

First published in:

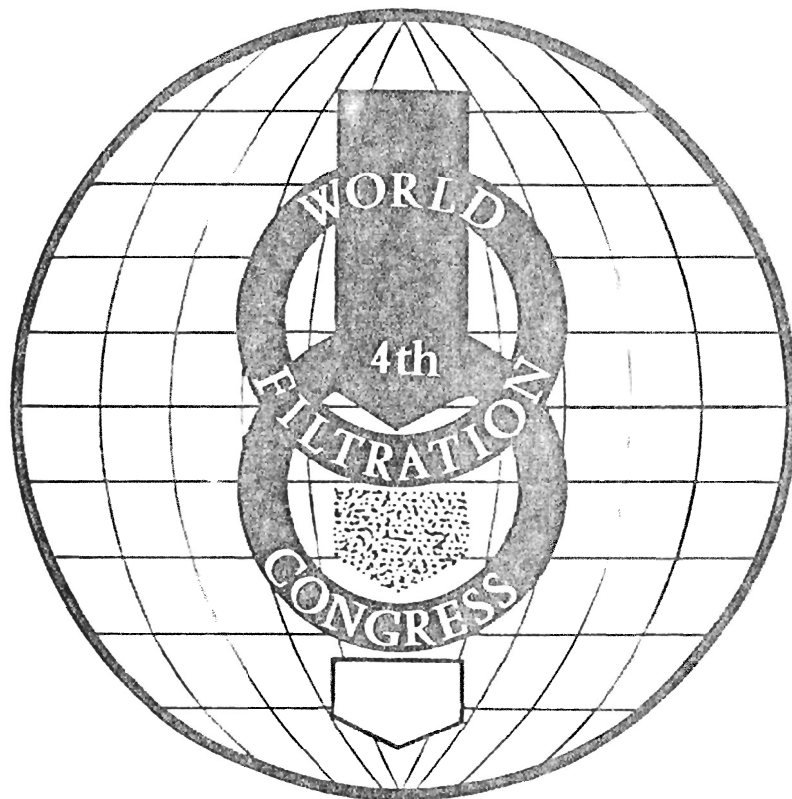
Technologisch Instituut-K.VIV  
Mechanical Separation and  
Particle Technology

Volume 3

798)

Nur zum persönlichen Gebrauch  
Vom Verfasser überreicht

## PROCEEDINGS Part II



22 - 25 April 1966  
Ostend, Belgium

Editors : R. Vanbrabant  
J. Hermia  
R.A. Weiler

The Royal Flemish Society of Engineers (K. V. V.) Antwerp, Belgium  
329th event of the European Federation of Chemical Engineers

EVA-STAR (Elektronisches Volltextarchiv – Scientific Articles Repository)  
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## ON THE DRAG EFFECT IN DECANTING CENTRIFUGES

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### ABSTRACT

On literature over separation in decanting centrifuges, calculations and declarations have been based hitherto on the application of Stokes's law. Experimentations have clearly shown that the separation can also be greatly influenced by other factors in the machine, i.e. the flushing of the already deposited particles by the current. To calculate this drag effect, new equations must be derived. The comparison with the measured values should yield good agreement.

To eliminate the probability that the separation would be determined eventually through the instability of the current, the most important criteria for stability should be represented and compared with the measured values.

### INTRODUCTION

The theory of equivalent clarification area based on Stokes's law, is still used today to explain the functions of the decanting centrifuge. Although it is well-known that this theory in many cases fails and one cannot simply use a correcting factor to account for the margin of error. Therefore, for the scale-up from the pilot experiment, it is necessary to have experience and feeling.

When one compares the actual degree of separation curve to the theoretical, then it can be seen that the degree of separation, when the critical flow is reached, abruptly slopes downwards, although theoretically it should be considerably more horizontal (Fig. 1).

The estimation is better, when a different process is overlaid at the point of the critical flow  $Q_{crit}$ . Up until this critical throughput, the separation can be described simply through the sedimentation process. Whereas for flow rates greater than the critical point, a different rule is more effective.

