Universität Karlsruhe (TH)
Institut für Baustatik

3D Finite Element Modeling of Fiber–Matrix Instabilities in Compression

Y. Lapusta, J. Harich, W. Wagner

Mitteilung 6(2003)

BAUSTATIK

Universität Karlsruhe (TH)
Institut für Baustatik

3D Finite Element Modeling of Fiber–Matrix Instabilities in Compression

Y. Lapusta, J. Harich, W. Wagner

Mitteilung 6(2003)

BAUSTATIK

	Prof. Dr.–Ing. W. Wagner	Telefon:	(0721) 608–2280
	Institut für Baustatik	Telefax:	(0721) 608–6015
©	Universität Karlsruhe	E–mail:	bs@.uni-karlsruhe.de
	Postfach 6980	Internet:	http://www.bs.uni-karlsruhe.de
	76128 Karlsruhe		

3D FINITE ELEMENT MODELING OF FIBER–MATRIX INSTABILITIES IN COMPRESSION

Y. Lapusta
S. Timoshenko Institute of Mechanics, Nesterov Str. 3, Kiev 03057, Ukraine and currently, Institut für Baustatik, Universität Karlsruhe (TH), Kaiserstr. 12, D-76131 Karlsruhe, Germany, e-mail: yl@bs.uni-karlsruhe.de

J. Harich
Institut für Baustatik, Universität Karlsruhe (TH), Kaiserstr. 12, D-76131 Karlsruhe, Germany

W. Wagner
Institut für Baustatik, Universität Karlsruhe (TH), Kaiserstr. 12, D-76131 Karlsruhe, Germany

Abstract. 3D finite element modeling of fiber-matrix instabilities in compression is presented. Calculations for simple particular 3D cases are carried out. The presented examples illustrate the selection of models with and without surface effects. The formulation can be readily extended to problems with transversely isotropic elastic fibers.

1. Introduction. The behavior of fiber-reinforced materials in compression along the reinforcement is an important issue, since a variety of compressive failure mechanisms are reported in the literature. A substantial amount of work has already been done in this field, reflected in numerous literature sources. Without pretending to be exhaustive we discuss here briefly some aspects of the question in order to outline the motivation of this contribution. Instabilities at the micro level of scale, or fiber-matrix instabilities (microbuckling) represent a limiting factor for compressive strength of fiber-reinforced composite materials. These phenomena, which are similar to longitudinal bending of a beam on an elastic foundation (Timoshenko and Gere, 1961), were first mentioned and modeled in 60s (Dow and Grunfest, 1960; Rosen, 1965; Schuerch, 1966; Sadovsky et al., 1967; Guz, 1969; etc). Rosen (1965) and Schuerch (1966) presented models which considered fibers and matrix as layers. Sadovsky et al. (1967) proposed a combination of a 1D consideration for a fiber with a 3D one for the matrix. Guz (1969) presented a 3D linearized approach to study the instability of a double-periodic system of fibers in an infinite elastic matrix. Reviews and literature on compressive behavior of fiber composites are given in Guynn et al. (1992), Guz (1992), Schultheisz and Waas (1996), Fleck (1997), and others. Results and bibliography on 3D models and methods of stability theory of composites are reviewed, for example, in Guz (1992) and Guz and Lapusta (1999). Closely related issues concerning buckling and accompanying phenomena at different scales are discussed, among others, in Wagner et al. (2001), Guz (2002), Lapusta and Wagner (2001), Tkachenko and Chekhov (2002), Zhuk et al. (2002), etc. Note that the majority of existing microbuckling models for fiber composites are limited to 2D considerations. 3D micromechanical modeling of fiber-matrix instabilities, especially by means of 3D finite elements, has obtained substantially less attention. However, the stress-strain fields around fibers are essentially three-dimensional. This means that 3D formulations are preferable if more rigorous results on fiber microbuckling are needed. This paper presents an attempt of 3D finite element modeling of fiber-matrix instabilities in compression. Calculations for some particular 3D cases are carried out to illustrate selection of models with and without surface effects. The formulation can be readily extended to problems with transversely isotropic elastic fibers.

