrons stronger in the case of an array of quantum dots.

For a single quantum dot ( $N = 2$ ) Eq. (56) reduces to

$$\delta G^{WL} = -\frac{e^2(1 - \beta)}{4\pi} \frac{1}{\left( 1 + \frac{\tau_D}{\tau_H} + \frac{\tau_D}{\tau_{\varphi}} \right)} \quad (62)$$

in agreement with Eq. (38).

For two identical quantum dots in series we obtain

$$\delta G^{WL} = -\frac{e^2}{9\pi} \left[ \frac{2 - \beta}{1 + \frac{2\tau_D}{\tau_H} + \frac{2\tau_D}{\tau_{\varphi}}} + \frac{\frac{2}{3} - \beta}{1 + \frac{2\tau_D}{3\tau_H} + \frac{2\tau_D}{3\tau_{\varphi}}} \right], \quad (63)$$

i.e. the magnetoconductance is just the sum of two Lorentzians in this case.

Finally, in the absence of any interactions ( $\tau_{\varphi} = \infty$ ) and at  $H = 0$  we obtain

$$\delta G^{WL} = -\frac{e^2}{\pi} \left[ \frac{1}{3} - \frac{\beta}{N} + \frac{1}{N^2} \left( \beta - \frac{1}{3} \right) \right]. \quad (64)$$

In the limit  $N \rightarrow \infty$  this result reduces to the standard one for a long quasi-1d diffusive wire<sup>28</sup> while for any finite  $N$  we reproduce the results for tunnel barriers<sup>27</sup> ( $\beta \rightarrow 1$ ) and open quantum dots<sup>29</sup> ( $\beta \rightarrow 0$ ).



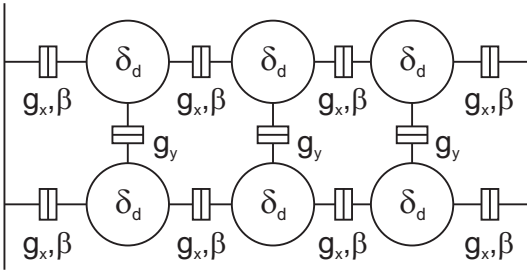


FIG. 8: 2d array of identical quantum dots. Here the number of barriers in the  $x$ -direction is chosen to be  $N = 4$ , and the number of quantum dots in the  $y$ -direction is  $M = 2$ . The barriers are characterized by the dimensionless conductances  $g_x$  and  $g_y$  and the Fano factor  $\beta$ .

## VI. GENERALIZATION TO 2D ARRAYS

Until now our analysis was only focused on structures with several quantum dots and 1d arrays. Generalization to the case of 2d and 3d systems is straightforward. Below we analyze an important case of 2d arrays.

Consider an array consisting of  $N - 1 \times M$  quantum dots. For simplicity, here we will only deal with the case of identical quantum dots, see Fig. 7. The WL correction to the conductance of this array reads

$$\delta G^{WL} = \frac{1}{N^2} \sum_{n=1}^N \sum_{m=1}^M \delta G_{nm}^{WL}, \quad (65)$$

where, similarly to Eq. (33),

$$\begin{aligned} \delta G_{nm}^{WL} = & -\frac{e^2 g_x \delta_d}{4\pi^2} \int_0^\infty dt \{ \beta [C_{n-1,n;mm}(t) \\ & + C_{n,n-1;mm}(t)] + (1-\beta) [C_{nn;mm}(t) \\ & + C_{n-1,n-1;mm}(t)] \} \end{aligned} \quad (66)$$

defines the WL correction for the barrier with ‘‘coordinates’’  $n, m$ . In order to find the Cooperons  $C_{nn';mm'}(t)$  one needs to solve the equation

