Occurrence probability and earthquake size of post shut-in events in geothermal projects

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Abstract

In recent geothermal projects that were associated with induced seismicity it has been observed that the largest earthquake or earthquakes occurred after shut-in, the moment when the pressurised fluid injection in the borehole is stopped. We use a probabilistic approach based on Omori's law and a Gutenberg-Richter magnitude distribution to demonstrate that the probability of exceeding a certain maximum magnitude after shut-in is as high or higher than before stopping the fluid injection. The amount of this increase is dependent on the exponent of Omori's law q. For the reference case of q=2 and a 10% probability at shut-in time t_S we obtain an increase to 14.6% for $t = 2t_S$. If we consider a constant probability level of occurrence for an event larger than a given magnitude at shut-in time, this maximum magnitude increases by 0.12 units for $t = 2t_S$. For the Fenton Hill experiment recent studies reveal q=7.5 that corresponds to only a small amount of probability increase for the post-injection phase.

Introduction

It is known that fluid injections at geothermal sites, which are performed to develop the reservoirs, can induce low magnitude earthquakes in critically stressed zones of the surrounding rock. Even after shut-in, that is, after the pressurised fluid injection into the borehole is stopped, a significant number of seismic events can occur. The understanding, characterisation, and forecasting of post-injection events is particularly important, because during recent geothermal projects such as Soultz-sous-Forêts (Charléty et al., 2007), Basel (Häring et al., 2008), and Landau it has been observed that the largest earthquakes tend to occur after shut-in. This makes it still more difficult to control such events. Those earthquakes have had a large impact in society and understanding their temporal occurrence was identified as one major goal of geothermal research (Mayer et al., 2007). There is speculation that the largest earthquakes are therefore causally related to the shut-in as if the stop of injection would lead to the larger earthquake. However, recent findings from Langenbruch & Shapiro (2010a) suggest that the presence of unstable pre-existing fractures may increase the seismicity rate and thus the probability of exceeding a certain magnitude even after stopping injection.

We demonstrate in this paper (a) that the largest earthquakes should occur after the shut-in of injection in the context of the "Seismicity Based Reservoir Characterization Theory (SBRC)"; (b) that even larger earthquakes have to be expected, if injection would be continued; (c) that the largest expected magnitude can be estimated. All three statements apply in a probabilistic sense only.

Theory

There are two fundamental laws in statistical seismology, namely the Omori law, which describes the decay rate of aftershock activity after tectonically driven earthquakes, and the Gutenberg-Richter relation describing the frequency magnitude distribution of earthquakes. It was observed and verified in recent works that both fundamental laws are also valid in the context of injection-induced seismicity (Shapiro et al., 2007; Langenbruch & Shapiro, 2010a).

We describe the fluid injection by a point source in a permeable fluid-saturated medium with pre-existing fractures and assume that the fluid is liberated from this source with constant strength until the shut-in time t_S.

According to Shapiro et al. (2007) this leads to a constant seismicity rate $\bar{\nu}_1$ for earthquakes with magnitudes larger than some lower threshold value m_1. For another magnitude M the Gutenberg-Richter earthquake size distribution suggests a constant seismicity rate $\bar{\nu}_M$:

$$\bar{\nu}_M = \bar{\nu}_1 \cdot 10^{-b(M-m_1)}. \qquad (1)$$

In general, the behaviour of seismicity triggering in space and time is controlled by the relaxation process of stress and pore pressure perturbation that was initially created at the injection source. This relaxation process can be approximated by linear pressure diffusion in the pore fluid of rocks. Following the Mohr-Coulomb failure criterion the resulting increase in pore pressure can lead to rock failure along pre-existing, sub-critically stressed cracks. If critical pore pressures leading to reactivation of individual pre-existing fractures are equally distributed between a lower bound C_{min}=0 Pa and a maximum value C_{max} larger than the overpressure between source and reservoir, Omori's law can be utilised to describe the decay rate of seismic activity after shut-in of injection in the following modified form (Langenbruch & Shapiro, 2010a):

$$\nu_1(t) = \bar{\nu}_1 \cdot \left(\frac{t_S}{t}\right)^q, \qquad (2)$$

with the constant seismicity rate $\bar{\nu}_1$ during injection, time $t \geq 0$ from injection start, the shut-in time t_S, and the exponent q between 1 and 2. Recently the analysis of seismicity data from geothermal projects suggests even higher values for q (Langenbruch & Shapiro, 2010a; see below).