2. Finite-element model. We consider a 3D fiber-matrix instability problem (Fig. 1). The system consists of a long fiber and a matrix. It is compressed along the fiber direction x_1. The matrix is assumed to be wide enough to exclude unwanted lateral surface effects and Euler buckling of the whole system.

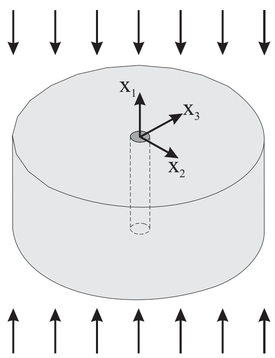

Fig. 1. Schematic representation of a system fiber-matrix under compression.

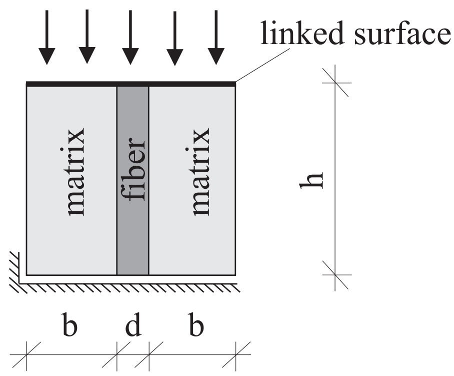

Fig. 2. Characteristic cell.

The lateral surfaces of the matrix are free of forces

$$F_1^m = 0, \quad F_2^m = 0, \quad F_3^m = 0. \tag{1}$$

A perfect adhesion of the fiber (denoted by superscript f) and the matrix (superscript m) is assumed, which means the continuity of forces and displacements across the interface

$$F_n^f = F_n^m, \quad F_s^f = F_s^m, \quad F_1^f = F_1^m,$$
$$u_n^f = u_n^m, \quad u_s^f = u_s^m, \quad u_1^f = u_1^m. \tag{2}$$

The value of the loading parameter leading to buckling of the system and the corresponding buckling half wavelength $0 < L < \infty$ must be calculated. To carry out finite element modeling, we consider characteristic cells as presented in Figs. 2-4. Two possible situations are considered. In the first one, we assume that the matrix has a circular or square cross section and the fiber is situated in its center (Figs. 3 and 4, $b = c$). In the second one we admit that the matrix has a rectangular cross-section and the fiber is situated in the vicinity of one of the matrix lateral surfaces (Fig. 4, $0 \leq c \leq b$).

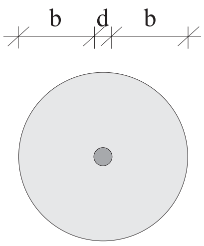

Fig. 3. Case/Geometry 1 (cross-section).

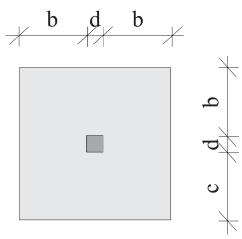

Fig. 4. Case/Geometry 2 (cross-section).

These characteristic cells are selected by accounting for the periodicity of the buckling form in the direction x_1 (period 2L=2h). The Euler buckling of the cell is excluded by the selection of its geometry, since the cell is taken wide enough in transverse directions.

An extended version of the finite element program FEAP (see e.g. Zienkiewicz and Taylor, 2000) is used. A standard displacement-based eight node isoparametric solid finite element with trilinear shape functions is employed to discretize the characteristic cells. The symmetry of the cells and of their eigenforms are taken into account (see e.g. Figs. 5, 6 and 10). Behavior of the fiber and the matrix is modeled by a linearly elastic material. For the calculations, the values $E^f = 0,4 \cdot 10^{12}$ Pa, $E^m = 0,2 \cdot 10^{10}$ Pa, $\nu^f = \nu^m = 0,3$, $d = 0,8 \cdot 10^{-5}$ (m) are taken. To save the calculation time, the transverse dimensions of the elementary cell are chosen to $b = h$. In case 2 (Fig. 4), we also place the fiber in the vicinity of one of the lateral surfaces by setting $0 \le c \le b$. This way we account for its influence on the fiber stability. The cell is compressed through prescribed displacements u_1 of the nodes at the linked surface (Fig. 2). These displacements are stepwise increased from zero. Due to the symmetry of the system and its eigenforms, we suppress the displacements u_2 of the symmetry plane (x_1, x_3). A bisection method is used to find the stability point within a certain prescribed accuracy. Then an eigenvalue analysis leads to the associated eigenforms.

