A Quantum Top Inside a Bose Josephson Junction

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We consider an atomic quantum dot confined between two weakly-coupled Bose-Einstein condensates, where the dot serves as an additional tunneling channel. It is shown that the thus-embedded atomic quantum dot is a pseudospin subject to an external torque, and therefore equivalent to a quantum top. We demonstrate by numerical analysis of the time-dependent coupled evolution equations that this microscopic quantum top is very sensitive to any deviation from linear oscillatory behavior of the condensates. For sufficiently strong dot-condensate coupling, the atomic quantum dot can induce or modify the tunneling between the macroscopic condensates in the two wells.

In the field of ultracold atoms, whose most spectacular achievement on relatively large scales is Bose-Einstein condensation (BEC), not only macroscopic systems are of interest, but also to confine several or even single atoms into optically created microtraps is becoming a potentially important experimental tool of what might be coined "nanobosonics." In nanoelectronics the control of electronic quantum dots is performed by biased conducting leads, attached to it. In nanobosonics the role of the "leads" is played by finite superfluid reservoirs of given particle number, which can be coupled to a particular atom by optical transitions. Trapping and manipulating single atoms [1, 2] opens up new perspectives in the coherent control of quantum states, and is of relevance for quantum computational tasks [3].

It has recently been demonstrated by Recati *et al.* [4] that an atomic quantum dot (AQD) (a single atomic two-level system), optically coupled to a superfluid BEC bath, can be mapped onto the spin-boson model. This system then exhibits a dissipative quantum phase transition, characteristic of this model [5]. Here, we study such a spin-boson model, but with a *time-dependent* bath: An AQD located inside a Bose Josephson junction (BJJ), i.e., a single bosonic atom coupled to two superfluid reservoirs. The setup under consideration is schematically depicted in Fig. 1. The Bose-Einstein condensate is trapped by the double-well potential $V_{\rm BEC}(\boldsymbol{r})$. The atom of the dot, which is in a hyperfine state different from that of the condensate, is confined by a very tight potential $V_{\rm AQD}$, to which condensate atoms are insensitive, and which causes a large gap for double occupation of the dot. The coupling of the dot to the condensates in the wells is performed in a tunable way via a Raman transition [4]. Due to their coherent nature, the weakly-coupled condensates exhibit quantum tunneling [6]. In the present paper, we investigate the mutual influence of the induced conventional Josephson oscillations between the two wells and the AQD, which provides an additional tunneling channel. We demonstrate, by numerically solving the time-dependent coupled evolution equations of AQD and condensates, that this additional channel can in certain cases

directly affect the macroscopic Josephson tunneling.

The Hamiltonian of our system consists of three parts

$$H = H_{\rm cond} + H_{\rm dot} + H_{\rm coupl}. \tag{1}$$

We will first describe these three parts in turn. The part $H_{\rm cond}$ characterizes the double-well trapped BEC:

$$H_{\rm cond} = \int d\boldsymbol{r} \left\{ \Psi^*(\boldsymbol{r}, t) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\rm BEC}(\boldsymbol{r}) \right] \Psi(\boldsymbol{r}, t) \right. $$
$$\left. + \frac{1}{2} g |\Psi(\boldsymbol{r}, t)|^4 \right\}, \tag{2}$$

where $\Psi(\boldsymbol{r}, t)$ is the condensate wavefunction and m the atomic mass. We assume that at low energies the interparticle interaction is given by the usual pseudopotential $V(\boldsymbol{r} - \boldsymbol{r}') = g\delta(\boldsymbol{r} - \boldsymbol{r}')$, where $g = 4\pi\hbar^2 a_s/m$, and a_s is the s-wave scattering length. The condensate is described within Gross-Pitaevskiĭ theory, sufficiently accurate at the very low temperatures we are considering [7].

