Tracking an Extended Object Modeled as an Axis-Aligned Rectangle

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Abstract: In many tracking applications, the extent of the target object is neglected and it is assumed that the received measurements stem from a point source. However, modern sensors are able to supply several measurements from different scattering centers on the target object due to their high-resolution capability. As a consequence, it becomes necessary to incorporate the target extent into the estimation procedure. This paper introduces a new method for tracking the smallest enclosing rectangle of an extended object with an unknown shape. At each time step, a finite set of noisy position measurements that stem from arbitrary, unknown measurement sources on the target surface may be available. In contrast to common approaches, the presented approach does not have to make any statistical assumptions on the measurement sources.

1 Introduction

Target tracking methods usually assume that the object extension is negligible in comparison to the sensor noise. However, due to the high-resolution capabilities of modern sensors, this assumption is not always justified. Therefore, target tracking algorithms have to take into account that position measurements may stem from different locations, named measurement sources, on the extended target object. Scenarios for tracking extended objects occur in military surveillance with radar devices [WD04, Koc08], but can also be frequently found in many other areas like robotics. An overview of extended object tracking methods is given in [WD04]. In [SG99], the motion of the extended object is modeled as one bulk that is determined by a finite set of individual components like points on the object. In [GS05], the target geometry is modeled with a spatial distribution and in [Koc08], an ellipsoidal object extension is modeled with random matrices.

In this paper, the extended target object, whose true shape is unkown, is modeled as an axis-aligned rectangle in two-dimensional space. At each time step, several noisy position measurements originating from unknown measurement sources on the target surface are received. In contrast to common approaches, we do not have to make any (statistical) assumptions on these measurement sources. This is a realistic point of view since the true shape and properties of the target surface are usually unknown. This approach was first introduced in [BH09] for circular discs and is applied here to axis-aligned rectangles.

2 Problem Formulation

The considered problem is to track an extended target object based on noisy position measurements stemming from the target surface. In this paper, the true shape of the target object is modeled as the *smallest axis-aligned two-dimensional rectangle* (in the following abbreviated with SER) that includes the extended target object. An axis-aligned rectangle with center $\left[x^c, y^c\right]^T$, width a, and height b is denoted with $\mathbf{Rec}(x^c, y^c, a, b) = \left\{\left[x, y\right]^T \in \mathbb{R}^2 \,|\, |x - x^c| \leq a \text{ and } |y - y^c| \leq b\right\}$. At each time step k, the parameters of the true rectangle are denoted with the four-dimensional vector $\underline{\tilde{p}}_k = \left[\tilde{x}^c_k, \tilde{y}^c_k, \tilde{a}_k, \tilde{b}_k\right]^T$.We treat the problem of estimating $\underline{\tilde{p}}_k$, which is not directly observable. Instead, at each time step, n_k two-dimensional position measurements $\{\underline{\hat{z}}_{k,j}\}_{j=1}^{n_k}$ may be available. Each of these individual measurements $\underline{\hat{z}}_{k,j}$ is a noisy observation of a two-dimensional point $\underline{\tilde{z}}_{k,j}$, called measurement source, which is known to lie in the true rectangle $\mathbf{Rec}(\underline{\tilde{p}}_k)$, i.e, $\underline{\tilde{z}}_{k,j} \in \mathbf{Rec}(\underline{\tilde{p}}_k)$ and $\underline{\hat{z}}_{k,j} = \underline{\tilde{z}}_{k,j} + \underline{\boldsymbol{w}}_{k,j}$, where $\underline{\boldsymbol{w}}_{k,j}$ denotes two-dimensional additive white observation noise[1] that models a random Cartesian displacement. The probability distribution of the measurement noise $\underline{\boldsymbol{w}}_{k,j}$ is known since it results from the particular sensor model. On the other hand, the position of the measurement source $\underline{\tilde{z}}_{k,j}$ is totally unknown. It is only assumed to be an element of the true circular disc. This can be seen as a set-valued uncertainty.

