Credit Default Swap Markets and Credit Risk Pricing – A Comparative Study –

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Yalın Gündüz, MSc.

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Referentin: Prof. Dr. Marliese Uhrig-Homburg

Korreferent: Prof. Dr. Detlef Seese

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To my parents, Ufuk and Güngör Gündüz

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Chapter 1

Introduction

In the 21^{st} century, the credit market has witnessed several major defaults including the defaults of Argentina and Enron. These two events had a wide spread impact on businesses, and were treated as case studies on how an entity approached default. As it is the case for many other default events, Argentina and Enron had their unique path that led them to bankruptcy. Argentina was affected negatively from the Russian default in 1998 and from the collapse of Brazil's exchange-rate-based stabilization program in 1999 (Zhang (2003)). The country entered the year 2001 with many financial problems. The government could not reverse the ongoing situation in Argentina, and after the increase of the severity of problems, the country announced default in December 2001. Unlike Argentina, the default of Enron was more of a surprise to financial markets. The auditing scandal of Arthur Andersen came out, and Enron's financial health was questioned only about a period from October to December 2001. When filed for Chapter 11 bankruptcy in December 2001, the credit ratings have only just been reduced to below investment grade by Moody's and Standard & Poor's (Longstaff, Mithal, and Neis (2005)).

What differentiates the defaults of Argentina and Enron from those in previous decades is the presence of an infant credit default swap market. These defaults probably did not lead to more serious financial difficulties or any chain reactions due to use of credit derivatives, which provide efficient allocation of risks (Deutsche Bank Research (2004)). Credit default swaps belong to the large family of credit derivatives, which are among the most successful financial innovations of the last decade. Recent practice in financial engineering has focused on these new instruments, which is reflected in a rapidly expanding market. Credit derivatives are contracts whose payoffs depend on the creditworthiness of corporate or sovereign entities. According to British Bankers' Association's Credit Derivatives Survey (2006), the global market is expected to reach USD 33.1 trillion by the end of 2008. Among other over-the-counter traded derivatives, such as interest rate, equity, and FX derivatives, the credit derivatives market is one of the fastest expanding branches of instruments in terms of volume (Bank for International Settlements (2005)).

Credit derivatives make risk management more efficient and flexible for financial institutions. By reducing the aggregate risk in the economy, credit derivatives may also diminish the chain effects of any individual defaults such as Argentina and Enron. Banks have higher power to alleviate financial difficulties, which results in a more stable banking sector. Moreover, the credit risk which has been primarily overtaken by banks in the past, can now be distributed to other financial institutions such as hedge funds or insurance companies. Therefore, risks are better allocated in the presence of credit derivatives. It is necessary to understand how markets of credit derivatives operate and how pricing can be accurately done. This study contributes in understanding some of the issues raised with the introduction of these multi-dimensional instruments, including the functioning of its markets and pricing.

Alternative credit derivative products have been developed to satisfy the different needs of counterparties. The term “credit derivative” refers to a wide variety of instruments that have a similar purpose but not necessarily the same features. The basic choices comprise of credit default products, credit spread products, and total return products. Credit default products are commonly used to offset default risk, whereas credit spread products offset the whole credit risk, i.e. the risk of increasing or decreasing spreads. The third type, total return products, transfers both credit and market risk between counterparties. There are more advanced forms of credit derivatives, which include credit-linked notes, basket credit derivatives, and asset-backed securities.

This study focuses on single-name credit default swaps (CDSs), which have a 33 per cent share in the expanding market, being the most frequently traded type of credit derivatives (British Bankers’ Association (2006)). A credit default swap is a contract that provides insurance against the risk of default of a specific entity. In this system, the buyer agrees to make periodic payments to the seller in exchange for compensation in case of a predefined default by the entity specified in the contract. Counterparties have to agree upon these periodic payments, the CDS premiums, which are a percentage of the contract’s notional amount in basis points upon contract initialization. The seller pays nothing, if default does not occur. If the specified entity defaults during the lifetime of the CDS, there are two forms of settlement: In a “physical” settlement, the buyer delivers the eligible bonds of the underlying defaulted entity to the seller (these “deliverable obligations” might cover a set of bonds with the same rank as the underlying bond), in exchange for the contract notional amount. In a cash settlement, however, the buyer keeps the underlying bond, but is compensated for the loss. In either case, the buyer's loss is fully covered. An earlier British Bankers' Association report shows that 86 per cent of the transactions contain a physical settlement clause, whereas the rest are based on cash settlements (British Bankers' Association (2004)).

A Numerical Example for a CDS Contract:

An example on the mechanics of CDS would be as follows: Let us suppose, e.g. on June 20, 2007, the insurance buyer agrees to enter into a 5-year CDS contract with the seller, written on a bond of DaimlerChrysler AG, with a CDS premium of 80 basis points, on a contract notional amount of USD 5 Million. The buyer may or may not own the corporate bonds of DaimlerChrysler. Let us assume the buyer owns 5,000 of underlying corporate bonds that mature on April 15, 2012, each having a par value of USD 1,000, so that the buyer would have fully covered protection against the loss (5,000 $\times$ 1,000). In exchange of the protection, the buyer has to pay quarterly installments of approximately 1/4 $\times$ 80 basis points of the notional (depending on the actual days in a quarter). In monetary terms, this corresponds to quarterly payments of 5,000,000 $\times$ 1/4 $\times$ 0.0080 = USD 10,000. The buyer will be paying this amount quarterly for 5 years. If default does not happen, the seller pays nothing. If it happens during the lifetime of the CDS, there can be either physical or cash settlement.

"Credit events" are default occurrences described in the Credit Derivatives Definitions by the International Swaps and Derivatives Association (ISDA) in 1999 (revised in 2003). They include the bankruptcy, obligation acceleration, obligation default, failure to pay, repudiation/moratorium, and restructuring of the underlying entity. All these instances require the seller to make the full notional payment to the buyer of the CDS. The default of Argentina and Chapter 11 filing for bankruptcy of Enron are examples of recent credit events. Nevertheless, the lack of consensus on the definition of a credit event is still seen as the major drawback of a CDS contract in practice. The British Bankers' Association (2004) reveals that of the major problems incurred in the CDS setting, the non-agreement on the nature of a credit event is ranked first. This issue underscores the importance of standardization for further development of the market.

A typical CDS contract possesses several other attributes. For instance, the maturity of the contract describes the coverage of the insurance in terms of years. The most common practice in the industry is to agree on a 5-year contract, whereas liquid entities may have CDSs in the range from 1 to 10 years. Another important contractual attribute is the rank of the underlying, which can be either senior or subordinated. Due to the difference in priority of payments to debt holders, subordinated CDSs command a higher insurance premium than senior CDSs. A third aspect is the restructuring clause applicable in the contract. The European and North American clauses differ in that North American contracts limit the set of deliverable bonds in case of default, which again should be reflected in CDS premiums (see Blanco, Brennan, and Marsh (2005); Houweling and Vorst (2005)).

Another major issue is the counterparty risk inherent in CDS trades. Nevertheless, this risk is reduced by the fact that trades are usually conducted between dealers of major institutions with relatively high credit ratings. As a consequence, the composition of market participants differs from that of the corporate bond market. According to Blanco, Brennan, and Marsh (2005), this structural difference is one reason why the CDS market provides timelier price information than the bond market. In addition, two further aspects work towards the lead effect in the price discovery process. First, short-sales constraints in the bond market are not present in the CDS market, as CDS contracts can be set up synthetically at any time. Moreover, the transfer of credit risk can be done in relatively higher volumes in a single contract. Second, the CDS market blends participants from different pools. Both aspects make the CDS market the easiest place to trade credit risk.

This study analyzes the credit default swaps in two aspects: First, the markets where credit default swaps are traded are discussed. In order to understand the markets of CDSs, alternative trading venues are analyzed with one of their most discriminating component, their liquidity. Until recently, the non-intermediated over-the-counter market (OTC) and the intermediated inter-dealer broker (IDB) market have been the alternatives. The analysis in Chapter 2 starts with describing the instrument, and continues by introducing the OTC and the IDB markets. The differences in transparency, immediacy, and level of trade execution are highlighted. As an alternative, how electronic brokerage might replace voice brokerage in the long run is discussed. Current hybrid market structures of brokers are given as an example for dual platforms. Afterwards, an empirical analysis is undertaken so as to look at the determinants of liquidity in the brokered market. It has been shown that various contract specifications are a determinant of the bid-ask spread. Finally, the liquidity in OTC and IDB markets are compared, where it has been found out that the higher transaction costs in the IDB market may account for the added value of the brokerage services.

The second aspect, which is considered in the remaining of the study is how credit default swaps are correctly priced. This is a challenging issue: Recalling the example of Argentina and Enron defaults, should the financial credit risk of an entity be modeled as a continuous process which may eventually deteriorate in time (case of Argentina) or as a surprise event which happens at short notice (case of Enron)? The CDS prices of entities fully reflect the financial health, as can be seen from Figures 1.1 and 1.2. Figure 1.1 depicts the CDS prices of Argentina from an interdealer broker for the year 2001. The evolution of the CDS price shows how the credit risk of the entity deteriorated over time. In contrast, Figure 1.2 shows how the credit market did not expect a deterioration of financial health until October 2001.

Figure 1.1: Premiums of Argentina CDSs (bps) vs. Time

The field of credit risk modelling has been trying to give a robust answer to the question of what the theoretically fair price of credit risk should be. Structural models assume the asset value of the firm to follow a Brownian motion, and this is in accordance from what we observe from the case of Argentina. There is a continuous process, and the entity defaults as a result of gradually deteri-

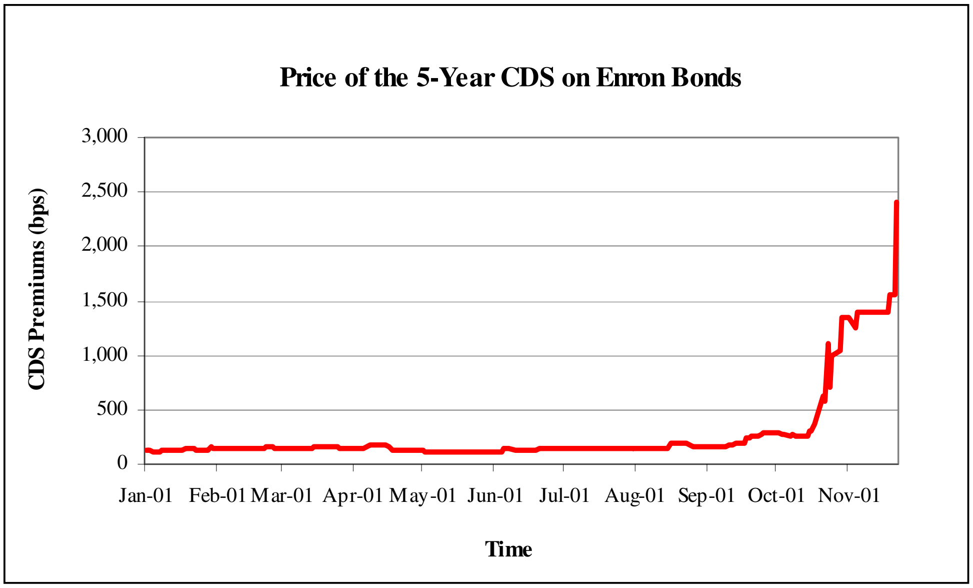

Figure 1.2: Premiums of Enron CDSs (bps) vs. Time

orating credit quality. On the other hand, reduced-form models accommodate surprise defaults, which we observe in the case of Enron. There is a jump probability modeled as a Poisson process. Unfortunately, neither the theory nor the empirical studies have so far reached a consensus on which type of framework better prices credit risk. Structural models have been criticized for their inaccurate predictions, whereas intensity models have been thought to lack economic intuition. Chapter 3 gives a brief overview on empirical studies with credit risk. First, tests of structural models with bond prices are introduced, with a further breakdown into exogenous and endogenous default barrier models. Then, studies with reduced-form models are discussed. The chapter continues with empirical studies with credit default swaps, both with those which models are tested, and with those in which various features of CDSs are highlighted.

The main question in Chapter 3 has been which type of framework better prices credit risk. The overview in Chapter 3 does not give a clear answer to this question since alternative studies with different datasets have yielded mixed results. In order to give this answer, this study puts forward that in order to compare the CDS pricing ability of different frameworks, the same procedures have to be applied to the same dataset. For this purpose, Chapter 4 starts from the basic structures of the structural and reduced-form frameworks, and provides a basis of comparison. The most basic structure of a structural model has been taken as the initial approach of Merton (1974). On the other hand, the constant intensity setup of Jarrow and Turnbull (1995) has been selected to represent the basic case of reduced-form models. A third approach has also been brought to analysis. Support Vector Machines (SVM) method has shown competitive empirical performance to traditional neural network approaches in recent studies. So, the out-of-sample CDS price prediction performances of three alternative approaches are tested. Structural models having the highest financial structure on one edge, and the reduced-form models having a lower financial structure, could be then compared with the SVM method with no financial structure at all. The analysis is carried out in two parts: First, companies are divided in risk classes that ought to have the same risk characteristics with the same rating, seniority, and currency. This "cross-sectional" analysis looks at whether prices of CDSs of companies in a risk class are a good indicator of the prices of CDSs of other companies in the same risk class. The second analysis looks at individual time series of prices of companies. This "time series" analysis hypothesizes that CDS prices are a good indicator of future CDS prices, namely the one-day, five-day, and ten-day-ahead prices. The three approaches are compared regarding these aspects, and out-of-sample prediction errors are tabulated. The results indicate that although the intensity model incorporates early default, the Merton model has competitive performance in cross-sectional and time series analyzes. The SVM method has failed in the cross-sectional setup, but has overperformed the financial models in the time series setup.

The setup in Chapter 4 accommodates the simplest versions of structural and reduced-form models. In order to correctly price CDSs, a more comprehensive setup can be built. Chapter 5 considers the state-of-the-art versions of structural and reduced-form models: By using leverage as a key credit risk variable, the two frameworks are brought into close proximity. On the structural side, the stochastic leverage model of Collin-Dufresne and Goldstein (2001) (CDG) is tested. In this model, the interest rate process follows Vasicek (1977) dynamics, and the asset value follows a geometric Brownian motion. Moreover, unique for the CDG model, the leverage follows a stationary process. If the leverage ratio of the firm is lower than a certain threshold, the firm issues new debt, and would not issue if the ratio is above the target. In order to understand whether the modeling structure makes a difference, this study contributes to the literature by developing a reduced-form model which is comparable to the CDG model in pricing the CDSs. The intensity model that this study utilizes has an adjusted discount rate as the affine sum of three variables: a constant, which represents the systematic risk; the short rate, which also follows a Vasicek process; and the log-leverage ratio, which follows exactly the same dynamics as in the CDG model. This setup creates a fair comparison possibility between the structural and reduced-form models in pricing CDSs. For the implementation, the interest rates which follow a Vasicek process are calibrated using a Kalman filter. In contrast to Chapter 4 which uses only CDS prices for calibration, bond prices, stock prices, and balance sheet information are utilized in this section to estimate model parameters. Finally, a simulation is used to generate paths of the leverage and interest rate processes in order to correctly price credit default swaps. In-sample fit to bonds and out-of-sample fit to CDSs are presented as the main results of the chapter. The prediction errors are then discussed in the light of structural and reduced-form model structures. It is shown that the out-of-sample prediction results with both models are similar, with the intensity model slightly dominating the CDG model, possibly due to more free parameters. Chapter 6 concludes with remarks and provides implications for future research.

Chapter 2

Credit Default Swap Markets

Financial markets are born, they evolve and become extinct over time. New necessities for products arise the need for a new market. After abundant liquidity is established, markets are constructed where trades for new financial products take place. The continuous growth of the market is maintained, if the product satisfies the needs of the counterparties. After time, in such cases where the product no longer meets the demands, it is of no surprise that the market eventually becomes extinct.

Credit default swap markets also follow this path of market maturity. Obviously, being still in its infant stages, CDS markets are still developing. The product has started to be traded in the early 1990s by direct over-the-counter trades via phone or quotation. As the market expanded, interdealer brokers emerged. Finally in 2007, Eurex initiated the first exchange-traded credit derivative products, which are based on the iTraxx Europe CDS index. Within this structure, it is highly interesting to understand how these markets operate, and how do individual venues contribute to the liquidity of trades. In this chapter, the basic over-the-counter and interdealer broker markets are contrasted and hybrid structures of brokers are highlighted. Then, a dataset of CDS prices is presented which will be further used in analyzing liquidity of these venues. Overall, this section of the study serves for understanding one of the many dimensions of the CDSs, the markets in which they are functioning.

2.1 Market Structures

2.1.1 The OTC and Interdealer Broker Market

Credit derivatives are most commonly traded on the over-the-counter (OTC) market, where interdealer trades are accomplished via the use of different matching technologies. Similar to other OTC derivatives, the most usual type of trade is transacted directly between two dealers over the telephone. Since the dealer has to search for a matching counterparty, this method is costly and time consuming. Compared to on-exchange derivatives markets, this kind of OTC market is opaque, non-anonymous, and highly fragmented. In recent years, increasingly more trades have been conducted through interdealer brokers (IDBs), who match buy- and sell-side dealers while offering some additional services beyond the pure matching function. The ISDA (2004) Operations Benchmarking Survey indicates that 34 per cent of the credit derivatives trades are arranged by brokers. This relatively high market share suggests that their services must provide some value to the dealers.

In other markets such as the government bond market, most of the interdealer brokerage firms operate either on a fully automated electronic trading system or a voice-based system. In a voice broking setting, although the brokers keep track of the quotes electronically, the dealers must still contact a broker over the telephone to place an order or to have a trade executed. Such an interaction between the dealer and the broker may provide information that may increase the speed and probability of matching customer orders. This will help to condition trades on a broader set of information which will improve the chance of order execution. What a broker can do is to surmise if there is more size behind the order than revealed and learn more about a dealer's trading incentives and true preferences. Meanwhile, the dealer can leave order contingencies with the broker. Obviously, one of the most important assets of an IDB is the network of dealers who are willing to offer liquidity. This hidden supply of liquidity is sought by the broker in order to complete a client's trade. Both electronic and voice brokers preserve the anonymity of the dealer. However, by using a voice broker, the dealer may opt to dispense with anonymity. This option has value depending on market conditions and the dealer's motivation for trading.

Because of this higher value added to the customer, it is not surprising that voice brokers charge a higher commission fee in practice. Barclay, Hendershott, and Kotz (2006) mention that for the US government bond IDB market, a voice broker's charge is roughly double the commission levied by electronic brokers. This is an explicit trading cost for the customer, who has to weigh this cost against the implicit cost incurred by any delay in trade execution. While the first type of cost is known before the trade, the second type is only ascertainable after a trade has been accomplished.

2.1.2 Hybrid Market Model of CreditTrade

According to the recent study of Gündüz, Lüdecke, and Uhrig-Homburg (2007), during the early years, when the market conditions were not yet ready for automation, the CDS brokerage market was only voice based. As the market became more liquid, CreditTrade and other major IDBs adapted a different, innovative strategy from IDBs operating in the FX or Treasury markets. By integrating voice brokerage and electronic brokerage under one roof, CreditTrade and others could internalize the competition and gained revenues from both matching technologies. In so doing, they succeeded in coping with potential competition from fully automated trading systems. These IDBs recognized at an early stage of the market's development that the fully automated trading of CDSs would not be achievable in the near future. By complementing voice brokerage, they could not only offer valuable intermediation services when necessary, but also enhance their efficiency through electronic brokerage and reach economies of scale. This market structure somewhat resembles an electronic trading system with an integrated upstairs market[1] run by the IDBs.

In CreditTrade's market model, the electronic system functions as follows: The firm offers a platform where dealers can enter quotes or hit existing quotes directly. This is a hit-and-take system where trades are triggered by dealers without using the voice broker. The firm's revenue is based on commissions per trade and a membership fee is not charged. Voice brokers do not have incentives to discourage customers from using the electronic platform, since the company charges a slightly lower commission for interdealer trades accomplished via its electronic platform compared to those executed by the voice bro-

[1] Bessembinder and Venkataraman (2004)'s empirical results support the work of Grossman (1992), who suggests that an upstairs market serves as a pool for unexpressed large orders.

kers. Commission schedules are defined in terms of basis points that increase with CDS premiums. The actual commission is computed as the product of the notional, maturity, and basis points (with reference to the strike interval of the premium), and is charged to both sides of the trade. Each client has a different decreasing scheme of basis points with increasing volume, which encourages more transactions. After a trade is executed, the details of the trade are processed, and a trade confirmation is sent to the buyers/sellers notifying them of their counterparty. Finally, the sides initiate their own post-trade processing, which is normally conducted by the Depository Trust and Clearing Corporation (DTCC).

With this dual approach to CDS trading, the dealers are offered a choice between two trading venues that differ with respect to trading costs, level of trade execution services, and market transparency. Unfortunately, electronic brokerage data from CreditTrade that could be used for analyzing the determinants of a dealer's choice of trading venue is not available. Nevertheless, recent literature allows us to derive a view of the market. As discussed before, the higher commissions charged for voice broking are due to the services supplied beyond the pure transactional service provided by the electronic trading system. The extent to which voice broking is used will vary depending on trade size, trade complexity, market conditions, and CDS features, such as currency. While the larger and more complex trades may be left with the voice broker, the electronic system can be used for simpler and smaller-sized transactions in the most widely traded CDS currencies, USD and EUR. However, if the volatility of the underlying market increases or the CDS market is exposed to asymmetric information, then dealers will be less willing to have their orders revealed on electronic quotation and will prefer trading via the voice broker.

Under these market conditions the IDBs can offer greater liquidity because their market provides access to a wide range of institutions that supply liquidity. The two trading venues also offer a choice between different degrees of market transparency. For instance, the voice brokerage system with less transparency may be preferred by traders with private information. Uninformed traders also stand to benefit by dealing through the voice brokers under conditions of asymmetric information; the IDB is able to certify them as uninformed, which results in trades at better prices. The next section presents a discussion on the effects of increased automation and transparency through electronic platforms.

2.1.3 Electronic Trading and Transparency in the CDS Market

The outlook on the CDS market is towards automation of the full trade process. Using Web-based technology, IDBs offer screen-based transaction services to facilitate the execution of trades and to disseminate pre-trade information via electronic platforms. However, none of the systems provides a forum for automated trade execution. Compared to similar interdealer markets, such as the FX market and the US Treasury bond market, which also rely on voice and electronic IDBs, the OTC market for credit derivatives has reached a relatively low level of automation for trade processes. Apart from trade execution, the ISDA (2004) Survey reports significant improvements in the automation of key functions for trading credit derivatives. For example, the auto-matching of trades as a method of trade confirmation was used for the very first time in 2004 and managed to corral four per cent of all trades. Nevertheless, the survey reveals that there are still potential gains to be had from automation with respect to improving the operational efficiency of the front office, where many process frictions occur due to data problems (e.g. errors in trade data, missing and/or untimely data). The more recent ISDA (2006) Survey indicates that among other derivatives, the credit derivatives market will be subject to most plans to increase automation in all trade phases. The rapid growth of volume in the OTC derivatives market has led to an industry consensus on the need to achieve "straight through processing" (STP), which means enabling the entire process from trade initiation to settlement to be conducted electronically. In January 2004, the ISDA issued a strategic plan calling for substantial industry automation of all OTC derivatives products by the end of 2006. Meanwhile, a number of firms offer valuable STP solutions to the industry for improving the operational efficiency of the OTC credit derivatives market.[2] Improved reference data management in particular represents a major step towards the achievement of STP.

The research on electronic platforms generally considers transparency to be an important design feature of financial markets. The implementation of electronic platforms by interdealer brokers yields more transparent markets, in which (anonymous) quotes can be tracked. Nevertheless, it is still debated in the literature whether higher transparency will lead to fairer markets with better liquidity and price discovery. The evidence on transparency as summarized by O'Hara (1995) includes cases where increased transparency reduces adverse selection costs so that dealers can spot other traders who are more informed; this in turn reduces spreads. On the other hand, some studies put forward that opaque markets may help to improve liquidity (Bloomfield and

[2] See i.e. http://www.finextra.com, "Creditex spin-off T-zero to provide STP for credit derivatives", published on July 29, 2005.

O'Hara (2000)). Another argument is that under increased transparency, the dealers in a quote-driven market would be less willing to reveal their strategies, which would also lead to lower liquidity (Madhavan, Porter, and Weaver (2005)). In addition, the model proposed by Madhavan (1996) predicts that transparency can worsen price volatility. The tradeoff between accessing information and revealing identities is hard to separate. Therefore, institutional traders would be at an advantage if they were able to reach fairer prices through a transparent market while remaining anonymous (which is feasible in many electronic interdealer brokerage platforms). Although there are no empirical studies on electronic CDS markets, some studies analyze the effects of market transparency in related markets, e.g. the bond market. Bessembinder, Maxwell, and Venkataraman (2006) have investigated the introduction of the TRACE[3] system and found that execution costs substantially decreased for bonds eligible for the electronic market.

Most of the evidence above is concerned with increased market transparency for the public. However, the market for CDSs is still a closed shop in which dealers are not willing to convey their quotes to outsiders. As in other markets, the vested interests of the dealers slow the train towards fully electronic trading, because trading profits will erode as markets become more centralized and more transparent. Hence, the discussion on market transparency is mostly restricted to this shop gaining deeper insight into the market. Undoubtedly, with the collection and distribution of quotes and prices, the market is becoming more transparent, although the gains are limited to a countable number of market participants. Obviously, one aim is to prevent outsiders who do not

[3] In July 2002, the National Association of Securities Dealers (NASD) began to report transactions in around 500 bonds through Trade Reporting and Compliance Engine (TRACE), which constituted a major step towards the market's transparency.

add value to the market from free riding on dealers' quotations. To promote further growth, closed shop trading should be reduced over time with an eye to opening the market to a wider public. Eroding profits should then be compensated for by the larger trading volume. An increase of market transparency and liquidity can be expected from the listing of exchange-traded credit derivative products based on the iTraxx Europe CDS index, which has been initiated by Eurex in March 2007. The iTraxx index consists of the 125 most liquid single name CDSs and was exclusively licensed to Eurex by the International Index Company (IIC) in July 2005. However, the Eurex platform has so far been unable to attract abundant liquidity. This could be most probably due to the unwillingness of the "closed club" of dealers to give out their quotes explicitly, as discussed before. In parallel to this, a short-term success of this platform is not expected by market participants.[4]

2.2 Credit Default Swap Data

In order to understand the functioning and pricing in markets of credit default swaps, daily indicative CDS bid-ask quotes are retrieved from CreditTrade, an interdealer brokerage company. For the period between January 2001 and January 2005, there are over 235,000 price quotes for liquid CDS contracts. The full set of prices comes from voice broking, and the number of daily observations rises from 70-80 in 2001 to around 300 in 2005. The dataset consists of 256 entities from a wide range of countries from Europe, the Middle East, and North and Latin America. The underlying entities are mainly corporates or banks, although CDSs written on sovereign entities also exist.

[4]Quoted from private communication with Eurex executive Ms. M. Dinc, 2007.

In a CDS contract, the financial variable of interest is the CDS premium. Obviously, the CDS premiums should be driven by the credit risk of the underlying entity; the higher the credit risk, the higher the CDS premium should be. This credit risk reflected in the premiums has been subject to recent investigation in the finance literature. Pricing of CDS will be analyzed in a subsequent chapter. Figure 2.1 plots the CDS bid-ask midpoints as a function of credit quality. As can be seen from the figure, the dataset is in line with the theoretical hypothesis that the higher the risk of default, the higher the insurance fee.

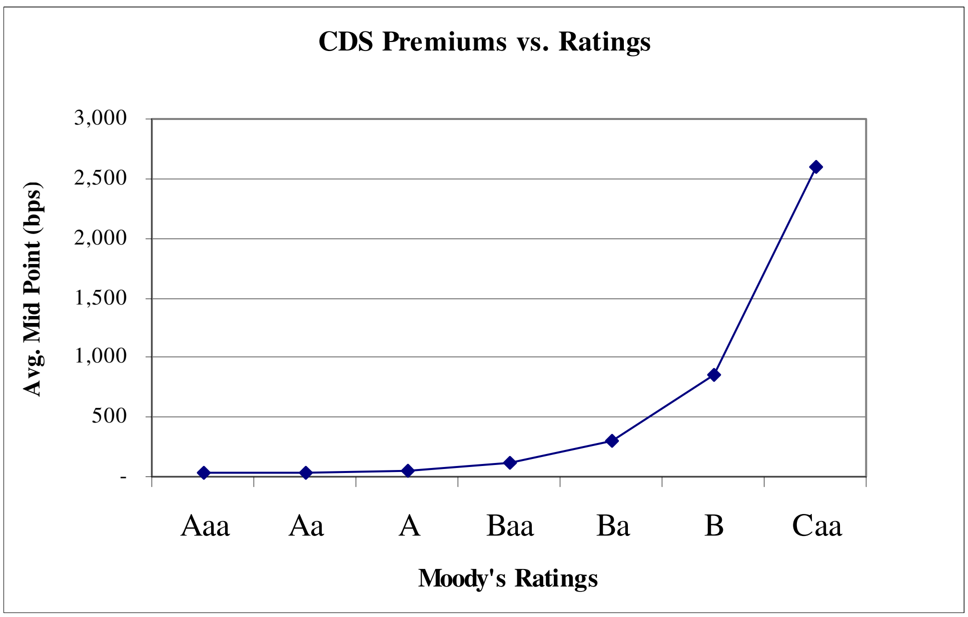

Figure 2.1: Average Bid-Ask Midpoints (bps) vs. Credit Quality (Moody's Ratings)

The descriptive statistics of the data set can be found in Table 2.1. It can be observed that the number of observations increase from around 24,000 in 2001 to 76,000 in 2004, indicating an expanding market. One direct measure for liquidity of the market is the size of the bid-ask spread, which shows the tightness of orders to buy and sell. The bid-ask spread is observed to decrease over time, attaining its lowest level in 2004.

Table 2.1: Number of Observations and Bid-Ask Spreads across CDS Types and Regions

			2001		2002		2003		2004		Total	
Type	Region	Curr.	Obs	Spread	Obs	Spread	Obs	Spread	Obs	Spread	Obs	Spread
Corp.	Europe	EUR	4409	21.38	26719	25.91	31890	13.21	33922	6.35	96940	14.68
Corp.	N.America	USD	8092	40.72	11749	44.81	15841	27.25	16286	17.10	51968	30.14
Bank	Europe	EUR	1917	9.90	10402	13.48	11262	8.00	11308	3.25	34889	8.20
Bank	N.America	USD	3646	13.17	4056	18.62	4000	11.42	3780	9.99	15482	13.37
Sov.	Europe	USD	777	2.86	1330	3.60	2530	2.80	2570	1.41	7207	2.46
Sov.	E.Europe	USD	1554	34.27	1746	25.10	3289	15.83	3341	10.82	9930	18.66
Sov.	L.America	USD	3369	77.70	4539	165.82	5252	72.21	5169	29.48	18329	84.35
Sov.	Mid.East	USD	259	143.20	158	118.39	253	61.58	257	22.46	927	83.22
Total/Average			24023	35.18	60699	37.14	74317	19.41	76633	10.00	235672	22.53

Moreover, for each subset of the credit ratings, seniority, maturity, region, and currency, the average midpoints of the bid-ask quotes for the CDS premiums were calculated. Table 2.2 shows the 5-year CDS premium midpoints with respect to several subsets. Note that sovereign CDSs are all denominated in USD, even for the European countries.