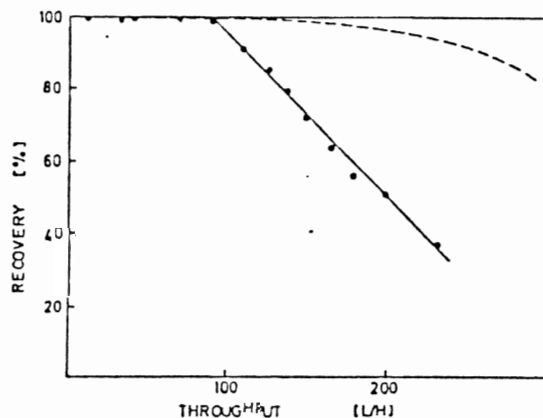


Fig. 1. The recovery as a function of the throughput

The following processes in the machine could have an influence on separation:

- the sedimentation of particles
- the particles on the canal bed can be pulled along (analogous to the sediment transport in the flowing fluids)
- the instability of the fluid
- the poor transportation of the solid material by the screw.

In the following work, the indetaking is to describe these processes and to estimate their effects on the separation in the decanting centrifuge.

### DESCRIPTION OF THE THEORY

The theory of the equivalent clarification area

The theory of equivalent clarification area and the limited precision are so often presented in literature [7-5], that the equation for calculating the limiting particle size can simply be stated as such:

$$x_G = \sqrt{\frac{9}{4\pi^3}} \cdot \sqrt{\frac{\eta}{\Delta\rho}} \cdot \sqrt{\frac{2}{n_H}} \cdot \sqrt{\frac{1}{L_e \cdot r_m^2}} \quad (1)$$

The limiting particle size becomes smaller as the effective clarification length  $L_e$  and the mean radius  $r_m$  grows larger. Other machine parameters do not influence the separation.

Whereas a few authors [3,4,5], in order to improve the agreement between theory and practice, have introduced correcting factors for:

- the slipping between the drum and the fluid
- the disturbance from the input and discharge zones
- the flow profile (no plug flow)
- the motion of the screw
- the reducing of the clarified volume by backflow of solid
- the property of the product such as the particle form and the concentration of the feed.

Faust [5] describes a new process. He applied the measured surface layer flow in the tubular centrifuge onto the decanting centrifuge. This means that there exists in the channel between the blades a fast flowing layer on the surface of the fluid, beneath which is a slow flowing zone. Through experimentation with a decanting centrifuge of perspex, it can be shown that the thickness of this surface layer is approximately equal the half of the overall fluid height. The throughput does not influence the layer thickness.

The limiting particle size can then be calculated as follows:

$$x_G = 1,7 \sqrt{\frac{n \cdot \Delta h \cdot Q}{k_w \cdot \omega_T \cdot r_m \cdot W_k \cdot \Delta \rho (\omega_H \cdot r_m \cdot i \cdot \Delta h \cdot b_k + Q)}} \quad (2)$$

#### The drag effect

Whereas one assumes, using the theory of equivalent clarification area, that the separation is stabilized through sedimentation, one must take into account that because of the flushing, the particles which have already settled can be carried away by the flow and over the weir out of the machine.

The process can be simplified as such description of this:  
Forces on a particle which has already settled (Fig. 2):

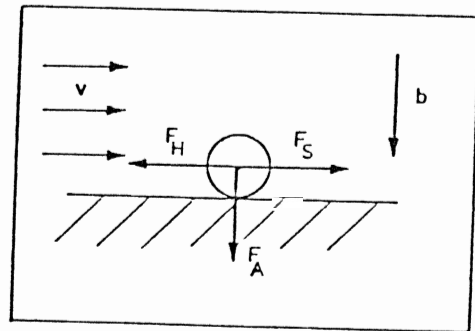


Fig. 2. Balance of forces around a particle already separated (drag force formulation)

- the contact pressure

$$F_A = \frac{\pi}{6} \cdot x_s^3 \cdot \Delta \rho \cdot b \quad (3)$$

- the drag force

$$F_S = k \frac{\pi}{4} \cdot x_s^2 \cdot \eta \cdot \kappa \quad (4)$$

- and, in analogy to Coulomb's law, the friction force

$$F_H = f \cdot F_A \quad (5)$$

From the equilibrium of forces for the horizontal direction, one obtains the following formula for the drag particle size:

$$x_s = \frac{3 \cdot \eta \cdot k}{2 \cdot f \cdot \Delta \rho \cdot b} \cdot \kappa \quad (6)$$