We assume that the induced earthquakes represent a Poisson process (Shapiro et al., 2010; Langenbruch & Shapiro, 2010b). If the seismicity rate is constant the Poisson process is called homogeneous. The probability that no earthquake in excess of M occurs between the initiation of injection and some time t can be written as

$$P_0(M,t) = \exp\left(-\bar{v}_M \cdot t\right) = \exp\left(-\bar{v}_1 \cdot 10^{-b(M-m_1)}\right) = \exp\left(-\bar{v}_1 \cdot \exp^{-\beta(M-m_1)}\right), \tag{3}$$

with $\beta = b \cdot \ln 10 \approx 2.3 \cdot b$. The distribution is of Gumbel type, which is not surprising as we look for the extreme value of $\bar{v}_M \cdot t$ earthquakes.

If the Poisson process varies with time it becomes inhomogeneous. The probability that no earthquake in excess of *M* occurs between the initiation of injection and some time *t* can be written as

$$P_0(M,t) = \exp\left(-\int_0^t v_M(\tau)\,d\tau\right). \tag{4}$$

For the shut-in time we get for the probability that magnitude *M* is not exceeded between time 0 and t_S:

$$\ln \frac{1}{P_0(M,t_s)} = \bar{v}_M \cdot t_s. \tag{5}$$

Using the decaying seismicity rate for the time after shut-in $t \geq t_S$ we get

$$\ln \frac{1}{P_0(M,t)} = \bar{v}_M \cdot t_s + \int_{t_s}^t v_M(\tau)\,d\tau = \bar{v}_M \cdot t_s \cdot \left(1 + \frac{1}{q-1} - \frac{1}{(q-1)\cdot(t/t_s)^{q-1}}\right). \tag{6}$$

Thus it is clear that the probability P_0 of not exceeding magnitude *M* is still decreasing after shut-in. If the injection is not shut off but continues some time beyond ($t \geq t_S$) we have the trivial relation

$$\ln \frac{1}{P_0(M,t)} = \bar{v}_M \cdot t_s \cdot \left(1 + \frac{t - t_s}{t_s}\right) = \bar{v}_M \cdot t. \tag{7}$$

Hence the probability to exceed magnitude *M* (that is $1 - P_0$) still increases after shut-in. Let us give an example (Fig. 1): If the probability to exceed magnitude *M* at the time of the shut-in is given by $1 - P_0$=10%, this probability increases to $1 - P_0$=14.6% considering all events occurring until $t = 2t_S$ and an exponent of *q*=2 (eq. 6). For *q*=1.5 the above probability changes to $1 - P_0$=15.4%. The corresponding value for a on-going injection is: $1 - P_0$=19% (eq. 7).
For *q*=1, first the limit of *q*-1 decreasing to zero has to be evaluated (see appendix):

$$\lim_{q \to 1} P_0(M, t \geq t_S) = \exp\left(-\bar{v}_M t_S \cdot \left(1 + \ln\left(t/t_S\right)\right)\right). \tag{8}$$

For $t = 2t_S$ it follows that $1 - P_0$=16.3%. However, this theoretical limit of *q* is not expected in nature.

Next we study how the probabilistically determined largest earthquake changes in magnitude given a constant probability level of occurrence. We assume a 50% chance that no earthquake larger than M_S occurs:

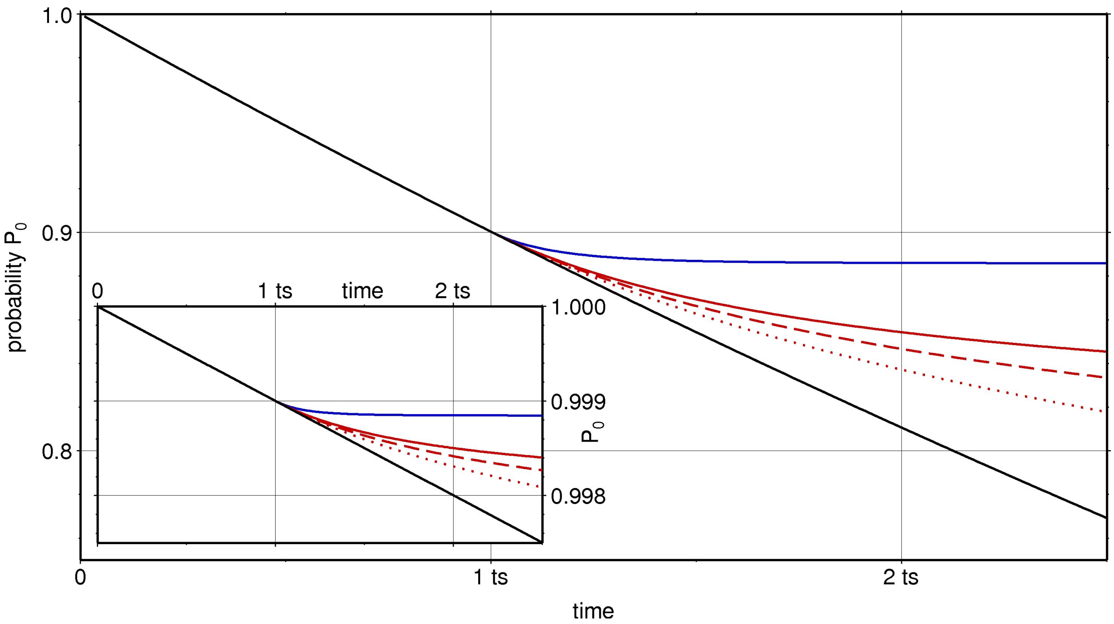

Figure 1. Probability P_0 of not exceeding a maximum magnitude M with time (shut-in time t_S). The solid black line corresponds to continued injection. Other lines show P_0 after shut-in for different q-values: q=1 (dotted red), q=1.5 (dashed red), q=2 (solid red), and as obtained for the Fenton Hill dataset q=7.5 (solid blue). Main graph: $P_0(t_S)$=90%, inset $P_0(t_S)$=99.9%.

$$\ln\frac{1}{P_0\left(M_s,t_s\right)}=\bar{v}_{M_s}\cdot t_s=\bar{v}_1\cdot 10^{-b\left(M_s-m_1\right)}\cdot t_s=\ln\frac{1}{0.5}\approx 0.7\,. \tag{9}$$

From this M_S can be calculated if $\bar{v}_1$ and b are known. Keeping the probability level constant – in our example 50% – we can ask how would the maximum magnitude increase beyond M_S if we (a) continue injection until $t \geq t_S$ and (b) stop injection and wait until $t \geq t_S$.

The general implicit formula for both cases is

$$P_0\left(M_s,t_s\right)=P_0\left(M_s+\Delta M,t\right). \tag{10}$$

In case (a) we get after some manipulations:

$$\Delta M=\frac{1}{b}\log\left(t/t_s\right). \tag{11}$$

Thus we have a 50% chance that no earthquake larger than M_S occurs until t_S and if we then extend the injection time from t_S to $t=2t_S$ we have a 50% chance to get no earthquake larger than

$$M_s+\Delta M=M_s+\frac{1}{b}\log\left(2\right)\approx M_s+\frac{0.3}{b}\approx M_s+0.2\,, \tag{12}$$

assuming a b-value of 1.5. The general shut-in case leads to

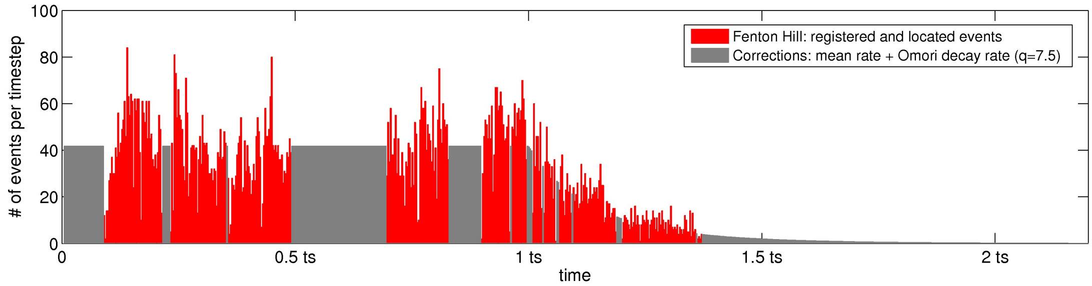

Figure 2. Temporal development of seismicity in the Fenton Hill geothermal project. Grey areas show the mean rate before shut-in time t_S for gaps in the observation due to a not operating monitoring system. After shut-in grey areas show the best fit for Omori's law with q=7.5.

$$\Delta M = \frac{1}{b}\log\left(1+\frac{1}{q-1}-\frac{1}{(q-1)\cdot(t/t_s)^{q-1}}\right). \tag{13}$$