Table 2.2: Average Midpoints of 5 Year-CDS Premiums Across Ratings with respect to Currency, Credit Type, Region, and Rank

Currency	Credit Type	Region	Rank	Avg. Midpoint (bps)
EUR	Bank	Europe	Senior	24.96
EUR	Corporate	Europe	Senior	95.67
EUR	Bank	Europe	Subordinate	49.72
USD	Bank	N. America	Senior	53.74
USD	Corporate	N. America	Senior	119.84
USD	Sovereign	Europe	Senior	9.16
USD	Sovereign	E.Europe	Senior	119.17
USD	Sovereign	Middle East	Senior	716.09
USD	Sovereign	L. America	Senior	784.01

Table 2.3 presents the descriptive data on rating classes across different maturities. The most liquid maturity is the 5-year CDS, followed by 10- and 3-year CDSs.

Table 2.3: Number of Observations and Bid-Ask Spreads across Ratings, Ranks and Maturities

Moody's Rating	Rank	Maturity(Years)									
		3		5		10		Other		Total	
		Obs	Spread	Obs	Spread	Obs	Spread	Obs	Spread	Obs	Spread
Aaa	Senior	-	-	3230	9.11	-	-	-	-	3230	9.11
Aa	Senior	767	8.07	25462	8.30	787	10.88	-	-	27016	8.37
Aa	Subord.	-	-	5457	8.90	-	-	-	-	5457	8.90
A	Senior	2534	9.93	55003	11.97	4649	9.25	717	38.73	62903	11.99
A	Subord.	-	-	6663	10.19	-	-	-	-	6663	10.19
Baa/Worse	Sen/Sub	6821	47.21	62773	28.61	5546	46.65	5431	82.49	80571	35.06
Non-rated	Sen/Sub	6529	32.73	31152	20.84	4299	29.08	7852	47.03	49832	27.23
Total/Average		16651	34.06	189740	18.24	15281	28.49	14000	60.36	235672	22.53

Interestingly, the EUR-denominated CDSs have lower average midpoints of bid-ask quotes than their USD counterparts. This result holds even when looking at the premiums within different rating classes, with the exception of Ba (Table 2.4). A first attempt to explain this phenomenon (which is also present in other datasets; see Houweling and Vorst (2005)) might come from looking at the different specifications of deliverable bonds in case of restructuring. The smaller the set of deliverable bonds, the lower the value of the "cheapest-to-deliver option", which reflects the extra premium for the buyer of CDS for the privilege of being able to deliver any bond from a basket of available deliverable obligations. Recent literature has realized the reflection of this contractual term into prices, which ought to be different in European and North American markets (e.g. Jankowitsch, Pullirsch, and Veza (2006)). The Modified Modified Restructuring (MMR), is the clause usually used in Europe, while Modified Restructuring (MR) is the valid clause for North American entities. MR clause is more restrictive and this reduces the value of the delivery option. Unfortunately, this argument contradicts the observations gleaned from the dataset. Since the deliverable bonds are limited for USD-denominated CDSs, which leads to lower delivery option values, the CDS premiums should be lower.

In the following chapters, more detailed analysis on regional features will be carried on.

Table 2.4: Average Midpoints of Senior, 5-Year CDS Premiums with respect to Rating and Currency

	Rating	EUR (bps)	USD (bps)
Investment	Aaa	14.77	60.12
Grade	Aa	22.18	40.25
	A	52.81	62.67
	Baa	107.83	121.49
Non-Investment	Ba	361.61	242.69
Grade	B	409.07	866.98

2.3 Empirical Evidence on Trading and Liquidity of the CDS Market

Within the described brokerage setting, the liquidity in the CDS market is analyzed in this section. In the first part, the determinants of liquidity in the IDB market are the focal point. Later, a comparison of liquidity across alternative trading venues is presented.

2.3.1 Liquidity of the IDB Market

Liquidity is one of the key attributes of financial markets and refers to different dimensions, such as depth, tightness, resiliency, and immediacy. Measuring liquidity is a complex task for which various instruments have been proposed (see Schwartz and Francioni (2004), pp. 60-63). These are based on transaction costs (e.g. bid-ask spread, market impact, total cost of trading), trading volume (e.g. turnover, number of shares, number of transactions), and volatility (e.g. price variance, resiliency, intraday mid-point returns). Most of the evidence in the literature on liquidity is provided for stock markets. There are also recent studies focusing on trading and liquidity in interdealer broker markets for government bonds using GovPX and CanPX data (Boni and Leach (2004); Huang, Cai, and Wang (2002); D'Souza, Gaa, and Yang (2003)). However, relatively little is known about the liquidity of derivatives markets. The few available studies concentrate on the effect of illiquidity on prices of currency options (Brenner, Eldor, and Hauser (2001)) or on interest rate options (Deuskar, Gupta, and Subrahmanyam (2006)).

Concerning the CDS market, there are recent works claiming that the liquidity of CDSs is relatively high compared to corporate bonds, since CDSs are contracts but not securities. Recent literature has investigated the liquidity premium in bond spreads (Janosi, Jarrow, and Yildirim (2002); Houweling, Mentink, and Vorst (2005)), while credit default swaps are modeled to have no liquidity premium due to several reasons. Longstaff, Mithal, and Neis (2005) provide the most comprehensive discussion on the issue, noting that CDSs are contracts that can be set up arbitrarily, while securities are fixed in supply. This makes CDSs invulnerable to the "squeezing" effects applicable to bonds. Furthermore, if liquidation is wanted, the counterparty simply enters into a new CDS in the opposite direction. Finally, it is quite easy to sell and buy protection with CDSs, while it is difficult and costly to short bonds (Longstaff, Mithal, and Neis (2005), pp. 2219-2220). Despite these reasons, which neglect the presence of a liquidity premium in prices for modeling purposes, it should be realized that these points show only a relative unimportance with respect to bonds. To the best of my knowledge, this study is the first to focus on the determinants of CDS liquidity.

The analysis examines bid-ask quotes in the absence of trade data.[5] Microstructure theory explains the bid-ask spreads by means of three components accounting for the different costs that a dealer faces. Viewing the spreads as a measure for the cost of transacting dates back to the work of Demsetz (1968). According to this early study, the quoted spread should be a fair compensation for dealers who offer immediacy by supplying resources to the market. Demsetz and others show that the spread depends on various proxies for trading activity, security risk, and competition. Secondly, the theoretical models of dealer markets put forward that spreads should increase with inventory holding costs (Ho and Stoll (1983); Biais (1993)). Lastly, information-based models of dealer markets imply an adverse selection part of the spread, which accounts for the risk of trading with informed investors (Bagehot (1971); Copeland and Galai (1983); Glosten and Milgrom (1985)). Easley, Kiefer, O'Hara, and Paperman (1996) provide strong empirical evidence that the risk of trading with an informed investor explains a large part of the variation in spreads of NYSE traded stocks. In addition, Flood, Huisman, Koedijk, and Lyons (1999) argue that these three components of the quoted spread can be extended by an additional search cost component to account for the asymmetries of counterparty search.

The quoted spreads should be driven by economic forces that are implied by the theoretical models of market microstructure, in a competitive CDS market. The characteristics of CDS contracts can be used to proxy for the risks a

[5]Bid-ask spreads have obvious limitations. However, it is difficult to define a single measure that reflects all dimensions of liquidity, leaving the bid-ask spread as a good proxy for analysis.

CDS dealer faces and thus be reflected in the spread. For instance, the CDS's maturity, notional amount, and premium will drive inventory costs, because these features indicate how much risk will be carried in the inventory. Thus, in a panel regression, the absolute bid-ask spreads is regressed on several contract features: The Moody's rating, rank, currency, notional amount, restructuring[6], and maturity have been selected as explanatory variables. Intuitively, credit risk proxied by ratings should be a determinant of absolute spreads. In an empirical investigation by Odders-White and Ready (2006) for the stock market, it is shown that poorer debt ratings are related to higher spreads due to both higher adverse selection and trading costs. On the other hand, Acharya and Johnson's (2007) study, one of the few empirical studies to address CDSs, finds no evidence that adverse selection affects prices or liquidity in the CDS market. Even then the argument of higher trading costs remains, so it is expected that the spread would widen as the credit quality indicated by ratings declines. Subordinated CDSs are also anticipated to be less liquid than senior CDS and carry a higher absolute bid-ask spread. In addition, a higher notional amount translates into a higher inventory cost and thus could also result in a higher bid-ask spread. There is no prior expectation for the liquidity differences regarding the currency and restructuring clauses of the contracts.

After extracting the data points with no rating or maturity information as well as sovereign entities (due to few data points with mostly low credit quality), 169,009 data points remain within the period of January 2001 to January 2005. For the whole sample the average bid-ask spread is 22 bid-ask midpoints (bps),

[6] The restructuring dummy has a value of "0" for the Old Restructuring (OR) clause where minimum restrictions on delivery option are present: "1" for Modified Modified Restructuring (MMR), the valid clause for Europe after June 2003, which constrains the old clause; and "2" for Modified Restructuring (MR), the valid clause for North American entities. The latter is the most restrictive clause overall and reduces the value of the delivery option.

with a standard deviation of 68 bps. The absolute spread could be as high as 2000 bps, as it was the case for CDSs written on Enron just before its default in 2001. The average bid-ask spread for the rating class Aaa is 9 bps, which increases to 58 bps for the B class.

The results of the panel regression, in which the absolute bid-ask spread is regressed on several independent variables, are reported in column (1) of Table 2.5. A low percentage of variation is explained (14.52 per cent) and a highly significant intercept is present, indicating that there are missing variables. Unfortunately, there is no volume data available from the IDB market, which would be helpful for understanding liquidity. Nevertheless, all explanatory variables are highly significant. Not surprisingly, the numerical value for the Moody's rating is the variable that contributes most to the explanatory power of the regression, explaining 7.15 per cent of variation. One notch of deterioration with respect to the rating increases the bid-ask spread by 3.98 bps on average. Currency and restructuring are other highly contributing variables. A move from EUR to USD increases the bid-ask spread by 20.72 bps. It is possible that currency may proxy for other undefined variables, such as settlement differences or the varying degrees of openness in different CDS market segments. In keeping with the expectations, the significant and positive parameter estimate for the notional amount indicates a move from 5 million notional to 10 million notional contracts, causing the bid-ask spread to increase by 12.99 bps. This coincides with an increasing inventory-holding cost component with higher notional amounts. A closer analysis of the dataset reveals that this finding is also related to a timing issue. The first two years in the dataset were dominated by 10 million notional contracts, and during this time the premiums were high. In the last two years, the average premium declined, as the set was dominated by

5 million notional contracts, and the CDS market was more liquid than during the first two years.[7] In order to investigate this timing issue, year dummies have been introduced to the regression (column (2)). It is observed that the absolute spread in the first two years (indicated by year dummies 1 and 2) are about 19 bps points higher than the base year, 2004.

For further analysis, separate regressions for each rating group have been undertaken. From columns (3)-(8) in Table 2.5, it can be seen that rank, currency, and restructuring estimates are consistent across rating classes. While subordinate absolute spreads are higher than that of senior CDSs, a move from EUR to USD causes the spreads to widen, except for class B (column (8)). Similarly, the restructuring variable coefficient is always negative; indicating that restricting the deliverable obligations decreases the spreads. Finally, although the percentage of variation explained seems to increase for low rating classes, this is not monotonous. Nevertheless, the regressions on non-investment grade CDS spreads (Ba and B) have a better R^2 than investment grade CDS spreads, on average.[8]

[7]Robustness checks executed by taking the log values of ratings and maturity lead to similar results.

[8]In order to check for robustness, regressions with the relative bid-ask spread as the dependent variable have been tested. The relative spreads are calculated by dividing the absolute spread by the midpoint of the quotes. In the regression, this affects the seniority, maturity, and rating variables. Due to the fact that subordinated CDSs have higher midpoints that enter into this calculation than the senior CDSs, parameter estimates with a reverse sign for the rank variable have been reached. The same is true for the rating variable in regression (1) in Table 2.5, since the lower ratings simply indicate a higher midpoint division. Similarly, these regressions had a negative estimate for the maturity variable, which is most likely not because of changing absolute spreads but instead due to increasing midpoints for higher maturities.

Table 2.5: Panel Regressions with Absolute IDB Spread as the Dependent Variable

	(1) Full Set	(2) Full Set	(3) Aaa	(4) Aa	(5) A	(6) Baa	(7) Ba	(8) B
Intercept	-11.92	-19.58	7.07	2.57	5.12	3.43	474.90	364.79
t	-25.28	-38.55	21.14	19.75	23.43	4.2	30.20	13.55
p	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**
Rank	3.99	3.99	Excluded	2.08	0.85	1.81	Excluded	Excluded
t	12.84	12.90		40.43	7.09	1.47		
p	**<0.0001**	**<0.0001**		**<0.0001**	**<0.0001**	0.1411		
Maturity	-0.04	-0.04	Excluded	0.05	0.01	-0.01	-6.47	-5.75
t	-7.39	-7.80		29.29	3.08	-0.69	-24.09	-12.83
p	**<0.0001**	**<0.0001**		**<0.0001**	**0.0021**	0.4902	**<0.0001**	**<0.0001**
Currency	20.72	13.28	13.03	9.37	9.89	14.70	112.52	-216.24
t	52.12	30.58	23.01	89.40	48.69	17.10	26.61	-27.14
p	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**
Restructuring	-7.12	-2.16	-3.64	-2.78	-2.09	-0.14	-78.14	Excluded
t	-30.11	-8.16	-10.19	-45.34	-17.62	-0.26	-26.30	
p	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	0.7926	**<0.0001**	
Rating	3.98	4.06						
t	122.18	124.83						
p	**<0.0001**	**<0.0001**						
Amount	12.99							
t	65.16							
p	**<0.0001**							
YearDummy1		18.68	-2.08	2.59	7.70	40.37	311.78	536.13
t		56.84	-3.3	33.19	54.78	56.81	48.85	47.03
p		**<0.0001**	**0.0010**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**
YearDummy2		19.64	0.84	4.65	10.32	27.10	68.93	503.92
t		71.54	2.72	70.85	81.27	48.97	29.19	66.89
p		**<0.0001**	0.0066	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**
YearDummy3		7.38	-0.38	1.56	3.92	8.66	39.36	280.51
t		34.36	-1.46	29.94	38.76	20.95	19.82	26.52
p		**<0.0001**	0.1443	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**	**<0.0001**
R-Square	**0.1452**	**0.1537**	**0.3304**	**0.5521**	**0.2924**	**0.1196**	**0.5609**	**0.9456**
Observations	169,009	169,009	3,230	32,473	62,720	63,577	5,813	1,196

Regression Equation for (1): $IDB_{BAS} = \beta_0 + \beta_1$ Rank $+ \beta_2$ Maturity $+ \beta_3$ Currency $+ \beta_4$ Restructuring $+ \beta_5$ Rating $+ \beta_6$ Amount $+ \varepsilon$.

Dependent Variable: IDB_{BAS} is the CreditTrade daily closing CDS absolute bid-ask spread. Explanatory Variables: Rank dummy, "0" for Senior, "1" for Subordinated contracts; Maturity, one of the values of "12", "36", "60", "84", or "120", in months; Currency dummy, "0" for EUR, "1" for USD denominated contracts; Restructuring, having a value of "0" for Old Restructuring (OR) clause; "1" for Modified Modified Restructuring (MMR); and "2" for Modified Restructuring (MR); Rating, a value between 1 and 16, "1" corresponding to "Aaa", and "16" to "B3", assigned by Moody's; Amount dummy, "0" for 5 million, "1" for 10 million notional amount contracts; Year Dummy1 for prices of 2001, Year Dummy2 for prices of 2002, Year Dummy3 for prices of 2003, where 2004 is the base year. Rank, Maturity, Amount and Restructuring variables were excluded in some regressions due to having single class.

2.3.2 Differences of Liquidity across Trading Venues

It is well documented in the literature that market structure has impacts on liquidity and trading costs (Stoll (2000)). In particular, bid-ask spreads may differ in terms of size and composition with respect to the alternative trading venues available in the CDS market. The early work of Garbade (1978) suggests that joining an IDB reduces search costs, resulting in lower trading costs. More recently Reiss and Werner (2005) pointed out that interdealer trades have two important motives, the risk sharing of dealers, and information asymmetries (Ho and Stoll (1983); Reiss and Werner (1998)). In their empirical investigation, they contrast interdealer trading on the London Stock Exchange (LSE) by using stock prices from the non-intermediated OTC market and brokered trading systems. They find that differences in liquidity across trading systems on the LSE are mainly driven by problems of adverse selection. On the other hand, Acharya and Johnson (2007) provide empirical evidence that adverse selection does not affect liquidity in the CDS market. However, their study is based on CDS benchmark products traded via an IDB. Hence, it is an open question whether this result transfers to the direct OTC market.

Assume that IDB and direct OTC CDS markets are well integrated due to the informational linkage. Then the asymmetric information component is the same in both venues. Assume moreover that inventory costs are similar across venues by virtue of their tendency to attract the same participants. According to Garbade's reasoning, this leaves only the trading cost differences between IDBs and the direct OTC market. An important extension to Garbade's argument should incorporate the additional services provided by IDBs. Although search costs decrease by joining an IDB, if brokers offer additional services, the trading cost component in bid-ask spreads should also reflect the premium for the added value of the brokerage function. Consider both search costs and the added value of the brokerage function: If the latter were to dominate, it would lead to IDB spreads being higher than the non-intermediated OTC market spreads, which forms the first hypothesis:

H1: The quoted absolute bid-ask spread in the IDB market is higher than in the direct, non-intermediated OTC market.

In order to analyze the spread differences in these markets, the monthly data of direct OTC quotes were retrieved from Bloomberg in addition to the quotes of CreditTrade presented in Section 2.2. The direct OTC dataset includes 12 mid-month observations for more than 200 entities for the year 2004.[9] In order to match the bid and ask quotes for a given entity on a day with the IDB data, the direct OTC quotes of multiple dealers are taken to reach a closing inside spread for each day. This resulting dataset had 1,883 matching observations. Around ninety per cent of the entities had 5-year contracts, whereas the remaining entities were written on 1-, 3-, 7-, and 10-year maturity contracts. All quotes belong to senior CDSs.

For testing the first hypothesis, Table 2.6 presents a significance test for the differences between the absolute bid-ask spreads of the two markets. In keeping with hypothesis *H1*, the difference of bid-ask spreads is highly significant, with the IDB spread being higher than the OTC spread. This suggests that the IDB spreads include a larger transaction cost component. Apparently, this conclusion rests on the idea that the information and inventory components in spreads are the same across the two venues. The asymmetric information and inventory holding costs affect both markets similarly if the markets are integrated and reflect the same information. This hypothesis of market integration

[9] In order to eliminate month-end effects, mid-month data are used.

can be tested by analyzing the percentage of non-overlapping bid-ask spreads and by also comparing the midpoints of the quotes. If both trading markets are integrated in prices, then it can be indeed concluded that the extra spread consists of only the trading costs incurred by the dealers who select the IDBs.

H2: Prices across IDB and direct OTC venues are the same, indicating well-integrated markets.

Table 2.6: Significance of the Differences of the Absolute Bid-Ask Spreads of Two Trading Venues

	Mean Difference	t-Statistic	p-Value
IDB_{BAS} - OTC_{BAS}	3.19	23.30	<0.0001
Number of Observations	1,883		

IDB_{BAS} is the interdealer broker daily closing CDS absolute bid-ask spread for the mid-month data, calculated by the absolute difference between ask and bid prices. OTC_{BAS} is the corresponding Bloomberg closing CDS absolute bid-ask spread for the same entity and maturity, calculated similarly.

Table 2.7 presents whether the direct OTC and IDB quotes are nested, overlapping, or non-intersecting. Seven possible alternatives for the bid-ask spreads are tabulated. The first three cases indicate when spreads are nested or equal. The fourth and fifth are overlapping, with higher quotes given in one market. The last two cases are the non-intersecting spreads, where an arbitrage possibility is present. As expected, most cases fall into the first five types. 9.02 per cent of the quotes denote arbitrage possibilities; a bid quote in one market being higher than the ask quote in the other market. However, it is worth noting that the indicative IDB quotes is compared with the inside direct OTC closing quotes. It therefore stands to reason that these may not indicate actual arbitrage possibilities. Overall, these results suggest that the markets are integrated.

Table 2.7: Fragmentation of Direct OTC and IDB Markets

	Case 1	Case 2	Case 3	Case 4
Case	**Cases Nested OTC Inside IDB**	**Cases Same OTC Matches IDB**	**Cases Nested IDB Inside OTC**	**Cases Overlapping IDB Ask Higher**
Definition	IDB Ask >= OTC Ask > OTC Bid >= IDB Bid	OTC Ask = IDB Ask > OTC Bid = IDB Bid	OTC Ask >= IDB Ask > IDB Bid >= OTC Bid	IDB Ask > OTC Ask > IDB Bid > OTC Bid
No. of Obs	737	28	201	367
Percentage	**39.14%**	**1.49%**	**10.67%**	**19.49%**

	Case 5	Case 6	Case 7	Total
Case	**Cases Overlapping OTC Ask Higher**	**Non-intersecting IDB Spread Higher**	**Non-intersecting OTC Spread Higher**	
Definition	OTC Ask > IDB Ask > OTC Bid > IDB Bid	IDB Ask > IDB Bid >= OTC Ask > OTC Bid	OTC Ask > OTC Bid >= IDB Ask > IDB Bid	
No. of Obs	380	98	72	1,883
Percentage	**20.18%**	**5.20%**	**3.82%**	**100.00%**

Additional support for the integrated structure of both markets is presented by directly comparing the midpoints for the same dataset. Table 2.8 presents the pricing differences of 1,883 pairs, which are taken to be the difference of direct OTC and IDB midpoints. It indicates that the prices are not significantly different. In summary, results of these two tables support the second hypothesis, which states that pricing is consistent across trading venues for the selected list of companies. This finding can be attributed to two reasons: Firstly, CDS trading mainly occurs between major institutions with good ratings, which suggests that dealer base overlaps are present. Secondly, the datasets used for the analysis are from a selection of major companies that have liquid contracts, denominated in EUR or USD. This result indicates that the information and inventory cost components of the two venues are the same. It also confirms the results obtained by Acharya and Johnson (2007), who do not find evidence of adverse selection with a similar dataset. In their study, Acharya and Johnson (2007) proxy the number of banking relationships as the measure of the prevalence of non-public information in the market. The regressions they hold do not show a relation between the number of banks indicating the number of informed players and liquidity.

Table 2.8: Comparison of the Midpoints of Direct OTC and Interdealer Broker Quotes

	Mean Difference	t-Statistic	p-Value
IDB_{MID} - OTC_{MID}	0.069	0.53	0.5962
Number of Observations	1,883		

IDB_{MID} and OTC_{MID} represent the average of bid and ask quotes (midpoints) of the interdealer broker and direct OTC markets (from Bloomberg), respectively.

Given that the markets are well integrated, one question remains to be addressed: What causes the differences in trading costs, or in other words, under which conditions are the additional services of a brokerage of value? Intuitively, it could be expected that illiquid, riskier CDS trades are conveyed to IDBs. This hypothesis would be in line with Barclay, Hendershott, and Kotz (2006)'s findings in the US Treasury bond market. Their results distinguish between on-the-run US Treasury securities that are traded by electronic IDBs with a market share of 80 per cent and off-the-run securities where voice broking is highly preferred (88 per cent). This is mostly because the intermediaries' extra effort is necessary for illiquid, complex, and larger-sized trades. The intermediaries' ability to match complex trades is sometimes referred to as "market color". Although the authors compare the automated and intermediated brokerage platforms, their results are relevant in that they indicate the necessity of intermediation in case of illiquidity. Hence, in the following, it is analyzed whether the spreads of the two markets are affected by the need of intermediation in cases of more complex trades. In the CDS market, a direct measure of trade complexity is the credit quality of the underlying since the CDS premiums are an indication of credit risk. It is expected that the deterioration of credit quality will widen the absolute bid-ask spreads, as well as the difference between IDB and direct OTC spreads. The first part of the argument was shown in the last section, while the second part forms the hypothesis below:

H3: The difference between the IDB and the direct OTC absolute spreads widens with decreasing credit quality.

To test this hypothesis, the difference between absolute spreads is regressed on the midpoint of the direct OTC quotes. The results in Table 2.9 indeed indicate a high significance of credit quality, which means that the higher the midpoint, the higher the spread difference between the IDB and the direct OTC market.[10] This suggests that riskier and more complex trades are conveyed to the IDBs, and can be interpreted as that the dealers value the additional transaction services beyond pure trade execution. Nevertheless, the percentage of variance explained is low, indicating missing variables. Obviously, one of these can be competitiveness differences between the quotes posted in two venues. IDB quotes can be less competitive than the direct OTC quotes, which actually can be varying for different credit qualities. In fact, these results might indicate that the more competitive direct OTC market may attract the high credit quality CDS, leaving the IDBs the less liquid and less competitive quotes. Unfortunately, a variable to proxy for this factor could not be constructed. Being still in its development stages, the CDS market does not provide volume data, which would be a natural candidate for proxying competitiveness. Similarly, the depth of quotes may be an important factor behind the liquidity differences. The presence of depth information would lead to an improved understanding of the overall picture concerning the liquidity in these two markets.[11]

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[10] Significance is also reached when IDB midpoints are used as the explanatory variable.

[11] A similar analysis has been carried with relative spreads. Due to the fact that relative

Table 2.9: Explaining the Difference of the Bid-Ask Spreads of Two Trading Venues with Credit Quality

Dependent Variable:	IDB_{BAS} - OTC_{BAS}		
	Parameter Estimate	t-Statistic	p-Value
Intercept	1.293	8.00	<0.0001
OTC_{MID}	0.032	18.73	<0.0001
R^2	0.157		
Number of Observations	1,883		

Regression Equation: IDB_{BAS} - $OTC_{BAS} = \beta_0 + \beta_1\ OTC_{MID} + \varepsilon$
IDB_{BAS} is the interdealer broker daily closing CDS absolute bid-ask spread for the mid-month data, calculated by the absolute difference between ask and bid prices. OTC_{BAS} is the corresponding Bloomberg closing CDS absolute bid-ask spread for the same entity and maturity, calculated similarly. OTC_{MID} is the average of daily bid and ask quotes (midpoints) for Bloomberg data.

To summarize, in this section, evidence concerning the liquidity of intermediated versus non-intermediated markets has been provided. It has been noted that the quoted bid-ask spread should account for adverse selection and inventory holding costs, as well as the costs of any transactional service, including search costs. The results show no significant difference in pricing across trading venues, indicating a well-integrated CDS market. Apparently, the overlapping of the set of dealers committed to both trading platforms might be a reason for this outcome. Despite the fact that the CDS market has a countable number of dealers and brokerage firms, this study indicates that quotes are not purely driven by market power but vary due to certain underlying economic forces. The quoted spread is higher in the intermediated market, which has been suggested to stand for the added value of the brokerage function. It is noteworthy to mention that this is only one of the possible explanations for

spreads are computed by a division of the midpoints, the significance of credit quality is in the opposite direction for Tables 2.6 and 2.9. This is most likely due to midpoints increasing stronger than the absolute spreads for low credit quality entities.

spread differences. Certainly, many factors come into picture considering trade volumes, depths, trade complexities and credit qualities. These results provide evidence for showing that brokerage has an effective place in the CDS market despite the higher costs of liquidity. The value of intermediation will exist as long as complex trades need special handling and explain why voice brokerage still has an important share in many markets in spite of increasing competition from electronic trading systems.

Chapter 3

Overview on Empirical Credit Risk Pricing

3.1 Credit Risk Modeling - Structural and Reduced-Form Models

In the last decade, credit risk modeling has received increasing attention within academia and practice. The pricing of credit risky instruments, such as bonds and credit default swaps, dates back to the 1970s. The introduction of the Black and Scholes (1973) and Merton (1973) valuation framework eventually led to the development of a new branch of finance. One of the extensions of this framework was the pricing application of Merton (1974) for corporate bonds. Merton made use of Black and Scholes (1973) and Merton (1973) equations to reach a "risk structure" of zero-coupon bonds. The pathway they opened is based on the central point that debt and equity can be interpreted as options on the firm value of a corporation.

Over time, different approaches to modeling credit risk have been developed, resulting in two main branches of research in the current academic literature. Following Merton's (1974) idea, the structural approach is based on modeling the evolution of issuer balance sheets. According to this approach, default occurs when the value of assets fall below a certain level and the issuer is unable or unwilling to meet its obligations. There have been various extensions of Merton (1974). Black and Cox (1976), Geske (1977), Leland (1994), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996) and Collin-Dufresne and Goldstein (2001) are among the important contributions to the defaultable claims framework. These models extended Merton's (1974) approach, most significantly in terms of (i) allowing default at any time during maturity, (ii) endogenously deriving the level of the default barrier, and (iii) introducing stochastic interest rates. Black and Cox (1976) have provided a closed form solution for the "first-passage time" models. In addition, their study was important for its derivation of an endogenous default barrier, which has since been extended by Leland (1994) and many others. Longstaff and Schwartz (1995) carried the debate to include stochastic interest rates. A relatively more recent model in Collin-Dufresne and Goldstein (2001) made use of stochastic interest rates and further included a stationary leverage ratio.

In contrast to structural models, reduced-form models specify the default probabilities exogenously. According to these models, default time is unpredictable and is calculated by means of a default intensity function. Instead of relying upon the diffusion process inherent in structural models, reduced-form approaches model the default time as the first occurrence of an event in a jump process. The simplest type of reduced-form models is one in which the default intensity is constant, as put forward by Jarrow and Turnbull (1995).

Many variations of this model have been developed. Important contributions among them include Jarrow, Lando, and Turnbull's (1997) version with rating-dependent intensities in a Markov chain setup, as well as the Cox processes used by Lando (1998) and Duffie and Singleton (1999) for default intensity modeling. A comprehensive review on both structural and reduced-form credit risk models can be found in Uhrig-Homburg (2002).

With all these different approaches available, the question becomes: How well do they represent a good benchmark for real prices? To date, there has been no common agreement in academia or practice as to which model framework better represents default risk. This is a crucial question for the pricing of credit default swaps presented in the last chapter. It is still an open issue of what type of method better prices CDSs. For this purpose, this chapter serves as an overview on the empirical literature in credit risk. Firstly, studies with structural and reduced-form pricing with bonds will be highlighted. Then, empirical applications concerning credit default swaps as the instrument will be discussed. It should be noted that only empirical studies with credit risk modeling are focused on. This leaves aside many other studies with corporate bonds where a structural or reduced-form model are not involved. Nevertheless, recent studies with CDSs are included, regardless of involving a credit risk model test or not. As a result, a broad overview of studies with CDSs will be presented.

3.2 Review on Empirical Studies in Credit Risk

3.2.1 Structural Models with Bonds as the Instrument

Structural models can be investigated in two main branches. Merton's initial idea defined the default barrier exogenously. Extensions of the Merton model which kept this assumption include Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Briys and de Varrenne (1997), Schöbel (1999) and Collin-Dufresne and Goldstein (2001). An alternative to exogenously defining the default barrier is when the default is modeled as a decision by the firm. In these models the barrier is endogenously determined as a result of the management decisions (e.g. Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996)).

Empirical studies with structural models differentiate most significantly in terms of (i) type of bonds used, i.e. whether they include call features or not, (ii) the estimation method for the parameters, most importantly the asset value and volatility, and (iii) usage of riskless term structure and the inclusion of stochastic interest rates into the model setup. With an emphasis on these issues, a review on empirical studies based on the two types of structural models will follow.