For the present dilute bosonic gas of finite extent, the quantum tunneling between the two wells is adequately
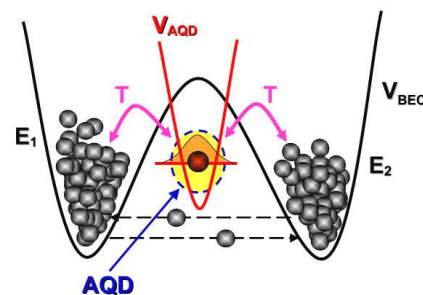

FIG. 1: [Color online] An atomic quantum dot between two weakly-coupled condensates, trapped in a double-well potential $V_{\rm BEC}$. The dot is a simple two-level system ≡ single atom present/not present and is created by the tight potential $V_{\rm AQD}$, located at the position of the top of the barrier. Atoms can be exchanged between wells either by direct tunneling (dashed arrows) or via the dot, coupled to the condensates by a transfer matrix T.

described within a two-mode approximation [8, 9]; one expands $V_{\mathrm{BEC}}(\boldsymbol{r})$ around each minimum, and introduces the local mode solution, $\phi_{1,2}(\boldsymbol{r})$, for each well separately. In first approximation the two modes can be considered to be orthogonal, $\int d\boldsymbol{r}\phi_1(\boldsymbol{r})\phi_2(\boldsymbol{r}) = 0$. The two-mode approximation then results in the following ansatz for the total condensate wavefunction ($\Psi_{1,2}(t) = \sqrt{N_{1,2}(t)}e^{i\theta_{1,2}(t)}$):

$$\Psi(\boldsymbol{r}, t) = \Psi_1(t)\phi_1(\boldsymbol{r}) + \Psi_2(t)\phi_2(\boldsymbol{r}). \qquad (3)$$

Since we are interested in tunneling events, i.e., in the time dependence of the wavefunctions $\Psi_{1,2}(t)$, it is convenient to write H_{cond} in the form

$$\begin{aligned} H_{\mathrm{cond}} &= \sum_{i=1,2} E_i^0 |\Psi_i(t)|^2 + U_i |\Psi_i(t)|^4 \\ &\quad - \kappa(\Psi_1^*(t)\Psi_2(t) + \Psi_2^*(t)\Psi_1(t)), \end{aligned} \qquad (4)$$

where $E_i^0 = \int\left[-\frac{\hbar^2}{2m}\mid\nabla\phi_i(\boldsymbol{r})\mid^2 + |\phi_i(\boldsymbol{r})|^2 V_{\mathrm{BEC}}(\boldsymbol{r})\right]d\boldsymbol{r}$ are the zero-point energies in the wells 1 and 2, respectively, the effective "on-site" interaction between the particles is given by $U_i = g\int|\phi_i(\boldsymbol{r})|^4\,d\boldsymbol{r}$, and finally $\kappa = -\int[\frac{\hbar^2}{2m}(\nabla\phi_1(\boldsymbol{r})\nabla\phi_2(\boldsymbol{r})) + \phi_1(\boldsymbol{r})V_{\mathrm{BEC}}(\boldsymbol{r})\phi_2(\boldsymbol{r})]d\boldsymbol{r}$ denotes the coupling matrix element [9].

The Hamiltonian of the dot itself is given by

$$\begin{aligned} H_{\mathrm{dot}} &= \int d\boldsymbol{r}[-\hbar\delta\hat{d}^\dagger(\boldsymbol{r}, t)\hat{d}(\boldsymbol{r}, t) \\ &\quad + \frac{U_{dd}}{2}\hat{d}^\dagger(\boldsymbol{r}, t)\hat{d}^\dagger(\boldsymbol{r}, t)\hat{d}(\boldsymbol{r}, t)\hat{d}(\boldsymbol{r}, t)]. \end{aligned} \qquad (5)$$

We assume that the dot operator factorizes according to $\hat{d}(\boldsymbol{r}, t) = \hat{d}(t)\phi_d(\boldsymbol{r})$, where $\phi_d(\boldsymbol{r})$ is the spatial wave function of the atom on the dot normalized to unity, $\int d\boldsymbol{r}|\phi_d(\boldsymbol{r})|^2 = 1$. The repulsive interaction between the dot atoms we consider to be much larger than any other energy scales in the system, $U_{dd} \to \infty$. The dot can then be described as a two-state system, the two states being that an atom is or is not trapped inside the dot. Finally, the dot interacts with the condensate as follows

$$\begin{aligned} H_{\mathrm{coupl}} &= g_{dc}\int d\boldsymbol{r}|\Psi(\boldsymbol{r}, t)|^2\hat{d}^\dagger(\boldsymbol{r}, t)\hat{d}(\boldsymbol{r}, t) \\ &\quad + \hbar\Omega\int\hat{d}\boldsymbol{r}(\Psi^*(\boldsymbol{r}, t)\hat{d}(\boldsymbol{r}, t) + \mathrm{h.c.}). \end{aligned} \qquad (6)$$