The shear gradient  $\kappa$  shows the changes in in velocity on the channel bed:

$$\kappa = \left( \frac{dn}{dy} \right)_{y=0} \quad (7)$$

For laminar flows it can be calculated by:

$$\kappa_l = \frac{3 \cdot Q}{b_k \cdot h_{Niv}} \quad (8)$$

and for turbulent flows

$$\kappa_t = 0,028 \cdot \frac{(b_k + 2 \cdot h_{Niv})^{0,25}}{(b_k \cdot h_{Niv})^2} \cdot \frac{Q^{1,75}}{v^{0,75}} \quad (9)$$

The drag particle size can be calculated by substituting Eq. (8) and (9) into Eq. (6).

- laminar flow

$$x_{s,l} = k_l \cdot \frac{\eta}{\Delta \rho} \cdot \frac{1}{f} \cdot \frac{Q}{b_k \cdot h_{Niv}^2 \cdot C \cdot g} \quad (10)$$

- turbulent flow

$$x_{s,t} = k_t \cdot \frac{1}{f} \cdot \frac{\rho_1 \cdot v^{0,25}}{\Delta \rho \cdot g \cdot C} \cdot \frac{(b_k + 2 \cdot h_{Niv})^{0,25}}{(b_k \cdot h_{Niv})^2} \cdot Q^{1,75} \quad (11)$$

The friction coefficient cannot be theoretically determined, so it is calculated with the help of the critical rate of mass flow.

This new model shows the following for the separation in a decanting centrifuge:

The separation can simply be calculated by the effect of sedimentation up until the critical mass flow rate has been reached. When the throughput is raised above the critical rate, the effect of the drag force of the current must be superimposed upon the effects of sedimentation. Accordingly depending upon how large the critical mass flow is (Fig. 3), the machines must be planned using the equivalent clarification area theory (1) or with the addition of the drag force taken into account.

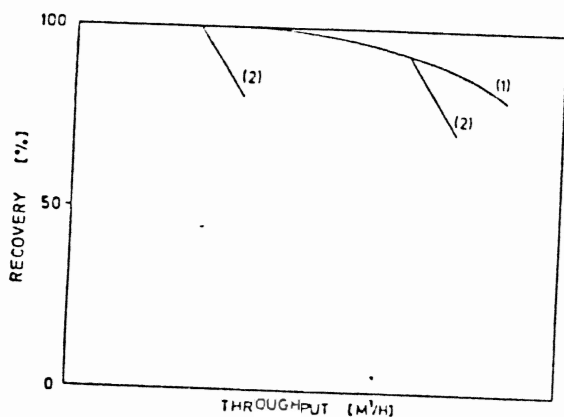


Fig. 3. The two cases: Theory of equivalent clarification areas and drag force effects

In contrast to the theory of equivalent clarification areas, only the width of the channel of the screw and the height of the fluid enter into the calculations of the drag particle size.

The measured results, however, show that these parameters have a large influence on the degree of separation.

#### Stabilization criteria

Another factor which explains the abrupt change in the behaviour of sedimentation is the instability of the current, which occurs after reaching the critical mass flow, for example, it could be a matter of the twining point between laminar and turbulent or between flowing and shooting current.

To describe this transition, the dimensionless numbers for the current flow mechanics are useful:

- the Reynolds number (laminar-turbulent)

$$Re = \frac{4 \cdot Q \cdot \rho_1}{(2 \cdot h_{Niv} + b_k) \cdot \eta} \quad (12)$$

where the critical Reynolds number is 2300.

- the Froude number (flowing-shooting)

$$Fr = \frac{Q}{b_k \cdot h_{Niv}^{1,5} \cdot r_m^{0,5} \cdot \omega_H} \quad (13)$$

The transition from flowing to shooting is where the Froude number equals 1.

This dimensionless numbers are derived from the description of current flow in the earth's gravitational field.

In a rotating system, there appears another transformation in the current which does not occur in a simple gravitational field: the building of Taylor vortices.

Taylor vortices occur when the outward directing centrifugal force can no longer compensate the pressure force. Then left and right oriented vortices begin to build. For the critical current profile, the potential eddies are stated [6,7] (Rayleigh-criteria).