Studies on Structural Models with Exogenous Default

Empirical studies that apply the theoretical framework developed by Merton (1974) have been scarce initially. This was mostly due to the availability of reliable data. When earlier empirical studies that test structural models in their ability to price corporate liabilities emerged, these works suffered from the applicable data problem. Prior to 1980s, most corporate bonds were callable, making it hard to work because observed spreads included also a call premium. Since it is arguable that on which proportion the observed spreads of corporate bonds depend on default risk and call options, these early studies have biases within.

The work of Jones, Mason, and Rosenfeld (1984) (henceforth JMR) is one of these early studies that uses callable bonds as the instrument. In the study, the monthly data of 27 firms between January 1975 and January 1981 were used. A simple capital structure was the most important decision variable in selecting these firms: As described by JMR, the firms had one class of stock, no convertible bonds, a small number of debt issues, and no preferred stock. They also had a small ratio of private debt and short term notes payable to total capital, and they typically had rated publicly traded debt. Ogden's (1987) work is best comparable to JMR in terms of data used since Ogden had a pre-condition of being callable and having a sinking fund while selecting the 57 bonds between 1973-1985 period. Nevertheless, the approaches of JMR and Ogden differ when JMR incorporate multiple issues of debt to the Merton model while Ogden allows only one bond. Ogden selected data of firms with simple capital structure and with primary market prices. Although Ogden thinks that a simple capital structure restriction will bias the sample towards smaller firms, he neglects this in favor of testing the ideal conditions of the model.

Both studies utilize an extended version of the Merton (1974) model with sinking fund provisions, callable debt, and - in the case of JMR - multiple debt issues. Since the resulting partial differential equation is not analytically solvable in both cases, numerical methods have to be applied. The two studies also have similarities in the estimation technique of the volatility σ_V of the firm value V to reach σ_V:

$$\sigma_E = \sigma_V E_V \frac{V}{E}, \tag{3.1}$$

where σ_E is the volatility of the equity value E and E_V is the partial derivative of the equity value with respect to the asset value V.[1] First, they estimate firm values from market values of equity and market (for JMR) or book values of debt (for Ogden), calculate the standard deviation of equity returns of the firm, and estimate E_V. Ogden handles the estimation of E_V by first assuming it to be equal to 1, then he uses Equation (3.1) for a revised estimate, whereas JMR use a similar method. Finally, JMR and Ogden make use of Equation (3.1) for an estimate of σ_V. Besides volatility data also interest rate data are required. JMR comment that assuming a flat term structure for default-free interest rates (which is similar to Merton's original assumption) would cause biased estimation. Thus, both JMR and Ogden make use of one year implied forward interest rates estimated from government bond data retrieved from the Wall Street Journal.

Both JMR and Ogden test Merton's model in its ability to explain credit spreads and reach the conclusion that the model computes lower spreads than observed in the market. JMR compare their results with a "naïve" model that only discounts the promised payments with the riskless rate. With an entire sample of 305 coupon bond prices, JMR find an absolute percentage error in predicting the risky bond prices with the Merton model of 8.5 per cent and a percentage error of 4.5 per cent. Both error values are above 10 per cent for non-investment grade bonds. These results are rather high in terms of percentage pricing errors. It came out that the Merton model has better predictions

[1] See Merton (1974) for derivation.

than the naïve model for lower quality bonds, while the models give similar results for higher qualities. Moreover, regression analyses reveal that variance estimation errors affect pricing differences substantially. Observing the indistinguishable results of the Merton model and the naïve model for investment grade bonds, JMR conclude by suggesting that introducing stochastic interest rates and taxes would improve the model's performance.

Ogden's experiences with the Merton model are also mixed. In his initial probit analysis for uncovering the relation between bond ratings and the Merton model's estimated risk measure (standard deviation, σ, and debt-to-firm value ratio, D/V) both variables have proven to be significant. A regression between observed and model spreads shows that predicted spreads are on average 104 basis points lower than the observed spreads, which indicates a poor performance. Ogden undertakes a second regression to understand the potential factors for the results, with firm size, rating grade, treasury yield, and slope of the term structure being the independent variables. While there is no evidence on the significance of bond rating, spread errors are negatively related to firm size. Ogden treats this finding as a standing point in parallel to Fisher's (1959) liquidity hypothesis which puts forward that bonds of larger firms are more liquid and have lower yields. Bond spreads should also be inversely related to firm size, *ceteris paribus*. Since model spreads are invariant to firm size, the spread errors are expected to be negatively related to firm size. Both term structure variables are also significant which suggests stochastic interest rates should be incorporated for better results.

It is interesting that both JMR and Ogden conclude their work by suggesting the introduction of stochastic interest rates. Since Merton's model does not include stochasticity, this option has become available only in latter theoretical studies, particularly after the work by Longstaff and Schwartz (1995). As we will see later, in recent empirical studies the presence of stochastic interest rates is tested with Longstaff/Schwartz's model, although it has not significantly improved the pricing. Eom, Helwege, and Huang (2004) suggest that the reason why JMR and Ogden necessitate stochastic interest rates is that the data they use was an interval in which Treasury rates were volatile. It is possible that subsequent studies such as Wei and Guo (1997) did not face the same necessity while using only 1992 data, with non-callable bonds in their analysis.

One of the first empirical studies that investigate the risk structure of pure discount bonds is Sarig and Warga (1989). The authors compare their findings to the behavior suggested by Merton's (1974) model but do not price the liabilities using a model. For the period of February 1985 to September 1987, data of 137 zero-coupon bonds of 42 companies were collected. In contrast to JMR and Ogden, the authors have omitted the callable bonds from their list. As a result credit spreads can be easily computed by subtracting the yield of a zero-coupon government bond from the yield of a zero-coupon risky bond with identical maturity. The resulting term structures of spreads were increasing for investment grade bonds, humped shaped for bonds rated BB, and decreasing for bonds rated B or C. Sarig/Warga comment that these findings match with the theoretical results of Merton, if leverage (the debt-to-firm value ratio, d) and rating are negatively correlated.

The importance of Sarig/Warga's study lies in being among the first testing implications of the Merton model with limited non-callable zero-coupon bonds data. A direct test of Merton's model based on zero-coupon bonds is Gemmill's (2002) work. In the study, zero-coupon bonds issued by closed end funds in UK are used as the source of data between the period of February 1992 and April

2001. Thus, Black/Scholes equations can be used to calculate model prices of bonds. Parameter estimation is also fairly simple given this database: The firm value, V, is proxied by market value of the closed end funds portfolio. Volatility can therefore be estimated from the time series data of firm value.

Gemmill confirms some results obtained in previous studies: Calculated spreads are lower than observed spreads for less riskier bonds and for bonds that are closer to maturity, by using zero-coupon bonds of closed-end funds as the dataset. Moreover, in parallel to Ogden he finds that observed spreads increase when interest rates fall and term structure steepens. However, in contrast to previous results, Merton's model performs surprisingly well in explaining credit spreads with an average of monthly deviations of around 6 per cent. Interestingly, the resulting term structures of credit spreads are consistently upward sloping although Merton's model predict and Sarig/Warga empirically confirm downward-sloping term structures for risky bonds. Gemmill comments that this could be possible if it is assumed that there will be a drift in leverage over time. The sample of bonds issued by close-end funds may be expected to have a leverage falling over time, causing the spread to fall on each bond as it approaches maturity. Gemmill suggests that this observation supports the argument of Collin-Dufresne and Goldstein (2001) who specify a target for leverage that has influence on model spreads.

In the eminent work of Longstaff and Schwartz (1995), interest rate risk is incorporated to the structural framework provided by Merton (1974). Thus, the constant interest rate assumption is relaxed. Longstaff/Schwartz also follow Black and Cox (1976) in allowing default earlier than maturity. In parallel to the findings of Sarig/Warga, the model predicts an increasing term structure of credit spreads for high quality bonds while low quality bonds show a humped shaped structure within their model. Recall that the original Merton model is in the same spirit which predicted a hump shaped structure for debt-to-firm value ratio being $d < 1$, and a decreasing structure for $d > 1$. In a first empirical analysis based on Moody's monthly yield averages for the period of 1977-1992, Longstaff/Schwartz find significant contribution of the correlation between asset returns and interest rates for pricing corporate bonds, when stochastic interest rates are introduced.

In their contribution to the literature, Wei and Guo (1997) compare the pricing ability of the Merton (1974) and the Longstaff and Schwartz (1995) models by utilizing Eurodollars as risky debt. Eurodollars are bank deposit liabilities that are not subject to US regulations, but they are issued in US dollars. Although they can be held anywhere outside US, the reason why they are called Eurodollars is that they were originally most liquid in Europe. The authors have chosen to work with Eurodollars since they are widely traded. What Wei/Guo would specifically like to test is whether the stochastic interest rates introduced with Longstaff/Schwartz's model, have been an improvement in the pricing ability over the Merton model. However, the authors only test the fitting ability (in-sample) of the models rather than their predictive power.

In order to implement the model Wei/Guo perform two steps. In a first step the term structures of risky and riskless interest rates are computed. For each Thursday in 1992 Wei/Guo use the seven days, one month, three months, six months and one year Eurodollar yield leading to a risky term structure consisting of five data points at each date. In addition, they derive the riskless term structure using T-bills ranging from seven days to one year. In a second step, model parameters are estimated. The Merton model parameters (volatility, σ_v^2; debt-to-firm value ratio, d) are estimated from the risky bond prices by inverting the Black/Scholes formula. Because of the number of parameters and the stochastic term structure, the Longstaff/Schwartz model's estimation occurs in two steps. First, the term structure parameters that follow a Vasicek (1977) process are estimated by non-linear least squares fitting of Treasury term structure. Secondly, the credit structure parameters are estimated by fitting the risky Eurodollar term structure of interest rates using a grid search method. At the end, estimates are available for in-sample credit term structure construction for each week in 1992.

As the sum of squared errors for the weekly calculations of the Merton model are smaller than for the Longstaff/Schwartz model, the Merton model can be commented to fit the observed credit structure better. This seems to be astonishing in the first place, since the Longstaff/Schwartz model with eight parameters might be expected to be more flexible in fitting the observed credit spreads. However, the Merton and the Longstaff/Schwartz models are non-nested and hence have superior characteristics to each other. Still, both do not exactly reach the shape of the observed credit structure. The credit structure of the Eurodollars has N-shape; however, both the Merton and the Longstaff/Schwartz models only reach a hump-shaped structure since an N-shaped structure cannot be obtained within these models. According to Wei/Guo, the Merton model has shown more reasonable fit over the Longstaff/Schwartz model since when time to maturity approaches infinity the authors reach a hump-shaped credit curve that converges to a constant. The Longstaff/Schwartz model converges to zero on the same circumstance, which limits the ability of the model to fit the credit term structure.

The Merton and Longstaff/Schwartz models are also objects of Lyden and Saraniti's (2000) work. They are the first to test these structural models using individual, non-callable corporate bonds. Using the database of Bridge Information Systems they select a sample of 56 firms with publicly traded common stock and a single outstanding bullet bond without embedded options within the period from 1990 to 1999. So as to estimate the parameters, Lyden/Saraniti first compute the firm value by adding total equity, book value of short term and other liabilities, and the market value of the bond. For asset volatility, the standard deviation of the time series of firm values is taken. The riskless rates are estimated with the Vasicek (1977) model from Treasury yields and the historical correlation between stock prices and the ten year on the run Treasury yield is used to proxy the correlation between asset value and riskless interest rates.

Consistent with previous work Lyden/Saraniti have reached the conclusion that both the Merton and the Longstaff/Schwartz models underestimate yield spreads with considerable difference. When testing the Merton model, the authors account for different possible debt structures. The assumption of equal priority given to all creditors in the event of default leads to the lowest mean absolute error (83 basis points), which is of course still extremely high. The corresponding mean error of 61 basis points documents the strong underprediction of market spreads by the Merton model, part of which is doubtlessly due to a liquidity premium. When regressing model errors on bond characteristics they confirm the finding of JMR and Ogden that the model particularly overprices longer term debt. Testing the Longstaff/Schwartz model in its ability to explain spreads occurs in two steps. First, Lyden/Saraniti introduce early default before incorporating stochastic interest rates. They do this by applying Longstaff/Schwartz model with zero volatility of risk-free rates and assume a recovery rate as 47.7 per cent, as suggested by Altman and Kishore

(1996). In the second step stochastic risk-free interest rates were introduced. Interestingly neither allowing early default nor volatile interest rates improves the model. Interest rates do have an impact, but bring accuracy problems. The model results deteriorate even more when industry specific recovery rates are incorporated. So, to rescue the model, the well-known problem of joint hypothesis must serve, stating that the rejection could either be due to an incorrect model or due to misestimated asset volatility. Overall, Lyden/Saraniti conclude by commenting that the Merton model has better performance in prediction than the Longstaff/Schwartz model - a result which confirms and extends the in-sample result of Wei/Guo to out-of-sample prediction.

The most comprehensive empirical analysis of structural models using corporate bonds is the recent study of Eom, Helwege, and Huang (2004) (henceforth EHH). Their comparison of five corporate bond pricing structural models includes the work of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001) and is the first work that tests the Geske, the Leland/Toft, and the Collin-Dufresne/Goldstein models.[2] Similar to Lyden/Saraniti, EHH concentrate on corporate bonds with simple capital structures. Restricting the sample to unsubordinated, non-callable bonds of non-financial and non-utility firms with standard cash flows and long maturities as well as to firms that have at least five years of stock price data and not more than five bonds, they end up with 182 bonds. They retrieve the 1986-1997 data of last trading day of each December from the Fixed Income Database. Bond maturities are mostly in the range of 5-10 years, although bonds that have maturities from one year to 30

[2]The Geske (1977) and Leland and Toft (1996) models are endogenous-default models that are analyzed in more detail in the next section. However, for the continuity of EHH study, a partition was not found useful.

years are present in the sample.

In order to estimate parameters for the structural models, EHH use various techniques. Asset volatility is estimated from (i) historical equity volatility, similar to JMR with Equation (3.1), (ii) 150-day future equity volatility, (iii) a GARCH (1,1) model, and finally (iv) implied volatility using previous-month bond prices. Unlike JMR, the firm value is computed by adding equity value and the book value of total debt. The firm's annual payout (dividends, bond coupons, share repurchases) is adjusted in the models as well. For the one-factor models of Merton, Geske and Leland/Toft, the risk-free rate is estimated using the Nelson and Siegel (1987) model. In both Longstaff/Schwartz and Collin-Dufresne/Goldstein models, interest rate dynamics are described by the Vasicek (1977) model, thus the authors implement these two models using Vasicek estimates for consistency. Moreover, in the Longstaff/Schwartz and the Collin-Dufresne/Goldstein models, the required correlation coefficient between asset returns and interest rates is proxied with the correlation between equity returns and changes in interest rates as in Lyden/Saraniti. EHH fix the recovery rate to 51.31 per cent following the literature on default recoveries. For all models except the Geske model, EHH can make use of analytical or quasi-analytical formulas to derive model prices for the coupon bond. The Merton model is extended to coupon bonds by simply pricing the coupon bond as portfolio of zero-coupon bonds each of which is priced using the standard Merton model. The more accurate treatment of coupons within the Geske model leading to a compound-option pricing problem is numerically solved with binomial trees.

Based on historical volatilities EHH find that the extended Merton model underpredicts spreads. Only with implied volatilities this result disappears.

However, a comparison of the model results using implied volatilities from the previous month's bond prices with a simple random walk model implying that the previous month's spreads is the best estimate for the current spread, leads to the conclusion that the random-walk model performs superior. Further analysis with the extended Merton model reveals that the specific form of the risk-free term structure (Nelson/Siegel or Vasicek) is not important, as the authors try the Vasicek model as an alternative source for the risk-free rate. The Geske model also underpredicts spreads while reaching closer values to the actuals. Specifically, the Merton and the Geske models generate very high spreads on bonds that are risky, and low spreads for the ones that are considered safe. In contrast, Leland/Toft's model overpredicts spreads. Since this overprediction of the model is closely related to the coupon-level, the authors suggest that the assumption of a continuous coupon payment, which can be basically interpreted as exercise of the option to continue operations, is responsible for this result. The Longstaff/Schwartz model also reaches high spreads for risky bonds but low spreads for safer bonds. Although the accuracy substantially worsens EHH argue that stochastic interest rates are important because spread estimates are sensitive to interest rate volatility estimates thereby allowing an additional source of volatility. Finally the Collin-Dufresne/Goldstein model overpredicts spreads on the average while helps to raise the spreads that are considered safe by Longstaff/Schwartz. With these findings, EHH conclude that the accepted argument in the literature which is that the structural models underpredict spreads consistently, is not correct. However, maintaining accuracy in prediction is important. Because of this reason, EHH suggest that raising the model spreads should not be seen as the central focus of further theoretical studies with structural models, rather it should be raising them for safer bonds while keeping them for riskier bonds.

A novel approach to testing structural models have recently arisen from the work of Schaefer and Strebulaev (2007). In their study the authors take a different path, and instead of predicting bond spreads which are usually underestimated by the Merton's (1974) model, they choose to look at whether the model can predict hedge ratios -of debt to equity- that are in line with those observed empirically. They motivate their reasoning that the price of a corporate bond can be thought to consist of a credit related component and a non-credit related component as suggested by prior research (i.e. Collin-Dufresne, Goldstein, and Martin (2001)). If structural models provide a good estimate of the credit related component, they will simultaneously predict the hedge ratios of actual prices while failing to explain their level. Therefore their starting point is to hypothesize that if they find a model that provides reasonable prediction of hedge ratios but a poor prediction of the price, they may identify the reasons of model failure.

With a sample of 1,360 bonds between 1996 and 2003 that do not have any embedded option-like features, Schaefer and Strebulaev (2007) first look at the sensitivity of corporate debt to changes in firm value. After having shown the significance of the sensitivities of corporate debt returns to the underlying equity and riskless debt, they check whether the magnitudes of the sensitivities are consistent with the Merton's model. By pursuing a simulation, the authors find that the Merton model simulated sensitivities are very close to actual sensitivities from data, although the model underestimates the observed level of credit spreads by more then 80 per cent. After calculating the asset volatility form historical equity and bond returns, they finally show that the Merton model provides reasonable predictions for hedge ratios but poor predictions for the bond price. Therefore, the authors suggest that the poor performance of the structural models might arise from the influence of non-credit factors, rather than not being able to capture credit exposure of debt. By checking other possible determinants of the bond returns, they document that the returns are affected by some factors (such as return on S&P 500 index, or changes in VIX index of implied volatility of options) that are not related to credit exposure. The authors therefore conclude the study by a further evidence in line with Collin-Dufresne, Goldstein, and Martin (2001) who find a significant non-credit component in credit spread.

Overall, it can be concluded that all tested structural models have difficulties in accurately predicting corporate credit spreads. The various extensions of the Merton model do not really do a better job. Surprisingly, the Longstaff/Schwartz model performs even worse with the incorporation of stochastic interest rates. While it is a common argument that models underpredict spreads for safe bonds there is no consensus on the pricing of riskier bonds. JMR found lower spreads for risky bonds with the Merton model whereas EHH have reached too high spreads for riskier bonds with the Longstaff/Schwartz model. In any degree of riskiness, the Leland/Toft model overpredicts spreads in the analysis of EHH. It is noteworthy to mention that many regression results with the errors indicate that longer term bonds have higher spread prediction errors. The criticism towards structural models that claim a common poor fit to empirical data therefore has solid examples. However, as the quality of data more and more improves and also as the techniques to estimate parameters become more powerful, one can have hope to implement structural models more successfully. Moreover, further efforts should investigate the differences of credit related and non-credit related components in the light of Schaefer and Strebulaev (2007) and concentrate on the explainable parts with structural models.

Studies on Structural Models with Endogenous Default

A renewed focus on structural models arises from the approach that endogenously determines the default boundary. These studies analyze the impact of firm's financing decisions and make detailed assumptions about the default event. Specifically, the firm has a management decision whether to continue paying coupons on the debt or to declare bankruptcy. This type of studies was initiated by Black and Cox (1976). Extensions of Black/Cox differ in the resulting default barrier, which is computed with optimization techniques. Uhrig-Homburg (2002) provides an extensive review on different types of endogenous models that extend the Black/Cox model with taxes, bankruptcy costs, and debt renegotiation using game theoretic elements.

Anderson and Sundaresan (2000) is among the first empirical studies that tests pricing ability of endogenous structural models. The purpose of their work is to discriminate among different endogenous models and compare it to the results of Merton. The two endogenous models they test are: (i) Leland (1994), which extends Black and Cox (1976) with bankruptcy costs and tax advantage of debt, and (ii) Anderson, Sundaresan, and Tychon (1996) (henceforth AST); which is a special case of both (a)Anderson and Sundaresan (1996) in which a game-theoretic model of the bankruptcy process is introduced, and (b) Mella-Barral and Perraudin (1997), in which the firm can be liquidated at any time at some given liquidation value. Throughout the study, perpetual coupon bonds are considered.[3]

Anderson/Sundaresan have chosen to work with indices instead of specific bond issues, because they think indices are more linked to economic factors than in-

[3] Anderson, Sundaresan, and Tychon (1996) is the continuous time version of the discrete model of Anderson and Sundaresan (1996).

dividual bonds which have a stronger connection to firm-specific factors. The monthly data of yield indices were collected for the time period from August 1970 to December 1996. This data constituted 30-year BBB rated bonds according to the rating system of Standard and Poor's. Similar to exogenous structural models the estimation of leverage and asset volatility is required, which is undertaken as follows: A leverage proxy, computed as monthly series, is obtained from annual aggregated balance sheet data and market values of equity. In contrast to previous work done in the literature, asset volatility is taken to be simply a multiple of equity return volatility from S&P 500, which is implicitly estimated by non-linear least squares from the bond yields along with other parameters. This can be viewed as a relaxation of the technique utilized by JMR and followers, given in Equation (3.1). In their effort to estimate parameters, Anderson/Sundaresan built a formulation for the yields on corporate bonds, Y_t:

$$Y_t = \lambda + Y(L_t, \sigma_{Et}, r_t, a, K, \theta, \beta) + u_t \tag{3.2}$$

and

$$u_t = \rho u_{t-1} + \varepsilon \tag{3.3}$$

where Y_t is the observed market yield for corporate bonds. The additive constant λ is included to capture a liquidity premium or some other effect that is not captured by the model. $Y(.)$ is the model-implied yield calculated from $Y = cP/B$ of the coupon bond with coupon c, principal P, and model value B of the defaultable bond computed from Leland, AST, and a perpetual version of Merton pricing formulas. As Equations (3.2) and (3.3) demonstrate, the yield depends on L_t, the leverage proxy described above (which is estimated directly from balance sheet information); σ_{Et}, the equity volatility (which is estimated from standard deviation of equity returns on the S&P 500); r_t, the riskless rate (proxied by moving averages of long-term Treasury yields); a, the equity/asset volatility multiple; K, the fixed bankruptcy cost; θ, the proportional recovery rate; β, the cash flow rate; and ρ, the autocorrelation coefficient. The constant parameters, the liquidity constant, bankruptcy cost, recovery rate, cash flow rate, and autocorrelation of residuals are estimated by the same nonlinear least squares with the asset volatility multiple. Naturally, the bankruptcy cost, the cash flow rate and the recovery rate are not present for the Merton model, whereas AST model uses all of the above variables except the recovery rate. For the Leland model, only a, λ, θ, and ρ are estimated and bankruptcy cost and the cash flow rate are left out.

According to Anderson/Sundaresan, implied parameter estimates are more plausible for the Leland and especially for the AST model compared to those from the Merton model. Nevertheless, it should be noted that only the asset volatility multiple that enters the Merton's model is estimated in-sample, whereas the endogenous models have more parameters to be estimated. Moreover, the estimated parameters have a single constant value for the whole estimation period, which is not on the same level of detail compared to weekly estimation, such as in Wei/Guo. It is also worth mentioning that the recovery rates estimated by the AST and the Leland models are over 90 per cent while the literature agrees on a rate around 50 per cent (i.e. Altman and Kishore (1996)).

The calculated spreads using the parameter estimates are compared to observed BBB-rated bond spreads. For all models, observed spreads are more dispersed than in-sample calculated spreads. But, the endogenous models tested have a better correlation with observed spreads than the Merton model. Anderson/Sundaresan conclude that the endogenous determination of default boundary has led to an improvement among structural models. Still, the results should be evaluated in the light of the parameter estimation process as described above. Another comment to include would be that the results of Anderson/Sundaresan show the estimate of liquidity constant is attributed to nearly all of the spread computed with the Merton model, leaving the model to account for only 4 bps. This raises the question of how much the models actually contribute to explaining spreads.[4] Finally, the fit of the models with firm specific data remains untested.

In their attempt to use structural models to explain credit spreads, Ericsson and Reneby (2004) have also chosen an endogenous model, a variation of the Leland (1994) model. They argue that the better fit of reduced-form models[5] in the literature arises from the estimation techniques used, rather than the model structure. Their aim is to show that structural models can perform as well when properly estimated. Using a maximum-likelihood method proposed by Duan (1994) and shown support in Ericsson and Reneby (2005), they estimate model parameters from a time series of stock prices and previous periods' bonds prices to predict current bond prices and spreads. The required capital structure information is retrieved from balance sheet data and as a proxy for the constant risk-free rate an interpolated Treasury yields is chosen. Ericsson/Reneby use a sample of 141 corporate bond issues that consists of nearly 5,600 dealer quotes. The out-of-sample, one month-ahead spread predictions result in finding a mean error of only 2 bps. When longer horizon out-of-sample prediction is applied, the average error increases to -18 per cent of the spread.

[4]Liquidity constant also constitutes a major portion of the explained spreads with the Leland and AST models.

[5]See Section 3.2.2 for the empirical results obtained with reduced-form models.

Comparing these results to prediction errors obtained with reduced-from models (specifically Duffee (1999) and Bakshi, Madan, and Zhang (2006)), the authors conclude that incorporating bond prices information into the estimation procedure of structural models provides valuable information and leads to a performance that is well comparable to reduced-form models.

In one of the more recent studies, Teixeira (2007) tests three structural models of default. These are the Merton (1974), Leland (1994), and Fan and Sundaresan (2000) models. The author uses 50 bonds of firms with simple capital structures between 2001 and 2004. The sample is limited to US non-financial firms that have no more than three bonds. Besides, only bullet, non-callable, non-putable or non-convertible bonds are allowed, whereas floating rate bonds and bonds with sinking fund provisions are excluded. The risk-free yield curve is estimated by fitting the Nelson/Siegel model. For the necessary parameters of asset value and asset volatility, the author solves the JMR equation and the equity valuation equation specific for the model simultaneously using numerical methods. This estimation method is an extension to JMR's original technique, first proposed by Ronn and Verma (1986). JMR's single equation is extended to solve two simultaneous equations for two unknowns, asset value and volatility, where the second equation is simply used to view equity as the call option on asset value. The results indicate that the fits of the models are quite poor. The Merton model has a spread error of -193 bps and a relative error of -78 per cent, whereas the Leland model has a spread error of 193 bps and a relative error of -75 per cent. The study fails in generating risk-neutral default probabilities close to actual probabilities as well. Merton and Leland risk-neutral probabilities are 12.3 per cent and 10.5 per cent, whereas the actual probability lies in the range of 0.11 per cent given rating and maturity.

Teixeira adds that the Fan/Sundaresan's model delivers a better performance with -62 per cent relative spread error. Nevertheless, in each of the three cases, spreads are underestimated.

Although some of the studies of endogenous default models reach more promising results than previous studies of structural models, it is questionable whether the endogenous default modeling itself is responsible for this improvement.[6] In contrast to most previous studies, the Anderson and Sundaresan (2000) and Ericsson and Reneby (2004) use bond price information to estimate model parameters for their endogenous default models, which at least partly explains better performance.

3.2.2 Reduced-Form Models with Bonds as the Instrument

Within the last decades, a new path of modeling default has been opened in credit risk analysis. This second approach is modeling default probabilities as an exogenous variable represented by a default intensity. These type of models remain silent about the cause of default, and model the unpredictable default by a jump process. This constitutes the main difference between structural and reduced-form models. Instead of a diffusing firm value term, there is the default intensity as the parameter for a Poisson process. Extensions of the constant intensity approach of Jarrow and Turnbull (1995) include Jarrow, Lando, and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), and Das and Sundaram (2000).

[6] Davydenko and Strebulaev (2007) explicitly investigate the influence of strategic behavior on corporate spreads and find that bond prices are affected by the possibility of debt restructuring. Though strategic variables are statistically significant in explaining credit spreads their economic significance is quite low.

Empirical studies with reduced-form models differentiate most significantly in terms of (i) their choice of the default intensity process, whether it is taken as a constant or as a Cox process, (ii) whenever Cox processes are used, the selection of the state variables as an unobservable Vasicek or Cox/Ingersoll/Ross process, or as an observable credit risk factor extracted from financial information, (iii) the methods used in estimating the default intensity, and (iv) the choice and estimation of the riskless term structure. Frühwirth and Sögner (2006) is among the few studies that tested a constant intensity approach. Although computationally tractable, a constant intensity translates into limited real-life examples. In the real world, the default probability of corporations change continuously. Duffee (1999) was among the first to see this at a relatively early time point, who conducted research with state variables depending on Cox/Ingersoll/Ross processes. Another contribution by him was incorporating Kalman filter into the estimation of the riskless and default intensity processes. Although his work was one of the first in-depth empirical studies with reduced-form models, a substantial attention to observable credit risk factors was not given until the work of Bakshi, Madan, and Zhang (2006). In their work, Bakshi/Madan/Zhang investigate the effects of choosing various credit risk factors from financial information of corporations, as the observable state variable. Two other studies are also critical in this section: The study of Duffie, Pedersen, and Singleton (2003) extends the analysis to multiple credit events of sovereign bonds, whereas Tauren (1999) has the intention to describe the dynamics of the spreads on their own. Overall, this section aims to show how the empirical studies with reduced-form models have used bonds intensively to explain credit risk premiums within.

In their contribution to the literature, Frühwirth and Sögner (2006) use daily prices of 51 German Mark or Euro denominated coupon bonds that do not have sinking funds or embedded options between January 1999 and June 2000 to estimate a constant default intensity and test the Jarrow/Turnbull model. These bonds are issued by banks and non-bank corporates which are rated as AA, A or BBB by Standard and Poor's during the period. In a three step procedure, Frühwirth/Sögner first analyze with a simulation whether the joint estimation of the default intensity and recovery rates is possible. The authors first calculate riskless zero-coupon bond prices by using the Svensson (1994) parameter estimates provided by the Deutsche Bundesbank. Then they estimate a constant default intensity and a recovery rate jointly by minimizing the errors between observed and the calculated bond prices with the Jarrow/Turnbull model. However, the performed simulation study has proven the weak performance of joint estimation. The estimates depend heavily on the initial values and thus the results are not stable numerically. Instead, the study is continued by estimating a constant default intensity keeping the recovery rate fixed at 50 per cent, following Altman and Kishore (1996) and Moody's (2000).

Then in-sample results from bond prices are analyzed to discover whether (i) cross-sectional estimation, using all bonds in a risk class, or (ii) separate estimation, using only one bond at a time among the total of 51, is a better alternative for default intensity estimation. The initial estimation method of the default intensity is by cross-sectional estimation within each rating class, where it is observed that default intensities differ regardless of the rating of bonds: As an example, the estimated intensity of banks rated A is lower than safer bank bonds that are rated AA. Frühwirth/Sögner try to find determinants of the default intensity by holding regressions on market and firm/bond specific variables. The results are not in favor of cross-sectional estimation:

The authors comment that intensities must be estimated separately since any kind of classification by rating, industry or issuer ends in poorer results than by bond basis. Frühwirth/Sögner therefore conclude that Jarrow/Turnbull's assumption of a constant default rate per issuer is not supported by their analysis.