The temporal evolution of the smallest rectangle, which includes the extended object, is modeled by means of a so-called *extended motion model* that captures both the motion and the change of extent of the target object (details are given in Section 4).

3 Static Extended Objects

First, we restrict the problem to estimating the SER of a static extended object based on measurements not corrupted by noise. We assume that at each time step k one (noise-free) position measurement $\underline{\hat{z}}_k = \underline{\tilde{z}}_k \in \mathbb{R}^2$ is given. Each $\underline{\hat{z}}_k$ represents a point known to lie in the true SER of the target object $\mathbf{Rec}(\underline{\tilde{p}})$, i.e., $\underline{\hat{z}}_k \in \mathbf{Rec}(\underline{\tilde{p}})$. In the following, we are going to derive a recursive procedure for determining the SER of all measurements $\underline{\hat{z}}_1, \ldots, \underline{\hat{z}}_k$ received so far. If we assume that the measurements cover the entire extended object in the course of time, this rectangle converges against the SER of the extended object. Let $\underline{p}^e_{k-1} = \left[x^e_{k-1}, y^e_{k-1}, a^e_{k-1}, b^e_{k-1}\right]^T$ be the parameters of the SER of the measurements $\underline{\hat{z}}_1, \ldots, \underline{\hat{z}}_{k-1}$. Then, the parameters $\underline{p}^e_k = \left[x^e_k, y^e_k, a^e_k, b^e_k\right]^T$ of the SER of $\mathbf{Rec}(\underline{p}^e_{k-1})$ and

[1]Note that all random variables are printed bold face in this paper.

$\hat{\underline{z}}_k = \left[x_k^m, y_k^m\right]^T$ are given by

$$\begin{bmatrix} x_k^e \\ a_k^e \end{bmatrix} = \begin{cases} 0.5 \cdot \begin{bmatrix} x_k^m + x_{k-1}^e - a_{k-1}^e \\ x_k^m - x_{k-1}^e + a_{k-1}^e \end{bmatrix} & \text{if } x_k^m > x_{k-1}^e + a_{k-1}^e \\ 0.5 \cdot \begin{bmatrix} x_k^m + x_{k-1}^e + a_{k-1}^e \\ -x_k^m + x_{k-1}^e + a_{k-1}^e \end{bmatrix} & \text{if } x_k^m < x_{k-1}^e - a_{k-1}^e \\ \begin{bmatrix} x_{k-1}^e \\ a_{k-1}^e \end{bmatrix} & \text{otherwise} \end{cases} \quad , \tag{1}$$

$$\begin{bmatrix} y_k^e \\ b_k^e \end{bmatrix} = \begin{cases} 0.5 \cdot \begin{bmatrix} y_k^m + y_{k-1}^e - b_{k-1}^e \\ x_k^m - y_{k-1}^e + b_{k-1}^e \end{bmatrix} & \text{if } y_k^m > y_{k-1}^e + b_{k-1}^e \\ 0.5 \cdot \begin{bmatrix} y_k^m + y_{k-1}^e + b_{k-1}^e \\ -y_k^m + y_{k-1}^e + b_{k-1}^e \end{bmatrix} & \text{if } y_k^m < y_{k-1}^e - b_{k-1}^e \\ \begin{bmatrix} y_{k-1}^e \\ b_{k-1}^e \end{bmatrix} & \text{otherwise} \end{cases} \quad . \tag{2}$$

Remark 1. In the following, $\mathcal{G} : \mathbb{R}^2 \times \mathbb{R}^3 \to \mathbb{R}^3$ denotes the total function defined by (1) and (2) that maps $\hat{\underline{z}}_k$ and $\underline{p}_{k-1}^e$ to $\underline{p}_k^e$.

