# Coherent single atom shuttle between two Bose-Einstein condensates

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We study an atomic quantum dot representing a single hyperfine "impurity" atom which is coherently coupled to two well-separated Bose-Einstein condensates, in the limit when the coupling between the dot and the condensates dominates the inter-condensate tunneling coupling. It is demonstrated that the quantum dot by itself can induce large-amplitude Josephson-like oscillations of the particle imbalance between the condensates, which display a two-frequency behavior. For noninteracting condensates, we provide an approximate solution to the coupled nonlinear equations of motion which allows us to obtain these two frequencies analytically.

When two phase-coherent quantum systems are brought close together, but are still separated by a tunneling barrier, particle current oscillations are induced. The exploration of the Josephson effect, predicted in 1962 [1], was for many years limited to superconducting materials, until in 1997 quantum oscillations through an array of weak links in superfluid $^3$He-B were observed [2].

Dilute atomic gases, which upon lowering the temperature to the sub-$\mu$K-range acquire phase coherence, allow to study macroscopic coherence effects in a highly controlled manner, with essentially single-atom accuracy. Calculations based on the two-mode description [3] of two weakly coupled condensates [4, 5] have exemplified the rich dynamics of particle imbalance oscillations inherent to Josephson junctions between BECs. In particular, upon increasing the particle interaction, two weakly-coupled condensates enter the novel collective state of macroscopic self-trapping, which is due to the self-interaction of the tunneling particles [4, 6], and has been confirmed experimentally [7, 8].

In the following, we explore the indirect particle exchange between two condensates, mediated by a single "impurity" atom coherently coupled to the two condensates, which is located in a tight trapping potential at the position of the barrier between the condensates. The conventional tunneling channel we assume to be strongly suppressed by raising the barrier between the two condensates. The dynamics of the type of impurity we consider – an atomic quantum dot (AQD) – when coupled by laser transitions to a single infinite superfluid reservoir, was studied in [9]. When the AQD is coupled to two well-separated condensates, it was demonstrated in [10] that when the dot-condensate coupling is smaller than or comparable to the intercondensate tunneling coupling, the effect of AQD on the particle oscillations between the condensates is negligibly small. The specific question we address in the present work is if the single impurity AQD can act as a *coherent* "shuttle" between the essentially isolated BEC reservoirs when particle transfer between wells is possible only via the coupling of the dot to the condensates: We term this the "strong coupling" limit. We find unexpected behaviour in this strong coupling limit; namely, apart from expected small and rapid oscillations of the particle imbalance due to single particles going to and fro between dot and condensates, large-amplitude Josephson-like oscillations between the condensates, mediated by the dot, occur at a smaller frequency.

Our model system consists of two condensates with a large (single-particle) potential barrier erected between them, and a single impurity atom, coupled to the condensates by a two-photon Raman transition. Assuming a symmetric double-well trap, the system is described by the following Hamiltonian,

$$\begin{aligned}\hat{H} &= U\left[|\Psi_1(t)|^4+|\Psi_2(t)|^4\right]-\kappa\left[\Psi_1^*(t)\Psi_2(t)+\text{h.c.}\right]\\ &\quad +T\sum_{i=1,2}\left\{\Psi_i(t)\hat{\sigma}_+ +\text{h.c.}\right\}-\hbar\delta\frac{1+\hat{\sigma}_z}{2}. \qquad (1)\end{aligned}$$