In a third step, optimal pooling intervals are determined such that out-of-sample pricing errors are minimized. Frühwirth/Sögner incorporate a past pooling time span up to 35 days, as well as using daily estimation. Utilizing optimal pooling intervals, "ex-ante" pricing errors, which are the out-of-sample testing results, show the mean absolute error between calculated and observed price is 24 to 331 bps in cross-sectional estimation, while 10 to 31 bps for separate estimation. In conclusion, the authors suggest the inability of estimating the default intensity and the recovery rate jointly can be used as a foundation of credit risk models such as Duffie and Singleton (1999) where the recovery rate is integrated into the default intensity. Their results suggest that focusing on models where default intensity is autoregressive in structure and is a function of liquidity and the riskless term structure should be the goal of further research.

Nevertheless, there are empirical literature already present partly at the direction they have pointed out. In his study, Duffee (1999) uses a model adapted from the reduced-form approach of Duffie and Singleton (1997, 1999). Following Pearson and Sun (1987), the interest rate process is assumed as the sum of a constant and two factors that follow square-root stochastic processes. The default intensity process is a single-factor square-root process plus two components that include default-free interest rate factors. Therefore any correlation between default intensities and riskless yields are targeted to be captured by the coefficients of the factors in the intensity process. Although incorporation of riskless interest rate factors to the default intensity is a major contribution of the study in the direction which Frühwirth/Sögner have mentioned, Duffee further brings an innovative approach to the estimation process as well. The author uses extended Kalman filter for the calibration of riskless and defaultable bonds. The model variables are estimated for the period from January 1985 through December 1995, with a total sample of 161 firms and around 40,000 prices. The estimation is undertaken first by estimating the two factors for the interest rate process using Treasury yields, and then by separately estimating the instantaneous default intensity for each firm, given the estimates of these factors. The estimates of the default intensities and yield spreads per rating class are computed afterwards. Duffee's in-sample results indicate a root mean squared error of only 10 bps. Duffee argues that his model captures a non-default component within spreads, even though his estimation process provided no information on liquidities. His model results also confirm the steeper slope of low grade firm spreads with respect to the higher grade firm spreads.

In their contribution to reduced-form approaches, Bakshi, Madan, and Zhang (2006) (BMZ henceforth) have followed Duffie/Singleton's stochastic default intensity framework. With an approach similar to Duffee, the risk-free factor is included in computing default risk-adjusted discounting process. The adjusted discounting factor is a sum of constant default factor, risk-free rate and a distress factor. By such formulation, BMZ maintain economy-wide and firm-specific variables to be presented simultaneously. The three factor model used by BMZ has a two-factor interest rate process and a single factor for the credit risk component. The long run mean of interest rates follows the Vasicek process while the short rate is an extended Vasicek process that incorporates the long run mean. Instead of an unobservable default intensity process, BMZ have chosen to work with an observable credit risk factor, which differentiates their study from its counterparts. BMZ constructed several proxies for distress factors and tested a group of different reduced-form models: leverage, book-to-market ratio, profitability, lagged credit spread and normalized stock price by money market account. Leverage is proxied as long term book value of debt divided by firm value (the sum of long term debt and market capitalization of common equity). Another proxy, the book-to-market ratio is defined as book value of equity to market value of equity, while profitability is computed as operating income divided by sales.

The data of BMZ consists of 93 firms and over 46,000 coupon bond prices between 1989 and 1998. BMZ first confirm by OLS regressions that cross-section of yields depends on firm specific distress factors. While estimating the interest rate parameters, they found out that the stochastic mean factor added to the one-factor Vasicek model improved performance. The firm-specific default parameters are estimated similarly, by minimizing the root mean squared pricing error. The in-sample fit shows the firm specific distress factors do not contribute substantially over the interest rate model. BMZ attribute this to their usage of low risk bonds as data set. Following, one-month out-of-sample predictions were undertaken. By dividing absolute valuation error to the bond price, the authors report absolute percentage pricing error, of which a maximum of 1.9 per cent is reached. Another error term is reported by BMZ by calculating the absolute deviation of the calculated yield from the observed yield: the absolute yield basis points error, which is between 27-33 bps for different model types when all rating classes are pooled. Ericsson and Reneby (2004) have pointed out the similarity of the out-of-sample prediction and their results with BMZ although they test a structural model: The errors in spreads are comparable in amounts and similarly rise as credit quality worsens. Finally, BMZ run OLS regressions to analyze pricing errors, and try to explain them with systematic factors such as industrial production growth rate, term premium, and default premium. Overall, the work of BMZ extends the initial work of Duffee and constant intensity approach of Frühwirth/Sögner with observed credit risk factors.

Duffie, Pedersen, and Singleton (2003) (DPS henceforth) extended the application of reduced-form models to sovereign bonds. They undertake the study for the special case of Russian default in 1998. Although sovereign default is a political decision at the end, bonds of sovereigns have been priced using similar models that are applicable to corporate bonds. Nevertheless, the authors comment that structural models, that capture the default incentives and solvency, will not be appropriate if empirical analysis should be undertaken. They note that nearly all of the studies in the literature have used reduced-form approach for pricing sovereign bonds (Pages (2000), Düllmann and Windfuhr (2000), Merrick (2001)).

In parallel to the literature, DPS apply Duffie and Singleton's (1999) reduced-form approach using weekly data between the period of February 1994 to August 1998, which is just before the default of Russian short-term discount bonds. DPS extend the Duffie/Singleton recovery-of-market value formulation to allow multiple types of credit events, specifically for (i) default or repudiation, (ii) restructuring and renegotiation, and (iii) regime switch of government. However, DPS comment that adequate data with the Russian bonds is not available to separate the credit event effects.

For the riskless yield, the authors use US Dollar swap yields with LIBOR-quality. The decision to use swap rates instead of Treasury yields was due that they are disburdened by repo specials and tax advantages, and that they can be reached on the basis of constant maturity. A two-factor affine model with stochastic volatility is used for the term structure model for which a maximum likelihood procedure is utilized to estimate its parameters. After reaching in-sample fits for the pre-crisis and default periods of Russia, DPS price two additional Russian bonds out-of-sample. A good fit is observed until August 1998, the credit event. Another observation is that the out-of-sample model prices are significantly lower than Eurobond prices after the credit event. DPS conclude that Russian yield spreads are affected from political events and that they are also correlated with the oil prices and foreign currency reserves. The extension of DPS to allow multiple types of credit events is one of the major contributions of the study.

A similar line of research to the aim of pricing corporate liabilities has the goal to explain credit spreads on their own. Although these two aims have close links, they distinguish in the fact that the estimation of the risk-free rate from government bonds is not taken into account for when the aim is directly explaining credit spreads. In his contribution to literature, Tauren (1999) attempts to focus directly on the spreads in a Duffie/Singleton framework. The selection of the default intensity is therefore in parallel to Duffee (1999), Bakshi, Madan, and Zhang (2006), and Duffie, Pedersen, and Singleton (2003). Focusing directly on spreads, Tauren formulates a stochastic differential equation for the expected loss rate, $h(t)L(t)$. In doing this, Tauren assumes that the expected loss rate and riskless rate are not correlated. 112 corporate bonds of 26 firms during the period of 1986 - 1994 are retrieved from Salomon Brothers for estimation purposes. The out-of-sample prediction data set consists of 22 corporate bonds from 8 companies. Tauren also collects the rating class yield indices from Standard & Poor's Bond Guides. The loss rate, L, is taken as 52 per cent as computed in Carty and Lieberman (1998) for senior unsecured public debt.

The starting point of Tauren is to apply the methodology developed by Chan, Karolyi, Longstaff, and Sanders (1992) (CKLS henceforth) to reduced-form models. CKLS have estimated and compared different models of short-term interest rate using Generalized Method of Moments (GMM). GMM brings the ease that specifying the distribution of error terms is not required and that autocorrelation and heteroscedasticity is allowed. To finalize the spread dynamics formula, Tauren inserts a negative survivorship bias to the drift, which produces an effect similar to mean reversion.

As the parameter estimates with CKLS model is reached the statistics show that the model is misspecified. This leads Tauren to test robustness of CKLS specification. One of the assumptions tested was the independence of riskless rate and the spreads. Tauren points out that the bonds in the sample have mostly call features, which may bring dependence on interest rates with call premia. Results show that as long as mean reversion is included, CKLS model is robust. A consequent subperiod analysis indicates that parameter estimates of the two half-intervals are identical. Finally, the out-of-sample prediction results show the mean-reversion included in CKLS model causes poorer performance. Consistent with Duffee (1999), the results suggest that long-term levels of spreads are higher for lower quality bonds than for higher quality bonds. The author comments that repeating the study with the inclusion of non-investment grade bonds would be a valuable effort.

As can be observed from the studies mentioned in this section, reduced-form modeling can involve various forms and techniques in application. Factors that affect the prediction ability are therefore numerous, and it is difficult to discriminate a model's capability from each other. Constant default intensity approach of Frühwirth and Sögner (2006) has shown the necessity of Cox processes for better results. In using a Cox process, one major distinction in application is whether the study uses observable distress factors for the default intensity such as in Bakshi, Madan, and Zhang (2006) or the default intensity is modeled as an unobservable process as in Duffee (1999). Application of reduced-form modeling to sovereign bonds with multiple credit events has also been accomplished with the study of Duffie, Pedersen, and Singleton (2003). Overall, reduced-form models can be accepted as viable tools as long as mathematical techniques support economical intuition.

3.2.3 Studies with Credit Default Swaps

Credit risk literature has recently started considering the expanding market of credit default swaps as a basis of research. The development of empirical studies has been parallel to the development of the market, as usable empirical data has been available in adequate time series only recently. Being unconstrained by liquidity effects, the price of a CDS is a pure measure of credit risk, and therefore is an appropriate tool to be used in studies with credit risk modeling. In the preceding sections, studies with bonds in which only credit risk models are tested were examined. However, in the scope of this thesis, it is necessary to understand the credit default swap as a product with its many dimensions. This section reviews both the studies that directly apply credit risk models to CDSs and those that empirically investigate the nature of CDS prices. Naturally, studies that test models may include regressions that analyze CDS features. Nevertheless, the first part contains studies that focus their study on mainly testing the models, whereas the second part includes studies which do not use models at all.

Studies that Test Credit Risk Models with CDSs

As a complementary tool to bonds as the empirical data source, prices of CDSs have been recently popular as datasets. For instance, the recent work of Longstaff, Mithal, and Neis (2005) (LMN henceforth) differs from other empirical papers on credit spreads by that they use the default intensities estimated from credit default swap premiums to analyze corporate bond spreads. In the first approach they use, they follow the no-arbitrage comparison in which the premium of default swap equals the credit spread of bonds. This assumption is thoroughly analyzed in Duffie (1999). Still, as investigated in the work by Duffie and Liu (2001), this approach can produce a biased measure of the default component. Therefore as a second approach, the authors generate a reduced-form model in the Duffie and Singleton (1997, 1999) framework. A major contribution of their work is the analysis they have undertaken by showing a non-default component is present in corporate bond spreads due to illiquidity measures.

LMN start their investigation with a case study of the default of Enron and then extend the analysis to 68 firms. Because of the problem of finding a matching five-year bond available at each date of five-year CDSs, the authors use a bracketing set approach. A set of bonds with maturities that bracket the five-year time span of the credit default swap are used, instead of working with a single Enron bond. When LMN compare the CDS premiums to the bond spreads, it is observed that the premium is 49 per cent of the total spread for AAA/AA-rated bonds, 53 per cent for A-rated bonds, 68 per cent for BBB-rated bonds and 84 per cent for bonds that are below investment grade. These are significant differences in absolute terms. Then they estimate the default intensity for each firm from the observed CDS premiums. In order to obtain default component for each bond, they regress the time series of yield spreads of each firm on the estimated default components for the firm. After comparing the ratio of default component to observed credit spreads, they comment that the "model independent approach", which is directly comparing default swap premiums to spreads, performs poorer results.

LMN put forward that the corporate spreads do not only consist of a default component, but also a non-default component as well. At the final stage, they investigate the characteristics of the non-default component in a cross-sectional and a consequent time series analysis. For this reason, they regress the non-default component of the spread on variables such as the coupon, the bid-ask spread, the principal, the age, the maturity, the financial institution dummy and the rating dummy for high investment grade bonds. It came out that the average bid-ask spread is significantly related to the non-default component, which signals that the size of the non-default component increases as liquidity decreases. The principal is negatively related, which confirms that large issue sizes that are more liquid have smaller non-default components. This argument that liquidity risk is priced in bond spreads is consistent with Fisher's (1959) early work. The following time series analysis aims to explain market wide measures of illiquidity, since the cross-sectional analysis better explains bond-specific measures. Finally, overall liquidity proxies for the market such as flows into money market funds are significant, as well.

Similar to the work of LMN, a two step procedure is used by Houweling and Vorst (2005) to analyze the default swap premiums and bond spreads. In the first step, under a no-arbitrage argument the authors directly compare observed bond spreads and observed default swap premiums. The next step is introducing a reduced-form model and calculating model default swap premiums. Their major contribution to the literature lies in suggesting swap or repo rates as a better benchmark for risk-free rate rather than government yields.

Houweling/Vorst use bond data between January 1999 and January 2001 from Reuters and Bloomberg. They use a sample of 1,131 fixed coupon, senior unsecured, bullet bonds with 258,000 price quotes. Their default data set is retrieved from commercial and investment banks, and internet trading services (Creditex and CreditTrade) which comprises of 225 reference entities and around 23,000 CDS prices. As a benchmark of the riskless curve, Treasury, swap and repurchase agreement (repo) rates are used for comparison. When Houweling/Vorst analyze the default swap data, it is observed that the average premiums decrease with credit quality as expected, whereas no apparent relation between maturity and premium could be found. In order to provide a basis for comparing observed bond spreads and CDS premiums, matching is done by either finding a bond that has a maturity discrepancy of at most 10 per cent from default swap maturity or by interpolating bond maturities. Pricing errors are calculated when a pair is formed by subtracting bond bid spreads from default swap ask quotes and vice versa. By doing so, similar sides of the market are compared. The interpolation method has proven better results, and in absolute terms premiums deviate from the market values about 68 per cent for AAA rated bonds and 19 per cent for BBB rated bonds.

In the second step of the study in which a reduced-form model is introduced, the default intensity is modeled as a polynomial function of time to maturity. In the case of constant default intensity model, the intensity increases with credit rating, and this suggests the success of credit ratings in assessing creditworthiness. It is also observed that the selected default free curve has an effect on the level of the default intensity. By constructing pairs similar to the approach described above, but this time using model default premium, new pricing errors are reached.

It is important to underline the approach difference between LMN and Houweling/Vorst studies: Houweling/Vorst have calculated default swap premiums from default intensities estimated from bond spreads. In contrary, LMN have estimated the default intensities from observed default swap premiums to calculate bond spreads. Moreover, Houweling/Vorst have not included an illiquidity premium in their model, while LMN have undertaken an in-depth analysis. The main result of Houweling/Vorst assesses that a reduced-form model performs better than the first method which directly compares spreads and premiums - a finding in parallel to LMN, but still there occurs an absolute deviation of 20 per cent to 50 per cent. The best performance for investment grade issuers among different default intensities and risk-free curves is by the quadratic model that uses the repo curve. A typical conclusion is that government curve should not be seen as the sole choice of riskless curve.

Another recent study that incorporates a liquidity component in both bond and CDS spreads is the work of Bühler and Trapp (2005). Unlike previous studies that assume perfect liquidity for CDS premiums, the authors point out to the fact that bid-ask spreads of CDSs might be large enough to justify a liquidity premium. Therefore the major contribution of the paper is to build a reduced-form model for the CDS prices with an additional liquidity component.

The study is designed so that bonds and CDSs of a given issuer have identical default risks but different liquidity risks.

For a dataset of 37 bonds of 10 telecommunications companies between August 2001 and May 2005, the authors assume a square-root process for the default intensity as in the study of LMN, and the liquidity process also follows a Brownian motion. Bühler/Trapp find that adding a CDS-specific liquidity component causes positive credit and liquidity premiums in the bond market. Moreover, as the default risk increases, the bond market's liquidity dries up, whereas the CDS market becomes more liquid. Through modeling liquidity components in both CDS and bond markets, the authors are able to explain both positive and negative values of the basis.

The LMN, Houweling/Vorst and Bühler/Trapp articles all study credit risk with reduced-form models. A more recent study, Chen, Fabozzi, Pan, and Sverdlove (2006), on the other hand, analyzes the sources of credit risk by comparing the structural models of Merton (1974), Rabinovitch (1989), and Longstaff and Schwartz (1995), with fixed and random recovery barrier models. The authors are inspired by the end result of Wei and Guo (1997) that the Longstaff/Schwartz and the Merton model do not outperform each other due to their non-nested constructions. Therefore, they try to build pairs of models which are nested, in which one model is the special case of the other, and look at what kind of constraint makes a difference. From the study of Wei/Guo in Section 3.2.1, one can recall that the Merton model has a single default and barrier, constant interest rates, and random recovery, whereas the Longstaff/Schwartz model has a continuous default barrier, stochastic interest rates, and fixed recovery. Since the models did not dominate each other, it was not possible to determine whether random recovery in the Merton model or the stochastic recovery in Longstaff/Schwartz model plays a more important role. With a CDS dataset of 3,496 trade observations between February 2000 and April 2003, the authors find that fixed recovery barrier and Longstaff/Schwartz model overestimate CDS spreads whereas the Merton, Rabinovitch and random recovery barrier models underestimate them. Their results indicate random recovery and stochastic interest rates are important assumptions, whereas they did not find support for the continuous default assumption. Their findings are consistent with the study of Wei/Guo.

An adjacent field of application which tests credit risk models is making use of sovereign crisis information. CDS prices of sovereign entities or of corporates that reside in these countries should show a clear demonstration of the crisis during turbulent times. A study that analyzes credit default swaps during such crisis times is Skinner and Diaz (2003) which uses the period of Asian crisis as a case study to compare Jarrow and Turnbull (1995) and Duffie and Singleton (1999) models. By choosing these two models recovery of treasury value and recovery of market value assumptions will be analyzed respectively. A binomial version of the models is utilized, and 31 default swaps between the period of September 1997 and February 1999 are used. The sample is divided into two sub-categories; the "crisis swaps" had a reference entity in Asian countries while the "non-crisis swaps" did not.

The main contribution of Skinner/Diaz is the comparison of the expected premium and the expected payment in case of default, for crisis and non-crisis CDSs. The Duffie/Singleton model reaches greater values for the expected payment than the premium consistently for non-crisis CDSs while for Jarrow/Turnbull model the groups are in balance. This is mostly because higher default intensities are reached by the Jarrow/Turnbull model, therefore calculated premiums and insurance payments are lower than those of Duffie/Singleton. The results show in general that for non-crisis default swaps the expected payment is larger than the expected premium, while for crisis default swaps, the situation is the contrary. The authors suggest that this may be due to a moral hazard problem, and point out that the credit event definition involves restructuring for their sample. The buyer of the insurance who can be same as the owner of the underlying bond, may therefore effect the insurance payments.

On the other hand, Zhang (2003) includes several features to his default swap valuation framework while focusing on the specific case of what the market expected before the Argentina default. The model is first to untie the default probabilities from the recovery rate, allow correlation between underlying state variables, and incorporate counterparty default risk, in one framework. Similar to Duffee (1999) and Bakshi, Madan, and Zhang (2006) the hazard rate is assumed to be linear in three state variables of economy. The data set consists of mid market quotes from J.P. Morgan of 10 contracts on Argentinean sovereign debt in a time span of January 1999 to December 2001. 149 weekly observations therefore total to 1490 default swap quotes. It is noted that mean absolute pricing errors of the model are within 10-20 bps while the performance deteriorates as default on December 2001 is approached. It is an observation that major rating agencies lagged the credit market in downgrading Argentina debt, with their optimistic view during the period.

Studies that Empirically Investigate Features of CDSs

In addition to the studies that tested credit risk models by using CDSs, there is a growing number of literature that investigate various features of this instrument. These studies fall mostly into one or more of the following categories: (i)Studies that basically compared the bond spread with the CDS spread under a no arbitrage comparison (Blanco, Brennan, and Marsh (2005); Hull, Predescu, and White (2004)). As seen in the last section, this analysis is also a first step towards understanding the relationship between bond and CDS spreads in testing a credit risk model (Longstaff, Mithal, and Neis (2005); Houweling and Vorst (2005)). (ii)Studies that have held regressions to understand the effects of various features on CDS premiums (Skinner and Townend (2002); Cossin, Hricko, Aunon-Nerin, and Huang (2002); Blanco, Brennan, and Marsh (2005)). These studies tried to find a significance between the underlying features and CDS prices. (iii)Studies that have undertaken an event study around certain credit risk events and looked at how CDS premiums changed (Hull, Predescu, and White (2004); Norden and Weber (2004)). In the studies taken for this review, this event was the rating announcements. All these studies try to be a part in the larger picture of how CDS features could be understood.

The first task of Blanco, Brennan, and Marsh (2005) (BBM henceforth) has been to compare the pricing of credit default swaps and bond spreads. Their results suggest that the no-arbitrage equivalence holds mostly, and that credit default swaps form an upper bound for the price of credit risk, whereas bond spreads are observed to construct a lower bound, due to the delivery option present in CDS premiums and the repo cost for short selling the cash bond. Another major attempt of BBM is to check which market is ahead in price discovery. New information is valued in credit default swap market prices more quickly than in bond prices hence the default swap market can be thought as more liquid.

BBM use daily closing price credit default swap data from CreditTrade like

Houweling/Vorst. They also use J.P. Morgan mid-market data for a confirmation of prices of CreditTrade. From Bloomberg, corporate bond data are retrieved and interpolated to fit the default swap data. At the end, the sample is left with 33 reference entities that default swap and bond data match. As the proxy for the risk-free yield, government bonds and swap rates are used at the first attempt.

Testing the no-arbitrage assumption, in average absolute terms CDS rates are 46 basis points higher than corporate bond spreads for treasury rates as the risk-free proxy and 15 basis points higher for swap rates as the proxy. BBM comment that these results are well comparable to the study of Houweling/Vorst, who found similar results for AA and A-rated bonds. In the second step, BBM aim to investigate the lead-lag relationships and the price discovery. The authors use two alternative methods (Hasbrouck (1995), Gonzalo and Granger (1995)) for measurement. The results suggest that CDS premiums clearly lead bond spreads. When final regressions are run with CDS price and bond spreads as the dependent variables, the price discovery leading of CDS premiums is confirmed. Variables like interest rates, term structure, stock market returns and stock market implied volatilities have a higher effect on bond spreads than CDS premiums while it is the opposite for firm-specific variables such as stock price returns and implied volatilities. Nevertheless, the adjusted R^2 is low (around 25 per cent) for the regressions, signaling the low percentage of explanation of the dependent variables.

The regressions held by Blanco/Brennan/Marsh had its early examples in Cossin, Hricko, Aunon-Nerin, and Huang (2002) (CHAH henceforth) and Skinner and Townend (2002). Although CHAH have also utilized only 75 observations to implement Das and Sundaram's (2000) reduced-form model, the major strength of their study lies in the regressions that aims to explain determinants of CDS transaction prices in a cross-sectional study. The sample set consists of 392 traded contracts during January 1998 to February 2000. Their results show that the credit rating is highly significant as a factor of credit risk explaining the 47 per cent of variance for US corporates. Structural variables such as asset value volatility, leverage, and interest rate are also significant and explain together the 31 per cent of the variance for US corporates. It is also an important observation that US and non-US corporates show different behaviors which suggest that the international markets are not homogeneous.

In another early empirical study with credit default swaps, Skinner and Townend (2002) suggest that CDSs can be examined as put options. They construct a sample of only 29 sovereign CDS quotes between September 1997 and February 1999. The authors subdivide their sample into Asian and non-Asian subgroups, with the thought of capturing a difference due to Asian currency crisis in the given period. When regressions are run, variables that affect the prices of put options such as riskless rate, volatility, underlying asset yield, and time to maturity came out to be significant in explaining CDS prices. Their R^2 is as high as 99.5 per cent when the full sample is tested. However, using the analogy of put options, Skinner/Townend have hypothesized that the CDS premium should increase with increasing maturity. In contrast, their results are robust to a negative relationship. As a reason, they highlight that the non-Asian swaps have an average maturity of 7 years with lower premiums, while Asian swaps are on average 3 years and have higher premiums due to Asian crisis. Their results suggest that in order to manage the risk traded in a crisis environment, buyers of a CDS could look for decreased premiums by entering into shorter term default swaps.

In a different path of examining CDSs, Hull, Predescu, and White (2004) (henceforth HPW) and Norden and Weber (2004) have analyzed the impact of credit rating announcements with event studies. In the first part of their study HPW adjust the no-arbitrage assumption following Duffie (1999) and Hull and White (2000) by dividing the difference of risky and riskless yield with accrued interest on par yield bond:

$$CDS\ Spread = \frac{Risky\ Yield - Riskless\ Yield}{1 + Accrued\ Interest} \tag{3.4}$$

By using the data between January 1998 and May 2002 from GFI, 370 CDS quotes were matched with corresponding bonds on the same day. The matching was done by simply regressing the yields of bonds on a given CDS quote day on the maturities of the bonds. The accrued interest in Equation (3.4) is assumed as $(Risky\ Yield/4)$ as all bonds paid interest semiannually. HPW compare the outcomes with Treasury rates and swap rates independently and reach the conclusion that swap rates are a better proxy for the risk free rate, given the no-arbitrage equation holds.

HPW then proceed with analyzing the effects of credit rating changes. The CDS market possesses an anticipation towards negative outlooks, reviews for downgrades and downgrades, while only reviews for downgrades provide information. On the other hand with a similar range of data, from July 1998 to December 2002, Norden/Weber find that both CDS and stock markets anticipate downgrades and review for downgrades. This finding is robust for all three big rating agencies: Standard & Poor's, Moody's and Fitch. The same results do not hold for positive rating events.

All the above studies with credit default swaps have either looked at the no-arbitrage relationship between bond and CDS spreads, or applied reduced-form models to correctly price a CDS. Alternatively, there have been studies that looked at price discovery and lead-lag relationships, or studies that have run regressions for the determinants of the CDS premiums. In any case, these studies should be treated as preliminary since the market is only at an infant stage. It is of no doubt that the number of empirical studies with CDSs will grow, and continue its field of research jointly with other topics in credit risk.

3.3 Summary

Obviously, there are numerous empirical studies in the field of credit risk. Structural models have on average failed to generate reasonable spreads, whereas alternative reduced-form formulations yielded different prediction results. Recently, studies with CDSs have emerged, bringing a new dimension to credit risk analysis. Still obviously, all the above studies have reached contradicting results. To date, there has been no common agreement in academia or practice as to which model framework (structural or reduced-form) better represents default risk. One main reason for the lack of consensus is that these empirical studies have provided controversial guidance in the validation process for theoretical models. The unevenness of the empirical studies can be attributed to three factors:

(i) When testing a given model empirically, the usage of different datasets produces widely varying prediction results. For instance, structural models were often criticized by early empirical studies as under-predicting credit spreads (Jones, Mason, and Rosenfeld (1984); Ogden (1987); Lyden and Saraniti (2000)), whereas more recent studies utilizing the same models suggest that this is not a consistent occurrence (Eom, Helwege, and Huang (2004)). It is unclear, however, whether this is due to the models or the datasets used.

(ii) Even though the prediction performances of structural and reduced-form models have been compared within each framework, there have been no empirical studies across modeling structures in which the same dataset and methodology were applied to both frameworks. For instance, although Eom, Helwege, and Huang's (2004) study is the most comprehensive study to date in which several structural models are compared in one setting, it does not address the results acquired with reduced-form approaches. Similarly, empirical studies conducted by Anderson and Sundaresan (2000) or Bakshi, Madan, and Zhang (2006) have approved a special model setup without any basis of comparison across frameworks. The only study that compares structural and reduced-form approaches is the work of Arora, Bohn, and Zhu (2005). This study compares the structural models of Merton and Vasicek/Kealhofer (the Moody's KMV approach) to a Hull/White type reduced-form model. Although the intention is acceptable, the methodology undertaken by this study is poor. The structural models are calibrated by stock prices whereas the reduced-form model is calibrated with bond prices. Although all three models in the end out-of-sample price CDSs, the methodology raises questions on the fairness and validity of such a comparison.

(iii) Moreover, although the error results between the compared models might turn out to be similar, the estimation technique and the sampling setup for prediction highly influences the forecasting ability. As an example, Ericsson and Reneby's (2004) out-of-sample prediction results, obtained by using an extension of Leland's (1994) endogenous structural model and employing an innovative estimation technique in Duan (1994), seem well comparable to Duffee's (1999) in-sample results with a reduced-form model utilizing Kalman filter estimation, both having a root mean squared error of around 10 bps. Apparently, looking at the prediction results alone would be misleading, since these do not express the detailed aspects of both settings.

The presence of all these issues therefore necessitates that further efforts be undertaken. Ideally, this would be accomplished by testing the structural and reduced-form models via the application of the same methodology to the same datasets, as will be presented in the following chapters.

Chapter 4

CDS Pricing with Basic Framework Structures

In this chapter, a comparative study between the two basic forms of structural and reduced-form frameworks is undertaken. This is in parallel to the work in Gündüz, Uhrig-Homburg, and Seese (2006) and Gündüz and Uhrig-Homburg (2007). In this first attempt to compare financial modeling frameworks in pricing CDSs, the model of Merton (1974) has been chosen to work with, which has the simplest form that a structural model can have. Meanwhile, the constant default intensity as outlined by Jarrow and Turnbull (1995) is employed, which can be regarded as the simplest form in the reduced-form framework. Although there are many extensions that have developed on top of these structures, a comparison of the two basic models has been concentrated on initially. This comparison would give a first indication on how structural and reduced-form frameworks vary in their structures. The results of both approaches are contrasted with the prediction results of a machine learning approach. The analysis is therefore three-fold:

(i) It applies the most financially structured models to a CDS dataset under the hypothesis that default is triggered by the asset value of the firm being below a certain threshold at maturity.

(ii) It applies the intensity-based Poisson jump process setting to the same dataset under the hypothesis that default is defined as a surprise event that can occur at any time during the lifetime.

(iii) It applies Support Vector Machines - a machine learning algorithm that does not have an economically-backed structure at all - under the hypothesis that whatever resides in the prices is the best source to train the function for predictions.

4.1 The Three Models

4.1.1 The Merton Model

Firstly, the Merton's (1974) model is applied to the CDS dataset. This model allows default only at maturity, and does not incorporate a stochastic process for the interest rate. In order to value a CDS, consider its two legs, the premium and the protection leg. The premium leg is the fee as a percentage of the contract amount that the buyer of the insurance has to pay to the seller until maturity or default, whichever comes first. The protection leg is the single payment that the seller of the contract is obliged to undertake in case of default of the entity upon which the contract is written. In Merton's setting, the premium leg is nothing but the discounting of each fair premium s^{theo} paid until maturity:

$$PremiumLeg = s^{theo} \sum_{i=1}^{n} e^{-r(i)T(i)} \tag{4.1}$$

where $T(i)$ is the time interval in yearly terms, and, as the usual practice is quarterly paid premiums, $T(1)$ is 0.25, $T(2)$ is 0.5, and so on; the maturity of the contract $T(n)$ is 5 years. $r(i)$ is the riskless interest rate for maturity $T(i)$ on the contract setup date. The protection leg constitutes the discounting of the probability of default at maturity, multiplied with the non-recoverable amount:

$$ProtectionLeg = (1 - \varphi)\Phi(-d_2)(e^{-r(n)T(n)}) \tag{4.2}$$

where $\Phi(-d_2)$ is the risk-neutral default probability in the Merton setting. The recovery rate in case of default, φ also enters the protection leg. It might have been a sound approach to estimate the recovery rate simultaneously with the default intensity; however, as mentioned in Chapter 3, recent applications undertaken by Houweling and Vorst (2005) and Frühwirth and Sögner (2006) have shown the insensitivity of the results based upon the selection of this variable. To simplify methods, the recovery rate can be fixed to a value obtained in empirical studies based on historically defaulted bonds. Following the results produced by Altman and Kishore (1996) and recent practice, for the senior class a value of 0.5 has been used.[1] As a last step, the theoretically fair CDS premium is reached by equating the premium and protection legs at time zero:

$$s^{theo} = \frac{(1 - \varphi)\Phi(-d_2)(e^{-r(n)T(n)})}{\sum_{i=1}^{n} e^{-r(i)T(i)}} \tag{4.3}$$

[1] Although the 0.5 figure is derived from the US market, recent efforts with European data have also relied on this figure (Houweling and Vorst (2005); Frühwirth and Sögner (2006)). Considering the fact that Basle 2 provisions accept a loss given default of 50 per cent for bank loans independent of the country chosen, this is not an unrealistic assumption.