Remark 2. All rectangles that enclose the measurements $\hat{\underline{z}}_1, \ldots, \hat{\underline{z}}_k$ are given by the so-called *solution set* [BH09]

$$\Delta(\underline{p}_k^e) := \left\{ \left[x^c, y^c, a, b\right]^T \mid \mathbf{Rec}(\underline{p}_k^e) \subset \mathbf{Rec}(\left[x^c, y^c, a, b\right]^T) \right\} \quad ,$$

which can be computed recursively according to $\Delta(\underline{p}_k^e) = \Delta(\underline{p}_{k-1}^e) \cap \Delta(\left[x_k^m, y_k^m, 0, 0\right]^T)$. In this manner, a set-theoretic estimator for the true parameters is obtained (see [BH09]).

The next step is to consider the problem of estimating a static extended object from measurements corrupted by stochastic noise. Now, we assume that at each time step one noisy position measurement $\hat{\underline{z}}_k$ is available. The measurement $\hat{\underline{z}}_k$ is a noisy observation of the measurement source $\tilde{\underline{z}}_k$ according to $\tilde{\underline{z}}_k \in \mathbf{Rec}(\tilde{\underline{p}}_k)$ and $\hat{\underline{z}}_k = \tilde{\underline{z}}_k + \underline{\boldsymbol{w}}_k$. The term $\underline{\boldsymbol{w}}_k$ denotes white measurement noise. Since the measurement source is unknown, no prior information about $\tilde{\underline{z}}_k$ is available. Thus, the knowledge about $\tilde{\underline{z}}_k$ is given by the random vector $\underline{\boldsymbol{z}}_k := \hat{\underline{z}}_k - \underline{\boldsymbol{w}}_k$. Then, the SER of $\tilde{\underline{z}}_1, \ldots, \tilde{\underline{z}}_{k-1}$ becomes a rectangle whose parameters are uncertain. We denote the random vector that specifies the parameter of this random rectangle with $\underline{\boldsymbol{p}}_{k-1}^e$. A random point $\underline{\boldsymbol{z}}_k$ can be fused with $\underline{\boldsymbol{p}}_{k-1}^e$ by evaluating $\mathcal{G}(\cdot)$ stochastically according to $\underline{\boldsymbol{p}}_k^e = \mathcal{G}(\underline{\boldsymbol{z}}_k, \underline{\boldsymbol{p}}_{k-1}^e)$ and $\underline{\boldsymbol{p}}_1^e = \left[\underline{\boldsymbol{z}}_1, 0, 0\right]^T$. In general, the distribution of $\underline{\boldsymbol{p}}_k^e$ cannot be computed in closed form for given distributions of $\underline{\boldsymbol{z}}_k$ and $\underline{\boldsymbol{p}}_{k-1}^e$, since $\mathcal{G}(\cdot)$ is not linear but only piecewise linear. Nevertheless, the distribution of $\underline{\boldsymbol{p}}_k^e$ can be approximated with a Gaussian distribution by employing the prediction step of a nonlinear stochastic state estimator [HH08, JU04]. In the following, we assume that $\underline{\boldsymbol{p}}_k^e \sim \mathcal{N}(\underline{p}; \hat{\underline{p}}_k^e, \mathbf{C}_k^e)$ and $\underline{\boldsymbol{z}}_k \sim \mathcal{N}(z; \hat{\underline{z}}_k, \mathbf{C}_k^z)$. A point estimate is given by $\mathrm{E}[\underline{\boldsymbol{p}}_k^e] = \hat{\underline{p}}_k^e$. In case the support of the measurement noise $\underline{\boldsymbol{w}}_k$ is bounded, $\hat{\underline{p}}_k^e$ approaches the true SER plus the bounded support. Otherwise, $\hat{\underline{p}}_k^e$ does not converge to a fixed point. In order to cope with this behavior, one can assume the noise to be bounded and subtract the (known) support afterwards from the estimated rectangle. This subtraction can be done in a purely geometric fashion. Another solution, which is employed in Section 5, is to incorporate further knowledge that allows to separate set-valued and stochastic uncertainties.