The above Hamiltonian is valid within the two-mode approximation [3] for the total condensate wave function, $\Psi(\boldsymbol{r},t)=\Psi_1(t)\phi_1(\boldsymbol{r})+\Psi_2(t)\phi_2(\boldsymbol{r})$, where the single-particle wave functions $\phi_{1,2}(\boldsymbol{r})$ (normalized to unity) describe the particles localized in their respective wells, and $\Psi_{1,2}(t)$ are time-dependent amplitudes representing the tunneling process. The pseudospin (equivalent to the two-level system represented by the dot) is defined by the Pauli matrix vector $\hat{\boldsymbol{\sigma}}(t)=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$, and $\hat{\sigma}_+=\frac{1}{2}(\hat{\sigma}_x+i\hat{\sigma}_y)=\hat{\sigma}_-^\dagger$ is a spin ladder operator. The "on-site" interaction between the particles is given by $U_i=g\int d\boldsymbol{r}\,|\phi_i(\boldsymbol{r})|^4$, $\kappa=-\int d\boldsymbol{r}\,[\frac{\hbar^2}{2m}(\nabla\phi_1(\boldsymbol{r})\nabla\phi_2(\boldsymbol{r}))+\phi_1(\boldsymbol{r})V_{\rm BEC}(\boldsymbol{r})\phi_2(\boldsymbol{r})]$ denotes the positive tunneling coupling [4], and $\delta$ is the detuning from the two-photon Raman transition coupling a single atom into the dot. The corresponding coupling parameter (transfer matrix element) is $T=\hbar\Omega_R\int d\boldsymbol{r}\,\phi_1(\boldsymbol{r})\phi_d(\boldsymbol{r})=\hbar\Omega_R\int d\boldsymbol{r}\,\phi_2(\boldsymbol{r})\phi_d(\boldsymbol{r})$, where the spatial wave function of the dot $\phi_d(\boldsymbol{r})$ is normalized to unity, and $\Omega_R$ is the Rabi frequency of the two-photon Raman transition. Note that while the overlap integrals of dot-condensate and condensate-condensate wave functions are both small and comparable in order of magnitude, the strong coupling limit of $T/\kappa \to \infty$ can be achieved by sufficiently increasing the Rabi frequency $\Omega_R$, i.e., well above the BEC single-particle energies in the overlap region, which enter the tunneling coupling $\kappa$.

From the Hamiltonian (1), we derive the coupled equations of motion for the condensate variables $\Psi_{1,2}$ and the pseudospin vector $\boldsymbol{s} = \langle \Psi_d(t)|\hat{\boldsymbol{\sigma}}|\Psi_d(t)\rangle$, respectively $s_\pm, s_z$, of the AQD; here, $|\Psi_d(t)\rangle$ is the temporal dot wave function. The condensate equations read ($\hbar \equiv 1$)

$$\begin{aligned} i\partial_t \Psi_1 &= U|\Psi_1|^2\Psi_1 - \kappa\Psi_2 + Ts_-, \\ i\partial_t \Psi_2 &= U|\Psi_2|^2\Psi_2 - \kappa\Psi_1 + Ts_-, \end{aligned} \tag{2}$$

while the equations of motion for the pseudospin are

$$\begin{aligned} i\partial_t s_- &= -\delta s_- - T(\Psi_1 + \Psi_2)s_z, \\ i\partial_t s_z &= -2T(\Psi_1^* + \Psi_2^*)s_- + 2T(\Psi_1 + \Psi_2)s_+. \end{aligned} \tag{3}$$

We now scale time with $T^{-1}$, and introduce the following set of dimensionless control parameters

$$\alpha = \frac{UN_0}{T}, \quad \beta = \frac{\delta}{T}, \quad \Gamma = \frac{\kappa}{T}, \tag{4}$$

where $N_0 = N_1(0) + N_2(0) = |\Psi_1(0)|^2 + |\Psi_2(0)|^2$ is the sum of the initial number of particles in the left and right wells, respectively; we also employ the scaling $\Psi_i \to \Psi_i/\sqrt{N_0}$. Decomposing $\Psi_i$ and $s_\pm$ into their real and imaginary parts, we then obtain seven equations for the coupled motion of the two condensates and the AQD pseudospin, which we solve numerically.