This premium ensures that the CDS contract has zero value on initiation, which in turn guarantees that the buyer and the seller are even under no-arbitrage assumptions.

4.1.2 The Constant Default Intensity Model

The basic structure of a constant default intensity model was introduced by Jarrow and Turnbull (1995). In contrast to more advanced intensity-based models, the stochastic process driving the riskless term structure and the default process are assumed to be independent in the Jarrow/Turnbull setting. While Jarrow/Turnbull assume a constant intensity under the real world measure, the intensity becomes time-varying when they turn to the risk-neutral world. In this application, it has been started directly with a constant risk-neutral intensity. Following Duffie and Singleton (2003), it can be shown that the pricing of CDS is composed of a premium and protection leg as below:

$$PremiumLeg = s^{theo} \sum_{i=1}^{n} e^{-(\lambda + r(i))T(i)} \tag{4.4}$$

$$ProtectionLeg = (1-\varphi) \sum_{i=1}^{n} e^{-r(i)T(i)} (e^{-\lambda T(i-1)} - e^{-\lambda T(i)}) \tag{4.5}$$

where λ is the constant default intensity parameter. Equating these two legs to extract the theoretically fair premium leads to:

$$s^{theo} = (1-\varphi)(e^{\lambda \Delta t} - 1) \tag{4.6}$$

when intervals $T(i+1) - T(i) = \Delta t$ are constant between premiums. Thus, in the case of quarterly payments, the interest rate parameters cancel out, and the constant intensity case is insensitive to interest rates. This is the most significant difference in the constant intensity setting from the Merton model in this application. A second important distinguishing feature is that the Merton model allows default only at maturity, whereas the setup permits early default in the intensity setting.

4.1.3 Support Vector Machines Regression (SVM)

In addition to comparing the two financial credit risk frameworks, a third alternative method is employed in this study. The recent developments in machine learning have opened a new pathway for computing empirical predictions: Support Vector Machines (SVM) is an innovative technique for data classification and regression. As an alternative to traditional neural network approaches, SVM, whose fundamentals were developed by Vapnik (1995), have become popular due to their promising empirical performance. Specifically, the SVM regression method proposes alternative kernel functions to be used in mapping into a high dimensional feature space. Literature on SVM regression applications on finance is sparse; there has been coverage especially on financial time series forecasting (Cao and Tay (2001); Müller, Smola, Rätsch, Schölkopf, Kohlmorgen, and Vapnik (1997)). To my knowledge, SVM has not been applied to credit derivatives pricing, and its results have accordingly not yet been compared with financial methods.

SVM has proven to be a good alternative to traditional neural network applications: The problem of building architecture for neural networks is replaced by the problem of choosing a suitable kernel for the SVM.[2] In this study, the

[2]Without the use of kernels, the problem of non-linear machine construction would have required two steps: First, a fixed non-linear mapping to transform the data into a "fea-

results of the financial models are compared to SVM regression models with linear, polynomial, Gaussian radial basis and exponential radial basis kernel functions. These four fundamental kernel functions are described below. The most basic kernel function is linear; it is simply the inner product of training points u and test points v:

$$K(u, v) = \langle u, v \rangle \tag{4.7}$$

An alternative approach would be to analyze polynomial kernel function with degree 2. This is a popular method for non-linear modeling:

$$K(u, v) = (\langle u, v \rangle + 1)^2 \tag{4.8}$$

The third type to have received significant attention in the literature is the Gaussian radial basis function, which is:

$$K(u, v) = exp\Big(- \frac{\parallel u - v \parallel^2}{2\sigma^2} \Big) \tag{4.9}$$

where σ is taken to be 0.5 after observing fits of alternative parameter choices used in the literature (Müller, Smola, Rätsch, Schölkopf, Kohlmorgen, and Vapnik (1997); Gunn (1998); Cao and Tay (2001)). A final choice would be exponential radial basis function, which is a similar alternative to Gaussian RBF.

$$K(u, v) = exp\Big(- \frac{\parallel u - v \parallel}{2\sigma^2} \Big) \tag{4.10}$$

ture" space where the analysis is easier, and then a linear machine to classify/regress it in the feature space. Kernel theory stipulates that an inner product in feature space has an equivalent kernel in input space; utilizing kernel functions therefore simplifies the algorithm. There is no more need to think about the mapping and evaluation of the feature map, but only about the inner products of test and training variables (for details see Cristianini and Shawe-Taylor (2000); Gunn (1998)).

A parameter value ζ, which allows slack in the system that permits the samples to be on the wrong side of the decision boundary (a penalty parameter of the error term), is also taken as 10, in all runs, after a search for the best-fitting value. Similarly, the ε insensitive band has been set to 10E-4.

4.2 Results of the Cross-Sectional Design

In order to pursue a cross-sectional analysis within a specific set of CDS prices, the dataset is divided into certain risk clusters that ought to exhibit identical risk characteristics. Thus the main hypothesis of the cross-sectional study is that within certain risk classes, the credit risk is priced the same. From the described data in Section 2.2, specific risk clusters were focused on, namely the contracts on Aa- and A-rated companies with 5-year maturity ranked senior, since these risk classes provided the adequate number of data points. Although the literature does not distinguish between North American and European entities, this breakdown would allow to analyze regional characteristics. An additional split according to the currency of the contract that the CDS is written on was not necessary, because European and North American entities had a natural breakdown into euros and US dollars, respectively.

Table 4.1 provides the average midpoints and spreads across ratings for the CDSs that are focused upon in this chapter of the study. The midpoints for AA-rated CDSs are lower than A-rated CDSs for both Europe and North America, in line with the theory. North American CDSs are consistently higher than their European counterparts in each of the risk classes and years. There are also relatively less observations for North American CDSs than European CDSs. The offer price minus the bid price for a given quote in the dataset has an average of 4.75 bps for the Aa-European class, whereas it is 6.81 bps for A-European CDSs. A similar rise is observed for North American CDSs. Notably, the average premiums and spreads are observed to steadily decrease over time.

Table 4.1: Descriptive Statistics of CDS Dataset between December 2002-December 2004. Midpoints of Bid-Ask Prices, Average Bid-Ask Spreads and Number of Observations across Ratings and Regions for 5-year, Senior CDS.

		2002			2003			2004			Total		
Rating	Region	Mid	Spread	Obs	Mid	Spread	Obs	Mid	Spread	Obs	Mid	Spread	Obs
Aa	Europe	30.83	9.17	400	22.06	6.06	5119	14.46	3.17	5382	18.63	4.75	10901
Aa	N.America	46.99	13.66	231	33.81	10.40	2374	26.45	10.03	2441	30.85	10.37	5046
A	Europe	65.06	13.18	858	48.91	8.69	10081	33.43	4.17	9273	42.49	6.81	20212
A	N. America	98.28	21.12	459	53.27	14.56	5493	34.82	9.96	5187	46.53	12.69	11139

Mid (bps): Average of the midpoint of each bid and offer price.
Spread (bps): Average of the difference of offer price - bid price.
Obs: Number of observations in the cluster.

In addition to the CDS dataset, riskless interest rates are required as a major variable in models. In doing this, USD- and Euro-denominated contracts have to be treated separately. The daily estimates of the Svensson (1994) model are used as the rates for the European region. Deutsche Bundesbank has estimated these parameters from government bonds, which is in detail explained in the Monthly Report of Deutsche Bundesbank (1997) in October (pp. 61-66). For the North American region, US Treasury Constant Maturity rates were linearly interpolated for quarterly intervals. The data are directly available from the corresponding web sites.[3]

4.2.1 Setting with the Merton's Model

The cross-sectional/out-of-sample prediction methodology is to first estimate the daily risk-neutral default probabilities for each firm in a "risk cluster" de-

[3] http://www.federalreserve.gov and http://www.bundesbank.de

scribed above from the observed CDS premiums of the firms in the estimation sample. Individual default probabilities ($\Phi_j(-d_2)$) were estimated for each firm j each day using:

$$\Phi_j(-d_2) = \frac{s_j^{obs} \sum_{i=1}^{n} e^{-r(i)T(i)}}{(1-\varphi)\left(e^{-r(n)T(n)}\right)} \tag{4.11}$$

where s_j^{obs} is the observed CDS premium for firm j. Afterwards, the Black/ Scholes parameter d_2 was averaged across the full set of companies in the estimation sample to reach a daily value. The estimation therefore results in an aggregate default probability for each class and day.[4] Figure 4.1 plots the daily risk-neutral default probability estimates for the Aa-rated North American and European CDSs. Interestingly, the North American CDSs have a higher default probability throughout the time horizon, which justifies the inclusion of the regional breakdown when setting up risk classes.

This daily value is used to predict the theoretical CDS premiums of a second set of firms in the prediction sample. Given the specific set of companies in the cluster (all with the same rating class, rank, currency and region), the division of the estimation sample and prediction sample are taken to be around the ratio of 2:1 - 4:1. Sample selection for estimation and prediction groups was fully random in order to preclude any biases due to sample choice.

[4] Minimizing the sum of squared differences was a possible alternative, which would have simply returned the default probability for the average of s^{obs} on any given day. The results from using this approach do not differ significantly.

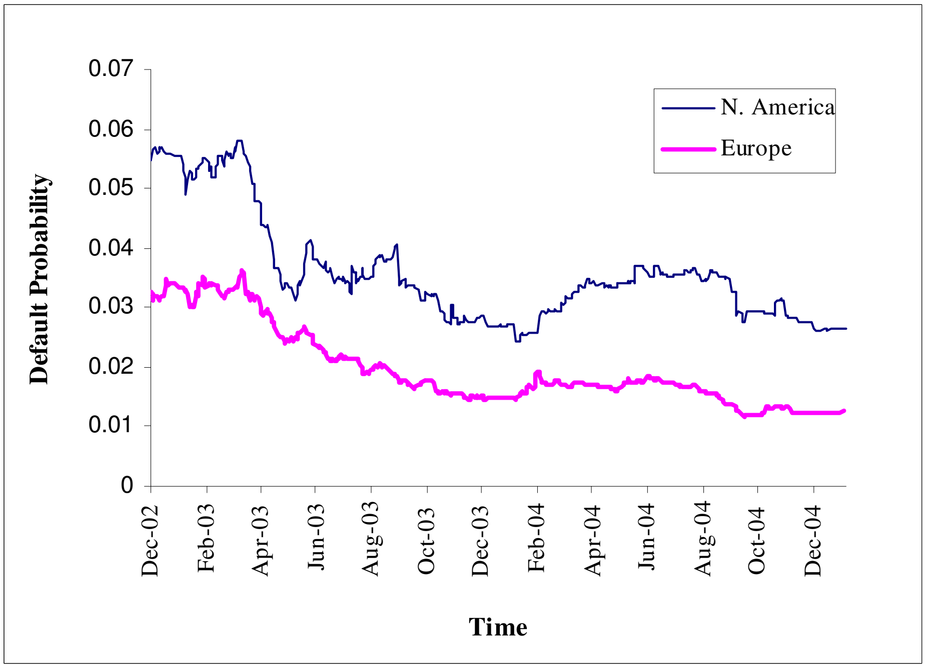

Figure 4.1: Default Probability Estimates with the Merton Model AA-rated North American vs. European Contracts

4.2.2 Setting with the Constant Intensity Model

To ensure that the parameter estimates of the constant default intensity setting are comparable to the Merton model, the firms in the estimation and prediction samples are kept the same. Similar to the Merton setup, the default intensity is estimated for each firm j each day from the first sample of firms using Equation (4.6):

$$\lambda_j = \frac{ln\left(\frac{s_j^{obs}}{(1-\varphi)} + 1\right)}{\Delta t} \tag{4.12}$$

Again, s_j^{obs} is the observed CDS premium for firm j. Next, an average daily default intensity for each risk class is obtained. This value is then plugged into Equation (4.6) to predict the fair value of the firms' CDS premium in the prediction sample.

At this point, it would be insightful to compare the estimates with a recent study. Table 4.2 compares the average default intensities with the results of Frühwirth and Sögner (2006), which is an application of the Jarrow and Turnbull model to European corporate/bank bonds. Our default intensity estimates from CDS prices and a larger dataset extend the estimates of this study. Moreover, some of the inconsistencies of their results have been overcome in our findings.[5]

Table 4.2: Comparison of Two Studies with Constant Default Intensity

European,	This Study		Frühwirth/Sögner
Senior Risk Class	No. of Obs	Intensity from CDS	Intensity from Bonds
Aa-rated Banks	4584	0.0036	0.0041
Aa-rated Corporates	3037	0.0041	0.0085
A-rated Banks	3366	0.0069	0.0035
A-rated Corporates	11179	0.0090	0.0116

4.2.3 Prediction Results with Cross-Sectional Design

Before comparing the out-of-sample prediction quality, some insights on the parameter estimates of the two modeling approaches can be provided. In order to make the intensity estimates comparable to the estimates advanced in Merton's model, the intensity estimates are used to calculate the 5-year risk-neutral default probability:

$$PD(T) = 1 - e^{-\lambda T} \tag{4.13}$$

This enables the direct comparison of the default probability estimates for the four risk classes. First, it is expected that the constant intensity model would

[5]The authors utilized a period between January 1999 and July 2000. In their estimates, A-rated banks had a lower average intensity than Aa-rated banks, which should supposedly be higher.

yield lower default probability estimates than the Merton model, since it incorporates early default. Figure 4.2 provides the trajectories for the Aa-North American risk class. Table 4.3 provides the means, deviations and the number of days that the Merton default probability is higher than the 5-year estimate obtained with the intensity model. It is observed that the means are close, and except for a few weekly intervals, the Merton probability estimate is higher, in line with the expectations. However, note that a higher default probability does not directly translate into a higher CDS premium. In fact, although not tabulated explicitly, the Merton model prediction for the premium is significantly lower than the intensity prediction for three out of four risk classes. The reason for this is that the default payment of φ is made available only at T in the Merton setting, whereas in the intensity setting the same level of payment can be made earlier.

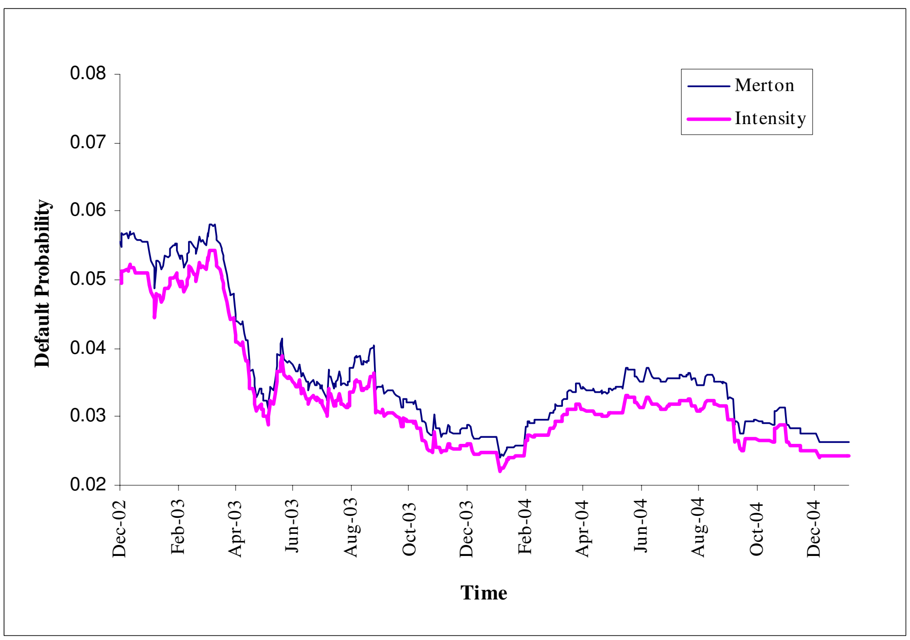

Figure 4.2: Default Probability Estimates with the Merton and the Constant Intensity Models for Aa-rated North American Contracts

The out-of-sample prediction errors are summarized in Table 4.4. The results

Table 4.3: 5-year Default Probability Estimates of the Merton and Intensity Models

	Mean PD (bps)	Std Dev PD (bps)	Obs	Merton PD > Intensity PD (%)
Aa Europe			523	97.1%
Merton	197	67		
Intensity	188	66		
Aa N.America			524	100%
Merton	358	90		
Intensity	328	83		
A Europe			526	93.2%
Merton	422	148		
Intensity	408	146		
A N.America			524	100%
Merton	479	198		
Intensity	452	196		

Mean PD (bps): Average default probability in basis points.

Std Dev PD (bps): Standard deviation of the default probability in basis points.

Obs: Number of observation days.

Merton PD > Intensity PD (%): Percentage of the total sample where Merton default probability is higher than 5-year constant intensity default probability.

indicate a low fit in basis points for all classes, while mean absolute percentage errors (MAPE) are high. The best fit in terms of MAPE is around 23-25 per cent. It can be observed that European/Euro-denominated CDSs have a better fit than North American/USD-denominated CDSs.

A comparison of the prediction errors produced by the Merton and the constant intensity models shows that the results are close. To test this observation statistically, the difference between the absolute prediction errors has been focused on in order to determine whether the models have predicted significantly differently. Table 4.5 summarizes the t-statistic results of the significance tests with the Yule-Walker estimation method. By so doing, the autocorrelation adjusted estimates of the time series for each rating class could be reached via backward

Table 4.4: Out-of-sample Prediction Errors of the Merton and the Constant Intensity Models in Cross-Sectional Design

	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size Estimation	Total Sample Size Prediction
Aa Europe					
Merton	-0.13	4.82	23.82%	7621	3135
Intensity	0.90	5.36	27.64%	7621	3135
Aa N.America					
Merton	9.13	9.30	43.16%	3475	1571
Intensity	9.87	10.01	46.16%	3475	1571
A Europe					
Merton	-2.92	10.18	25.57%	14545	5393
Intensity	0.46	10.20	27.49%	14545	5393
A N.America					
Merton	2.00	10.69	25.61%	9046	2093
Intensity	1.98	10.68	25.60%	9046	2093

Mean Error(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} s_f^{theo} - s_{f,h}^{obs}}{F.H}$ Mean AbsError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} |s_f^{theo} - s_{f,h}^{obs}|}{F.H}$

Mean AbsPercError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} \frac{|s_f^{theo} - s_{f,h}^{obs}|}{s_{f,h}^{obs}} \times 100}{F.H}$

s^{theo} is the theoretical CDS premium predicted by the Merton and Intensity models on day f, where F is the number of available days in the time series. s^{obs} is the observed CDS premium on day f for firm h, where $h = 1..H$ represents the number of firms in the prediction sample.

elimination of insignificant autoregressive lags. In one out of four classes, the Merton model has lower average errors, while there is no statistical difference in the remaining three. This result is rather surprising; by allowing default only at maturity, the Merton model appears to be more restrictive. This may be due to the treatment of default probability as a single parameter rather than breaking it down into Black/Scholes parameters like asset value and asset volatility. Structural models have often been criticized for their weakness of lacking a robust estimation process in these parameters, which hinders their predictive performance. The cross-sectional results signify that in the absence of an estimation process for the asset value and asset volatility parameters, Merton's model can perform at least as well as a reduced-form model.

Table 4.5: Significance Tests for the Difference of Absolute Errors with the Merton and the Constant Intensity Models in Cross-Sectional Design

	Mean Difference	t-statistic	p-value	
Aa Europe	-0.54	-1.64	0.1011	
Aa N. America	-0.72	-4.68	<0.0001	***
A Europe	0.02	0.04	0.9713	
A N.America	0.01	0.41	0.6816	

Mean Difference (bps): Difference of Absolute Errors for prediction (Merton - Intensity) computed per day per firm for risk class. Absolute Error on day f, for firm h is $\mid s_f^{theo} - s_{f,h}^{obs} \mid$
*** Significance at 95 per cent level

4.2.4 Setting and Prediction Results with SVM Regression Method

In order to design cross-sectional samples for SVM comparable to the Merton/Intensity setups, two datasets are required for training the SVM function, plus two additional datasets for test input and test output. Therefore, the firms in the estimation samples in the previous sections have also been selected for training input, training output and test input samples. For instance, if the estimation in the Merton/Intensity setups includes data from 23 firms (as in AA-North America), then these 23 firms were divided into 3 groups; the training input, the training output and the test input samples. Specifically, an out-of-sample SVM prediction is maintained as follows: First, within each risk class, the firms' CDS premiums within each of the four samples (three estimative and one predictive) are averaged to obtain a daily value. In order to train the function, the daily average of the training input sample is mapped to the average of the training output sample. Afterwards, the daily average of the premiums in the test input sample are used to predict a theoretical daily premium based on the SVM function. Finally, the predicted value is compared to the test output sample daily average values, so that the out-of-sample prediction errors can be computed.

The results from the cross-sectional approach can be found in Table 4.6. It can be observed that the SVM algorithm yields poor results in comparison to the financial models in most cases. Some kernels produce results that are too inaccurate to be considered an alternative, e.g. the polynomial kernel. Among all kernels, the linear kernel has the best MAPE in three out of four risk classes. In Table 4.7, the difference between the absolute errors of the Merton/Intensity models and the best performing linear kernel SVM is tabulated. The financial methods are a better predictor in two risk classes, whereas there is insignificance in the remaining two. For the indecisive risk classes, the linear kernel MAPE results come close to the financial methods, one being the A North American class, which produces the best result of all. Due to the overall poor fit of SVM kernels, the efforts with the full set of kernels is discontinued, and instead the linear kernel is focused on in the rest of the study.[6]

[6] Alternative to the cross-sectional setup, a panel setting was analyzed as well. There are 2,650 data points in the training input and output sets from the data of five companies for each set, respectively; 2,120 data points in test input and output sets were used from the data of four companies for each set. This is a setting that is computationally more expensive, and whose prediction results are inferior to those yielded by the cross-sectional design. The results are therefore not presented.

Table 4.6: Out-of-sample Prediction Errors of SVM Algorithms in Cross-Sectional Design

	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size Estimation	Total Sample Size Prediction
Aa Europe					
Linear	-0.55	5.60	26.77%	7621	3135
Polynomial (Deg:2)	-1.05	5.25	24.69%	7621	3135
Gaussian RBF	-6.87	9.31	41.04%	7621	3135
Exponential RBF	-7.21	9.45	42.57%	7621	3135
Aa N.America					
Linear	3.14	17.75	81.18%	3475	1571
Polynomial (Deg:2)	49.85	157.68	657.85%	3475	1571
Gaussian RBF	-3.33	15.72	68.55%	3475	1571
Exponential RBF	-4.01	14.44	61.89%	3475	1571
A Europe					
Linear	6.82	12.57	37.17%	14545	5393
Polynomial (Deg:2)	6.57	12.51	37.30%	14545	5393
Gaussian RBF	-0.59	1.15	166.51%	14545	5393
Exponential RBF	-9.51	18.22	41.49%	14545	5393
A N.America					
Linear	-9.70	12.07	23.11%	9046	2093
Polynomial (Deg:2)	152.67	152.67	310.13%	9046	2093
Gaussian RBF	-17.10	18.02	35.06%	9046	2093
Exponential RBF	-4.01	14.44	61.89%	9046	2093

Mean Error(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} s_f^{theo} - s_{f,h}^{obs}}{F.H}$ Mean AbsError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} |s_f^{theo} - s_{f,h}^{obs}|}{F.H}$

Mean AbsPercError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} \frac{|s_f^{theo} - s_{f,h}^{obs}|}{s_{f,h}^{obs}} \times 100}{F.H}$

s^{theo} is the theoretical CDS premium predicted by SVM algorithms on day f, where F is the number of available days in the time series. s^{obs} is the observed CDS premium on day f for firm h, where $h = 1..H$ represents the number of firms in the prediction (test output) sample.

Table 4.7: Significance Tests for the Difference of Absolute Errors between the Merton/Intensity models and Linear Kernel SVM in Cross-Sectional Design

	Mean Difference	t-statistic	p-value	
Aa Europe				
Merton - SVM	-0.85	-1.65	0.0998	
Intensity - SVM	-0.28	-0.60	0.5494	
Aa N.America				
Merton - SVM	-8.46	-19.25	<0.0001	***
Intensity - SVM	-7.75	-24.29	<0.0001	***
A Europe				
Merton - SVM	-2.32	-2.03	0.0426	***
Intensity - SVM	-2.30	-3.01	0.0026	***
A N.America				
Merton - SVM	-0.49	-0.14	0.8897	
Intensity - SVM	-0.49	-0.14	0.8892	

Mean Difference (bps): Difference of Absolute Errors for prediction (Merton - SVM) and (Intensity - SVM) computed per day per firm. Absolute Error on day f, for firm h is $\mid s_f^{theo} - s_{f,h}^{obs} \mid$
*** Significance at 95 per cent level

4.3 Results of the Time Series Design

4.3.1 Credit Risk Models

As an alternative to cross-sectional estimation and prediction, the models are analyzed in a time series design. This effort hypothesizes that every firm in the sample has a constant default probability/intensity. In contrast to the cross-sectional design, in which the daily default probabilities/intensities are averaged, now the default probabilities/intensities are estimated from a fixed interval, and one-, five-, and ten-day-ahead default probabilities/intensities are predicted separately for each firm.

A rolling estimation and prediction is applied to both the Merton and intensity settings. Frühwirth and Sögner (2006) analyzed the impact of the length of the estimation period on the prediction errors. Within a 5-25 day period, the 14-day mark gave one of the best results. Parallel to these findings, the rolling estimation period is set at 14 days. In order to estimate the default probabilities and predict the CDS premium one day ahead, the approach below has been adapted. First, default probabilities are estimated by minimizing the sum of squared errors between the observed and theoretical CDS premiums:

$$\Phi(-\hat{d_2})_{t+14} = \arg \min_{\Phi(-d_2)} \sum_{k=t}^{t+13} (s_k^{obs} - s_k^{theo}(\Phi(-d_2)))^2 \tag{4.14}$$

where s_k^{obs} is the observed CDS premium on the k^{th} day within the 14-day period, and s_k^{theo} is the theoretically fair price computed from Equation (4.3). For each firm's 14-day period, a default probability estimate is reached, and this figure is plugged into Equation (4.3) to obtain a theoretically fair CDS premium for the 15th day. By comparing the observed and theoretical CDS premiums for one day ahead in a rolling procedure, out-of-sample prediction error statistics are computed.

In Table 4.8, mean errors (ME), mean absolute errors (MAE) and mean absolute percentage errors (MAPE) for the prediction process are given.[7] As can be observed from Table 4.8, MAEs and MAPEs are significantly lower in the time series prediction in comparison to cross-sectioning. The Merton model predicted the four datasets with a MAPE of around 6 per cent. Furthermore, all the error statistics decline in increasing credit quality.

With the constant intensity model, a similar analysis is applied to the same dataset. The sum of squared errors was minimized in 14-day periods to reach an

[7] Only consecutive 14-day periods of observation were used to ensure the continuity of the time series. The estimation sample is simply 14 more for each firm in the risk class and have not been explicitly tabulated.

estimate of the default intensity, as can be seen in Equation (4.15), where s_k^{theo} corresponds to the theoretically fair price of the CDS premium from Equation (4.6).

$$\hat{\lambda}_{t+14} = \arg\min_{\lambda} \sum_{k=t}^{t+13} (s_k^{obs} - s_k^{theo}(\lambda))^2 \tag{4.15}$$

Again, Table 4.8 shows that a fit superior to cross-sectional estimation has been reached. A similar pattern of decreasing errors with increasing credit quality is also indicated by the figures. Moreover, the time series prediction results of the Merton and intensity settings appear even closer than in the cross-sectional setup. Nevertheless, a test for significance has revealed that the intensity model outperformed its counterpart in three out of four risk classes in absolute error terms. Panel A of Table 4.10 tabulates these results. Although the mean difference of absolute errors is close, low standard deviation and large sample size caused high significance.

A second step would be to look at further horizon out-of-sample results. In a similar setup, 14-day time series are utilized to predict five-day- and ten-day-ahead CDS premiums. These results can be found in Table 4.9. As expected, the prediction quality deteriorates stepwise, to a MAPE of around 7-9 per cent for five-day-ahead and to around 9-11 per cent for ten-day-ahead predictions. When viewed side by side, the differences between the models become less pronounced. The significance tests in Table 4.10 Panels B and C reveal that for five-day-ahead prediction, the intensity model outperforms the Merton model in only two cases now (Aa-Europe, A-Europe), and in one (A-North America), the Merton model even provided smaller absolute errors. Turning to the ten-day-ahead prediction, there is a balance between the Merton and intensity models, with each proving superior for one class apiece (Aa-North America and AA-Europe, respectively), and the two remaining classes are indifferent

Table 4.8: One-Day-Ahead Out-of-sample Prediction Errors of the Merton and the Constant Intensity Models in Time Series Design

	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size Prediction
Aa Europe				
Merton	0.30	1.06	5.71%	10373
Intensity	0.28	1.00	5.25%	10373
Aa N.America				
Merton	0.33	1.56	5.30%	4850
Intensity	0.33	1.54	5.17%	4850
A Europe				
Merton	0.49	2.78	6.15%	19200
Intensity	0.46	2.75	6.04%	19200
A N.America				
Merton	0.93	2.97	6.61%	10672
Intensity	0.93	2.98	6.59%	10672

Mean Error(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} s_f^{theo} - s_{f,h}^{obs}}{F.H}$ Mean AbsError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} |s_f^{theo} - s_{f,h}^{obs}|}{F.H}$

Mean AbsPercError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} \frac{|s_f^{theo} - s_{f,h}^{obs}|}{s_{f,h}^{obs}} \times 100}{F.H}$

s^{theo} is the theoretical CDS premium predicted by the Merton and Intensity models on day f for firm h, where $f = 1..F$ is the number of available days for prediction (preceded by 14 consecutive days of CDS premiums for estimation), with $h = 1..H$ being the number of firms in the risk class. s^{obs} is the observed CDS premium on day f for firm h.

for the models. Again, these results have mostly arisen from a very small mean difference, complemented by a very low standard error and large sample size.