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \frac{1}{\tau_H} + \frac{1}{\tau_\varphi} \right) C_{nn';mm'} + \frac{\delta_d}{4\pi} [(2g_x + 2g_y) \\ & \times C_{nn';mm'} - g_x C_{n-1,n';mm'} - g_x C_{n+1,n';mm'} \\ & - g_y C_{n,n'-1;mm'} - g_y C_{n,n'+1;mm'}] = \delta_{nn'} \delta_{mm'} \delta(t). \end{aligned} \quad (67)$$

which is directly analogous to Eq. (15). In the zero frequency limit the solution of this equation with appropriate boundary conditions reads

$$\begin{aligned} C_{nn';mm'}(\omega \rightarrow 0) = & \frac{2}{MN} \sum_{q_x=1}^{N-1} \sum_{q_y=0}^{M-1} \\ & \frac{\sin \frac{\pi q_x n'}{N} \left[ \cos \frac{\pi q_y m'}{M} + \cos \frac{\pi q_y (m'-1)}{M} \right]}{\frac{1}{\tau_H} + \frac{1}{\tau_\varphi} + \frac{g_x \delta_d}{2\pi} \left( 1 - \cos \frac{\pi q_x}{N} \right) + \frac{g_y \delta_d}{2\pi} \left( 1 - \cos \frac{\pi q_y}{M} \right)} \\ & \times \frac{\sin \frac{\pi q_x n}{N} \left[ \cos \frac{\pi q_y m}{M} + \cos \frac{\pi q_y (m-1)}{M} \right]}{\delta_{0,q_y} + 1 + \cos \frac{\pi q_y}{M}}. \end{aligned} \quad (68)$$

Combining Eqs. (65), (66) and (68), we obtain

$$\begin{aligned} \delta G^{WL} = & -\frac{e^2 g_x \delta_d}{2\pi^2 N^2} \sum_{q_x=1}^{N-1} \sum_{q_y=0}^{M-1} \\ & \frac{\beta \cos \frac{\pi q_x}{N} + 1 - \beta}{\frac{1}{\tau_\varphi} + \frac{1}{\tau_H} + \frac{g_x \delta_d}{2\pi} \left( 1 - \cos \frac{\pi q_x}{N} \right) + \frac{g_y \delta_d}{2\pi} \left( 1 - \cos \frac{\pi q_y}{M} \right)}. \end{aligned} \quad (69)$$

This result is valid provided an external magnetic field  $H$  dephases electrons predominantly inside quantum dots. This is the case provided the field is not too low<sup>30,31</sup>,  $H \gtrsim H^*$ , where  $H^* = \pi c g \delta \tau_{\text{fl}} / e d^2$ . At lower fields,  $H < H^*$ , one can apply the standard theory<sup>2</sup> developed for homogeneous metals, in which case  $1/\tau_H^{\text{met}} = 4eDH/c$ , where  $D$  is now defined in Eq. (61). Substituting the value  $H^*$  into this expression we get  $1/\tau_H^* \sim \tau_{\text{fl}}/\tau_D^2$ . Comparing this energy scale with  $1/\tau_\varphi$  and  $E_{\text{Th}}$  we immediately arrive at the condition

$$\max\{1/\tau_\varphi, E_{\text{Th}}\} \gg g^2 \delta^2 \tau_{\text{fl}}, \quad (70)$$

for which Eq. (69) is applicable at all values of  $H$ .

Turning to concrete examples we first consider the simplest case with  $N = 2, M = 2$ , which is a symmetric version of the system of Fig. 3. Eq. (69) then yields

$$\begin{aligned} \delta G^{WL} = & -\frac{e^2 g_x \delta_d}{8\pi^2} \frac{1 - \beta}{\frac{1}{\tau_\varphi} + \frac{1}{\tau_H} + \frac{g_x \delta_d}{2\pi}} \\ & - \frac{e^2 g_x \delta_d}{8\pi^2} \frac{1 - \beta}{\frac{1}{\tau_\varphi} + \frac{1}{\tau_H} + \frac{g_x \delta_d}{2\pi} + \frac{g_y \delta_d}{2\pi}}. \end{aligned} \quad (71)$$

For  $H = 0$  and  $\tau_\varphi \rightarrow \infty$  this expression reduces to

$$\delta G^{WL} = -\frac{e^2(1-\beta)}{4\pi} \left( 1 + \frac{g_x}{g_x + g_y} \right). \quad (72)$$

Next we consider an extended isotropic ( $g_x = g_y$ ) 2d array and stick to the diffusive regime

$$E_{\text{Th}} \lesssim \frac{1}{\tau_\varphi} + \frac{1}{\tau_H} \lesssim \frac{1}{\tau_D}.$$

In this case we find

$$\delta G^{WL} = -\frac{e^2}{2\pi^2} \frac{M}{N} \left[ \ln \frac{\tau_H \tau_\varphi}{\tau_D (\tau_H + \tau_\varphi)} + 2.773 - \pi \beta \right]. \quad (73)$$

The leading term in this equation matches with the standard WL correction for a 2d diffusive metallic film in the parallel magnetic field<sup>2</sup>.