We first concentrate on the strong coupling case $\Gamma \to 0$. Our major result is that large amplitude Josephson-like oscillations of the particle imbalance $n(t) = (N_1(t) - N_2(t))/N_0$, with amplitude $n(0)$, can be induced by the quantum dot, which can coherently transfer atoms one by one from left to right and vice versa even when conventional tunneling is completely switched off (Fig.1 (a), black curve). With increasing $\alpha$, macroscopic self-trapping, defined by an average $\langle n(t)\rangle \neq 0$, occurs (Fig. 1 (b), black curve). Thus, there is a critical value $\alpha_c$, depending on $\beta$, such that for $\alpha > \alpha_c$ particle imbalance oscillations are self-trapped and for $\alpha < \alpha_c$, Josephson-like oscillations of the particle imbalance with $\langle n(t)\rangle = 0$ occur. We have studied in detail the dependence of $\alpha_c$ on $\beta$ and present the results for 0- and $\pi-$junctions in the phase diagram Fig. 2. We observe that, for small $\beta$, $\alpha_c$ increases linearly in $\beta$. For large $\beta$, $\alpha_c \sim 1/\beta$, implying that the critical interaction $U_c \propto T^2/\delta$ in this limit.

We stress that the self-trapping crossover obtained here is very different from the well-known one [4], as it occurs also if ordinary tunneling is blocked. In the conventional self-trapping scenario, the latter fact would imply (trivially) that the system is always self-trapped, with zero oscillation amplitude. Here, by contrast, the coupling to the AQD can induce Josephson-like oscillations for sufficiently small $\alpha$, cf. Fig. 1. To further emphasize the difference to the conventional scenario, varying the particle number, we find that for large $\beta$ the critical $\alpha_c$ becomes essentially independent of $N_0$, while for small $\beta$, $\alpha_c$ *decreases* approximately linearly in $N_0$ with increasing $N_0$ [keeping $n(0)$ fixed], which is opposite to what one would expect for macroscopic self-trapping driven by the total interaction energy.

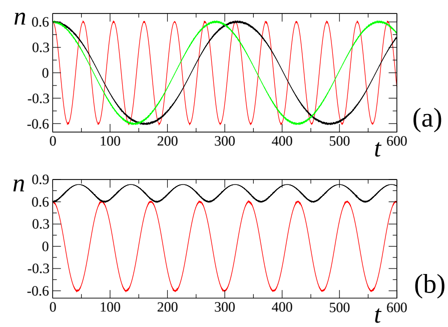


FIG. 1: (Color online) Large amplitude Josephson-like oscillations around zero particle imbalance, $n = 0$, for $N_0 = 1000$, $\beta = 10$, initial $n(0) = 0.6$ and initial phase difference $\phi(0) = \pi$ for $\alpha = 0.01$ in (a) and $\alpha = 0.1$ in (b) [where crossover to a self-trapped state has occured for $\Gamma = 0$]. Black solid curve: $\Gamma = 0$, red (thin grey) curve: $\Gamma = 0.05$. In (a), we display in addition the noninteracting case ($\alpha = 0$) with $\Gamma = 0$ in green (thick grey). We assume throughout our calculations that there is initially one particle in the dot, $s_z(0) = 1$ and that the initial particle imbalance $n(0) = 0.6$; time is in units of $T^{-1}$.

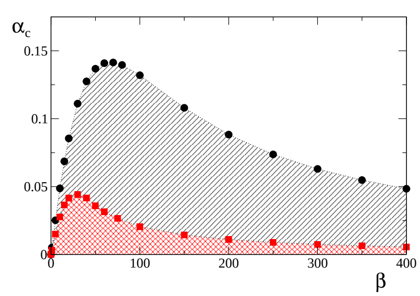


FIG. 2: (Color online) The critical value of the scaled interaction $\alpha = \alpha_c$ for self-trapping of the effective Josephson oscillations, as a function of the scaled detuning $\beta$, for a $\phi(0) = 0-$junction (black dots) and $\phi(0) = \pi-$junction (red squares), with $N_0 = 1000, \Gamma = 0, n(0) = 0.6$. Within the shaded areas, the system shows Josephson-like oscillations, while inside the white area, it is self-trapped.