Overall, the comparison shows that it is hard to distinguish between the Merton and intensity model in a time series setup as well. Nevertheless, the errors are much lower than in the cross-sectional analysis. This better fit in the time series analysis over cross-sectioning signifies that credit risk may not be uniformly priced in a given risk class. This result parallels the findings of Frühwirth and Sögner (2006), who have applied the constant intensity model to out-of-sample bond price prediction and concluded that any kind of cross-sectioning would

Table 4.9: Five- and Ten-Day-Ahead Out-of-sample Prediction Errors of the Merton and the Constant Intensity Models in Time Series Design

Panel A. Five-Day-Ahead Out-of-Sample Prediction Errors				
	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size Prediction
Aa Europe				
Merton	0.47	1.39	7.51%	10270
Intensity	0.44	1.34	7.11%	10270
Aa N.America				
Merton	0.50	2.09	7.13%	4798
Intensity	0.50	2.09	7.08%	4798
A Europe				
Merton	0.76	3.73	8.32%	19004
Intensity	0.72	3.71	8.23%	19004
A N.America				
Merton	1.41	4.00	9.02%	10567
Intensity	1.42	4.02	9.04%	10567

Panel B. Ten-Day-Ahead Out-of-Sample Prediction Errors				
	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size Prediction
Aa Europe				
Merton	0.67	1.71	9.30%	10150
Intensity	0.64	1.67	8.96%	10150
Aa N.America				
Merton	0.69	2.62	9.02%	4733
Intensity	0.70	2.61	9.00%	4733
A Europe				
Merton	1.10	4.69	10.64%	18768
Intensity	1.05	4.67	10.54%	18768
A N.America				
Merton	2.00	4.99	11.47%	10437
Intensity	2.01	5.01	11.51%	10437

Mean Error(bps)=$\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} s_f^{theo}-s_{f,h}^{obs}}{F.H}$ Mean AbsError(bps)=$\frac{\sum_{f=1}^{F}\sum_{h=1}^{H}|s_f^{theo}-s_{f,h}^{obs}|}{F.H}$

Mean AbsPercError(bps)=$\frac{\sum_{f=1}^{F}\sum_{h=1}^{H}\frac{|s_f^{theo}-s_{f,h}^{obs}|}{s_{f,h}^{obs}}\times 100}{F.H}$

s^{theo} is the theoretical CDS premium predicted by the Merton and Intensity models on day f for firm h, where $f = 1..F$ is the number of available days for prediction (preceded by 18(or 23) consecutive days, with the first 14 consisting of the CDS premiums for estimation) and $h = 1..H$ representing the number of firms in the risk class. s^{obs} is the observed CDS premium on day f for firm h.

Table 4.10: Significance Tests for the Difference of One-Day-Ahead, Five-Day-Ahead, and Ten-Day-Ahead Absolute Prediction Errors with the Merton and the Constant Intensity Models in Time Series Design

Panel A. Significance Tests for the Difference of One-Day-Ahead Absolute Prediction Errors				
	Mean Difference	t-statistic	p-value	
Aa Europe	0.06	11.67	<0.0001	***
Aa N.America	0.03	3.23	0.0012	***
A Europe	0.03	5.09	<0.0001	***
A N.America	-0.01	-0.94	0.3488	

Panel B. Significance Tests for the Difference of Five-Day-Ahead Absolute Prediction Errors				
	Mean Difference	t-statistic	p-value	
Aa Europe	0.05	8.45	<0.0001	***
Aa N.America	0.01	1.28	0.2010	
A Europe	0.02	3.32	0.0009	***
A N.America	-0.02	-2.49	0.0128	***

Panel C. Significance Tests for the Difference of Ten-Day-Ahead Absolute Prediction Errors				
	Mean Difference	t-statistic	p-value	
Aa Europe	0.04	6.62	<0.0001	***
Aa N.America	0.01	0.97	0.3297	
A Europe	-0.01	-0.96	0.3348	
A N.America	-0.02	-2.78	0.0054	***

Mean Difference (bps): Difference of Absolute Errors for prediction (Merton - Intensity) computed per day per firm. Absolute Error on day f, for firm h is $\mid s_f^{theo} - s_{f,h}^{obs} \mid$

*** Significance at 95 per cent level

provide poorer estimates than a bond-by-bond analysis.

4.3.2 SVM Regression Method

In the last step, the machine learning approach was compared to the financial models in a time series setup. To this end, the concentration was given to the linear kernel due to its overall best performance in the cross-sectional setting.

In order to use an analogous setup with the same number of observations as in the financial models, the time series of prices of each firm was divided into estimation and prediction samples. A ratio of 3:1 for estimation and prediction sample sizes was applied to each firm, which indicates that the first three quarters of the time series was used to train the SVM function. With a rolling time horizon in this estimation sample, the consecutive 14-day observations were used as the training input dataset, whereas the observation on the following day was used as the training output. After the function was trained, the unused last quarter of the time series was utilized for prediction. This time the remaining consecutive 14-day observations were used as test input to predict the observation on the following day as the test output. By virtue of such a setup, the comparability of the out-of-sample design to the design used for the financial models is ensured.

Interestingly, the results presented in Panel A of Table 4.11 are very promising. For one-day-ahead prediction, the SVM method exhibited a surprisingly good fit in terms of mean absolute percentage errors, which are around 2-3 per cent. Similar to financial models, as the prediction horizon extends, this figure worsens. Panels B and C present the five-day- and ten-day-ahead prediction errors; these deteriorate to 4-6 per cent and 6-8 per cent, respectively. Again, these figures indicate that the time series design achieved results superior to those of the cross-sectional design for SVM as well.

Furthermore, in comparison to the financial model results with the same setting, this time the SVM method also yielded very strong results. Each of the one-day-, five-day- and ten- day-ahead prediction results signifies a better fit than either of the financial models presented in Table 4.8. In all four risk classes, SVM errors were significantly lower than both the Merton and intensity one-

Table 4.11: One-Day-, Five-Day-, and Ten-Day-Ahead Out-of-sample Prediction Errors with Linear Kernel SVM in Time Series Design

Panel A. One-Day-Ahead SVM Out-of-Sample Prediction Errors	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size
Aa Europe	0.09	0.32	2.48%	10373
Aa N.America	0.34	0.87	3.18%	4850
A Europe	0.005	0.90	2.32%	19200
A N.America	-0.02	1.02	2.88%	10672

Panel B. Five-Day-Ahead SVM Out-of-Sample Prediction Errors	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size
Aa Europe	0.25	0.60	4.65%	10270
Aa N.America	0.62	1.38	5.14%	4798
A Europe	0.85	2.52	6.23%	19004
A N.America	0.35	1.96	5.84%	10567

Panel C. Ten-Day-Ahead SVM Out-of-Sample Prediction Errors	Mean Error (bps)	Mean Abs Error (bps)	Mean Abs Perc Error (%)	Total Sample Size
Aa Europe	0.47	0.92	7.12%	10150
Aa N.America	0.99	1.86	7.15%	4733
A Europe	0.83	3.17	8.61%	18768
A N.America	0.97	2.73	8.68%	10437

Mean Error(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} s_f^{theo} - s_{f,h}^{obs}}{F.H}$ Mean AbsError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} |s_f^{theo} - s_{f,h}^{obs}|}{F.H}$

Mean AbsPercError(bps)= $\frac{\sum_{f=1}^{F}\sum_{h=1}^{H} \frac{|s_f^{theo} - s_{f,h}^{obs}|}{s_{f,h}^{obs}} \times 100}{F.H}$

s^{theo} is the theoretical CDS premium predicted by the SVM algorithm with a linear kernel on day f for firm h, where $f = 1..F$ is the number of available days for test output (approximately $1/4^{th}$ of the full sample) and $h = 1..H$ representing the number of firms in the risk class. s^{obs} is the observed CDS premium on day f for firm h.

day-ahead absolute prediction errors (Table 4.12).[8] This result is interesting and suggests that further work on the subject is warranted. However, it should be kept in mind that the best performing case among different kernels has been chosen, whereas structural and reduced-form models were presented in their simplest forms. It therefore remains to be seen if these results would vary if more sophisticated financial models were applied.

Table 4.12: Significance Tests for the Difference of One-Day-Ahead Absolute Prediction Errors between the Merton/Intensity models and Linear Kernel SVM in Time Series Design

	Mean Difference	t-statistic	p-value	
Aa Europe				
Merton - SVM	0.31	4.36	<0.0001	***
Intensity - SVM	0.21	2.85	0.0044	***
Aa N.America				
Merton - SVM	0.30	2.20	0.0277	***
Intensity - SVM	0.26	1.88	0.0607	*
A Europe				
Merton - SVM	0.91	3.56	0.0004	***
Intensity - SVM	0.84	3.30	0.0010	***
A N.America				
Merton - SVM	0.91	7.58	<0.0001	***
Intensity - SVM	0.88	7.31	<0.0001	***

Mean Difference (bps): Difference of Absolute Errors for prediction (Merton - SVM) and (Intensity - SVM) computed per day per firm. Absolute Error on day f, for firm h is $\mid s_f^{theo} - s_{f,h}^{obs} \mid$

*** Significance at 95 per cent level

* Significance at 90 per cent level

4.4 Summary

This chapter compared basic versions of structural (Merton) and reduced-form (constant intensity) models as a first attempt. In this regard, four aspects of

[8] The five-day- and ten-day-ahead prediction errors are also significantly better than the financial models, which have not been tabulated.

the study stand out: First, while cross-sectional results indicated a better fit of the Merton model in only one out of four cases, the one-day-ahead time series analysis revealed the significance of lower absolute prediction errors with the constant intensity model in three classes. Five-day- and ten-day-ahead predictions produced mixed results, signifying that one framework's performance does not significantly outperform the other. The most distinctive feature of the models is the default timing, which revealed in the cross-sectional setup that the Merton model estimated higher default probabilities on average, as the constant intensity model allows early default. The second major feature is the inclusion of interest rates in the Merton model, whereas the intensity model is insensitive to this parameter. Despite these factors, the error results are rather close. This could be attributable to treating the default probability in the Merton setting as the firm value variable on its own, rather than breaking it down into Black/Scholes parameters, such as asset value and volatility. Therefore, it has been decided to extend the results of this chapter with an analysis where structural model parameters are estimated as well. This extension will investigate whether the differences in prediction power between frameworks arises from this choice.

Secondly, estimation and out-of-sample prediction using solely CDS data was unique to this study and requires special attention. The usage of CDS data allowed to concentrate the prediction ability of credit models directly on the default risk premiums that constitute the prices. Without the presence of liquidity and other non-default premiums in CDS prices, the models could be applied to investigate the credit risk factors. Nevertheless, further efforts should include bond and stock price data to extend the estimation process for both modeling classes. The analysis in the next chapter will take this point into consideration and make use of bond, stock price, and balance sheet information in the prediction of CDS prices.

Third, the results from this study confirm recent results in the literature indicating that cross-sectioning is inferior to separate estimation. The high prediction errors from cross-sectional analysis in comparison to lower errors in the time series analysis revealed that credit risk is priced separately for each individual firm, rather than the joint classification provided by rating classes/regions. Taking this into consideration, the extension in the next chapter will analyze credit risk on firm basis.

Fourth, although most of the cross-sectional predictions with SVM algorithms have ended in poor results, it is important to underline that one-day-, five-day-, and ten-day-ahead time series prediction results with the linear kernel SVM have achieved significantly lower error figures than financial methods. A thorough analysis for applying alternative kernel functions should be pursued that investigates cross-sectional and time series mappings of the data. Nevertheless, for the sake of brevity, the analysis with SVM is ceased, and a focus on financial models are given in the next chapter.

Overall, the results of this chapter will be extended in the next section by applying different versions of structural and reduced-form models. Introducing stochastic interest rates for structural models while modeling intensity to be stochastically dependent on state variables for reduced-form models should be the next step in the comparison of both frameworks. The next chapter will therefore provide a comparison of more sophisticated models, using bond, stock price and balance sheet data as the source of the empirical investigation.

Chapter 5

CDS Pricing with Advanced Framework Structures

In this chapter, the analysis will be taken a step further, and more advanced forms of structural and reduced-form models will be compared in their ability to correctly price CDSs, by making use of additional bond, stock price, and balance sheet data. Which components should the model setups accommodate? Structural models have components that are rarely part of a reduced-form framework. Only alternative is to make use of structural variables that are also explanatory in an intensity setup. This leads to recalling possible structural variables: The original Merton model included leverage and asset volatility as key parameters. These have been extended to many others in more advanced settings. Focusing on the initial key variables, the question that needs to be answered is whether they are explanatory for CDS premiums as well. The answer could be found in recent research: In Ericsson, Jacobs, and Oviedo (2007), leverage alone explains 45 per cent of CDS premium levels, and together with equity volatility and interest rates, this raises to 60 per cent. It is well known since the study of Collin-Dufresne, Goldstein, and Martin (2001) that changes in leverage are significant in explaining change in bond spreads. These findings suggest that leverage can be chosen as a key variable both in structural and reduced-form models.

Given this choice, the model of Collin-Dufresne and Goldstein (2001) (henceforth CDG) with a stationary leverage ratio on the structural side and a best-comparable intensity model with the leverage process as the state variable on the reduced-form side seem to be a perfect match. This selection had a very distinctive reason. The main aim is to construct a comparable model setup with both frameworks, so that the fundamental model prediction capabilities could be compared. How could this ideally be done? The CDG model incorporates stochastic interest rates and a leverage process as the second source of uncertainty. Thus, a reduced-form model should be set up according to these characteristics. Following this reason, the adjusted discount rate has been selected to be an affine sum of the default intensity and the short rate as in Lando (1998). First, the short rate is taken to be stochastic to keep the comparable structure with the CDG setup. Second, the default intensity is assumed to be the affine sum of a constant and a state variable, which this state variable is nothing but the leverage process, exactly the same as in the CDG model. This setup is therefore believed to accommodate a best comparable structure between structural and reduced-form frameworks, enabling to understand the impact of model structures in the prediction of credit default swap prices.

5.1 Structural Approach - Collin-Dufresne / Goldstein (2001) Model

5.1.1 The Model

Besides treating the interest rate process as a stochastic Ornstein-Uhlenbeck process, the Collin-Dufresne and Goldstein (2001) model accommodates a stationary leverage ratio. With the typical asset value process following a geometric Brownian motion, the model setup includes a stochastic leverage process. The steps that result in the structural defaultable claim pricing model of Collin-Dufresne/Goldstein (CDG) would be as follows:

The dynamics of the short rate is a Vasicek (1977) process:

$$dr_t = \kappa_r(\theta_r - r_t)dt + \sigma_r dW_1^Q \tag{5.1}$$

Here r is the risk-free interest rate, κ_r is the mean reversion rate, θ_r is the long-run mean, σ_r is the volatility of the short rate, and W_1^Q is a Brownian motion under the risk-neutral measure.[1] The asset value follows a GBM with

$$dV_t/V_t = (r_t - \delta)dt + \sigma_v dW_2^Q \tag{5.2}$$

where δ is the payout ratio, σ_v is the asset volatility and W_2^Q is the Brownian motion under the risk-neutral measure. Moreover, in the CDG setup, the logarithm of the default threshold K, which can be taken as the total liabilities of the firm, is considered to follow a stationary process. In the log-representation,

[1] For an in-depth description of risk-neutral and physical measures, see Björk (2004).

with $k = lnK$ and $y = lnV$, this stationary process is modeled as:

$$dk_t = \kappa_l(y_t - \nu - k_t)dt \tag{5.3}$$

Here, κ_l is the mean-reversion rate of the leverage to its long-run mean and ν is a buffer parameter for default distance of log-asset value to log-default threshold. When k_t is less than $(y_t - \nu)$ the firm acts to increase k_t, and vice versa. Therefore, the firm issues new debt when leverage ratio falls below a target level, and would not issue new debt to replace maturing debt when the ratio is above the target. CDG define the log-threshold k as the logarithm of book value of debt. In their paper they state that "even though the default threshold need not be equal to the outstanding book value of debt, it seems reasonable to assume both are related."

It is noteworthy to mention that, CDG also make use of an extended version of the above stationary leverage ratio formula in their study. The drift of the log-default threshold can be taken as a decreasing function of the spot interest rate, since debt issuances drop during high interest periods. This formulation is consistent with the findings of Malitz (2000) for the high interest period of early 1980s. The extended version of the formula in the CDG study is,

$$dk_t = \kappa_l(y_t - \nu - \phi(r_t - \theta_r) - k_t)dt \tag{5.4}$$

where ϕ is the sensitivity parameter to interest rates, and θ_r is the long-run mean for short-term interest rates as defined before. In order to reduce the complexity of the model, this study will implement the basic version where the sensitivity of log-default threshold to interest rates is neglected.

By defining log-leverage as,

$$l_t = k_t - y_t = ln\frac{K}{V} \tag{5.5}$$

the firm defaults at the first passage time of the firm value reaching the default boundary, or equivalently, the log-leverage ratio l reaching zero. This idea is in line with Black and Cox (1976) and Longstaff and Schwartz (1995) where default happens the first time when the firm value reaches an exogenously specified boundary. The CDG Model is therefore an advanced form of structural models, extending the basic idea of Merton (1974) with (i) stochastic interest rates, (ii) first-passage time, and (iii) stationary leverage ratios.

From Ito's Lemma, l_t follows the one-factor Markov dynamics:

$$dl_t = \kappa_l(\theta_l(r_t) - l_t)dt - \sigma_v dW_2^Q \tag{5.6}$$

where we have

$$\theta_l(r_t) = \frac{\delta + \frac{\sigma_v^2}{2} - r}{\kappa_l} - \nu = -\frac{r}{\kappa_l} - \bar{\nu} \tag{5.7}$$

5.1.2 Pricing Corporate Debt

CDG assume a recovery-of-treasury payment, which is that a risky coupon bond recovers a φ proportion of the otherwise identical riskless coupon bond. The difference from the recovery-of-face value assumption is that instead of the φ amount being received at the time of default, it is received at maturity.

Let $v^{theo}(r_0, l_0, T)$ be the theoretical price of a risky discount bond that matures at T. Then with the recovery-of-treasury assumption, a transformation will be necessary to switch from the expectation under risk-neutral measure E^Q to the expectation under T-forward measure E^{F_T}.[2]

$$
\begin{aligned}
v^{theo}(r_0, l_0, T) &= E^Q\left(e^{-\int_0^T r(s)ds} \cdot \mathbf{1}_{\{\tau>T\}} + e^{-\int_0^\tau r(s)ds} \cdot \varphi \cdot b(r_\tau, T) \cdot \mathbf{1}_{\{\tau\leq T\}}\right) \\
&= E^Q\left(e^{-\int_0^T r(s)ds} \cdot \left(\mathbf{1}_{\{\tau>T\}} + \varphi \cdot \mathbf{1}_{\{\tau\leq T\}}\right)\right) \\
&= E^Q\left(e^{-\int_0^T r(s)ds} \cdot \left(1 - (1-\varphi) \cdot \mathbf{1}_{\{\tau\leq T\}}\right)\right) \\
&= b(r_0, T) \cdot E^{F_T}\left(1 - (1-\varphi) \cdot \mathbf{1}_{\{\tau\leq T\}}\right) \\
&= b(r_0, T) \cdot \left(1 - (1-\varphi) \cdot Q^{F_T}(r_0, l_0, T)\right) \qquad (5.8)
\end{aligned}
$$

where $b(r_0, T)$ is the price of a riskless bond. It remains to determine $Q^{F_T}(r_0, l_0, T)$ which is the time-0 probability of default occurring before maturity T, under the T-forward measure (See Geman, El-Karoui, and Rochet (1995) and Jamshidian (1989)). Similarly, considering a coupon-paying risky bond with N coupons on payment dates t_j, we have,

$$
\begin{aligned}
v^{theo}(r_0, l_0, T) &= \sum_{j=1}^{N} C \cdot b(r_0, t_j) \cdot \left(1 - (1-\varphi_{coup}) \cdot Q^{F_{t_j}}(r_0, l_0, t_j)\right) \\
&\quad + b(r_0, T) \cdot \left(1 - (1-\varphi) \cdot Q^{F_T}(r_0, l_0, T)\right) \qquad (5.9)
\end{aligned}
$$

where C is the coupon fraction and φ_{coup} is the recovery rate of coupons in case of default. Generally, certain assumptions must hold in order to price each coupon as a zero-coupon bond as in Equation (5.9). This is called the "portfolio of zeroes" approach. However, since the pricing is under recovery-of-treasury approach, there is no need to concern about the applicability. Additionally, in practice, CDG note from Helwege and Turner (1999) who claim that future coupon payments are of low priority, and are rarely recovered in default. Therefore φ_{coup} is set to 0, letting only the principal payment receive compensation at default. The recovery rate on principal is fixed at 0.5, following the results

[2]See Björk (2004) for measure transformations.

produced by Altman and Kishore (1996) and recent practice.

For the critical variable of $Q^{F_T}(r_0, l_0, T)$, CDG follow Longstaff and Schwartz (1995) and implement a version of Fortet's (1943) implicit formula for the first passage time density. Utilizing this framework, Eom, Helwege, and Huang (2004) make use of the derivation of CDG and arrive at the below formulation (pp. 537-539):

$$Q^{F_T}(r_0, l_0, T) = \sum_{i=1}^{n} q(t_{i-1/2}; t_0) \tag{5.10}$$

In deriving this formula, t_0 is set equal to 0 and the time is discretized into n intervals as $t_i = iT/n$,

for $i = 1, 2, ..., n$,

$$q(t_{i-1/2}; t_0) = \frac{N(a(t_i; t_0)) - \sum_{j=1}^{i-1} q(t_{j-1/2}; t_0) N(b(t_i; t_{j-1/2}))}{N(b(t_i; t_{i-1/2}))} \tag{5.11}$$

The sum on the right hand-side of the equation becomes zero when $i = 1$. N is the cdf of Normal distribution. a and b are defined as

$$a(t_i; t_0) = -\frac{M(t_i, T|X_0, r_0)}{\sqrt{S(t_i|X_{t_j})}} \tag{5.12}$$

$$b(t_i; t_j) = -\frac{M(t_i, T|X_{t_j})}{\sqrt{S(t_i|X_{t_j})}} \tag{5.13}$$

$X = V/K$ is the inverse of the leverage ratio, where M and S are

$$M(t, T|X_0, r_0) = E_0^{F_T}[lnX_t] \tag{5.14}$$

$$S(t|X_0, r_0) = var_0^{F_T}[lnX_t] \tag{5.15}$$

$$M(t,T|X_u) = M(t,T|X_0,r_0) - M(u,T|X_0,r_0)\frac{cov_0^{F_T}[lnX_t, lnX_u]}{S(u|X_0,r_0)},\ u \ \epsilon\ (t_0,t) \tag{5.16}$$

$$S(t|X_u) = S(t|X_0,r_0) - \frac{(cov_0^{F_T}[lnX_t, lnX_u])^2}{S(u|X_0,r_0)},\ u \ \epsilon\ (t_0,t) \tag{5.17}$$

What remains is to have closed form solutions for $E_0^{F_T}[lnX_t]$ and $cov_0^{F_T}[lnX_t, lnX_u]$ which are computed in Eom/Helwege/Huang (pp. 538-539).

$$\begin{aligned} E_0^{F_T}[lnX_t] &= e^{-\kappa_l t}\Big[lnX_0 + \bar{\nu}(e^{\kappa_l t} - 1) \\ &+\Big(\frac{1}{\kappa_l - \kappa_r}(e^{(\kappa_l - \kappa_r)t} - 1)(r_0 - \theta_r + \frac{\sigma_r^2}{\kappa_r^2} - \frac{\sigma_r^2}{2\kappa_r^2}e^{-\kappa_r T}) \\ &+\frac{1}{\kappa_l + \kappa_r}\frac{\sigma_r^2}{2\kappa_r^2}e^{-\kappa_r T}(e^{(\kappa_l + \kappa_r)t} - 1) + \frac{1}{\kappa_l}(\theta_r - \frac{\sigma_r^2}{\kappa_r^2})(e^{\kappa_l t} - 1)\Big) \\ &-\frac{\rho\sigma_v\sigma_r}{\kappa_r}\Big(\frac{e^{\kappa_l t} - 1}{\kappa_l} - e^{\kappa_r T}\frac{e^{(\kappa_l + \kappa_r)t} - 1}{\kappa_l + \kappa_r}\Big)\Big] \end{aligned} \tag{5.18}$$

$$
\begin{aligned}
cov_0^{F_T}[lnX_t, lnX_u] \quad = \quad & e^{-\kappa_l(t+u)}\Big[\frac{\sigma_v^2}{2\kappa_l}(e^{2\kappa_l u} - 1) \\
& + \frac{\rho\sigma_v\sigma_r}{\kappa_l + \kappa_r}\Big(\frac{e^{2\kappa_l u} - 1}{2\kappa_l} - \frac{e^{(\kappa_l - \kappa_r)u} - 1}{\kappa_l - \kappa_r}\Big) \\
& + \frac{\rho\sigma_v\sigma_r}{\kappa_l + \kappa_r}\Big(\frac{1 - e^{(\kappa_l - \kappa_r)t}}{\kappa_l - \kappa_r} + \frac{e^{2\kappa_l u} - 1}{2\kappa_l} \\
& + e^{(\kappa_l + \kappa_r)u}\frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l - \kappa_r}\Big) \\
& + \frac{\sigma_r^2}{2\kappa_r}\Big(- \frac{(e^{(\kappa_l - \kappa_r)t} - 1)(e^{(\kappa_l - \kappa_r)u} - 1)}{(\kappa_l - \kappa_r)^2} \\
& - \frac{\kappa_r}{\kappa_l^2 - \kappa_r^2}\frac{e^{2\kappa_l u} - 1}{\kappa_l} + (e^{(\kappa_l + \kappa_r)u} - 1)\frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l^2 - \kappa_r^2} \\
& + \frac{1}{\kappa_l^2 - \kappa_r^2}(1 - 2e^{(\kappa_l - \kappa_r)u} + e^{2\kappa_l u})\Big)\Big]
\end{aligned}
$$

where

$$\bar{\nu} = (\nu - (\delta + \sigma_v^2/2)/\kappa_l) \tag{5.19}$$

From these equations one can obtain $Q^{F_T}(r_0, l_0, T)$ required for pricing the bond.

5.2 Reduced-Form Approach - The Stochastic Leverage Model

5.2.1 The Model

In this attempt to compare the CDS price prediction ability of structural and reduced-form models of default, it has been decided to make the modeling structures of the approaches as close as possible. This would then enable us to understand whether the model structure has an impact on prediction power. In order to maintain this, it has been chosen to use the stochastic leverage component in the CDG model, directly in the intensity setup. The below formulation will explain the reduced-form setting in detail.

In an intensity model, the critical choice is the selection of the state variables driving the credit risk. There have been many empirical studies with reduced-form models that either estimated a stochastic process for the unobserved intensity (Duffee (1999), Driessen (2005)), or made use of a credit risk factor as part of the adjusted discount rate (Bakshi, Madan, and Zhang (2006)). This study accommodates the second approach, where the leverage process has been defined as the credit risk factor that drives the intensity. By letting the stochastic leverage process to be the main driver of credit risk in both models, it could be possible to have a basis of comparison.

5.2.2 Pricing Corporate Debt

In order to be comparable with the CDG model recovery assumption, it has been chosen to work with Lando's (1998) doubly stochastic model with recovery-of-treasury value. The theoretical default risky discount bond price $v^{theo}(r_0, l_0, T)$ that matures at T is:

$$\begin{aligned} v^{theo}(r_0, l_0, T) &= E^Q\left(e^{-\int_0^T r(s)ds} \cdot \mathbf{1}_{\{\tau > T\}} + e^{-\int_0^\tau r(s)ds} \cdot \varphi \cdot b(r_\tau, T) \cdot \mathbf{1}_{\{\tau \leq T\}}\right) \\ &= E^Q\left(e^{-\int_0^T r(s)ds}\left(\mathbf{1}_{\{\tau > T\}} + \varphi \cdot \mathbf{1}_{\{\tau \leq T\}}\right)\right) \\ &= E^Q\left(e^{-\int_0^T r(s)ds}\left(\varphi + (1-\varphi)\mathbf{1}_{\{\tau > T\}}\right)\right) \\ &= \varphi \cdot b(r_0, T) + (1-\varphi) \cdot E^Q\left(e^{-\int_0^T r(s)ds}\mathbf{1}_{\{\tau > T\}}\right) \\ &= \varphi \cdot b(r_0, T) + (1-\varphi) \cdot E^Q\left(e^{-\int_0^T R(s)ds}\right) \end{aligned} \tag{5.20}$$

Equation (5.20) is the reduced-form version of Equation (5.8). Notice that the two equation series only differ in the last two lines. In deriving the formula, there was a switch to the forward measure in the structural model. On the other hand, the default probability is incorporated into the adjusted discount rate in the Lando setting. Here, the important decision is how to formulate the adjusted discount rate at the last part of Equation (5.20). In choosing the adjusted discount rate setup, the classical Lando (1998) formulation could be kept in mind. In his model, the adjusted discount rate is the sum of short rate r and default intensity λ. Recalling the arguments of keeping the structural and reduced-form models comparable, the following formulation is proposed.

$$R(t) = a + r(t) + c \cdot l(t) \tag{5.21}$$

With this setting, the Lando-type form is preserved, by letting $a + c \cdot l(t)$ to be equal to default intensity λ. The adjusted discount rate is the sum of the short rate, and a credit risk component. This component consists of a constant part

a and a stochastic part, $c \cdot l(t)$. Here, $l(t)$ is nothing but the log-leverage ratio, which has a process exactly the same as in the structural part (see Equation (5.6)). The two stochastic differential equations parallel to the structural setup are:

$$dr_t = \kappa_r(\theta_r - r_t)dt + \sigma_r dW_1^Q \tag{5.22}$$

and

$$dl_t = \kappa_l(\theta_l(r_t) - l_t)dt - \sigma_v dW_2^Q \tag{5.23}$$

where we have

$$\theta_l(r_t) = \frac{\delta + \frac{\sigma_v^2}{2} - r}{\kappa_l} - \nu = -\frac{r}{\kappa_l} - \bar{\nu} \tag{5.24}$$

Let v be the defaultable bond price as in the last part of Equation (5.20) in expectation brackets. One can interpret this as the price of a defaultable zero-coupon bond with zero recovery. Following the property of affine structures as discussed in Duffie and Singleton (2003), the closed form solution of the expectation is,

$$v = E_t^Q\left(e^{-\int_t^T R(s)ds}\right) = e^{A(t,T)-B(t,T)r_t-C(t,T)l_t} \tag{5.25}$$

for which the following PDE can be derived:

$$\frac{\partial v}{\partial t}+\kappa_r(\theta_r-r)\frac{\partial v}{\partial r}+\kappa_l(\theta_l(r)-l)\frac{\partial v}{\partial l}+\frac{1}{2}\sigma_r^2\frac{\partial^2 v}{\partial r^2}+\frac{1}{2}\sigma_v^2\frac{\partial^2 v}{\partial l^2}-\rho\sigma_v\sigma_r\frac{\partial^2 v}{\partial r \partial l} = (a+r+cl)v \tag{5.26}$$

where $\theta_l(r)$ is as in Equation (5.24). In doing this calculation, the boundary conditions of $A(T,T) = 0$, $B(T,T) = 0$, and $C(T,T) = 0$ are necessary. By taking the partial derivatives of v in Equation (5.25) with respect to t, r, l and second derivatives with respect to rr, ll, and rl and replacing into the PDE, the below closed form solution could be reached where,

$$C(t,T) = \frac{c}{\kappa_l}(1 - e^{-\kappa_l(T-t)}) \tag{5.27}$$

$$\begin{aligned} B(t,T) &= \Big[e^{-\kappa_r(T-t)}\Big(\frac{c}{\kappa_l\kappa_r} + \frac{c}{\kappa_l(\kappa_l-\kappa_r)} - \frac{1}{\kappa_r}\Big)\Big] \\ &\quad + \Big[\frac{1}{\kappa_r} - \frac{c}{\kappa_l\kappa_r} - \frac{c}{\kappa_l}\frac{e^{-\kappa_l(T-t)}}{\kappa_l-\kappa_r}\Big] \end{aligned} \tag{5.28}$$

$$A(t,T) = -a(T-t) - \Xi - \Upsilon + \Gamma + \Lambda - \Pi \tag{5.29}$$

with

$$\widetilde{\theta}_l = \frac{\delta + \frac{\sigma_v^2}{2}}{\kappa_l} - \nu \tag{5.30}$$

$$W = \frac{c}{\kappa_r\kappa_l} + \frac{c}{\kappa_l(\kappa_l-\kappa_r)} - \frac{1}{\kappa_r} \tag{5.31}$$

$$Z = \frac{1}{\kappa_r} - \frac{c}{\kappa_l\kappa_r} \tag{5.32}$$

$$\Xi = \widetilde{\theta}_l c\Big[(T-t) - \frac{1-e^{-\kappa_l(T-t)}}{\kappa_l}\Big] \tag{5.33}$$

$$\Upsilon = \kappa_r\theta_r\Big[W\frac{1-e^{-\kappa_r(T-t)}}{\kappa_r} + Z(T-t) - \frac{c}{\kappa_l^2(\kappa_l-\kappa_r)}\Big(1-e^{-\kappa_l(T-t)}\Big)\Big] \tag{5.34}$$

$$\Gamma = \frac{\sigma_v^2c^2}{2\kappa_l^2}\Big[(T-t) - \frac{2(1-e^{-\kappa_l(T-t)})}{\kappa_l} + \frac{1-e^{-2\kappa_l(T-t)}}{2\kappa_l}\Big] \tag{5.35}$$

$$\begin{aligned}\Lambda &= \frac{\sigma_r^2}{2}\Big[\frac{W^2}{2\kappa_r}(1-e^{-2\kappa_r(T-t)}) + \frac{2WZ}{\kappa_r}(1-e^{-\kappa_r(T-t)}) + Z^2(T-t) \\ &\quad -\frac{2Wc(1-e^{-(\kappa_l+\kappa_r)(T-t)})}{\kappa_l(\kappa_l-\kappa_r)(\kappa_l+\kappa_r)} - \frac{2Zc(1-e^{-\kappa_l(T-t)})}{\kappa_l^2(\kappa_l-\kappa_r)} \\ &\quad +\Big(\frac{c}{\kappa_l(\kappa_l-\kappa_r)}\Big)^2\frac{1-e^{-2\kappa_l(T-t)}}{2\kappa_l}\Big] \end{aligned} \tag{5.36}$$

$$\begin{aligned}\Pi &= \rho\sigma_v\sigma_r\Big[\frac{Wc(1-e^{-\kappa_r(T-t)})}{\kappa_l\kappa_r} + \frac{Zc(T-t)}{\kappa_l} - \frac{c^2(1-e^{-\kappa_l(T-t)})}{\kappa_l^3(\kappa_l-\kappa_r)} \\ &\quad -\frac{Wc(1-e^{-(\kappa_l+\kappa_r)(T-t)})}{\kappa_l(\kappa_l+\kappa_r)} - \frac{Zc(1-e^{-\kappa_l(T-t)})}{\kappa_l^2} \\ &\quad +\frac{c^2(1-e^{-2\kappa_l(T-t)})}{2\kappa_l^3(\kappa_l-\kappa_r)}\Big] \end{aligned} \tag{5.37}$$

5.3 Empirical Methodology and Results

The structural and reduced-form models are calibrated to corporate bond prices, leverage ratios and US Treasury rates. For both of the models, the interest rate process parameters $(\kappa_r, \theta_r, \sigma_r)$, the initial values of the leverage ratio and the short rate (l_0, r_0), the leverage process parameters (κ_l, θ_l), and the correlation between the interest rate process and the asset value process (ρ) enter similarly. Each of the models will use their theoretical bond prices $v^{theo}(r_0, l_0, T)$ to estimate their unique asset volatility (σ_v) figure. In the reduced-form setup, two additional constant parameters a and c are also necessary. After calibration of the models to market prices, CDS prices can be predicted out-of-sample, without making use of any information used prior. This setup will allow to see how well the CDG and the reduced-form model perform in correctly pricing CDSs. Utilizing bond, stock and balance sheet information is a large step after the analysis provided in Chapter 4 with only using CDS data for prediction. However, it is necessary to take an advanced step towards the assessment of more state-of-the-art types of models. The following section will introduce the datasets used in analysis.

