n-D-Topological Data Structures: Some Theoretical and Pragmatic Considerations for GI-Science

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1. Introduction

Topology is a central concept for GI-Science and -Systems. However, there still seems to be a gap between the use of topology in 2D-GIS and in higher-dimensional geo-applications such as 3D city models, geology, and geophysics dealing with the analysis of 3D geo-scientific phenomena. In CityGML, playing a central role in data exchange for city models, topology is only implicitly defined. Unified topological modelling, however, should be generally usable and independent of thematic contexts.

The purpose of this paper is to present some approaches to dimension-independent modelling and managing of topology in GI-Science. Different concepts for topological data structures are introduced. An approach of how to manage topology with an object-oriented geo-database is given. Finally, an outlook is given on further research concerning topology in GI-Science.

2. Topological Data Structures

In the following sections we will present our current work on different implementation approaches of topological data models: Relational chain complexes, developed from the pure mathematical viewpoint, G-Maps with their interesting algebraic properties, and our current implementation of a spatial database DB4GeO which uses simplicial complexes.

2.1 Relational Chain Complexes

A relational chain complex is a simple relational database representation of a chain complex from algebraic topology (Hatcher 2002). As spatial data usually is some realization of such a chain complex one can consider such a relational representation canonical.

A complex, associated to a topological space, algebraically expresses that an object is circumscribed by its boundary elements. Such a combination of elements is then called a cycle like, for example, the chain of `ncw` (next clock-wise) and `nccw` (next counter-clock-wise) references in the winged-edge data structure (Baumgart 1975)–often called "ring" or "loop". A "shell", however, is also a cycle in the above sense.

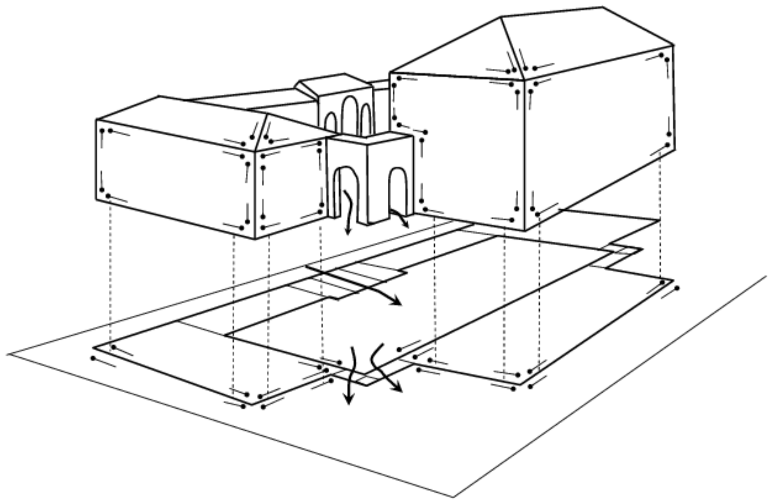

Fig. 1. Sketch of Osnabrück Palace using G-Maps. The darts at the vertices are the primary data element. Each topological primitive consists of a set of darts–called orbit. Primitives are incident iff the orbits intersect.

A relational complex schema can be defined with the dimension dynamically changing at run-time or statically fixed as in the classical Vertex–Edge–Face–Volume schemata. The dynamic version has two interesting features: First, it exposes a simple common schema for topological data of arbitrary dimension, thereby unifying all 4D, 5D, or any-D modelling approaches. Second, it turns the data structure itself into a topological space and links spatial modeling with the mathematical theory of topological constructions. For example, extrusion can be generalized to topological product spaces, different levels of details (LODs) may be considered quotient spaces. In particular, the unification of edges, vertices, hypervolumes etc. into one data type allows a seamless mapping between different LODs.

Another reason for considering relational complexes canonical is efficiency: Storing arbitrary topological data for a set of size n always costs $\mathcal{O}(n^2)$ storage space in the worst case (Paul 2009), and relational complexes are already space optimal in the general case. So further optimization is only possible when restricted to special cases.