We observe that the dimensionless control parameter for the occurrence of the Josephson-like oscillations is the

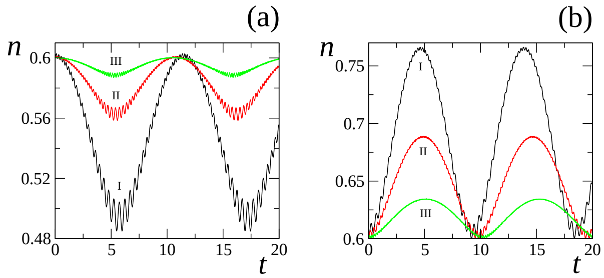


FIG. 3: (Color online) Particle number dependence of induced oscillations for vanishing conventional tunneling rate, $\Gamma = 0$, for $N_0 = 100$ (black, I), $N_0 = 300$ (red, II), and $N_0 = 1000$ (green, III). Parameters are $\beta = 10$, $\alpha = 1$, The initial phase difference is $\phi(0) = 0$ in (a) and $\phi(0) = \pi$ in (b); time is again in units of $T^{-1}$.

ratio of the dot energy over the self-interaction energy of the condensates: $\beta/\alpha = \delta/UN_0$. Given that the typical mean-field energy $UN_0 \sim$10 nK for particle numbers $N_0 \sim 1000$, for $\beta/\alpha = 10^1 \cdots 10^4$, the necessary detuning is of order $\delta = \frac{\beta}{\alpha} N_0\,[\mathrm{Hz}]$, which is experimentally feasible.

If we further increase $\alpha$ (decrease $\beta/\alpha$), the amplitude of the self-trapped oscillations becomes very small; Fig. 3 (green curves). However, the magnitude of oscillations again increases when the number of particles $N_0$ is decreased. In this regime of smaller particle numbers, one can clearly distinguish a two-frequency behavior of the dot-induced oscillations (which is not discernible in Fig. 1). In order to understand the origin of these two frequencies, we provide below an analytical derivation of the oscillations in the limit of noninteracting condensates.

We have also analyzed the case of a finite, but still small $\Gamma$ (Fig. 1, red curves). The small rapid oscillations on top of the envelope of the oscillations rapidly vanish if one increases $\Gamma$ from zero, and thus occur in the strong coupling limit only. Furthermore, we observe from Fig. 1 that the change in the effective tunneling oscillation frequency, increasing the tunneling coupling $\Gamma$, strongly depends on the value of $\alpha$. If we are in a state of ordinary oscillations around zero population imbalance, Fig. 1(a), the oscillation frequency changes strongly; conversely, in the self-trapped state, we have nearly no change in oscillation frequency. Furthermore, from Fig. 1(b), we conclude that by a slight increase of $\Gamma$, we may switch the system from a self-trapped state to one with Josephson-like oscillations.

For noninteracting condensates [11], $\alpha = U = 0$, an analytical approximation to the coupled equations of motion is possible, from which we are able to reproduce the numerically established dot-induced oscillations in that limit. Defining the new variables $\psi = \Psi_1 + \Psi_2$ and $\bar{\psi} = \Psi_1 - \Psi_2$, we find immediately from Eqs. (2) and (3), that $\bar{\psi}$ decouples according to $i\partial_t\bar{\psi} = \kappa\bar{\psi}$. Hence, $\bar{\psi}(t) = \bar{\psi}(0)e^{-i\kappa t}$, and we are left with the equations

$$\begin{aligned} i\partial_t\psi &= -\kappa\psi + 2Ts_-, \qquad i\partial_t s_- = -\delta s_- - T\psi s_z, \\ \partial_t s_z &= -4T\,\mathrm{Im}\,[\psi^* s_-]. \end{aligned} \tag{5}$$