5.3.1 Data

CDS Data

An extended version of the dataset described in Section 2.2 has been used in this analysis. Time series of CDS prices were extended until the end of December 2005. This was required in order to have enough observations. After this extension, mid-month observations were selected for 5-year, senior CDSs, for the period of January 2003 - December 2005 due to the matching with bond and balance sheet data. The mid-month value is typically on the 15^{th} of each month. In case that the 15^{th} is a non-working day, the next working day is selected. The indicative bid and ask quotes are averaged to reach a daily CDS premium, in the end attaining 36 mid-month observations for the three year period. The credit quality of the issuers varies between Aa and Ba rated by Moody's. The lowest CDS midpoint of 9 bps is within the series of the Aa-rated WAL-MART, whereas the highest midpoint is as much as 412.5 bps for the Baa/Ba-rated HILTON.

Interest Rate Data

The 3-, 6-monthly, and 1, 2, 3, 5, 7, 10-yearly yields are retrieved for the interest rate calibration process. Because the sample consists of US companies, Constant Maturity Treasuries from the Federal Reserve Board have been used.

These are historical series of on-the-run US Treasury yields that also have been utilized in recent research. The series are actually the average yields on US Treasury securities adjusted to a constant maturity. Yields are interpolated by the US Treasury from the daily yield curve. This curve, which relates the yield on a security to its time to maturity, is based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. This method therefore provides a yield for a 10-year maturity, for instance, even if no outstanding security has exactly 10 years remaining to maturity. A detailed explanation of how these series are constructed can be found on the web page of Federal Reserve Board. The time span used to calibrate the model is January 1998 to December 2005.

Corporate Bond Data

REUTERS was the main source for the bond dataset, which has been constructed after considering the removal of the bonds with the following properties:

- callable, putable, or convertible bonds
- perpetual bonds
- index-linked bonds
- floating rate notes
- foreign currency bonds (bonds should be in the same denomination as the CDS)
- any rank else than senior unsecured bonds

- financial companies' bonds

Bonds with non-standard properties are excluded due to the necessity of including intricate techniques in bond price calculations. Senior unsecured bonds are utilized since CDSs have only these as deliverable obligations. Financial companies are excluded due to having significantly different capital structure. The time span of the bonds match the CDS dataset, being from 2003 to 2005.

Balance Sheet and Stock Market Data

Leverage values are constructed by dividing the total liabilities to the sum of market value of equity and total liabilities. This approach has also been followed by recent studies (see Eom, Helwege, and Huang (2004)). Quarterly total liabilities figures were retrieved from REUTERS balance sheet pages, while market value of equity (MVE) is the product of number of outstanding shares times the closing stock price on a given day. MVE figures can be retrieved daily, whereas total liabilities figures are available only quarterly. In order to avoid loss of data, a method similar to Eom/Helwege/Huang has been used. For the mid-month dates where bond and CDS data are available, the leverage ratios are computed, by making use of the latest available quarterly liabilities figure from balance sheets. For a consecutive three month period after the quarterly announcement, the leverage ratio is constructed from a constant liabilities figure and an MVE figure unique for the day.

Since European companies are not obliged to report their balance sheets on a quarterly basis, their leverage ratios could not be constructed without rough interpolation and therefore only US companies were focused on. The remaining dataset had 16 firms with 48 bonds. In Table 5.1, the descriptive statistics for the leverage ratio, total liabilities, and market capitalization can be found, whereas in Table 5.2 below, the short list of the firms and their bonds used in the study with details such as the issue date, maturity date and coupon amount can be observed. In the last column the ratings given by Moody's during the observation period are presented.

Table 5.1: Descriptive Statistics for the Leverage Ratios

Firm	Avg. Lev. Ratio(%)	Max. Lev. Ratio(%)	Min. Lev. Ratio(%)	Avg. Total Liab. (Mil.USD)	Avg. Market Capital. (Mil.USD)
CITIZENS	61.01	71.84	53.12	6,005	3,840
DEERE & CO	61.00	69.78	54.80	22,847	14,758
DELL	13.86	20.28	11.41	13,861	86,054
FEDERATED	66.88	78.55	48.22	9,434	4,728
HP	36.87	42.87	31.25	36,703	63,534
HILTON	47.08	60.37	37.85	5,960	6,880
IBM	4.00	4.62	3.34	5,960	144,175
INT.PAPER	57.47	61.81	52.67	24,479	18,173
MGM MIRAGE	69.01	79.79	42.60	9,577	4,870
MOTOROLA	35.05	51.87	22.77	18,515	36,940
NORDSTROM	50.65	71.36	21.87	2,865	3,477
NORFOLK	58.40	65.44	47.88	14,873	10,941
NORTHROP	58.43	84.87	44.70	18,135	13,650
TARGET	33.47	42.77	26.14	19,491	39,581
WAL-MART	22.53	29.32	17.50	64,975	224,869
WALT DISNEY	36.40	44.89	31.58	27,000	47,812

5.3.2 Estimation of the Parameters

Interest Rate Process Parameters

There can be alternative estimation techniques for calibrating the Vasicek process in Equation (5.1). It has been chosen to work with the method of Kalman filter, since it allows making use of cross-sectional and time series in-

Table 5.2: Short List of Firms/Bonds Used in Analysis

Firm	Bond ID	Issue Date	Maturity Date	Coupon	First Coup.	Rating
CITIZENS	1	23/05/01	15/05/11	9.250	15/11/01	Baa2-Ba3
CITIZENS	2	11/03/02	15/08/08	7.625	15/08/02	Baa2-Ba3
CITIZENS	3	12/11/04	15/01/13	6.250	15/07/05	Baa2-Ba3
DEERE & CO	1	17/04/02	25/04/14	6.950	25/10/02	A3
DELL	1	27/04/98	15/04/08	6.550	15/10/98	A2-A3
FEDERATED	1	14/07/97	15/07/17	7.450	15/01/98	Baa1
FEDERATED	2	14/06/99	01/04/09	6.300	01/10/99	Baa1
FEDERATED	3	06/06/00	01/06/10	8.500	01/12/00	Baa1
FEDERATED	4	27/03/01	01/04/11	6.625	01/10/01	Baa1
HP	1	16/12/02	17/12/07	4.250	17/06/03	A3
HP	2	26/06/02	01/07/07	5.500	01/01/03	A3
HILTON	1	15/04/97	15/04/07	7.950	15/10/97	Baa3-Ba1
HILTON	2	22/12/97	15/12/09	7.200	15/06/98	Baa3-Ba1
HILTON	3	22/12/97	15/12/17	7.500	15/06/98	Baa3-Ba1
HILTON	4	11/05/01	15/05/08	7.625	15/11/01	Baa3-Ba1
HILTON	5	22/11/02	01/12/12	7.625	01/06/03	Baa3-Ba1
IBM	1	01/10/98	01/10/08	5.400	01/04/99	A1
IBM	2	03/12/98	01/12/08	5.400	01/06/99	A1
IBM	3	15/01/99	15/01/09	5.500	15/07/99	A1
IBM	4	22/01/99	22/01/09	5.390	22/07/99	A1
IBM	5	26/01/99	26/01/09	5.400	26/07/99	A1
IBM	6	01/08/02	15/08/07	4.200	15/02/03	A1
IBM	7	27/12/02	15/12/06	3.000	15/06/03	A1
IBM	8	30/01/03	15/01/09	3.500	15/07/03	A1
IBM	9	06/02/03	15/02/13	4.200	15/08/03	A1
IBM	10	01/02/05	01/02/08	3.800	01/08/05	A1
INT.PAPER	1	27/08/01	01/09/11	6.750	01/03/02	Baa2
MGM MIRAGE	1	15/09/00	15/09/10	8.500	15/03/01	Ba1-Ba2
MGM MIRAGE	2	17/09/03	01/10/09	6.000	01/04/04	Ba1-Ba2
MGM MIRAGE	3	27/02/04	27/02/14	5.875	27/08/04	Ba1-Ba2
MGM MIRAGE	4	23/03/04	27/02/14	5.875	27/08/04	Ba1-Ba2
MGM MIRAGE	5	30/11/04	01/09/12	6.750	01/03/05	Ba1-Ba2
MOTOROLA	1	14/01/02	01/11/11	8.000	01/05/02	Baa2-Baa3
MOTOROLA	2	13/11/00	15/11/10	7.625	15/05/01	Baa2-Baa3
NORDSTROM	1	20/01/99	15/01/09	5.625	15/07/99	Baa1
NORFOLK	1	26/04/99	15/04/09	6.200	15/10/99	Baa1
NORFOLK	2	23/05/00	15/05/10	8.625	15/11/00	Baa1
NORFOLK	3	06/02/01	15/02/11	6.750	15/08/01	Baa1
NORTHROP	1	14/04/00	15/10/09	8.000	15/04/00	Baa2-Baa3
NORTHROP	2	12/07/01	15/02/11	7.125	15/08/01	Baa2-Baa3
NORTHROP	3	16/08/04	16/11/06	4.079	16/11/04	Baa2-Baa3
TARGET	1	26/03/01	01/04/07	5.500	01/10/01	A2
TARGET	2	10/10/01	01/10/08	5.400	01/04/02	A2
TARGET	3	02/05/03	15/05/18	4.875	15/11/03	A2
WAL-MART	1	10/08/99	10/08/09	6.875	10/02/00	Aa2
WALT DISNEY	1	28/06/99	28/06/10	6.800	01/08/99	Baa1
WALT DISNEY	2	20/06/02	20/06/14	6.200	20/12/02	Baa1
WALT DISNEY	3	27/10/93	27/10/08	5.800	01/02/94	Baa1

formation at the same time, and therefore is an appropriate tool for short rate process estimation. The literature on Kalman filter estimation of the short rate processes have mostly focused on Vasicek (1977) and Cox, Ingersoll, and Ross (1985) processes due to their linear structure (i.e. Duan and Simonato (1999); Geyer and Pichler (1999); Babbs and Nowman (1999); Chen and Scott (2003)). A detailed explanation of the Kalman filter technique can be found in Appendix A.

The method results in time series of the short rate r_t, plus the Vasicek process parameters κ_r, θ_r, σ_r, and the market price of risk parameter η. In Table 5.3, the estimated values for the risk-neutral parameters can be found.

Table 5.3: Kalman Filter Estimates of the Interest Rate Process

Parameter	Value
κ_r	0.247
θ_r	0.061
σ_r	0.012
η	-0.205

The risk-neutral (under Q) and physical (under P) processes of the short rate are: $dr = \kappa_r(\theta_r - r)dt + \sigma_r dW^Q$ and $dr = \kappa_r(\tilde{\theta}_r - r)dt + \sigma_r dW^P$ where $\theta_r = \tilde{\theta}_r - \frac{\sigma_r \eta}{\kappa_r}$

The mean reversion rate is in accordance with the values found in the literature. The same is true for the volatility parameter. The risk-neutral long-run mean is relatively high at 6.1 per cent. This converts to a physical mean of 5.1 per cent. Considering that the US interest rates varied between 1 and 6 per cent during the observation period, this value is within feasible range. Further efforts with the structural and reduced-form models described in the following sections will use the short rate series and Vasicek parameter estimates generated by the Kalman filter.

Leverage Process Parameters and the Correlation Coefficient

For the leverage process parameters (κ_l, θ_l), the approach of Eom/Helwege/Huang is followed (pp. 540-541). The authors use a regression method in order to estimate κ_l and θ_l. First, notice that θ_l is a function of $\bar{\nu}$, where $\bar{\nu}$ has been defined in Equation (5.7). Under the physical measure P,

$$dV_t/V_t = (\mu_v - \delta)dt + \sigma_v dW^P \tag{5.38}$$

and

$$dln(V_t/K_t) = [\mu_v + \kappa_l\bar{\nu} - \kappa_l(ln(V_t/K_t)]dt + \sigma_v dW^P \tag{5.39}$$

where μ_v is a constant, and dW^P is a standard Brownian motion under the physical measure P. Let

$$\alpha_l = \mu_v + \kappa_l\bar{\nu} \tag{5.40}$$

A regression of the change in the log-leverage ratio against log-leverage ratio lagged one period will generate parameter estimates $\hat{\alpha}_l$ and $\hat{\kappa}_l$:

$$ln(V_t/K_t) - ln(V_{t-1}/K_{t-1}) = \beta_0 + \beta_1 ln(V_{t-1}/K_{t-1}) + \epsilon \tag{5.41}$$

As a result, β_0 will be equal to the α_l in Equation (5.40), and β_1 will be equal to $-\kappa_l$ in Equation (5.39). In addition, the $\hat{\mu_v}$ can be estimated from the mean return of the asset value over the prior 5 years. Then $\bar{\nu}$ can be estimated as follows:

$$\hat{\bar{\nu}} = (\hat{\alpha}_l - \hat{\mu_v})/\hat{\kappa}_l \tag{5.42}$$

In the implementation, monthly market leverage ratios are regressed on one month lagged ratios for the period of 2001-2005.

The correlation coefficient ρ arises from the presence of correlated Brownian motions in the processes. The correlation between asset returns and the interest rate process is estimated from correlation between equity returns and changes in the interest rates. Over the 2001-2005 sample horizon, the correlation between changes in the 3-monthly interest rates series and daily closing stock returns is computed.

Asset Volatility and Reduced-Form Model Specific Parameters

From the target parameter set $(\kappa_r, \theta_r, \sigma_r, \kappa_l, \bar{\nu}, \rho, \sigma_v)$, the remaining variable is the asset volatility, σ_v. It has been chosen to retrieve "bond-implied" volatility, by making use of bond prices. By minimizing the sum of squared errors over each observation day and each bond price, one can reach the implied asset volatility for the structural CDG model:

$$\min_{\sigma_v} \sum_{i=1}^{ObsDays} \sum_{j=1}^{Bonds} (v_{i,j}^{theo}(l_t, r_t) - v_{i,j}^{obs})^2 \tag{5.43}$$

On the reduced-form side, there are two additional parameters to be estimated. These are the adjusted short rate parameters a and c. It is an option to simultaneously estimate a and c from the minimization of sum of squared errors formula, per firm:

$$\min_{\sigma_v, a, c} \sum_{i=1}^{ObsDays} \sum_{j=1}^{Bonds} (v_{i,j}^{theo}(l_t, r_t) - v_{i,j}^{obs})^2 \tag{5.44}$$

Note that the number of free parameters used to calibrate the models to bond prices differs across approaches. In CDG there is the asset volatility as the only free parameter, whereas in the intensity case there will be three parameters. The results will be analyzed taking into account the number of free parameters.[3]

5.3.3 Estimation Results

Parameter Estimates from Bond and Stock Prices, and Balance Sheet Information

As described in Section 5.3.2, the bond and stock prices, as well as the balance sheet information are the source for the CDG and intensity model parameter estimates, which will be used for the prediction of CDS prices at a latter step. By inserting the interest rate process parameters, the estimated leverage process parameters $\kappa_l, \bar{\nu}$, and the correlation coefficient ρ into the CDG and intensity formula, the SSE method enables to pull out the asset volatility σ_v from bond prices. In the intensity case, there are additionally the adjusted discount rate parameters a and c. Firstly, the parameter estimates common in both models can be found in Table 5.4.

The parameter estimates are mostly in a reasonable range. First, the mean-reversion rate of the leverage κ_l, has a value around 5-10 per cent, although very low figures as well as higher figures are also estimated from regressions. These values fall in a consistent range with prior studies: To Fama and French (2002) who reach a value around 7-10 per cent in their regression analysis and to Shyam-Sunder and Myers (1999) who have a sample weighted towards large

[3] Although not documented, an alternative version of the intensity model has also been tested in the runs. This model estimated the a and c parameters common to all firms, instead of individual estimation. The out-of-sample prediction results were inferior to both the firm-specific intensity setup and the CDG model.

Table 5.4: Parameters Common to Both Models

Firm	κ_l	$\bar{\nu}$	ρ
CITIZENS	0.031	0.727	0.003
DEERE & CO	0.136	0.438	0.078
DELL	0.009	0.082	0.070
FEDERATED	0.061	0.216	0.133
HP	0.279	0.972	0.090
HILTON	0.105	0.567	0.176
IBM	0.052	2.795	0.049
INT.PAPER	0.197	0.595	0.119
MGM MIRAGE	0.169	0.282	0.137
MOTOROLA	0.030	1.024	0.048
NORDSTROM	0.002	-2.398	0.116
NORFOLK	0.030	0.432	0.094
NORTHROP	0.028	0.261	-0.086
TARGET	0.083	1.022	0.154
WAL-MART	0.027	1.053	0.120
WALT DISNEY	0.078	1.011	0.151

and financially conservative firms and reach a value around 40 per cent. The correlation coefficient between the stock returns and change in interest rates is comparable to the figure of Eom/Helwege/Huang who report that they have relatively low correlation values all below 15 per cent. They also note that the correlation variable has not been found to effect the spreads significantly. Although there are no comparable figures for the value of $\bar{\nu}$ in the literature, one can compute the long-run means and infer the soundness of these estimates.

The CDG model and the intensity model have their unique asset volatility figures. For the CDG model, the asset volatilities which are inferred from bond prices are between 15-40 per cent except a few outliers. These values are compared to option-implied volatilities computed from at-the-money call options with a maturity of June 2007 for all of the above listed companies. This comparison reveals that option-implied volatilities are in the range of 15-35 per cent, indicating that the bond-implied figures are economically reasonable as well. Nevertheless, there are significant outliers such as the values for IBM and DELL. The estimated and option-implied volatilities for the CDG model can be found in Table 5.5.

Table 5.5: CDG Model Estimation Figures

Firm	σ_v Estimated	σ_v Option-implied	θ_l	$\theta_{K/V}$
CITIZENS	0.213	0.226	-1.70	0.36
DEERE & CO	0.177	0.370	-0.66	0.58
DELL	0.594	0.312	-3.38	>1
FEDERATED	0.141	0.237	-0.71	0.58
HP	0.392	0.240	-1.08	0.45
HILTON	0.345	0.292	-0.85	0.75
IBM	0.913	0.173	-3.37	>1
INT.PAPER	0.240	0.192	-0.75	0.55
MGM MIRAGE	0.178	0.347	-0.46	0.68
MOTOROLA	0.350	0.237	-2.02	>1
NORDSTROM	0.147	0.320	-13.04	<0.01
NORFOLK	0.176	0.299	-1.42	0.40
NORTHROP	0.102	0.168	-1.33	0.32
TARGET	0.365	0.230	-1.38	0.48
WAL-MART	0.429	0.181	-1.13	0.41
WALT DISNEY	0.384	0.208	-1.39	0.64

In Table 5.5, the last two columns indicate the long-run mean of the log-leverage process and the leverage itself by applying Ito's lemma to log-leverage process in Equation (5.6). In calculating the long-run mean of the log-leverage from $\theta_l = -\bar{\nu} - r/\kappa_l$, the short rate r is assumed at a constant 3 per cent. It can be observed that the long-run means are at a reasonable level. Except three outliers, all long-run leverage ratios are between 0 and 1. The higher figures for DELL and IBM arise from a very high asset volatility estimate and a very low mean-reversion rate, which end up in a figure higher than one. For the other companies, the value for the long-run mean of leverage falls close to

Table 5.6: Intensity Model Parameter Estimates

Firm	a	c	σ_v	$\theta_{K/V}$
CITIZENS	0.045	0.001	0.501	>1
DEERE & CO	0.034	0.033	0.014	0.52
DELL	0.020	0.001	0.828	>1
FEDERATED	0.043	0.043	0.211	0.71
HP	0.024	0.003	0.335	0.35
HILTON	0.165	0.158	0.001	0.43
IBM	0.022	0.002	0.112	0.03
INT.PAPER	0.024	0.001	0.987	>1
MGM MIRAGE	0.078	0.062	0.844	0.63
MOTOROLA	0.095	0.058	0.003	0.13
NORDSTROM	0.034	0.015	0.042	<0.01
NORFOLK	0.056	0.057	0.007	0.24
NORTHROP	0.036	0.025	0.247	0.26
TARGET	0.055	0.035	0.001	>1
WAL-MART	0.015	0.001	0.232	0.31
WALT DISNEY	0.106	0.071	0.356	0.56

the empirically used monthly leverage figures as well, which can be seen from the values in Table 5.1.

Using bond prices as a source for the parameter estimation, the intensity model described in Section 5.2.2 is also calibrated. The sum of squared errors method in Equation (5.44) has yielded the results in Table 5.6. The interest rate process parameters, the leverage process parameters $\kappa_l, \bar{\nu}$ and the correlation coefficient ρ are estimated exactly the same as in the CDG model. From the intensity model for bond prices, one can extract the asset volatility σ_v, and the intensity parameters a and c.

In Table 5.6 the a and c figures convert mostly into reasonable values for the default intensities. The next section will present a more detailed analysis of how these figures transfer into default probabilities. Note also that the long-run mean for the leverage figures calculated by equating the short rate r to 3 per cent lie in a reasonable range. For instance, for IBM, the actual leverage inputs are also in the range of 3-4 per cent. The model was able to capture these successfully. Except four outliers which have long-run means above the standards, all the long-run means are near the range of original input leverage parameters.

In-Sample Fits to Bond Prices

To assess the estimation results, the in-sample fit to bond prices can be found, in Table 5.7. In the table, the mean error (ME), the mean absolute error (MAE), and the mean absolute percentage errors (MAPE) are computed. The results indicate that there is a good fit to prices with rather low error figures. A better fit is observed with the intensity model, also indicated by the significance test. However, note that the intensity model has three free parameters in estimation whereas the structural model has only one. The Akaike Information Criterion (AIC) is an ideal measure in case there are free parameter differences between the models whose fits are tested. But also after considering the free parameters, the AIC values in the lower panel show that the intensity model has a better (lower) value, and thus a better fit. Further analysis in Section 5.4 will show whether the better in-sample fit to bond prices carries over to an out-of-sample fit to CDS prices.

Default Probabilities from Bond Prices

Before taking a step towards prediction of CDS prices, it might be insightful to compute the default probabilities indicated by the parameter estimates. With the structural model, the forward risk-neutral probability of default is

Table 5.7: Structural and Intensity Models - In-Sample Fit to Bond Prices

	Bonds		Structural			Intensity		
Firm	No. of Bonds	No. of Prices	ME (pts)	MAE (pts)	MAPE (%)	ME (pts)	MAE (pts)	MAPE (%)
CITIZENS	3	86	1.89	7.94	7.23%	0.06	3.75	3.41%
DEERE & CO	1	36	0.35	3.48	3.00%	-0.24	3.04	2.63%
DELL	1	36	0.05	1.13	1.04%	-0.02	1.01	0.92%
FEDERATED	4	144	1.12	3.96	3.47%	-0.06	1.94	1.69%
HP	2	66	0.26	1.02	0.99%	0.00	0.82	0.79%
HILTON	5	180	-0.37	4.22	3.98%	0.01	2.00	1.88%
IBM	10	331	0.23	1.19	1.17%	-0.01	1.03	1.01%
INT.PAPER	1	36	-0.05	2.51	2.28%	-0.21	2.32	2.10%
MGM MIRAGE	5	119	1.53	4.99	4.89%	-0.06	2.04	2.00%
MOTOROLA	2	72	0.47	2.57	2.23%	-0.08	2.52	2.21%
NORDSTROM	1	36	2.45	3.38	3.21%	-0.02	1.13	1.08%
NORFOLK	3	108	0.54	2.00	1.76%	-0.06	1.46	1.28%
NORTHROP	3	89	3.62	4.85	4.25%	0.04	1.17	1.02%
TARGET	3	104	0.64	2.46	2.44%	0.12	1.48	1.46%
WAL-MART	1	36	0.05	1.75	1.55%	-0.04	1.25	1.10%
WALT DISNEY	3	108	-0.07	2.21	2.06%	-0.09	1.92	1.79%
Average			0.79	3.10	2.85%	-0.04	1.80	1.65%
AIC				232.65			145.25	
Significance Test	Mean Difference			t-statistic			p-value	
Difference of Str. - Int.	1.34			20.27			0.000	

"Mean Error (ME)" is the difference between the model and the observed bond price.
"Mean Absolute Error (MAE)" is the absolute value of the difference between the model and the observed bond price.
"Mean Absolute Error (MAPE)" is the percentage value of the division of MAE by the observed bond price.
"AIC" is the Akaike Information Criterion calculated from $2k + nln(RSS/n)$ where k is the number of free parameters for the model, n is the number of observations, and RSS is the residual sum of squares.
"Difference of Structural - Intensity" is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day.

$Q^{F_T}(r_0, l_0, T)$ mentioned in Equation (5.10). One can easily compute the 5-year probability of default and compare it with the actual default probabilities for the same rating class reported by Moody's. Moody's actual default rates correspond to a period of 1970-2003. Model-implied 5-year default probabilities are the average values of the full observation period (36 mid-month observations). Table 5.8 presents this comparison. Similar to the structural side, one can look at default probabilities as a result of the estimation process with the reduced-form setting as well. The default probability PD in the intensity setting is:

$$PD = 1 - E^Q\left(e^{-\int_0^T \lambda ds}\right) \tag{5.45}$$

Afterwards, the risk-neutral probability can be converted into the forward probability easily. With this formulation, the 5-year model-implied default probabilities with the intensity model can be found in the second column of Table 5.8.

For both models, the default probability figures seem indistinguishable. One model is not consistently higher or lower than its counterpart. Actually, the model-implied default probabilities draw a clear picture. Although not strictly monotonous, the higher the actual probability of default, the higher is the model-implied probability. For example, the Ba rated companies are estimated to have the highest PD's, whereas the less riskier rating classes have significantly lower values. The model-implied probability is the highest for CITIZENS, HILTON, and MGM MIRAGE, which have the lowest ratings in the sample. Another important point is that the risk-neutral probabilities are always higher than real world probabilities, in line with the theory and other empirical findings.

Table 5.8: Model-Implied and Actual Probabilities of Default

Firm	Structural Model-implied 5 year PD	Intensity Model-implied 5 year PD	Rating (Moody's)	Actual PD in Rating Class
CITIZENS	19.37%	20.11%	Baa2-Ba3	2.16%-11.17%
DEERE & CO	9.46%	8.51%	A3	0.54%
DELL	11.43%	8.93%	A2-A3	0.54%
FEDERATED	12.21%	12.17%	Baa1	2.16%
HP	10.35%	9.99%	A3	0.54%
HILTON	24.79%	22.44%	Ba1-Baa3	2.16%-11.17%
IBM	8.17%	7.43%	A1	0.54%
INT.PAPER	13.04%	11.19%	Baa2	2.16%
MGM MIRAGE	24.33%	22.14%	Ba1-Ba2	11.17%
MOTOROLA	13.23%	14.97%	Baa2-Baa3	2.16%
NORDSTROM	11.67%	11.13%	Baa1	2.16%
NORFOLK	9.63%	11.73%	Baa1	2.16%
NORTHROP	9.28%	10.57%	Baa2-Baa3	2.16%
TARGET	10.07%	8.11%	A2	0.54%
WAL-MART	8.18%	7.29%	Aa2	0.24%
WALT DISNEY	14.53%	14.45%	Baa1	2.16%

The observation that the model-implied probability increases with decreasing credit quality can be also seen from Table 5.9. If the default probabilities are averaged across companies with respect to rating classes, a clear stepwise increase in the model-implied default probabilities in comparison to the rating implied actual default probabilities can be observed. The applicable rating class is taken as the rating at the beginning of the observation period (January 2003).