Let us briefly discuss an effect of anisotropy. In the limit of small  $g_y \ll g_x/N^2$  the system reduces to a set of  $M$  essentially independent 1d arrays and, hence,  $\delta G^{WL} = M \delta G_{1d}^{WL}$ , where  $\delta G_{1d}^{WL}$  is defined in Eqs. (55,56). In the opposite limit of large  $g_y \gg g_x M^2$  electron diffusion in the direction perpendicular to the current becomes fast, and one can treat the system as a 1d array of  $N - 1$  composite quantum dots each of them consisting of  $M$  original dots. In this limit we get  $\delta G^{WL} = \delta G_{1d}^{WL}$ .

Finally, let us note that our Eq. (69) also allows to reproduce recent results<sup>31</sup> for WL correction to the conductivity of bulk granular metals. In order to handle this limit, in Eq. (69) one should formally set  $M, N \rightarrow \infty$  (which yields  $\delta G^{WL} \propto M/N$  and allows to define the conductivity) and then put  $\beta = 1$  and  $g_x = g_y$ .

## VII. SUMMARY

In this paper we have developed a theoretical approach based on a combination of the scattering matrix formalism with the non-linear  $\sigma$ -model technique. This approach allows to analyze weak localization effects for an arbitrary system of quantum dots connected via barriers with arbitrary distribution of channel transmissions. This general model can be used to describe virtually any type of disordered conductors. Employing our approach we have evaluated WL corrections to the system conductance in a number of important physical situations, e.g., for the case of two quantum dots connected to each other and to external leads in an arbitrary way (Sec. V B), as well as for 1d (Sec. V C) and 2d (Sec. VI) arrays of identical quantum dots. In a number of specific limits our general results reduce to those derived earlier by means of other approaches.

The results obtained here remain valid either in the absence of interactions or provided the interaction effects on weak localization are taken into account within a phenomenological scheme which amounts to introducing electron decoherence time  $\tau_\varphi$  as an additional parameter. The method proposed here also serves as a good starting point for a more general and systematic analysis of electron-electron interaction effects. This analysis will be worked out in our forthcoming publications.

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## APPENDIX A: RELATION BETWEEN GREEN FUNCTION $\check{G}$ AND $\check{Q}$ .

Here we will closely follow the method proposed by Nazarov<sup>20</sup>. Let us select one of the barriers in our array and denote (coordinate-independent)  $Q$ -fields in the left (right) dot with respect to this barrier as  $\check{Q}_L$  ( $\check{Q}_R$ ). Provided  $\check{Q}_{L,R}$  are slow functions of time, in the barrier vicinity one can neglect the term  $i\partial/\partial t$ . In addition, one can linearize the electron spectrum in the vicinity of the Fermi energy and replace  $\nabla^2/2m \rightarrow \pm iv_m \partial/\partial x$ , where  $v_n = p_n/m$  is the electron velocity in a given channel. As a result, for the left dot one gets

$$\left( i\alpha v_n \frac{\partial}{\partial x} + \frac{i}{2\tau_e} \check{Q}_L \right) \check{G}_{nm}^{\alpha\beta} = \delta(x-x') \delta_{nm} \delta_{\alpha\beta}. \quad (\text{A1})$$

Defining the diagonal matrix  $\hat{v} = v_n \delta_{nm}$ , and making use of the normalization condition (1), we can write the solution in the form

$$\begin{aligned} \check{G}_L^{\alpha\beta}(x, x') = & \\ & \frac{1}{4} \left[ e^{\hat{v}^{-1}x/2\tau_e} (1 - \alpha\check{Q}_L) + e^{-\hat{v}^{-1}x/2\tau_e} (1 + \alpha\check{Q}_L) \right] \\ & \times \left[ -i\alpha\delta_{\alpha\beta}\hat{v}^{-1}\theta(x-x') + \check{R}_L^{\alpha\beta} \right] \\ & \times \left[ e^{\hat{v}^{-1}x'/2\tau_e} (1 + \alpha\check{Q}_L) + e^{-\hat{v}^{-1}x'/2\tau_e} (1 - \alpha\check{Q}_L) \right]. \quad (\text{A2}) \end{aligned}$$