4 Dynamic Extended Objects

In order to track an extended object that moves and varies its shape over time, the current random rectangle $\mathbf{Rec}(\underline{\boldsymbol{p}}_k^e)$ (and implicitly the random solution set $\Delta(\underline{\boldsymbol{p}}_k^e)$) has to be propagated through an (extended) motion model to the next time step. Furthermore, at each time step a finite set of noisy position measurements $\{\underline{\hat{z}}_{k,j}\}_{j=1}^{n_k}$ with measurement noises $\underline{\boldsymbol{w}}_{k,j}$ is fused with the predicted random rectangle. In this paper, we consider linear *extended motion models* of the form

$$\underline{\boldsymbol{p}}_k = \mathbf{A}_k \underline{\boldsymbol{p}}_{k-1} + \mathbf{B}_k(\underline{\hat{u}}_{k-1} + \underline{\boldsymbol{v}}_{k-1}) \ , \tag{3}$$

which map the parameters $\underline{\boldsymbol{p}}_{k-1}$ of the rectangle at time step $k-1$ to the parameters $\underline{\boldsymbol{p}}_k$ at time step k. The term $\underline{\boldsymbol{v}}_{k-1}$ denotes zero mean Gaussian white noise and $\underline{\hat{u}}_{k-1}$ is a deterministic system input. Since (3) is linear, the prediction $\underline{\boldsymbol{p}}_k^p$ is Gaussian distributed, in case $\underline{\boldsymbol{p}}_{k-1}^e$ is Gaussian distributed and $\underline{\boldsymbol{v}}_{k-1}$ is Gaussian noise. The mean and covariance matrix of $\underline{\boldsymbol{p}}_k^p$ can be computed with the Kalman filter prediction step, i.e., $\underline{\hat{p}}_k^p = \mathbf{A}_k \underline{\hat{p}}_{k-1}^e + \mathbf{B}_k \underline{\hat{u}}_{k-1}$and $\mathbf{C}_k^p = \mathbf{A}_k \mathbf{C}_{k-1}^e \mathbf{A}_k^T + \mathbf{C}_{k-1}^v$. In the strict sense, the entire set $\Delta(\underline{\boldsymbol{p}}_k^e)$ would have to be propagated through the motion model, i.e., $\Delta(\underline{\boldsymbol{p}}_k^p) = \mathbf{A}_k \Delta(\underline{\boldsymbol{p}}_{k-1}^e) + \mathbf{B}_k(\underline{\hat{u}}_{k-1} + \underline{\boldsymbol{v}}_{k-1})$, which only holds if $\mathbf{A}_k = \mathbf{I}$. However, the error made if this does not hold is typically insignificant. The measurements $\{\underline{\hat{z}}_{k,j}\}_{j=1}^{n_k}$ can be fused with $\underline{\boldsymbol{p}}_k^p$ by computing the SER of $\mathbf{Rec}(\underline{\boldsymbol{p}}_k^p)$ and the random points $\{\underline{\boldsymbol{z}}_{k,j}\}_{j=1}^{n_k}$ with $\underline{\boldsymbol{z}}_{k,j} := \underline{\hat{z}}_{k,j} - \underline{\boldsymbol{w}}_{k,j}$. This can be done recursively by setting $\underline{\boldsymbol{p}}_{k,0}^e := \underline{\boldsymbol{p}}_k^p$ and computing $\underline{\boldsymbol{p}}_{k,j}^e = \mathcal{G}(\underline{\boldsymbol{z}}_{k,j}, \underline{\boldsymbol{p}}_{k,j-1}^e)$ for $j = 1, \ldots, n_k$. The parameters of the random rectangle are then given by $\underline{\boldsymbol{p}}_k^e := \underline{\boldsymbol{p}}_{k,n_k}^e$.