Table 5.9: Model-Implied and Actual Probabilities of Default, Breakdown into Rating Classes

Rating (Moody's)	Structural Model-implied 5 year PD	Intensity Model-implied 5 year PD	Actual PD in Rating Class
Aa	8.18%	7.29%	0.24%
A	9.90%	8.59%	0.54%
Baa	12.87%	13.29%	2.16%
Ba	24.56%	22.29%	11.17%

5.4 Prediction of Credit Default Swap Prices

5.4.1 Prediction Methodology

The final aim with both types of models is to predict the prices of CDSs out-of-sample. The fair price of a credit default swap (CDS) with recovery-of-treasury assumption would be:

$$CDS(T^*) = \frac{E^Q\left(e^{-\int_t^{\tau} r(s)ds}(1 - \varphi \cdot e^{-\int_{\tau}^{T} r(s)ds}) \cdot \mathbf{1}_{\{\tau < T^*\}}\right)}{E^Q\left(\sum_{i=1}^{n} e^{-\int_t^{t_i} r(s)ds} \cdot \mathbf{1}_{\{\tau > t_i\}}\right)} \tag{5.46}$$

The denominator is the cumulation of n discount factors which are at time points t_i. The numerator gives the recovered amount in case of default prior to the maturity (T^*) of the CDS. The recovery leg (the numerator) has to be equal to the premium leg (the denominator) under no-arbitrage assumptions, which will yield the theoretically fair price of $CDS(T^*)$.

A simulation algorithm has been used in order to reach the fair price of a credit default swap. Paths of the short rate and the leverage ratio are simulated where default occurred at the first time when the log-leverage is larger than zero (leverage is greater than or equal to 1). For a typical 5-year horizon of the maturity of the CDS, the simulation algorithm generates paths and at each time point the log-leverage is checked for whether it has a value higher than zero:

(i) At first step, the short rate is simulated using an Euler discretization of the Vasicek process: Start with r_t=r_0, and generate r_{t+1} through

$$r_{t+1} = r_t + \kappa_r(\theta_r - r_t)\Delta t + \sigma_r\sqrt{\Delta t}\epsilon_t^1 \tag{5.47}$$

where $\epsilon_t^1 \sim N(0, 1)$.

(ii) Substitute the simulated r_{t+1} into

$$\theta_l(r_{t+1}) = -\bar{\nu} - \frac{r_{t+1}}{\kappa_l} \tag{5.48}$$

(iii) Generate l_{t+1} through Euler discretization of the leverage process:

$$l_{t+1} = l_t + \kappa_l(\theta_l - l_t)\Delta t - \sigma_v\sqrt{\Delta t}(\rho\epsilon_t^1 + \sqrt{1-\rho^2}\epsilon_t^2) \tag{5.49}$$

Here, note that the Brownian motions of the two processes are correlated with a factor of ρ and $\epsilon_t^2 \sim N(0, 1)$.

a. If $l_{t+1} < 0$ (log leverage having a negative sign) then no default occurs. The CDS premiums up to this time point are cumulated, when a quarter is complete (typical quarterly payments is assumed).

This accumulation constitutes the "Premium Leg" of a CDS.

$$PremLeg_i = PremLeg_{i-1} + \left(e^{-\sum_0^{t_i} r_{t_i}\Delta t}\right) \tag{5.50}$$

Here, t_i is the i^{th} premium date. Simulation continues with step (iv).

b. If $l_{t+1} \geq 0$, default happens. Simulation is terminated and the recovery leg is computed to constitute the numerator of the fair price of a CDS. $\tau = t + 1$ and

$$RecLeg = \left(e^{-\sum_0^{\tau} r_t\Delta t}(1 - \varphi \cdot b(r_\tau, T - \tau))\right) \tag{5.51}$$

In addition, the accrued premium since the last premium payment is calculated and added to the premium leg. In this implementation, the recovered bond maturity (T) is taken to be the longest dated bond's maturity. According to the intuition, with no recovery on coupons, the longest available bond should be delivered in case the "cheapest-to-deliver" option is available. Recall from earlier chapters that the delivery option denotes the possibility of the buyer to deliver the cheapest bonds available in case of default. By assuming no recovery on coupons, the bond with longest maturity should be the deliverable obligation.

(iv) Go back to step (i) to generate r_{t+2}.

For simulating the fair price of a CDS in the reduced-form case, Euler discretizations for the short rate and leverage process as in Equations (5.47) and (5.49) have been used. Following Schönbucher (2003), a uniform random variate U is generated as the trigger level. Let γ be the default countdown process, which is initiated by letting $\gamma(0) = 1$. Different from the CDG model described above, step (iii) is replaced by:

(iii) Generate l_{t+1} through Euler discretization of the leverage process:

$$l_{t+1} = l_t + \kappa_l(\theta_l - l_t)\Delta t - \sigma_v\sqrt{\Delta t}(\rho\epsilon_t^1 + \sqrt{1-\rho^2}\epsilon_t^2) \tag{5.52}$$

Compute the associated default intensity as:

$$\lambda(t+1) = a + cl_{t+1} \tag{5.53}$$

Then at each time step, the default countdown process is decreased by,

$$\gamma(t+1) = \gamma(t)e^{\lambda(t+1)\Delta t} \tag{5.54}$$

a. If $U < \gamma(t+1)$ then no default occurs. Similar to the structural side, the CDS premiums up to this quarter are cumulated, when a quarter is complete. This is the premium leg of the CDS.

$$PremLeg_i = PremLeg_{i-1} + \left(e^{-\sum_0^{t_i} r_{t_i}\Delta t}\right) \tag{5.55}$$

b. If $U \geq \gamma(t+1)$, default happens and the recovery leg is computed.

$$RecLeg = \left(e^{-\sum_0^{\tau} r_t\Delta t}(1 - \varphi \cdot b(r_\tau, T-\tau))\right) \tag{5.56}$$

Accrued premiums are taken into account since the last premium payment date, as well.

5.4.2 CDS Prediction Results

In the final step, model implied CDS prices can be generated using the methodology described in Section 5.4.1. Once the parameter estimates are available, it is straightforward to use the steps described in the mentioned section. This out-of-sample prediction of CDS prices can be evaluated using deviations from the observed prices. The mean errors (ME), the mean absolute errors (MAE), and the mean absolute percentage errors (MAPE) can be computed to understand the deviation from the market prices. In Table 5.10, the out-of-sample prediction error figures for the CDG and the intensity model can be found.

The results indicate that both models have mostly underpredicted CDS premiums with an average of 25 bps. The absolute errors for the structural and reduced-form models are 33 and 30 bps respectively. At first sight, the structural model has a higher percentage error (49 per cent) than the intensity model (37 per cent). Parallel to this, a comparison of the absolute errors of two models indicate that the difference between the two models is statistically significant, the intensity model yielding lower error figures. However, it should not be forgotten that the intensity model had three free parameters for fitting to bond prices while the structural model had only one. After checking the Akaike Information Criterion values, it can be observed that the figures of the two models are quite close, with the intensity model having a slightly better (lower) value. Overall, it can be concluded that constructing comparable approaches have yielded comparable results in pricing CDSs, as well.

The errors can further be analyzed by classifying to ratings and number of bonds used in the estimation. The rating, which the company possesses at the beginning of the observation period (January 2003) was taken as the ap-

Table 5.10: CDG and Intensity Models - Out-of-Sample Fit to CDS Prices

	CDG			Intensity		
Firm	ME (pts)	MAE (pts)	MAPE (%)	ME (pts)	MAE (pts)	MAPE (%)
CITIZENS	-126.19	130.91	58.47%	-115.87	115.87	51.95%
DEERE & CO	-2.69	7.52	26.80%	-5.95	7.77	23.75%
DELL	17.94	19.33	113.91%	10.39	12.95	75.15%
FEDERATED	-11.49	15.37	32.37%	-12.41	16.93	29.25%
HP	-4.52	8.16	22.59%	-3.98	8.65	22.67%
HILTON	-55.98	59.91	31.51%	-84.41	84.58	50.02%
IBM	-3.93	6.95	25.63%	-3.49	7.36	25.58%
INT. PAPER	-29.36	29.71	42.84%	-32.68	32.98	46.89%
MGM MIRAGE	-85.61	91.97	49.50%	-88.07	88.50	43.61%
MOTOROLA	-42.18	42.18	52.82%	-40.81	40.81	43.53%
NORDSTROM	-29.00	29.00	72.98%	-13.50	13.85	27.55%
NORFOLK	-4.84	10.33	30.91%	-4.35	7.30	19.36%
NORTHROP	-32.24	34.47	84.30%	-9.59	10.22	21.99%
TARGET	8.94	10.01	47.38%	-3.55	7.90	29.08%
WAL-MART	-10.64	10.64	59.34%	3.16	5.38	35.54%
WALT DISNEY	0.48	13.81	29.94%	0.92	17.51	39.13%
Average	-25.71	32.52	48.83%	-25.26	29.91	36.56%
AIC		235.90			232.39	

Significance Test	Mean Difference	t-statistic	p-value
Difference of CDG - Intensity	2.61	3.03	0.003

"Mean Error (ME)" is the difference between the model and the observed CDS price.
"Mean Absolute Error (MAE)" is the absolute value of the difference between the model and the observed CDS price.
"Mean Absolute Error (MAPE)" is the percentage value of the division of MAE by the observed CDS price.
"AIC" is the Akaike Information Criterion calculated from $2k + nln(RSS/n)$ where k is the number of free parameters for the model, n is the number of observations, and RSS is the residual sum of squares.
"Difference of Structural - Intensity" is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day.

plicable class. Panel A of Table 5.11 shows that mean absolute errors almost always increase as the rating worsens. This is reasonable, since the higher rated companies have lower CDS premiums on average. Both models have difficulty especially in reaching the high CDS premiums for low rated classes, where almost always underprediction is observed. Although not monotonous for the structural model, the intensity model's underprediction continuously increases as the credit rating worsens. Moreover, it is observed from the significance tests that although the intensity model outperforms the structural model in better rated CDSs, the structural model performs better in the pricing of Ba-rated firms.

In Panel B of Table 5.11 the error figures are averaged with respect to the number of bonds used in the estimation. For both models, the lowest MAE figures are with 1, 4, and 10 bonds. It can also be inferred that using a single bond in estimation almost always results in a higher MAPE than using more bonds. However, a trend depending on the number of bonds can not be observed for either of the approaches. Significance tests indicate that the intensity model performs better for 1 and 3 bonds, whereas the structural model is better for 5 bonds. Other cases do not show any significance.

These results are comparable to prior research results in two ways. First, the testing of Collin-Dufresne and Goldstein model has few examples in the literature. Among them, the study of Eom, Helwege, and Huang (2004) (EHH), which compares the CDG model with four other structural models, is the most noteworthy one. EHH make use of bond data only, which is the major difference from this study. They find that the CDG model suffers from an accuracy problem, where predicted bond spreads are either too small or incredibly large. In particular, they note that a more accurate term structure model than Va-

Table 5.11: Structural and Intensity Models - Out-of-Sample Fit, Breakdown to Ratings and No. of Bonds Used in Estimation

Panel A	Structural			Intensity			t-test Structural-Intensity		
Rating	ME (pts)	MAE (pts)	MAPE (%)	ME (pts)	MAE (pts)	MAPE (%)	Mean Diff.	t-stat	p-value
Aa	-10.64	10.64	59.34%	3.16	5.38	35.54%	5.25	4.77	0.000
A	3.15	10.39	47.26%	-1.32	8.93	35.25%	1.47	2.13	0.034
Baa	-34.35	38.22	50.58%	-28.54	31.93	34.95%	6.29	5.23	0.000
Ba	-70.80	75.94	40.50%	-86.24	86.54	46.81%	-10.60	-2.51	0.014

Panel B	Structural			Intensity			t-test Structural-Intensity		
No. of Bonds	ME (pts)	MAE (pts)	MAPE (%)	ME (pts)	MAE (pts)	MAPE (%)	Mean Diff.	t-stat	p-value
1	-10.75	19.24	63.17%	-7.72	14.59	41.78%	4.65	4.77	0.000
2	-23.35	25.17	37.70%	-22.40	24.73	33.10%	0.44	0.37	0.713
3	-30.77	39.91	50.20%	-26.49	31.76	32.30%	8.15	5.10	0.000
4	-11.49	15.37	32.37%	-12.41	16.93	29.25%	-1.56	-0.50	0.624
5	-70.80	75.94	40.50%	-86.24	86.54	46.81%	-10.60	-2.51	0.014
10	-3.93	6.95	25.63%	-3.49	7.36	25.58%	-0.41	-0.61	0.548

"Mean Error (ME)" is the difference between the model and the observed CDS price.
"Mean Absolute Error (MAE)" is the absolute value of the difference between the model and the observed CDS price.
"Mean Absolute Error (MAPE)" is the percentage value of the division of MAE by the observed CDS price.

sicek's model could be of use. As a result, they reach an percentage error of out-of-sample spread prediction in the level of 269.78 per cent and an absolute percentage error of spread prediction of 319.31 per cent. These results extend the EHH study to the prediction of CDS prices, and on average yield a relative mean absolute error of 48.83 per cent.

Secondly, the results can be compared with recent studies that predict CDS prices using other types of structural and intensity models. For instance, Ericsson, Reneby, and Wang (2006)'s CDS premium prediction mean errors, with Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000) are in the range of 10 - 52 bps, whereas Arora, Bohn, and Zhu (2005) have reached 27 - 102 bps with the Merton (1974) model and -80 - 2 bps with Vasicek/Kealhofer model.[4] These results are well comparable with the mean error of -25.71 bps and mean absolute error of 32.52 bps. The error figures signify that the CDS price prediction ability of the structural model is competitive with respect to other models used in the literature. On the other hand, Bakshi, Madan, and Zhang's (2006) observable credit risk factor approach in an intensity model has yielded out-of-sample absolute bond yield prediction errors in a range of 26-49 bps when log-leverage is selected as the factor. These results extend Bakshi, Madan, and Zhang's (2006) results with bond prices to a CDS price prediction.

5.4.3 Robustness Check

The analysis in the last section showed that the prediction power of the models are close. At a further step, looking at whether significant differences in approaches are revealed in a time out-of-sample analysis, can be insightful. In order to check this, the estimation results from the full observation period were used to compute the theoretical CDS prices of mid-month January 2006. By doing this, it is ensured that the out-of-sample analysis does not include the time horizon of estimation.

Table 5.12 shows the mean absolute errors and mean absolute percentage errors for this time point. It is observed that the prediction power deteriorates - an expected outcome with time out-of-sample analysis. The percentage errors

[4] See Crosbie and Bohn (2003), Kealhofer (2003a), Kealhofer (2003b), and Vasicek (1984)

have increased to 79 and 47 per cent for January 2006. The significance tests this time indicates indifference between the prediction power of the structural and intensity models in terms of absolute errors, as can be seen from the lower panel of the table. Once again, the results show that the two models do not outperform one another.

Table 5.12: Structural and Intensity Models - Time Out-of-Sample Fit

Firm	January 2006			
	Structural		Intensity	
	MAE (bps)	MAPE (%)	MAE (bps)	MAPE (%)
CITIZENS	161.33	81.89%	109.76	55.71%
DEERE & CO	0.42	1.99%	1.35	6.45%
DELL	63.47	409.47%	21.55	139.00%
FEDERATED	4.38	10.43%	1.64	3.92%
HP	8.09	35.18%	7.62	33.15%
HILTON	59.06	46.51%	94.40	74.33%
IBM	5.65	33.25%	4.57	26.86%
INT. PAPER	41.31	57.37%	35.18	48.86%
MGM MIRAGE	120.74	73.17%	60.77	36.83%
MOTOROLA	14.30	59.57%	17.16	71.49%
NORDSTROM	29.14	88.31%	19.31	58.52%
NORFOLK	17.81	74.21%	8.12	33.84%
NORTHROP	16.92	76.92%	0.18	0.81%
TARGET	20.24	161.95%	6.44	51.49%
WAL-MART	4.91	37.80%	4.84	37.24%
WALT DISNEY	8.36	23.88%	23.43	66.94%
Average	36.01	79.49%	26.02	46.59%
Significance Test	t-statistic		p-value	
Difference of Structural - Intensity	1.67		0.116	

"Mean Absolute Error (MAE)" is the absolute value of the difference between the model and the observed CDS price.
"Mean Absolute Error (MAPE)" is the percentage value of the division of MAE by the observed CDS price.
"Difference of Structural - Intensity" is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm.

5.5 Summary

This chapter has provided a comparison of the pricing of CDSs by alternative frameworks. On one hand, with a structural approach that has a stationary leverage ratio, it has been aimed to extract credit information from bond, stock and balance sheet information in order to correctly price CDSs. On the other hand, it has been examined whether a comparable reduced-form model with the leverage process as the state variable could better price a CDS. The results show that the overall out-of-sample prediction performance is equally well with both models. The intensity model has yielded slightly better prediction results possibly due to three free parameters. After incorporating this information, the Akaike Information Criterion showed that the results with both models are quite close, with the intensity model yielding slightly lower values. Moreover, the time out-of-sample analysis indicated insignificance between the prediction powers of the two models. Attributing the default intensity not as an unobservable latent factor but instead as an observable credit risk factor, the leverage process, has yielded competitive results in comparison to the structural model.

This analysis has been carried for the most liquid entities' bond and CDS prices. The CDSs utilized had a five-year maturity, which are the most liquidly traded contracts. This should be taken into account when judging the results that showed indifference between the two frameworks. It remains for future research to look at the capability of the models to fit to the full term structure of CDSs and bonds.

Other issues worth noting are the basis between bond and CDS prices, and liquidity. As seen in the studies of Blanco, Brennan, and Marsh (2005) and

Hull, Predescu, and White (2004) the no-arbitrage equality between CDS premiums and bond spreads may not perfectly hold. This may be partly due to liquidity. Recent studies such as Longstaff, Mithal, and Neis (2005) have investigated the bond and CDS price differences including a liquidity premium in bond prices. However, in this analysis liquidity differences are not explicitly taken into account. Although it is interesting to check whether extensions with liquidity yield better performance of the models on an absolute level, liquidity should affect market prices in a way that enters both models similarly. Therefore significant differences is not expected in relative terms.

Chapter 6

Conclusions

Credit default swaps are already a critical instrument for hedging and risk transfer purposes in financial markets worldwide. The market volume is increasingly expanding, which is a result of the product creating numerous financial opportunities. Until the arrival of credit derivatives, risk management in an asset portfolio was based on few broad activities. The traditional methods to safeguard against losses were to maintain a level of economic capital to cover any unexpected losses on loans, and to limit the size of any loan to any customer so as to maintain diversification of the portfolio risk. In the presence of CDSs, credit risk can be managed or traded independently of the ownership of the underlying asset. Properly functioning CDS markets are therefore a major benefit for any institution that would like to hedge or transfer credit risk.

This study has contributed in understanding how markets of CDSs function and how a theoretically fair price could be reached. First, the direct OTC and interdealer broker markets have been analyzed in depth. Markets were compared regarding one of their most crucial ingredients, their liquidity. Most of the contractual variables were shown to be determinants of the bid-ask spread of the brokered market. The two markets seem to be integrated in prices, which leaves only the trading cost differences in the bid-ask spreads. Moreover, the liquidity differences between the two market venues were attributed to the added value provided by the brokers. These two alternatives have recently been complemented by the first exchange traded credit derivatives by Eurex. These are futures on CDS indices of the 125 most traded entities. With the inclusion of exchanges into the trading arena, the market venues for CDS are likely to attract more and more counterparties.

There are certain implications for further research concerning this first part of the study. First, once they become liquid, the electronic platform data of the interdealer brokers will be a unique source in understanding differences in market venues. Then, it would be possible to look at three alternatives; the direct conversations in the OTC market, the voice brokerage offered by IDBs and the electronic platforms. Such a study would have implications on the effects of transparency on liquidity. It remains an open question for further research on how an open limit-order book in the CDS market would be different from other markets. The first exchange-traded CDS indices have not yet reached abundant liquidity in the Eurex system, as reported in different sources. Exchange traded CDS data would be an indispensable source for looking at default correlations. This analysis with single-name CDSs can be extended to basket-CDSs with the inclusion of correlation.

The second part of the study deals with pricing CDSs in a theoretically fair way. With the Basel II Capital Accord in effect, there has been increased attention to credit risk modeling. The Basel II, being a revision of the original Basel Capital Accord, has reformed the way how solvency requirements are computed. During this process, credit risk models have been under focus, since financial institutions are now allowed to make use of their own internal rating systems for their risk exposure in credit contracts. This study provides an insight on how these credit risk models can give guidance in reaching the theoretically fair price of credit risk, in case they need to be applied for institutional use.

Structural models which are based on diffusion processes and reduced-form models which depend on Poisson jump processes have been utilized in the credit risk literature in various studies. Structural models have the strength of being intuitive, however, they have yielded poor results in predicting credit risky instrument prices. Reduced-form models, on the other hand, have reached a relatively better performance, despite the fact that they have less economic intuition. Due to natural differences, there could be no basis of comparison between the prediction power of these frameworks, since they could take various forms, and were applied to different datasets. One of the most important contributions of the study is trying to build a comparable structure between structural and reduced-form models. Up to date only Arora, Bohn, and Zhu (2005) tried to pursue an empirical study that compares the two frameworks. However, their study had limitations, such as that bond data was used to calibrate the intensity model (Hull/White) whereas equity price data was necessary in calibrating the structural Merton and Vasicek/Kealhofer models. This raises doubts with respect to the comparability of the two approaches.

The analysis in Chapter 4 provided a comparison on how basic framework structures could price CDSs. The basic Merton model and the constant intensity model have been compared in their ability to generate cross-sectional and time series out-of-sample predictions. The Support Vector Machines (SVM) method has been included in this section to look at whether the absence of financial structure affects prediction power. The results indicate that the Merton and the constant intensity models show quite close performance. The cross-sectional results demonstrated a better fit of the Merton model in one out of four cases, whereas the one-day-ahead time series analysis revealed better results for the constant intensity model in three classes. Five-day- and ten-day-ahead predictions produced mixed results, showing that one framework's prediction power does not outperform the other. This result might be partly attributable to not breaking down the default probability in the Merton model into Black/Scholes parameters. The basic comparison study can be extended by including the asset value process and the asset volatility for this version of the Merton model. This has been the most significant difference from the original model that is not used in the study.

The Support Vector Machines method has also implications on further work. Although not suitable for cross-sectional analysis, the SVM Regression method has proven to be a useful tool in time series settings. Alternative kernels should be used until the most suitable fit has been reached in a given setup. A further extension might include SVM in an advanced comparison analysis and use bond prices, stock prices and balance sheet information as inputs for training the SVM function. Alternatively, the SVM Classification method can be used for preprocessing the data, where one can classify CDS entities into risk classes.

In Chapter 5, the analysis has been taken a step further. The simplistic Merton and constant intensity models were replaced by more advanced forms of structural and reduced-form models. Inspired by the fact that leverage has been an explanatory variable for CDS levels, it has been used as a key credit factor in both frameworks. The structural Collin-Dufresne/Goldstein model, which contains a stationary leverage ratio has been compared to an analogous intensity model, in order to determine whether the model structure has an impact on prediction power. By creating this model pair, the study provides an answer to the challenging question that which type of framework better prices credit risk. Given comparable settings, they similarly price this risk.

Both models contained a 1-factor Vasicek model for the riskless interest rate process, which was calibrated by a Kalman filter. This method has proven to be a useful approach in estimating the term structure parameters. Once the process involves an affine term structure model, Kalman filters can be applied without any hesitation. Further, in the comparison of advanced model structures, this study has used exactly the same bond and stock price datasets to calibrate the models in predicting CDS prices. In this way, the CDG model, which models the evolution of the asset value, could be brought in close proximity to an intensity model which takes into account Poisson processes. The out-of-sample prediction performances by both models indicated that the intensity model is slightly better than the CDG model, possibly due to more free parameters. The Akaike Information Criterion that takes this into account has yielded quite similar results with both models. The advanced comparison study can be further extended with alternative setups. It is a task for further research to maintain even better accuracy of predictions, and to find the best performing structural and reduced-form models.

The implicit testing of credit risk models also has implications for benchmarking purposes. The models that fit the observed prices best have not only provided an accurate estimate for the fair price, but have also been used as a benchmark for market participants in their actions. For equity and FX derivatives, applying the Black/Scholes option pricing framework has been widely accepted as the benchmark model. This price may be used for looking at hedge ratios and arbitrage possibilities and treated as fundamental information. Black/Scholes prices are well comparable to actual prices in the market, and provide guidance in market actions. Although the results from this study did not point out a specific credit risk model among others, a successful model in future efforts can serve as a benchmark model in credit risk.

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Appendix A

Kalman Filter Estimation

In 1960, R.E. Kalman has published his famous article which describes a recursive solution to the discrete-data linear filtering problem (Kalman (1960)). Since then, the Kalman filter has been subject to extensive research and applications. The Kalman filter is a set of mathematical equations which enables recursive computation to estimate the state of a process. The filter has been proven to be very powerful and precise, and has been widely applied.

One of the application fields is the estimation of the unobserved short rate series. Applying the filter to affine term-structure models, numerous studies have aimed to estimate term-structure parameters. The affine term structure models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985) have received attention in application of the Kalman filter with single or multi-factor variations. These studies have mostly reached a good fit in estimating the term-structure parameters in-sample. The procedure described below consists of estimating process parameters and short rate time series of a single factor Vasicek model using the Kalman filter.

A.1 State-Space Formulation

In this section, the basic structure of a linear Kalman filter is described. Following, the application to term-structure models will be discussed.

Proposition 5.1 *Suppose that there is an n-dimensional state vector* $x_t = [x_t^1, ..., x_t^n]$ *corresponding to the number of factors and an m-dimensional vector* $z_t = [z_t^1, ..., z_t^m]$ *corresponding to the number of cross-sectional measurement observations. Then the following set of equations represent the time update equation and the measurement equation of the Kalman filter, which should have the form*

$$x_t = Ax_t + Bx_{t-1} + \omega_{t-1}, \quad \textit{time update equation} \tag{A.1}$$

$$z_t = C_t + Hx_t + \varsigma_t, \quad \textit{measurement equation} \tag{A.2}$$

The random variables ω_t *and* ς_t *represent the process and measurement noise respectively. They are assumed to be independent from each other and have normal probability distributions,*

$$\omega_t \sim N(0, Q), \tag{A.3}$$

$$\varsigma_t \sim N(0, R) \tag{A.4}$$

These equations constitute the backbone of the recursive estimation using the Kalman filter. The below propositions describe the time update and measurement update recursive equations for the filter.

Proposition 5.2: Time Update *The filter computes the best prediction for*

x_t based on information available until x_{t-1}, which is described by $\mathcal{F}_{t-1}$.

Time update projection:

$$\hat{x}_{t|t-1} = E[x_t|\mathcal{F}_{t-1}] = A + B\hat{x}_{t-1}, \tag{A.5}$$

Update error covariance:

$$\hat{P}_{t|t-1} = Var[x_t|\mathcal{F}_{t-1}] = B\hat{P}_{t-1}B' + Q_t \tag{A.6}$$

Up to now only time has been incremented but no measurement has been taken. The below proposition incorporates the effect of measurement equation to the recursive solution of the filter. First the "Kalman gain" is computed which determines the weight given to the new observation. Then new estimates of the state and error covariance are derived.

Proposition 5.3: Measurement Update *Including the contribution of the measurement, the measurement update equations are as follows:*

Computing the "Kalman gain":

$$K_t = \hat{P}_{t|t-1}H'\Big(H\hat{P}_{t|t-1}H' + R\Big)^{-1} \tag{A.7}$$

Update state after measurement z_t:

$$\hat{x}_t = E[\hat{x}_t|\mathcal{F}_t] = \hat{x}_{t|t-1} + K_t(z_t - E[z_t|\mathcal{F}_{t-1}]) \tag{A.8}$$

Update error covariance after measurement:

$$\hat{P}_t = Var[\hat{x}_t|\mathcal{F}_t] = (I - K_tH)\hat{P}_{t|t-1} \tag{A.9}$$

After the Kalman recursions, finally a maximum likelihood estimator can be used to estimate the parameters of the process desired.

Proposition 5.4: Maximum Likelihood Estimation *The log-likelihood function consists of the following elements:*

$$\varpi_t = z_t - \hat{z}_t \tag{A.10}$$

$$G_t = Cov[\varpi] = H\hat{P}_{t|t-1}H' + R \tag{A.11}$$

The log-likelihood equation:

$$logL = \sum_{t=1}^{T}\Big(\frac{N_t log(2\pi)}{2} - \frac{log|G_t|}{2} - \frac{1}{2}\varpi_t' G_t^{-1}\varpi_t\Big) \tag{A.12}$$

where T *is the full range of the time series and* $N_t = dim(\varpi_t)$*.*

In the end, the parameters that are contained in A, B, C and H are estimated through the optimization of the maximum likelihood function.

A.2 Estimation of the Short Rate Process through Kalman Filter

Given the above state-space formulation, the Kalman filter can be applied to the estimation of the short rate process time series and parameter estimation. Being a method that takes into consideration both time series and cross section of yields, Kalman filtering is an appropriate tool in estimating the unobserved short rate time series and process parameters. As described in the last section, the Kalman filter makes use of measurement and update equations in order to iteratively find the best parameter estimates. Both the CDG model and the intensity model derived contains the Vasicek process as the driving source for the short rate as below:

$$dr_t = \kappa_r(\theta_r - r_t)dt + \sigma_r dW_1^Q \tag{A.13}$$

where κ_r is the mean reversion rate, θ_r is the long-run mean, σ_r is the volatility of the short rate, and W_1^Q is a Brownian motion under the risk-neutral measure.

The unobserved series of short rate and the parameters of this Vasicek process will be estimated using the filter. For this purpose, the closed form solution of the Vasicek bond pricing formula are used as the measurement equations, whereas the transition process of the Vasicek equations are used as the time update equations. The following sections describe the data and the results of the estimation. Only the one-factor version of the Vasicek process is applied, therefore, n, the number of factors described in Proposition 5.1 is only 1. Eight yields at each time point are used as an observation for the Kalman filter, so m described in the state space formulation is 8. Moreover the length of time series, T, is 2188 days, spanning a period of over eight years.

A.3 Calibration of the Interest Rate Process

In the Vasicek setup, the closed-form bond pricing formula constitutes the measurement equation, making use of the cross-sectional information of each of the eight yields on a given day. The bond pricing formula of Vasicek is:

$$b(r_0, T) = e^{C(T) - H(T)r_0} \tag{A.14}$$

where $C(T)$ and $H(T)$ correspond to the measurement equation vectors on a given day, which are in the form:

$$H(T) = \frac{1}{\kappa_r}(1 - e^{-\kappa_r T}), \tag{A.15}$$

$$C(T) = \frac{(H(T) - T)\Big(\kappa_r^2(\tilde{\theta}_r - \frac{\sigma_r \eta}{\kappa_r}) - \frac{\sigma_r^2}{2}\Big)}{\kappa_r^2} - \frac{\sigma_r^2 H^2(T)}{4\kappa_r} \tag{A.16}$$

Here, the η is the market price of risk parameter, and the $\tilde{\theta}_r$ is the long run mean under the physical probability measure. The risk-neutral long run mean θ_r of the process is reached by:

$$\theta_r = \tilde{\theta}_r - \frac{\sigma_r \eta}{\kappa_r} \tag{A.17}$$

Moreover, the transition equations, which will comprise the time update equations, in the Vasicek setup are as follows:

$$A(t) = \tilde{\theta}_r(1 - e^{-\kappa_r \Delta t}) \tag{A.18}$$

$$B(t) = e^{-\kappa_r \Delta t} \tag{A.19}$$

$$Q(t) = \frac{\sigma_r^2}{2\kappa_r}(1 - e^{-2\kappa_r \Delta t}) \tag{A.20}$$