The height h of the characteristic cell is stepwise increased and the critical displacement u_1 is calculated as a function of the buckling half wavelength L (which is equal to h). Thus functions $u_1/L = f(L/d)$ can be found. Since sufficiently long fibers buckle with a certain wavelength corresponding to the shortening minimum, a minimization of the calculated value u_1/L with respect to the parameter L/d is performed.

3. Results. Figs. 5, 6 and 10 show examples of the calculated eigenforms and the chosen 3D mesh.

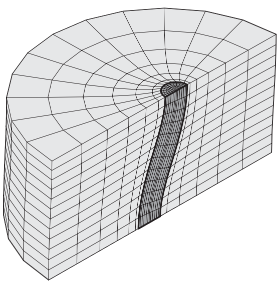

Fig. 5. Buckling form and mesh for case 1.

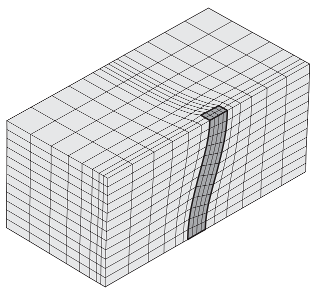

Fig. 6. Buckling form and mesh for case 2.

Figs 7 and 8 show values u_1/L with respect to the ratio L/d. To analyze free surface effects, we have also performed calculations for different values of spacing c between the fiber and the matrix boundary $(0 \le c \le b)$. Note that for this case, Lapusta and Wagner (2001) presented a semi-analytical study which distinguished between two different instability mode orientations with respect to the free boundary. These are: (a) mode towards the boundary, which is symmetric with respect to the plane that contains the fiber's axis and is perpendicular to the boundary (mode 1), and (b) mode for which the fiber buckles out of the mentioned plane (mode 2).

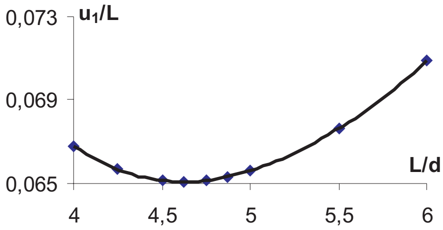

Fig. 7. u_1/L calculated for case/geometry 1.

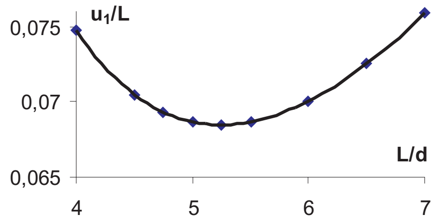

Fig. 8. u_1/L calculated for case/geometry 2.

Since the mentioned above paper demonstrated that only mode 1 was critical, we have limited our consideration to the case of mode 1. In this case, we use symmetry and calculate the eigenform as shown in Fig. 10.

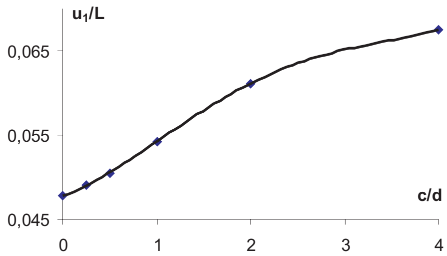

Fig. 9. u_1/L vs. spacing between the fiber and a boundary c/d.

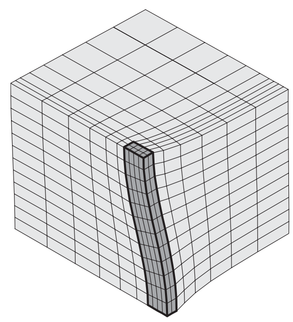

Fig. 10. Buckling form and mesh for a fiber touching the free boundary.