Here, g_{dc} is the dot-condensate interaction constant, and the second term describes the coupling of the condensate atoms to the lowest vibrational state in the AQD via a Raman transition with characteristic Rabi frequency Ω. Spontaneous emission is suppressed by a large detuning from the excited electronic states, which is absorbed into the effective dot energy $\hbar\delta$ [4].

To represent the evolution equations following from the Hamiltonian (1) in a physically transparent form, we introduce a new set of parameters $U_{id} = g_{dc}\int d\boldsymbol{r}|\phi_i(\boldsymbol{r})|^2|\phi_d(\boldsymbol{r})|^2$, $U_{12d} = g_{dc}\int d\boldsymbol{r}\phi_1(\boldsymbol{r})\phi_2(\boldsymbol{r})|\phi_d(\boldsymbol{r})|^2$, and $T_i = \hbar\Omega\int d\boldsymbol{r}\phi_i(\boldsymbol{r})\phi_d(\boldsymbol{r})$, with $U_{12d} = U_{12d}^*$ and $T_i = T_i^*$. In the single-occupation limit, the temporal wavefunction of the dot is just a superposition of singly and non-occupied states, $|\Psi_d(t)\rangle = \alpha_0(t)|0\rangle + \alpha_1(t)|1\rangle$, where $|\alpha_0|^2 + |\alpha_1|^2 = 1$. The dot operators then correspond to Pauli matrices: $\hat{d}(t) \to \hat{\sigma}_-(t)$ and $\hat{d}^\dagger(t) \to \hat{\sigma}_+(t)$, introducing the pseudo-spin ladder operators $\hat{\sigma}_\pm = \frac{1}{2}(\hat{\sigma}_x \pm i\hat{\sigma}_y)$ in terms of the Pauli matrices $\hat{\sigma}_{x,y,z}$. We can thus write for the coupling term

$$\begin{aligned} H_{\mathrm{coupl}} &= \sum_{i=1,2}\left[U_{id}|\Psi_i|^2 + (U_{12d}\Psi_1^*\Psi_2 + \mathrm{h.c.})\right]\frac{1 + \hat{\sigma}_z(t)}{2} \\ &\quad + \{T_i\Psi_i\hat{\sigma}_+(t) + \mathrm{h.c.}\} \end{aligned} \qquad (7)$$

One can now derive the coupled equations of motion for the condensate (3) and the spin $\boldsymbol{s}(t) = \langle\Psi_d(t)|\hat{\boldsymbol{\sigma}}|\Psi_d(t)\rangle = \langle\Psi_d|\hat{\boldsymbol{\sigma}}(t)|\Psi_d\rangle$, from the total Hamiltonian Eq. (1). The equations for the condensate are

$$\begin{aligned} i\hbar\partial_t\Psi_1 &= \left[E_1^0 + U_1 N_1(t) + U_{1d}n_d(t)\right]\Psi_1 \\ &\quad + (U_{12d}n_d(t) - \kappa)\Psi_2 + T_1 s_-, \\ i\hbar\partial_t\Psi_2 &= \left[E_2^0 + U_2 N_2(t) + U_{2d}n_d(t)\right]\Psi_2 \\ &\quad + (U_{12d}n_d(t) - \kappa)\Psi_1 + T_2 s_-, \end{aligned} \qquad (8)$$

while the dot equations are

$$\begin{aligned} i\hbar\partial_t s_- &= [-\hbar\delta + U_{1d}N_1(t) + U_{2d}N_2(t) + U_{12d}\Psi_1^*\Psi_2 \\ &\quad + U_{12d}\Psi_2^*\Psi_1]s_- - (T_1\Psi_1 + T_2\Psi_2)s_z, \\ i\hbar\partial_t s_z &= 2(T_1\Psi_1^* + T_2\Psi_2^*)s_- - 2(T_1\Psi_1 + T_2\Psi_2)s_+. \end{aligned} \qquad (9)$$

