DEPHASING DUE TO NONSTATIONARY $1/f$ NOISE

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Motivated by recent experiments with Josephson qubits we propose a new phenomenological model for $1/f^\mu$ noise due to collective excitations of interacting defects in the qubit's environment. At very low temperatures the effective dynamics of these collective modes are very slow leading to pronounced non-Gaussian features and nonstationarity of the noise. We analyze the influence of this noise on the dynamics of a qubit in various regimes and at different operation points. Remarkable predictions are absolute time dependences of a critical coupling and of dephasing in the strong coupling regime.

Despite a large number of investigations, the detailed explanation of the origin of $1/f$ noise stays an open problem in condensed matter physics. The diversity of systems that exhibit $1/f$ fluctuations suggests that its physical origin is not universal.[1] Still, because of its ubiquity, the study of underlying mechanisms, relevant at least in some cases, is interesting.

In the context of quantum-information processing[2] the field has attracted new attention. In particular in solid state devices, e.g. Josephson qubits, low frequency noise, typically with a $1/f$ power spectrum, is considered as the most important limitation for the preservation of the phase coherence. The microscopic origin of this noise depends thereby on the considered system. In the case of charge noise, for instance, it is believed to be due to the activity of random traps for charges in the dielectric substrate and in the oxide barrier of the Josephson junction.[3] Similarly, neutral defects in the junction barriers can cause critical current fluctuations and low-frequency flux noise may take its origin from trapped vortices and magnetic impurities. In practice, $1/f$ noise appears difficult to suppress and, since dephasing is dominated by low-frequency noise, it is particularly destructive.

Decoherence of a qubit due to $1/f$ noise has already been studied in literature using various models for the noise: One possibility is to assume Gaussian-distributed fluctuations, e.g. of a large collection of microscopic modes, which in some cases may be modeled by a harmonic oscillator bath.[5] Alternatively, one can consider a microscopic model with independent bistable fluctuators, each one characterized by a switching rate γ_i and an effective bias voltage v_i induced at the qubit.[6,7] If the switching rates are distributed according to $p(\gamma) \sim 1/\gamma$ the resulting power spectrum exhibits a $1/f$ divergence at low frequencies. If only a few fluctuators (microscopic modes) dominate, the effect of the noise differs.

In this work, we present a new model of $1/f$ noise and discuss its effects on the phase coherence of a qubit. In contrast to the mentioned proposals we consider situations where the mutual interaction of the localized defects in the background cannot be neglected, e.g. at very low temperatures. Since neutral and charged defects behave as elastic and electric dipoles respectively [7] the strength of the interaction is long-range in both cases decaying as $1/r^3$ as a function of the distance r. Although weak, this interaction may play a crucial role at very low temperatures, leading to formation of highly cooperative clusters of interacting defects.[9] The resulting slow dynamics can be described as a perpetual search of the absolute free-energy minima: the system spends most of the time in local free energy minima corresponding to metastable configurations, occasionally flipping between these states. The exponential relation between the typical trapping time τ and the energy barrier to overcome E, $\tau(E) \simeq \tau_0 e^{E/T}$, implies that the distribution of the trapping times, $P(\tau)$ is generally broad, decaying very

slowly for large times: $P(\tau) \simeq \tau^{-1-\mu}$. Here μ is a positive dimensionless number that can be written as the ratio of temperature T and a typical height E_0 of the energy barriers, $\mu \sim T/E_0$.

The fluctuating extra bias induced at the qubit is described by an asymmetric telegraph noise, where the times between two successive plateaux are distributed according to an algebraic distribution

$$P(\tau) = \frac{\mu}{\tau_0} \left(\frac{\tau_0}{\tau_0 + \tau} \right)^{1+\mu} . \tag{1}$$