The formation of Taylor eddies can be predicted by using the Taylor number:

$$Ta = \frac{4 \cdot u_{Niv}^2}{v^2} \cdot r_{Niv}^4 \cdot \frac{u_T / u_{Niv}}{(1 - \frac{u_T}{u_{Niv}})} \cdot \frac{(1 - \frac{u_T / u_{Niv}}{r_{Niv} / r_T})}{1 - \frac{r_{Niv}}{r_T}} \quad (14)$$

The critical Taylor number can be described in case of where  $r_T - r_{Niv} \ll (r_T + r_{Niv})/2$  and  $Q \ll u_T/u_{Niv} \ll 1$ :

$$Ta_c = \frac{3416}{1 + \left(\frac{u_T}{u_{Niv}}\right)} \quad (15)$$

Ludwig [8] introduced the following formula in 1964 for the stability criteria of an incompressible, frictionless current with screw-formed streamlines:

$$(1 - c_\omega) \cdot (1 - c_\omega^2) - \left(\frac{5}{3} - c_\omega\right) \cdot c_z^2 > 0 \quad (16)$$

with the dimensionless velocity gradients:

$$c_\omega = \frac{dv_\omega/dr}{v_\omega/r} \quad (17)$$

and

$$c_z = \frac{dv_z/dr}{v_\omega/r} \quad (18)$$

Through experimental results, he could provide proof for this stability criteria. By applying the Taylor number and the stability criteria from Ludwig to decanting centrifuges, it must be noted that Ludwig produced the screw-formed streamlines, not through the use of a screw, but rather the axial slinding of an inner rotor. Also, the derivation of the Taylor number for cylindrical sections does not take axial current into account.

Both criteria can be applied strictly for just one annular clearance (no free surface).

#### METHOD OF TESTING

For experimentation, there are many different sizes of test machines with different structures from which to chose (drum radius from 0.07 m to 0.7 m and clarification length from 0.15 m to 1.7 m). For an experimental material, chinese coal, PVC, PMMA, Celite, aluminium hydroxide, mud and sludge were used.

During the experimentation, the absolute rotational speed, the height of the weir and the mass flow were changed.

By regulating the mass concentrate in the centrate and in the feed, the degree of separation could be determined.

The optimal differential speeds were found through pre-tests, so that any

side effects due to the movement of the screw could be eliminated.

The precision of the measurements were confirmed by many repetitions of the same experiment.

With very fine test materials (such as chinese coal and  $Al(OH)_3$ , the disturbance of separation in the machine due to accumulating of solid materials could be precisely determined.

As long as the influence of the solid backflow could not be eliminated by the operating parameters of the machine, the results cannot be extracted for evaluation.

#### TEST RESULTS

With the aid of the different test results, the dependance of the critical throughput can be determined from the main rotational speed and the height of the weir. In table 1, the actual results are compared with theoretical ones.

The throughput is proportional to		
Stokes	*	$n_H^2$
lam. drag effect	$h_{Niv}^2$	$n_H^2$
turb. drag effect	$h_{Niv}^{1,07}$	$n_H^{1,14}$
experiments	$h_{Niv}^{0,9-1,2}$	$n_H^{1,6-2}$

Table 1. Influence on separation

Since the Stokes's law has no direct influence of pond depth, there is no dependance introduced.

Calculating with the mean radius one finds a dependance on the niveau height with the exponent -0,3.

Reasoning: As the weir height is increased, thus is the mean radius decreased whereby the centrifugal force is also reduced.

Whereas the measured dependance of the critical mass flow on the niveau height shows good agreement with the turbulent drag effect, the influence of the rate of rotation can only be approximated.

The test results do not allow for a

plain, welldefined correlation.