These yield directly $\partial_t(|\psi|^2 + s_z) = 0$, so that

$$|\psi|^2 + s_z = C = |\psi(0)|^2 + s_z(0) \tag{6}$$

is a conserved quantity. Next, we write $\psi = |\psi|e^{i\varphi}$ and $s_- = |s_-|e^{i\lambda}$ and utilize that the variables $|\psi|, |s_-|, s_z$ are directly connected to each other via the conservation of both $|\psi|^2 + s_z$ and $4|s_-|^2 + s_z^2$. We get

$$\partial_t|\psi| = -2T|s_-|\sin(\varphi - \lambda), \tag{7}$$

$$\partial_t\varphi = \kappa - 2T\frac{|s_-|}{|\psi|}\cos(\varphi - \lambda), \tag{8}$$

$$\partial_t\lambda = \delta + T\frac{|\psi|s_z}{|s_-|}\cos(\varphi - \lambda). \tag{9}$$

Up to now we have treated Eqs. (2) and (3) without any approximations. In the following, we consider the limit $|s_-|^2/|\psi|^2 \ll 1$, which is natural given that we assume the condensates to be sufficiently large for the Gross-Pitaevskiĭ description to apply. Furthermore, we assume that $s_z$ oscillates around an average value $\bar{s}_z$, so that the average value of $|\psi|^2$ is $C - \bar{s}_z$, accordingly. Similarly, we denote the average value of $\partial_t\varphi$ by $\bar{\omega}$. Differentiating $|\psi|^2$ twice we obtain

$$\frac{\partial_t^2|\psi|^2}{|\psi|^2} = 8T^2\frac{|s_-|^2}{|\psi|^2} + 4T^2 s_z + 2(\partial_t\varphi - \kappa)(\kappa - \delta). \tag{10}$$

Neglecting the first term on the right hand side and replacing $|\psi|^2$ and $\partial_t\varphi$ by their averaged values, we get

$$\partial_t^2|\psi|^2 \approx 4T^2(C - |\psi|^2)(C - \bar{s}_z) + 2(\bar{\omega} - \kappa)(\kappa - \delta)|\psi|^2, \tag{11}$$

which results in the analytical solutions

$$\begin{aligned} |\psi|^2 &= C - \bar{s}_z - A_0\cos[2T\sqrt{C}(t - t_0)], \\ s_z &= \bar{s}_z + A_0\cos[2T\sqrt{C}(t - t_0)], \end{aligned} \tag{12}$$

and the self-consistency condition

$$-2T^2\bar{s}_z = (\bar{\omega} - \kappa)(\kappa - \delta). \tag{13}$$

The quantities $|\psi|^2$ and $s_z$ therefore oscillate around their average values with the frequency $\omega_1 = 2T\sqrt{C}$. For the remaining determination of the constants $A_0, t_0, \bar{s}_z$ and the average frequency $\bar{\omega}$, we use from now on the simplifying assumption of $\psi(0) > 0$, i.e. $\varphi(0) = 0$, and $s_-(0)$ being real. In that case, the amplitude is given by $A_0 = s_z(0) - \bar{s}_z$, whereas $t_0 = 0$. Moreover, a good approximation for $\bar{\omega}$ is then given as the average of two extrema of $\partial_t\phi$ in Eq. (8) according to $\bar{\omega} = \kappa - \mathrm{sgn}[s_-(0)]T\left(\frac{|s_-(0)|}{|\psi(0)|} + \frac{|s_-(\pi/2T\sqrt{C})|}{|\psi(\pi/2T\sqrt{C})|}\right)$. After a lengthy, but straightforward calculation we finally obtain

$$\bar{\omega} = \kappa - 2T^2\frac{(\kappa - \delta)s_z(0) + 2\omega_1 s_-(0)}{(\kappa - \delta)^2 + \omega_1^2}, \tag{14}$$

which also directly determines $\bar{s}_z$ via Eq. (13). The final result can then be written as

$$\psi(t) = \sqrt{C - \bar{s}_z - A_0 \cos(2T\sqrt{C}t)}\, e^{i\bar{\omega}t},$$
$$\bar{\psi}(t) = \bar{\psi}(0)e^{-i\kappa t}, \quad \text{where} \quad A_0 = s_z(0) - \bar{s}_z. \quad (15)$$