The heights and lengths of the plateaux are distributed according to narrow distributions $\mathcal{P}(v)$, which is assumed to have zero mean, and $\rho(\tau_\uparrow)$ respectively. The coupling strength to the qubit is characterized by the dimensionless parameter $g^2 = \langle (v\tau_\uparrow)^2 \rangle$. For $\mu > 2$, when $\overline{\tau} = \int_0^\infty d\tau \tau P(\tau)$ and $\overline{\tau^2}$ are finite our model is equivalent to usual Poissonian telegraph noise as proposed in[6,7]. However, at ultra-low temperatures, i.e. $\mu < 2$, important differences emerge.[a] We thereby have to distinguish two additional classes of noise: (i) for $1 < \mu < 2$, $\overline{\tau}$ is still finite but $\overline{\tau^2}$ diverges. Consequently, the rate of occurring plateaux fluctuates strongly around the average $\overline{\tau}^{-1}$. In particular, due to the diverging $\overline{\tau^2}$ the statistics of the noise are only approximately Gaussian, even in the weak coupling regime. (ii) for $0 < \mu < 1$, when both $\overline{\tau}$ and $\overline{\tau^2}$ diverge, it is even impossible to define an average rate of occurring plateaux, i.e. the noise is scale invariant. In this case, the two point noise correlator decays as

$$\overline{X(t+t')X(t)} - \overline{X(t+t')} \ \overline{X(t)} \quad \simeq \quad \overline{X(t)}^2 (t/t')^{1-\mu} , \tag{2}$$

as opposed to the exponential decay in the case $\mu > 2$. For $0 < \mu < 1$, the power spectrum thus exhibits a $1/f^\mu$ divergence at low frequencies. Furthermore, the t/t'-scaling in (2) implies an intrinsic *nonstationarity* of this noise.

In this work we focus on the influence of such a noise on the phase coherence of a qubit. The full Hamiltonian of a dissipative quantum two-level system can be written in the form:

$$\mathcal{H} = -\frac{1}{2}\epsilon \sigma_z - \frac{1}{2} \left(\sin \eta \, \sigma_x + \cos \eta \, \sigma_z \right) X + \mathcal{H}_{\mathrm{env}}(X) , \tag{3}$$

where ϵ and η are control parameters which we assume to be time-independent and X is the fluctuating extra bias induced by the environment described by $\mathcal{H}_{\mathrm{env}}$. For definiteness we consider in the following two special working points for the qubit: (i) the optimal point ($\eta = \pi/2$), where the linear longitudinal coupling to the noise is tuned to zero and consequently decoherence is reduced considerably,[4] and (ii) the case of longitudinal coupling ($\eta = 0$). Below we will show that transverse linear coupling is - regardless of the considered noise model - equivalent to quadratic longitudinal coupling and a remaining transverse noise component with suppressed low-frequency tail.

Optimal point ($\eta = \pi/2$): In general, transverse noise leads to relaxation, but also contributes to pure dephasing in higher orders. The effect of low-frequency transverse noise ($\omega \ll \epsilon$) can be treated in the adiabatic approximation: the state of the qubit follows the effective magnetic field and (3) can be diagonalized to $-\sigma_z \sqrt{\epsilon^2 + X^2}/2 \approx -\sigma_z \left[\epsilon + X^2/(2\epsilon) \right]/2$. Therefore, low-frequency transverse noise contributes to pure dephasing as quadratic longitudinal noise. On the other hand, higher frequencies, $\omega \sim \epsilon$, mainly contribute to relaxation and the adiabatic approximation breaks down. Situations where both low and high frequencies are present therefore need further analysis.

The unitary transformation $U(t) = \exp\{i\Theta(t)\sigma_y/2\}$ with $\Theta(t) = \arctan(X(t)/\epsilon)$ transforms the Hamiltonian (3) to a spin frame that follows the effective field $\epsilon\hat{z} + X(t)\hat{x}$:

$$\tilde{\mathcal{H}} = U(t)\mathcal{H}U^\dagger(t) + i \, U(t)\frac{d}{dt}U^\dagger(t) = -\frac{1}{2}\sqrt{\epsilon^2 + X^2(t)} \, \tau_z + \frac{1}{2}\dot{\Theta}(t) \, \tau_y , \tag{4}$$

[a]Our predictions also differ from those for an ensemble of independent Poissonian fluctuators.[6,7] A detailed comparison is not straightforward and will be addressed elsewhere.

where the Pauli matrices τ_i remind that the spin is now measured in the time-dependent basis, defined by $\tau_z(t) = U(t)\sigma_z U^\dagger(t)$. Note that for small amplitude of the noise $|X(t)| \ll \epsilon$ the new basis is almost identical to the original basis.