It is easily verified that the Eqs. (9) can be written in the vector form of a Bloch equation

$$\hbar\partial_t\boldsymbol{s} = \boldsymbol{\omega}(t) \times \boldsymbol{s}, \qquad (10)$$

where the time-dependent frequency vector reads

$$\boldsymbol{\omega}(t) = \begin{pmatrix} 2T_1\sqrt{N_1(t)}\cos\theta_1(t) + 2T_2\sqrt{N_2(t)}\cos\theta_2(t) \\ -2T_1\sqrt{N_1(t)}\sin\theta_1(t) - 2T_2\sqrt{N_2(t)}\sin\theta_2(t) \\ -\hbar\delta + U_{1d}N_1(t) + U_{2d}N_2(t) + \omega_{12}(t)\cos\phi(t) \end{pmatrix}. \qquad (11)$$

where $\omega_{12}(t) = 2U_{12d}\sqrt{N_1(t)N_2(t)}$ and $\phi(t) = \theta_2(t) - \theta_1(t)$. It follows that the AQD inside the BJJ is equivalent to a quantum top. In the case of time-independent $\boldsymbol{\omega}$, Eq. (10) can be solved analytically. The presence of Josephson tunneling between the condensates however generally results in a time-dependent $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$, and the equations need to be solved numerically [10].

For simplicity, in what follows we consider the case of a fully symmetric system: $E_1^0 = E_2^0 \equiv 0$, $U_1 = U_2 \equiv U$, $U_{1d} = U_{2d}$, $T_1 = T_2 \equiv T$, $U_{12d} = U_{21d}$. In order to compare our results with previous work on BJJ [9], we introduce dimensionless parameters: $t \to 2\kappa t$, $\Lambda = UN_0/\kappa$, where $N_0 = N_1(0) + N_2(0)$ is the initial total number of particles in the condensates; note that the *conserved*
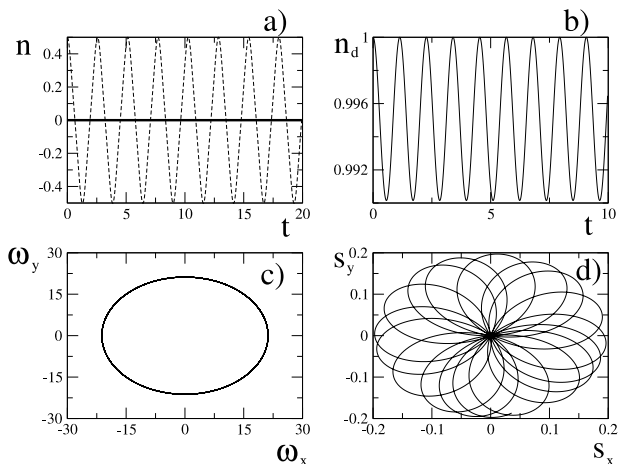
FIG. 2: Results for weak coupling $T_{\rm rel} = 0.01$ ($N_0 = 1500$ throughout Figs. 2–4, as used in experiment [13]) and $\Lambda = 10$. $n(0) = 0$ (Fig.2a – solid line), $n(0) = 0.5$ (Fig. 2a – dashed line). Fig. 2b displays the dot occupation for $n(0) = 0$; Fig. 2c shows the precessional behaviour of ω (in units of 2κ), and Fig. 2d the corresponding spin nutation for $n(0) = 0$.
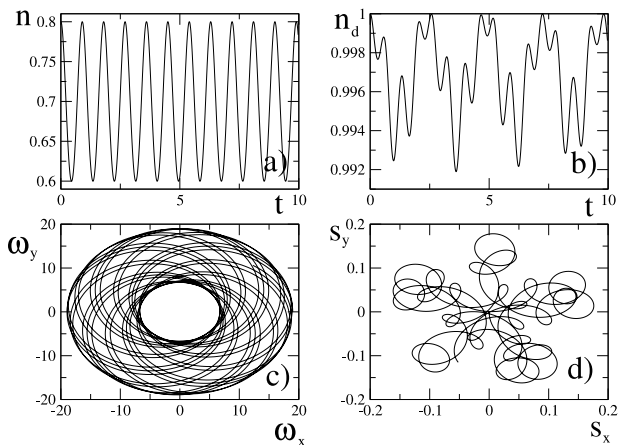