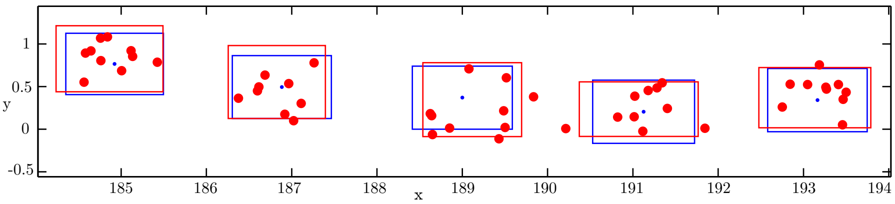
5 Incorporating Knowledge About the Number of Measurements

In case of noise-corrupted measurements, it is necessary to incorporate further knowledge about the size of the target object since it is not possible to tell set-valued and stochastic uncertainties apart. A realistic assumption is that the number of measurements received from the target object at a particular time step depends on its size [Koc08]. Here, we assume that a conditional probability density $f(n_k|r_k)$ that specifies the number of measurements depending on the current perimeter[2] of the true SER of the extended object is available. In order to incorporate this knowledge into the estimation procedure, we additionally maintain a random variable $\boldsymbol{r}_k^e$ that captures the knowledge about the true perimeter obtained from the number of measurements n_k. The state vector is then given by $\left[(\underline{\boldsymbol{p}}_k^e)^T, \boldsymbol{r}_k^e\right]^T$, which can be propagated through the extended motion model according to

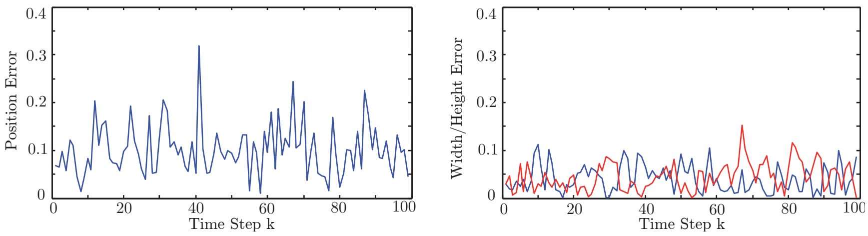
$$\begin{bmatrix} \underline{\boldsymbol{p}}_k^p \\ \boldsymbol{r}_k^p \end{bmatrix} = \begin{bmatrix} \mathbf{A}_k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{p}}_{k-1}^e \\ \boldsymbol{r}_{k-1}^e \end{bmatrix} + \begin{bmatrix} \mathbf{B}_k \\ b_k^{(3)} + b_k^{(4)} \end{bmatrix} (\underline{\hat{u}}_{k-1} + \underline{\boldsymbol{v}}_{k-1}) \ ,$$

where $b_k^{(3)}$ resp. $b_k^{(4)}$ denotes the third/fourth row of $\mathbf{B}_k$. We assume that $\left[(\boldsymbol{p}_{k-1}^e)^T, \boldsymbol{r}_{k-1}^e\right]^T$ is Gaussian distributed such that $\left[(\underline{\boldsymbol{p}}_k^p)^T, \boldsymbol{r}_k^p\right]^T$ is also Gaussian distributed and can be computed with the Kalman filter prediction step. The prediction $\left[(\underline{\boldsymbol{p}}_k^p)^T, \boldsymbol{r}_k^p\right]^T$ can be

[2] It would also be possible to consider the area of the true SER, but this would result in nonlinear constraints.



(a) Snippet of the state-space for several time steps: True (blue) and estimated (red) rectangle.



(b) Estimation error.