As we can see from the expression of the transformed Hamiltonian (4) linear transverse noise is indeed equivalent to quadratic longitudinal noise - as predicted in the adiabatic approximation. But due to the explicit time dependence of the transformation $U(t)$ an additional transverse component proportional to the time derivative of the original noise X appears, $\dot{\Theta}(t) = (\dot{X}(t)/\epsilon)/(1 + (X(t)/\epsilon)^2) \approx \dot{X}(t)/\epsilon$. One might think that the transformed Hamiltonian is therefore even more complicated than the original one since we are now dealing with both longitudinal and transverse fluctuations. But a closer look at the expression reveals that the low-frequency transverse noise is suppressed, $S_{\dot{\Theta}}(\omega) = 2\langle \dot{\Theta}(t)\dot{\Theta}(t')\rangle_\omega \approx \left(\frac{\omega}{\epsilon}\right)^2 S_X(\omega)$. Hence, in the experimentally relevant situation where the spectrum S_X is less singular than $1/\omega^2$ at low frequencies, $S_{\dot{\Theta}}$ is a regular function at low frequencies with $S_{\dot{\Theta}}(\omega = 0) = 0$. As a consequence the transformed transverse noise does not contribute considerably to dephasing but only to relaxation. Dephasing is dominated by the longitudinal part of the noise proportional to X^2 that still contains all low-frequency contributions. We can therefore add up the two contributions in (4) independently and extract simple expressions for dephasing and relaxation rates:

$$T_1^{-1} = \frac{1}{4}S_{\dot{\Theta}}(\omega = \epsilon) \simeq \frac{1}{4}S_X(\omega = \epsilon) \tag{5}$$

$$T_2^{-1} = \frac{1}{2}T_1^{-1} + T_\varphi^{-1} \tag{6}$$

where T_1 and the first contribution to T_2 are due to the second term in (4) and can be calculated simply using the golden rule. The pure dephasing time T_φ is defined as the characteristic decay time due to quadratic longitudinal noise $X^2(t)/(2\epsilon)$ in (4). In the case of a non-singular noise spectrum it is given by $T_\varphi^{-1} = S_{X^2}(\omega = 0)/(4\epsilon^2)$. For Gaussian $1/f$ noise it has been calculated in Ref.[8] In this case the dephasing law differs from a simple exponential and one should add two decay laws rather than the rates in (6). For the model of nonstationary $1/f$ noise considered in this work the evaluation of T_φ can be reduced straightforwardly to the case of linear longitudinal coupling: $X(t)$ and $X^2(t)/(2\epsilon)$ both describe an asymmetric telegraph noise, the major difference being a change in the amplitude and thus in the coupling strength.[b] The coupling strength at the optimal point is thereby considerably reduced in comparison to linear longitudinal coupling.

Longitudinal coupling ($\eta = 0$): We now focus on the case of pure dephasing, $\eta = 0$ in (3). Therefore, we follow the time evolution of the off-diagonal entries of the qubit's density matrix, $\langle\sigma_+(t)\rangle = \overline{D(t_p,t)}e^{i\epsilon t}\langle\sigma_+(0)\rangle$, where the dephasing factor is defined by $\overline{D(t_p,t)} = \overline{e^{i\Phi(t_p,t)}}$ with $\Phi(t_p,t) = \int_{t_p}^{t_p+t} X(t')dt'$. Because of the intrinsic nonstationarity of the noise (2) we expect to find an explicit dependence of dephasing on the *preparation time* t_p. In the following we will therefore investigate this explicit dependence of $\overline{D(t_p,t)}$ on t_p. This can be achieved using CTRW techniques and taking special care of initial conditions at t_p.[12]

As already discussed previously we have to distinguish three different cases depending on the value of μ: When $\mu > 2$ the noise is equivalent to Poissonian telegraph noise[6] and therefore stationary (for $t_p > \tau_0$). Consequently, dephasing is insensitive to the absolute time t_p.