FIG. 3: Condensate in the self-trapped MST state, $n(0) = 0.8$, $\Lambda = 10$, $T_{\rm rel} = 0.01$. Relative population oscillations are shown in Fig. 3a, and the dot occupancy in Fig. 3b. In Fig. 3c, we display the projection of the frequency ω (10) on the $x-y$ plane in units of 2κ, and in Fig. 3d the pseudospin.

quantity is $N_{\rm tot} = N_1(t) + N_2(t) + n_d(t)$. For numerical convenience, we fix the dot energy at $\hbar\delta = 2\kappa$. In addition, the interactions between AQD and condensate $U_{1d}/(2\kappa)$ and $U_{12d}/(2\kappa)$ are assumed to be vanishingly small ($U_{1d} \ll U$, $U_{12d} \ll U$), and $\omega_z \simeq -\hbar\delta = {\rm const}$. Our main parameters are then the dimensionless strength of coupling of dot to condensate $T_{\rm rel} = T/\kappa$, quantifying the relative importance of tunneling channels via dot and directly by conventional Josephson tunneling, respectively; and Λ, measuring the relative importance of mean-field interaction in and tunneling between the wells.

In the following, we present results for the fractional

population imbalance

$$n(t) = \frac{N_1(t) - N_2(t)}{N_0}, \qquad (12)$$

the occupation of the dot $n_d(t)$ and the trajectories of the pseudospin s on the Bloch sphere, as well as the projection of the frequency-vector ω on the $x-y$ plane.

We first consider the situation when the dot does not have a notable effect on the tunneling between the wells (Figs. 2 and 3); we fix $\Lambda = 10$, and only change the initial condition for the particle imbalance, $n(0)$. The most simple situation is the stationary one of an initial population imbalance $n(0) = 0$ and initial phase difference $\phi(0) = 0$ (for definiteness in all figures $n_d(0) = 1$, i.e., there is initially exactly one atom in the dot). These conditions result in an AQD coupled to a time-independent BEC [4], i.e., to the problem of a spin in a constant magnetic field, however without dissipation. The pseudospin generally undergoes nutation (also if we put $n(0) = 0.5$ – Fig. 2a – dashed line), as shown in Fig. 2d, while the vector ω precesses, Fig. 2c. However, there is an exception to this general behavior: For $\Lambda = 1$, $n(0) = 0$ there occurs a simple precession of the pseudospin [12] (not shown), while the occupation of the dot exhibits linear oscillations. For $\Lambda \neq 1$ the precession is lost, an effect due to the finite number of particles in the system.