Here  $\check{R}_L^{\alpha\beta}$  is an arbitrary operator. Requiring  $\check{G}_L^{\alpha\beta}$  not to grow exponentially far from the barrier we arrive at the following constraints:

$$\begin{aligned} \begin{pmatrix} 1 + \check{Q}_L & 0 \\ 0 & 1 - \check{Q}_L \end{pmatrix} \begin{pmatrix} \check{R}_L^{++} & \check{R}_L^{+-} \\ \check{R}_L^{-+} & \check{R}_L^{--} \end{pmatrix} = 0, \\ \begin{pmatrix} \check{R}_L^{++} & \check{R}_L^{+-} \\ \check{R}_L^{-+} & \check{R}_L^{--} \end{pmatrix} \begin{pmatrix} 1 - \check{Q}_L & 0 \\ 0 & 1 + \check{Q}_L \end{pmatrix} = 0. \quad (\text{A3}) \end{aligned}$$

Similarly, for the right dot we obtain

$$\begin{aligned} \begin{pmatrix} 1 - \check{Q}_R & 0 \\ 0 & 1 + \check{Q}_R \end{pmatrix} \begin{pmatrix} \check{R}_R^{++} & \check{R}_R^{+-} \\ \check{R}_R^{-+} & \check{R}_R^{--} \end{pmatrix} = 0, \\ \begin{pmatrix} \check{R}_R^{++} & \check{R}_R^{+-} \\ \check{R}_R^{-+} & \check{R}_R^{--} \end{pmatrix} \begin{pmatrix} 1 + \check{Q}_R & 0 \\ 0 & 1 - \check{Q}_R \end{pmatrix} = 0. \quad (\text{A4}) \end{aligned}$$

Note that the elastic mean free time  $\tau_e$  drops out of Eqs. (A3,A4), thus indicating a very general nature of these constraints. The Green functions on the left and right barrier sides are related to each other by the  $S$ -matrix

$$\hat{S} = \begin{pmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{pmatrix} \quad (\text{A5})$$

of this barrier. This relation has the form

$$\sqrt{\hat{v}} \check{G}_R^{\alpha\beta} \sqrt{\hat{v}} = \hat{M} \sqrt{\hat{v}} \check{G}_L^{\alpha\beta} \sqrt{\hat{v}} \hat{M}^\dagger, \quad (\text{A6})$$

$$\hat{M} = \begin{pmatrix} \hat{t} - \hat{r}'\hat{t}'^{-1}\hat{r} & \hat{r}'\hat{t}'^{-1} \\ -\hat{t}'^{-1}\hat{r} & \hat{t}'^{-1} \end{pmatrix} \quad (\text{A7})$$

being the transfer matrix which satisfies  $\hat{M}\sigma_z\hat{M}^\dagger = \sigma_z$ .

Eqs. (A3), (A4) and (A6) for  $\check{R}_{L,R}^{\alpha\beta}$  can be resolved making use of the fact that in the barrier vicinity, i.e. for  $|x|, |x'| \ll v_n\tau_e$ , the Green function takes the form

$$\begin{aligned} \check{G}_{L,R}^{\alpha\beta} = & \begin{pmatrix} \check{R}_{L,R}^{++} & \check{R}_{L,R}^{+-} \\ \check{R}_{L,R}^{-+} & \check{R}_{L,R}^{--} \end{pmatrix} \\ & + \frac{i\hat{v}^{-1}}{2} \begin{pmatrix} -\text{sign}(x_1 - x_2) - \check{Q}_{L,R} & 0 \\ 0 & \text{sign}(x_1 - x_2) - \check{Q}_{L,R} \end{pmatrix}. \quad (\text{A8}) \end{aligned}$$