Figure 1: Tracking an extended object: Simulation run.

updated with $\{\hat{\underline{z}}_{k,j}\}_{j=1}^{n_k}$ in the following three steps:

1. Compute the joint pdf of $\left[(\underline{\boldsymbol{p}}_k^e)^T, \boldsymbol{r}_k^e\right]^T := \left[(\underline{\boldsymbol{p}}_{k,n_k}^e)^T, \boldsymbol{r}_k^e\right]^T$ with $\underline{\boldsymbol{p}}_{k,0}^e := \underline{\boldsymbol{p}}_k^p$ and $\underline{\boldsymbol{p}}_{k,j}^e = \mathcal{G}(\underline{\boldsymbol{z}}_{k,j}, \underline{\boldsymbol{p}}_{k,j-1}^e)$ for $j = 1, \ldots, n_k$.
2. Compute the posterior pdf $f(\underline{p}_k^e, r_k^e \mid n_k)$ with Bayes' rule according to
$$f(\underline{p}_k^e, r_k^p \mid n_k) = c \cdot f(n_k \mid r_k) \cdot f(\underline{p}_k^e, r_k^e) \enspace ,$$
where c is a normalization constant.
3. Compute the posterior pdf $f(\underline{p}_k^e, r_k^e \mid n_k, \{\boldsymbol{a}_k^e + \boldsymbol{b}_k^e \leq \boldsymbol{r}_k^e\})$. This (truncated) pdf can be approximated with a Gaussian distribution, if $f(\underline{p}_k^e, r_k^e \mid n_k)$ is Gaussian (see the algorithm for linear inequality constraints in [Sim06]). Note that the above constraint arises from the fact, that there is no rectangle with perimeter $\boldsymbol{r}_k^e$ in the set of feasible rectangles $\Delta(\underline{\boldsymbol{p}}_k^e)$ if $\boldsymbol{a}_k^e + \boldsymbol{b}_k^e > \boldsymbol{r}_k^e$.

6 Simulation

Figure 1 depicts the result of a simulation run where the true extended object is in fact an axis-aligned rectangle. At each time step, the measurement sources are sampled uniformly from the true rectangle and the measurement noise $\underline{\boldsymbol{w}}_{k,j}$ is Gaussian with zero mean and covariance matrix diag$(\left[0.01, 0.01\right]^T)$. Note that the introduced estimator does not exploit any knowledge about the distribution of the measurement sources which is assumed to be unkown. At the first time step, the true rectangle is located at position $\left[1, 1\right]^T$ and has a width of 0.5 and a height of 0.3. We assume that the number of measurements n_k produced by the true rectangle with perimeter r to be approximately

Gaussian distributed, i.e., $\boldsymbol{n}_k \sim \mathcal{N}^*(n_k; 10r, 0.6)$. The symbol $\mathcal{N}^*$ denotes the Gaussian distribution with truncated negative values. The extended motion model is given by Equation (3) with $\mathbf{A}_k = \mathbf{B}_k = \text{diag}(\left[1, 1, 1, 1\right]^T)$, input $\hat{\underline{u}}_{k-1} = \left[2, 0, 0, 0\right]^T$ and input noise $\mathbf{C}_k^v = \text{diag}(\left[0.02, 0.02, 0.001, 0.001\right]^T)$. Furthermore, the unscented transformation [JU04] was used to evaluate $\mathcal{G}(\cdot)$.

Figure 1a depicts a snippet of the state-space including the true rectangle (blue) for several time steps. Note that the true rectangle does not necessarily enclose all measurements. In general, the higher the measurement noise, the more measurements lie outside of the rectangle. At each time step, the estimated rectangle are plotted red. Figure 1b depicts the absolute estimation error of the width (blue) $|\text{E}[\boldsymbol{a}_k^e] - \tilde{a}_k|$ and height (red) $|\text{E}[\boldsymbol{b}_k^e] - \tilde{b}_k|$ over 100 time steps. Figure 1b shows the estimation error $\|\text{E}[[\boldsymbol{x}_k^e, \boldsymbol{y}_k^e]^T] - [\tilde{x}_k^c, \tilde{y}_k^c]^T\|_2$ for the center of the rectangle over the first 100 time steps.

7 Summary and Future Work

In this paper, a novel method for tracking the smallest enclosing axis-aligned rectangle of extended objects was proposed. Future work consists of extending the proposed method to rectangles with an arbitrary orientation.

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