The fact that $N_{\rm tot}$ is a finite quantity constitutes one major difference to the system of a single spin coupled to superconducting leads considered in [11]. Furthermore, while deviation from simple precessional behavior also occurs in that system, the effective fractional population imbalance $n(t)$ is essentially zero. Regimes related to large $n(t)$ of order unity, to be discussed below, are thus not accessible for the superconducting Josephson junction – single spin system. In addition, the tunneling (quasi-)particles are treated as noninteracting in the latter case. Here, by contrast, including interactions between the fundamental bosons is crucial. In particular, as a consequence of interactions, and as discussed in detail in [8, 9], depending on Λ and the initial conditions, a condensate in a double-well potential can exhibit a novel quantum state – macroscopic self-trapping (MST), successfully observed experimentally [13]. MST is only present for the *self-interacting* matter waves, and is characterized by a nonzero time average of the population imbalance $n(t)$. The transition to the MST state is a gradual crossover, and we observe that our quantum top is very sensitive to this crossover. In the plots of Fig. 2, far away from the self-trapped state, the coupling strength $T_{\rm rel}$ does not influence in a qualitative way the behavior of the quantum top. The pseudospin behavior however drastically changes as we approach MST It appears that the AQD is sensitive to the deviation from linear oscillatory behavior of the condensates occurring in this regime. The linear oscillation of the dot occupation is then destroyed (Fig. 3b), and the pseudospin
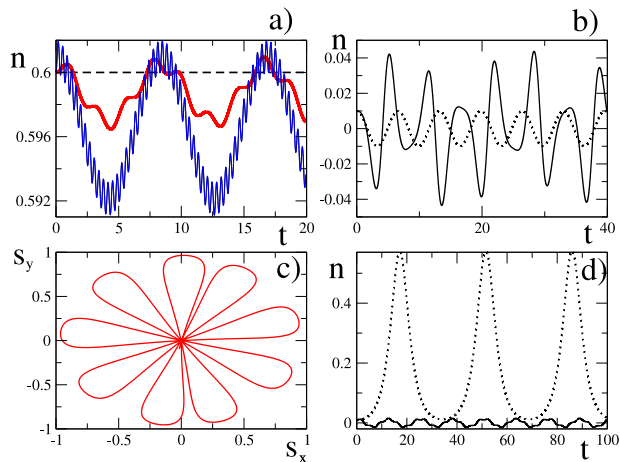
FIG. 4: [Color online] Results for a π-junction. In Fig.4a, self-trapped MST state with $n(0) = 0.6$, $\Lambda = 1.25$; decoupled dot, $T_{\rm rel} = 0$ (black dashed line); $T_{\rm rel} = 0.1$ (thick red line), $T_{\rm rel} = 1$ (thin wavy blue line); the pseudospin then nutates, Fig. 4c ($T_{\rm rel} = 0.1$). The AQD induces strong modifications both for small oscillations between wells and in the MST state, Figs. 4b and 4d; $n(0) = 0.01$, and $T_{\rm rel} = 0$ (dotted curve), $T_{\rm rel} = 1$ (solid curve). Weak interaction coupling, $\Lambda = 0.1$ in Fig. 4b, strongly coupled MST state, $\Lambda = 1.1$, in Fig. 4d.

undergoes multiple-frequency rotations (Figs. 3c and 3d). In the MST state, the pseudospin can thus behave in a rather irregular manner already for small values of the relative coupling $T_{\rm rel}$.

There is another potentially interesting regime, which occurs when the effect of the dot on the tunneling between the wells becomes significant (Fig. 4), i.e., with increasing value of the coupling to the wells $T_{\rm rel}$. Consider, for instance, a π-junction [9], $\phi(0) = \pi$, $n(0) = 0.6$, $\Lambda = 1.25$ (Fig. 4a, dashed line). The coupling to the dot leads to small oscillations between the wells (results for different $T_{\rm rel}$ are shown in Fig. 4a), and the pseudospin undergoes nutation, as apparent from Fig. 4c. For weak interactions (in the so-called Rabi regime [6, 9]) and very small particle imbalance, the effect of the dot becomes more pronounced (Fig. 4b). When the coupling to the dot is very weak, we observe the nutation of s and precession of ω. Increasing $T_{\rm rel}$ leads to significant modifications of the tunneling picture (Fig. 4b, solid line), with strongly non-sinusoidal oscillations of the population imbalance. Finally, we observe that, changing $T_{\rm rel}$ from small to large values, the dot can switch the BJJ from the MST state to a small population imbalance state (Fig. 4d).

In conclusion, we have shown that two weakly-coupled condensates, with an AQD situated at the location of the top of the barrier between them, can exhibit several regimes of oscillatory behavior. The AQD behaves as a quantum top whose behavior is very sensitive to the tunneling mode between the condensates. Even for small couplings and stationary condensates the "spin" of the dot nutates, an effect due to the finite number of parti-

cles in the system, which vanishes for an infinite system. Nutation is a characteristic feature of the quantum top in regimes far away from the MST state. Conversely, moving towards the self-trapped regime, we obtain strong deviations from nutational behavior, and the quantum top motion becomes strongly irregular. However, when the AQD itself modifies in a significant way the oscillations between the wells, nutation can emerge also in a MST state. Finally, the dot can act as a switch for the BJJ from MST to small population imbalance oscillations.

We treated the condensate on a mean-field level. In future studies, it would be of interest to study the influence of condensate quantum fluctuations on the AQD [14], in the limiting case that the dot provides the dominant tunneling channel between the condensates.

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[12] The term nutation has its standard meaning of top theory in the present paper, i.e., nutation is generally understood to be the (quasi-)periodic "wobble" motion of a spinning top, as opposed to a simple precession.

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