For $1 < \mu < 2$ it turns out that nonstationarity only plays a role in the strong coupling regime $g > 1$. For weak coupling $g < 1$ the noise is qualitatively equivalent to the Poissonian case $\mu > 2$, i.e. only subdominant corrections appear.[12] For $g > 1$ the dephasing law differs from a simple exponential and the dephasing time, defined as $\overline{D(t_p,\tau_\phi)} = e^{-1}$, scales as $\tau_\phi \simeq \tau_0[1/e + (\tau_0/(\tau_0 + t_p))^{\mu-1}]^{-1/(\mu-1)}$. The dependence of τ_ϕ on t_p thus only matters for values of μ close to 1 and disappears as μ increases to higher values (see Figure 1).

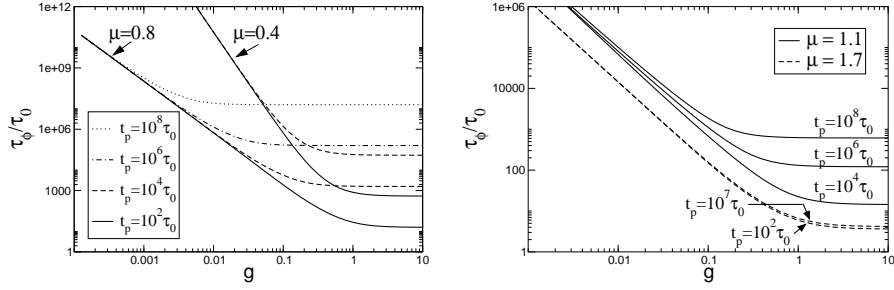[b]In addition $X^2(t)/(2\epsilon)$ has a finite mean. A thorough discussion of this case will be presented elsewhere.

Figure 1: Dephasing time τ_ϕ as a function of the coupling strength g for $\mu < 1$ (left) and $\mu > 1$ (right).

As expected nonstationarity turns out to be most prominent in the case $0 < \mu < 1$. Again, we have to distinguish two regimes of weak and strong coupling separated by a critical coupling constant g_c. But here g_c is not of order one, but depends explicitly on the age of the noise: $g_c(t_p) \simeq \lambda(\mu)t_p^{-\mu/2} \ll 1$. Therefore, the range of the strong coupling regime increases with the age of the system implying that any qubit subject to a noise with $0 < \mu < 1$ eventually ends up in the strong coupling regime.

For weak coupling $g < g_c(t_p)$ dephasing is essentially insensitive to the age of the noise. Away from a (short) initial aging regime $\overline{D(t_p, t)}$ decreases as $\exp(-(t/\tau_\phi)^\mu)$, where the dephasing time scales as $\tau_\phi \sim \tau_0 g^{-2/\mu}$ (in the marginal case $\mu = 1$ we get logarithmic corrections $\tau_\phi \sim \tau_0 g^{-2}|\ln g|$). Indeed, for $t < \tau_\phi$, $\overline{D(t_p, t)}$ is accurately given within a Gaussian approximation of the phase: $\overline{D(t_p, t)} \simeq \exp(-\frac{1}{2}\overline{\Phi(t_p, t)^2})$. Only for $t > \tau_\phi$ the Gaussian approximation breaks down and the exponential decay goes over to a much slower algebraic one, $\overline{D(t_p, t)} \propto 1/t^\mu$.

In the strong coupling regime $g > g_c(t_p)$ the dephasing depends strongly on the age of the noise. The Gaussian approximation of the phase breaks down even at short times and the decay is not exponential any more. The dephasing time becomes proportional to t_p, $\tau_\phi \simeq C(\mu)t_p$, and saturates as a function of g. We emphasize that for $\mu \to 1$, $C(\mu)$ takes smaller and smaller values implying that the dephasing time can be orders of magnitude shorter than the preparation time.

In summary, we have proposed a new model of $1/f^\mu$ noise describing phenomenologically a slow collective environment. The present model shows a cross-over from nonstationary $1/f^\mu$ noise to a memoryless Poissonian telegraph noise as a function of temperature. We analyzed the decay of coherence of a qubit subject to this noise at different operation points. For low temperatures and strong coupling the dephasing time was found to depend explicitly on the age of the noise.

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