For the reference value of q=2 and again $t = 2t_S$, we find

$$\Delta M = \frac{\log 1.5}{b} \approx 0.12\,. \tag{14}$$

With q =1.5 this changes to

$$\Delta M = \frac{\log 1.6}{b} \approx 0.14\,. \tag{15}$$

For the lower value of q=1 the limit of equation 13 has to be evaluated:

$$\lim_{q\to 1}\Delta M = \frac{1}{b}\log\left(\lim_{q\to 1}\left(1+\frac{1}{q-1}-\frac{1}{(q-1)\cdot(t/t_s)^{q-1}}\right)\right). \tag{16}$$

With the results from above it follows:

$$\lim_{q\to 1}\Delta M = \frac{1}{b}\log\left(1+\ln(t/t_s)\right). \tag{17}$$

For the b=1.5 and $t = 2t_S$ given above that is

$$\lim_{q\to 1}\Delta M \approx 0.152\,. \tag{18}$$

Fenton Hill

We apply the above theory to real data that was recorded during the Fenton Hill (New Mexico, USA) Hot Dry Rock injection experiment in 1983 (House, 1987). After 62 h the injection was stopped and the seismic monitoring went on for 23 h, i.e t/t_s =1.4. Figure 2 shows the temporal evolution of the observed seismicity. Gaps in the monitoring are filled in with the mean seismicity rate before shut-in. The decay of seismicity for the post-injection phase can be well approximated by the modified Omori law. A value of q=7.5 results in the best fit to the observed post-injection seismicity. Highly unstable fracture systems result in low q-values and a high seismicity rate close before and after the shut-in. This may also be the reason for large magnitude events close before or after shut-in. With increasing stability of the fracture system the q-value increases (Langenbruch & Shapiro, 2010a). We use the value of q=7.5 to calculate the probability P_0 of not exceeding magnitude M_S. Assuming P_0 = 99.9% at shut-in time, P_0 becomes 99.88% for $t = 2t_S$ (eq. 6, see Fig. 1).

Conclusion

We have shown that based on a modified Omori law and a Gutenberg-Richter distribution the probability P_0 of not exceeding a maximum magnitude during injection and after its termination can be determined. The decay of seismicity after shut-in and thus P_0 strongly depends on the exponent q of the modified Omori law. Two characteristic values have been calculated for the post-injection phase and the case of an on-going injection: (a) the continuing decrease of P_0 and (b) the increase of the maximum magnitude given a constant probability level of occurrence. For (a) we find an increase of $1-P_0$, i.e. the probability of exceeding a maximum magnitude, from a given value of 10% at shut-in time t_S to 14.6% for time $t = 2t_S$ (q=2). At the same time a continued injection would result in $1-P_0$=19%. The maximum magnitude (b) thus increases for $t = 2t_S$ by 0.2 for continued injection and 0.12 for the shut-in case (that corresponds to an increase of seismic energy by a factor of 1.5). Thus for low probabilities to exceed the maximum magnitude at shut-in time and a high q-value only a small increase in the risk is to be expected, while higher probabilities in combination with lower q-values result in a significant enlargement of the probability of exceeding the magnitude threshold.

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Appendix

For the evaluation P_0 for q=1, we substitute r=q-1:

$$\lim_{r\to 0} P_0\left(M, t \geq t_S\right) = \exp\left(-\bar{\nu}_M t_S \cdot \left(1 + \lim_{r\to 0}\left(\frac{1}{r} - \frac{1}{r\cdot\left(t/t_S\right)^r}\right)\right)\right). \tag{A1}$$

With

$$\lim_{r\to 0}\left(\frac{1}{r} - \frac{1}{r\cdot\left(t/t_S\right)^r}\right) = \lim_{r\to 0}\frac{1-\left(t/t_S\right)^{-r}}{r} \tag{A2}$$

and applying L'Hôpital's rule

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}, \tag{A3}$$

it follows:

$$\lim_{r \to 0} \frac{1-(t/t_S)^{-r}}{r} = \lim_{r \to 0} (t/t_S)^{-r} \cdot \ln(t/t_S) = \ln(t/t_S) \tag{A4}$$

and

$$\lim_{r \to 0} P_0(M, t \geq t_S) = \exp\left(-\bar{\nu}_M t_S \cdot \left(1+\ln(t/t_S)\right)\right). \tag{A5}$$

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