2.2 d-Generalized Maps and Cell-Tuple Structures

Another topological data structure–the d-Generalized Maps (d-G-Maps) (Lienhardt 1994) and the closely related cell-tuple structures (Brisson 1994)–can be used for multi-representation databases (Thomsen et. al. 2008). For practical purposes these can be considered equivalent (Lévy 1999), and they are well suited as a dimension-independent approach for 2D and 3D data modelling.

In 3D space tuples describing a G-Map or cell-tuple structure contain unique combinations between nodes, edges, faces, and solids which are connected by 4 involutionary operations each consisting of all possibilities of exchanging a single node, or edge, face,

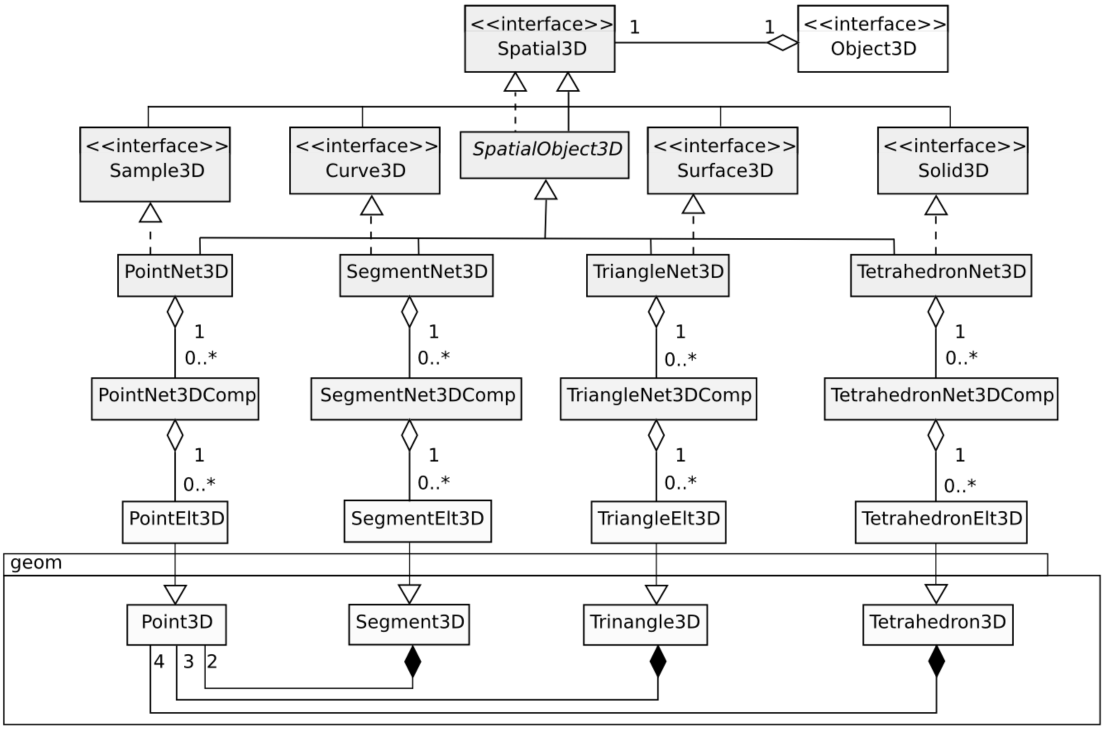

Fig. 2. The essential classes of simple and complex geo-objects provided by the geometry library.

or solid of a tuple, thus forming an abstract simplicial complex. Navigation through the topology is supported by sequences of different involution operations.

G-Maps and cell-tuple structures may be made persistent in an Object-Relational Database (ORDB). Figure 1 shows the example of Osnabrück Palace as part of a 3D city model represented with G-Maps.

Whereas the implementation of elementary transformations by database transactions is straightforward, querying the connectivity of a subset of cells of a model (e.g. a group of rooms within a 3D building) needs more consideration. Based on “multiple groupings” (Fradin et al. 2002) closed loops through the corresponding sets of cell-tuples can be used for access, navigation and retrieval of connected parts.