Using that $\mathrm{Re}[\psi\bar{\psi}^*] = |\psi_1|^2 - |\psi_2|^2$, the particle imbalance oscillates according to

$$n(t) = \frac{\sqrt{C - \bar{s}_z - A_0\cos(\omega_1 t)}}{N_0} \mathrm{Re}\,[e^{i\Omega t}\bar{\psi}^*(0)]. \quad (16)$$

It executes small oscillations of the amplitude at the frequency $\omega_1 = 2T\sqrt{C}$, cf. Fig. 3, while the major oscillations of the envelope are given by

$$\Omega = 2\kappa - 2T^2 \frac{(\kappa - \delta)s_z(0) + 2\omega_1 s_-(0)}{(\kappa-\delta)^2 + \omega_1^2}. \quad (17)$$

When $T \to 0$, we have simply conventional Josephson oscillations at the frequency $2\kappa$. Conversely, in the strong coupling limit, and with one atom initially in the dot, the envelope frequency $\Omega$ depends in a simple way on $\beta$: $\frac{\Omega}{T} = \frac{2\beta}{\beta^2+4C}$, with a maximum at $\beta = 2\sqrt{C}$. Comparing the frequency $\Omega$ with our numerical results shows excellent agreement. For the set of parameters used in Fig. 1 (a), for example, the analytically obtained periods are $284\,T^{-1}$ and $52\,T^{-1}$ for $\Gamma = 0$ and $\Gamma = 0.05$, respectively. The first period corresponds exactly to the green curve. Comparing the latter with the red curve shows that the agreement remains very good even for small interactions (finite $\alpha$), provided $\Gamma$ is also finite.

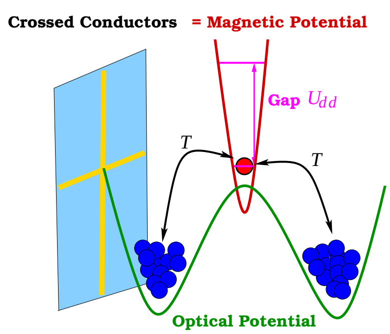


FIG. 4: (Color online) Possible experimental setup for an atomic quantum dot coupled to BECs on a microchip (light blue). A standing laser wave (green) creates deep potential wells perpendicular to the chip surface, equivalent to an optical potential for all hyperfine states of the atoms. Two crossed conductors on the chip (yellow) create a tight magnetic potential for one hyperfine species (red) causing a large gap $U_{dd}$ for double occupation of the dot. An additional laser (not shown) couples the impurity atom (red) and the condensates (dark blue), resulting in the transfer coupling $T$.

We finally discuss a possible experimental realization of our theoretical proposal. For that purpose, we suggest a combined microtrap–standing optical wave setup, in which a standing laser beam and crossed conductors on a microchip create the required trapping architecture [12, 13], see Fig. 4. We note that all atoms feel the optical potential used in this setup. However, the magnetic potential acting on one particular hyperfine state dominates the optical potential at the AQD location, provided that the "contact interaction gap" $U_{dd}$ (the energy barrier for double occupation of the dot) dominates the other relevant energy scales, $U_{dd} \gg T, UN_0$.

In conclusion, we have shown that an atomic quantum dot can act as a coherent "shuttle" between two essentially isolated BECs, transferring atoms from left to right and vice versa such that Josephson-like oscillations are established. Using an analytical approximation in the noninteracting limit, we obtained explicit expressions for the two frequencies characterizing the dot-induced oscillations. We have numerically established a phase diagram for non-self-trapped and self-trapped phases of the system, and analyzed experimental feasibility. An extension of the present work is to consider arrays of atomic quantum dots in optical lattices, where phenomena like self-trapping [14] and the influence of the array on the Mott insulating state can be studied.

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