#### Calculating of the degree of separation process

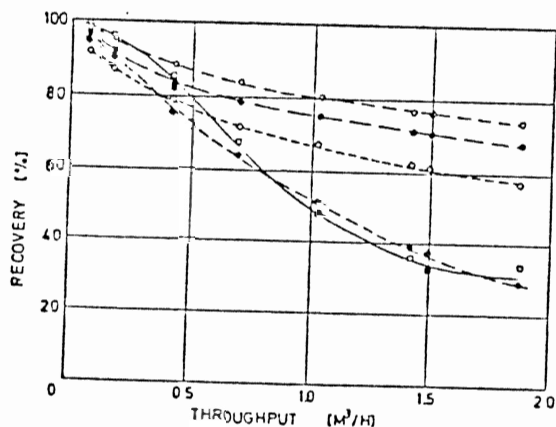
With the help of the size distribution, the separation curve and the particle size, the degree of separation can be described in dependance upon the mass flow.

$$\eta_E = \int_{x_{\min}}^{x_{\max}} T(x) dQ_A(x)$$

Through the separation degree  $T(x)$ , the following processes can be considered:

- Sedimentation: All of the particles are not fed onto the surface of the fluid, but are distributed equally throughout the entire height of the fluid. Thus particles which are smaller than the limiting particle size can still be separated.
- Drag effect: In the sediment, smaller particles are closed in by larger particles and therefore can not be flushed away. For this reason, the particles which are smaller than the drag particle size will be separated out.

In Fig. 4, the actual degree of separation is compared to the theoretical separation curve.



○ Theory of equivalent clarification area  
 ● Faust, □ lam. drag effect, ◆ turb. drag effect, □ experiment  
 Fig. 4. Calculated and measured degree of separation in dependance on the mass flow.

As it was shown earlier, the measured results cannot be described through the theory of equivalent clarification area, nor with the adaption made by Faust.

The measured values show good agreement with the drag force model for the turbulent current.

The validity of this effect is underlined through the calculation of the Re- and the Ta-number: both yield a turbulent current.

#### The stability criteria

The stability criteria which was discussed above have been made use for the different test results.

For the Reynolds-number, one receives values for the critical mass flow which are considerably larger than the critical Reynolds-number. The Reynolds-number is not constant for one product and one machine when the parameters are changed. This leads us to conclude that the abrupt sloping away of the separation curve is not due to the laminar-turbulent turning point. Because of the higher values there exists a turbulent current in the machine.

The calculated Froude-numbers are always considerably smaller than 1. Thus, a flowing current is built in the channel.

The calculating of the Taylor-number does not provide for the assumption of the producing of Taylor vortices in the decanting centrifuge. The larger Taylor-numbers support the producing of a turbulent current in the screw channel.

When one uses the stability criteria from Ludwig, than it becomes clear that the current is stabil - even under the assumption of a thin surface layer current.

In summary, the suddenly sloping separation curve is not caused by the instability of the current in the screw channel.

These ascertainments were proven through analysis results using a special test machine. This concerns a reconstructed decanting centrifuge where the current in the screw channel can be viewed during operation.

The following observations were made:

- In the screw channel, a surface layer current develops when the weir height

is very large and the mass flow is simultaneously decreased.

- Normally there exists a turbulent flow in the screw channel.
- Solid material on the channel bed is flushed away by the flowing fluids.

#### SUMMARY

Through comparison of theory with measured values and through the examination of processes in the screw channel, it can be shown that the separation in the decanting centrifuge in many cases is not limited by sedimentation effects, but rather by the carrying away of the already settled particles on the screw channel bed.

The influence of the current instability can be eliminated through the calculated dimensionless numbers.

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#### Symbols used:

$b$	centrifugal acceleration	$m/s^2$
$b_k$	channel width	$m$
$c_\varphi, c_z$	dimensionless velocity gradient	
$f$	friction factor	-
$F_A$	pressing force	$N$
$F_H$	adhesive force	$N$
$Fr$	Froude number	-
$F_S$	drag force	$N$
$k_w$	correcting factor for feed concentration	-
$L_e$	effective clarification length	$m$
$g$	gravitational acceleration	$m/s^2$
$h_{Niv}$	liquid height	$m$
$n_H$	main rotational speed	$1/min$
$Re$	Reynolds number	-
$r_T$	drum radius	$m$
$r_m$	mean radius	$m$
$Ta$	Taylor number	-
$u$	velocity of the flow	$m/s$
$u_T, u_{Niv}$	peripheral speed	$m/s$
$x_G$	limiting particle size	$m$
$x_S$	drag particle size	$m$
$\Delta\rho$	differenz between density of solids and liquid	$kg/m^3$
$\Delta h$	height of flowing layer	$m$
$\eta$	viscosity of liquid	$Ns/m^2$