Figure 11 shows some results of convergence study for a fiber touching the free boundary (case 2 with $c = 0$, Figs. 4, 10). In this Figure, the deviation $D(\%)$ of the critical shortenings u_1/L for different meshes with respect to the results obtained with the highest possible accuracy admitted in this paper (right edges of the curves) is given. A comparison of cases 1 and 2 shows that geometry 2 can be used in modeling case 1 within approximately 5 % accuracy for the critical shortenings. Free surface effect for case 2 can be substantial and decrease the value of the critical shortening by up to about 40 %.

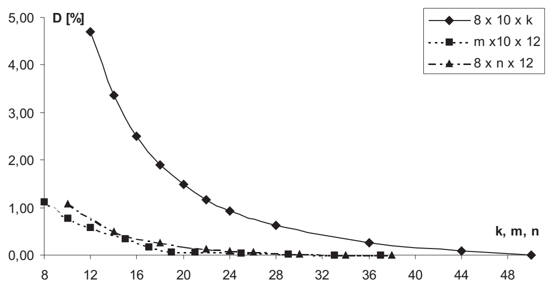

Fig. 11. Convergence with respect to number of elements k,m,n.

Acknowledgements. A part of this work has been carried out with the financial support of the DFG (grant WA 746/12-2), which is gratefully appreciated.

REFERENCES

Timoshenko S.P., Gere J.M. (1961). Theory of Elastic Stability, Second Edition. New York, NY: McGraw-Hill Book Company.

Dow N.F., Grunfest I.J. (1960). Determination of most needed potentially possible improvements in materials for ballistic and space vehicles. General Electric Co, Space Sci Lab, TISR 60 SD 389.

Rosen B.W. (1965). Mechanics of Composite Strengthening. Fiber Composite Materials, American Society of Metals, 37-75.

Schuerch H. (1966). Prediction of compressive strength in uniaxial boron fiber-metal matrix composite materials. *AIAA Journal*, **4** (1), 102-106.

Sadovsky M.A., Pu S.L., Hussain M.A. (1967). Buckling of Microfibers. *Journal of Applied Mechanics*, December 1967, 1011-1016.

Guz A.N. (1969). Construction of a theory of stability of unidirectional fiber composites. *Prikladnaya Mekhanika* **5**(2), 62-70 (in Russian).

Guynn E.G., Bradley W.L., Ochoa O.O. (1992). A parametric study of variables that affect fiber microbuckling initiation in composite laminates: Part 1 - Analyses. *Journal of Composite Materials* **26**(11), 1594-1616.

Guz A.N. (Edit.) (1992). Micromechanics of composite materials: Focus on Ukrainian research. *Applied Mechanics Reviews, Special Issue*, **45**(2), 13-101.

Schultheisz C.R., Waas A.M. (1996). Compressive failure of composites. *Prog. Aerospace Sci.* **32**, 1-78.

Fleck N.A. (1997). Compressive failure of fiber composites. *Advances in Applied Mechanics*, **33**, 43-117.

Guz A.N., Lapusta Y. (1999). Three-dimensional problems of the near-surface instability of fiber composites in compression (model of a piecewise-uniform medium) (Survey). *International Applied Mechanics* **35**(7), 641-670.

Wagner W., Gruttmann F., Sprenger W. (2001). A finite element formulation for the simulation of propagating delaminations in layered composite structures, *International Journal for Numerical Methods in Engineering*, **51**, 1337-1359.

Guz A.N. (2002). Critical phenomena in cracking of the interface between two prestressed materials. 1. Problem formulation and basic relations. *International Applied Mechanics*, **38**(4), 423-431.

Lapusta Y., Wagner W. (2001). On various material and fibre-matrix interface models in the near-surface instability problems for fibrous composites. *Composites Part A: Applied Science and Manufacturing*, **32**, 413-423.

Tkachenko E.A., Chekhov V.N. (2002). Stability of an elastic layer stack between two half-spaces under compressive loads. *International Applied Mechanics*, **38** (11), 1381-1387.

Zhuk Ya.A., Soutis C., Guz I.A. (2002). Stiffened composite panels with a stress concentrator under in-plane compression. *International Applied Mechanics*, **38** (2), 240-252.

Zienkiewicz O.C., Taylor R.L. (2000). The Finite Element Method, Butterworth Heinemann, 5th edition.