The operators  $\check{R}_{L,R}^{\alpha\beta}$  turn out to be diagonal in the channel indices in the basis for which the matrices  $\hat{t}$  and  $\hat{r}$

are diagonal as well. Defining the channel transmission values  $T_n = |t_n|^2$ , we get

$$\check{R}_{L,nm}^{\alpha\beta} = -\frac{i}{v_n} \delta_{nm} \left[ 1 + \frac{T_n}{4} (\{\check{Q}_L, \check{Q}_R\} - 2) \right]^{-1} \times \left( \begin{array}{cc} \frac{T_n(\check{Q}_R + [\check{Q}_R, \check{Q}_L] - \check{Q}_L \check{Q}_R \check{Q}_L)}{8} & -\frac{r_n^*(1 - \check{Q}_L)}{2} \\ \frac{r_n(1 + \check{Q}_L)}{2} & \frac{T_n(\check{Q}_R - [\check{Q}_R, \check{Q}_L] - \check{Q}_L \check{Q}_R \check{Q}_L)}{8} \end{array} \right), \quad (\text{A9})$$

$$\check{R}_{R,nm}^{\alpha\beta} = -\frac{i}{v_n} \delta_{nm} \left[ 1 + \frac{T_n}{4} (\{\check{Q}_L, \check{Q}_R\} - 2) \right]^{-1} \times \left( \begin{array}{cc} \frac{T_n(\check{Q}_L + [\check{Q}_R, \check{Q}_L] - \check{Q}_R \check{Q}_L \check{Q}_R)}{8} & \frac{r_n'(1 + \check{Q}_R)}{2} \\ -\frac{r_n'^*(1 - \check{Q}_R)}{2} & \frac{T_n(\check{Q}_L - [\check{Q}_R, \check{Q}_L] - \check{Q}_R \check{Q}_L \check{Q}_R)}{8} \end{array} \right). \quad (\text{A10})$$

In order to find WL correction (30) it is sufficient to determine the Green function (A9) only in the left dot:

$$\check{R}_{L,nm}^{++} = \frac{T_n \delta_{nm}}{2v_n} \begin{pmatrix} 0 & 2\Delta u_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & g^K \Delta u_1 & 0 & 0 \\ -\Delta v_2 g^K & 0 & 2\Delta v_2 & 0 \end{pmatrix},$$

$$\check{R}_{L,nm}^{--} = \frac{T_n \delta_{nm}}{2v_n} \begin{pmatrix} 0 & 0 & 0 & 0 \\ -2\Delta u_2 & 0 & 0 & 0 \\ 0 & \Delta v_1 g^K & 0 & -2\Delta v_1 \\ -g^K \Delta u_2 & 0 & 0 & 0 \end{pmatrix},$$

$$\check{R}_{L,nm}^{+-} = \frac{ir_n^* \delta_{nm}}{2v_n} \check{A}, \quad \check{R}_{L,nm}^{-+} = \frac{ir_n \delta_{nm}}{2v_n} \check{A}, \quad (\text{A11})$$

where  $\Delta u_{1,2} = u_{1,2R} - u_{1,2L}$ ,  $\Delta v_{1,2} = v_{1,2R} - v_{1,2L}$  and

$$\check{A} = i \begin{pmatrix} -2i & -2u_{1L} & 0 & 0 \\ 2u_{2L} & 0 & 0 & 0 \\ -ig^K & -v_{1L}g^K - g^K u_{1L} & 0 & 2v_{1L} \\ v_{2L}g^K + g^K u_{2L} & ig^K & -2v_{2L} & -2i \end{pmatrix}.$$

Here  $R_{L,nm}^{\alpha,\beta}$  has been expanded to the first order in  $u_{1,2}$  and  $v_{1,2}$ . From Eqs. (A8) and (A11) we find

$$G_{L,nn;34}^{++} = -\frac{v_{1L}}{v_n}, \quad G_{L,nn;34}^{--} = \frac{-v_{1L} - T_n \Delta v_1}{v_n},$$

$$G_{L,nn;34}^{+-} = -r_n^* \frac{v_{1L}}{v_n}, \quad G_{L,nn;34}^{-+} = -r_n \frac{v_{1L}}{v_n};$$

$$G_{L,nn;43}^{++} = \frac{v_{2L} + T_n \Delta v_2}{v_n}, \quad G_{L,nn;43}^{--} = \frac{v_{2L}}{v_n},$$

$$G_{L,nn;43}^{+-} = r_n^* \frac{v_{2L}}{v_n}, \quad G_{L,nn;43}^{-+} = r_n \frac{v_{2L}}{v_n}. \quad (\text{A12})$$

Substituting these expressions into Eq. (31), after some transformations we arrive at Eq. (32).

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