2.3 Advantages of the presented Topological Approaches

The presented approaches are *dimension-independent* approaches and can be used for 2D *and* 3D GIS applications. Not only are they dimension independent but, even more, spatial dimension can become a dynamic feature of the data structure and could even change at run-time.

In contrast to CityGML (2010) these approaches provide a generic spatial model with spatial information strictly kept separate from other semantics.

2.4 Managing G-Maps in DB3D/DB4GeO

For the management of spatial information we are currently using our service-based geo-database DB4GeO/DB3D (Breunig et. al. 2004, Bär 2007) which has its scientific roots in GeoToolKit (Balovnev et. al. 2004). DB4GeO/DB3D is based on an object-oriented DBMS, has a service-based architecture, and is exclusively implemented in

Java. REST (Fielding 2000) is used as a communication platform between clients and the database server. The geometric and topological data model of DB4GeO/DB3D is based on simplicial complexes, i.e. on points, segments, triangles, and tetrahedra.

DB4GeO provides a “geometry element” (Elt) object for every simplex. However, the geo-object model of DB4GeO is not yet capable of differentiating (thematically) topological objects within one meshed component. This will be addressed in our forthcoming development.

The targeted topological framework shall provide the possibility to build “larger” thematically defined topological units such as blocks in geology or 3D-city-model environments and support the management of hierarchic and temporal cellular complexes. An application scenery for this concept is depicted in Figure 3.

The figure shows one of our data sets (Lautenbach, Berlekamp 2002), a model of the Piesberg landfill site near the city of Osnabrück, Germany, which we use as a test case for the topology module as it features different thematically defined units inside a continuous net component which changes in time.

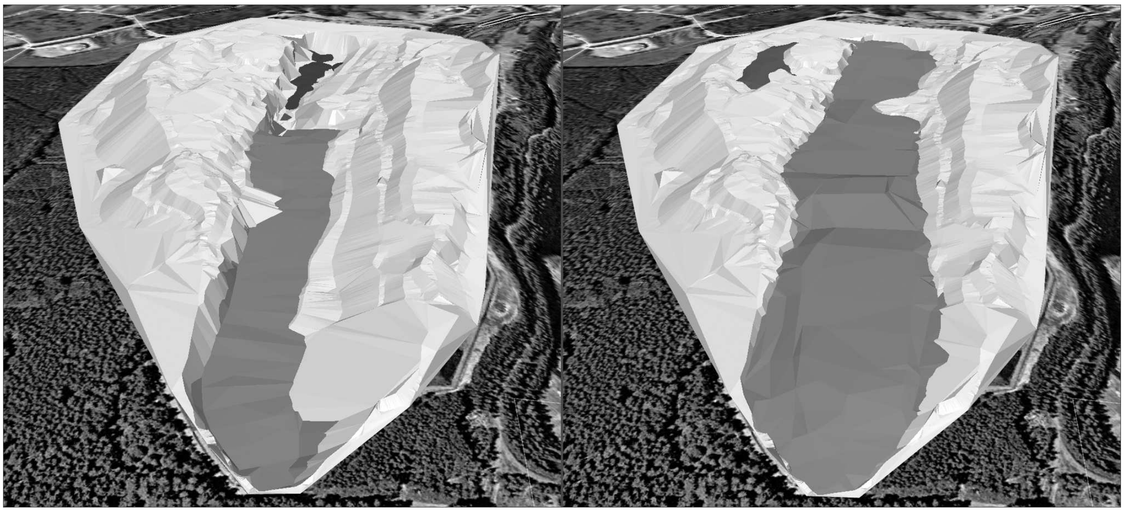

Fig. 3. 3D model of Piesberg (light gray) and its landfill (dark gray) with cells of different usage in 1982 and 1993, visualized with GOCAD© by Paradigm and Google Earth™

3. Outlook

In our future research we will focus on topology considering planning alternatives and versions in building information models. Furthermore, we will examine the management of topological objects in geological applications, and unify the different approaches we currently assess.

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