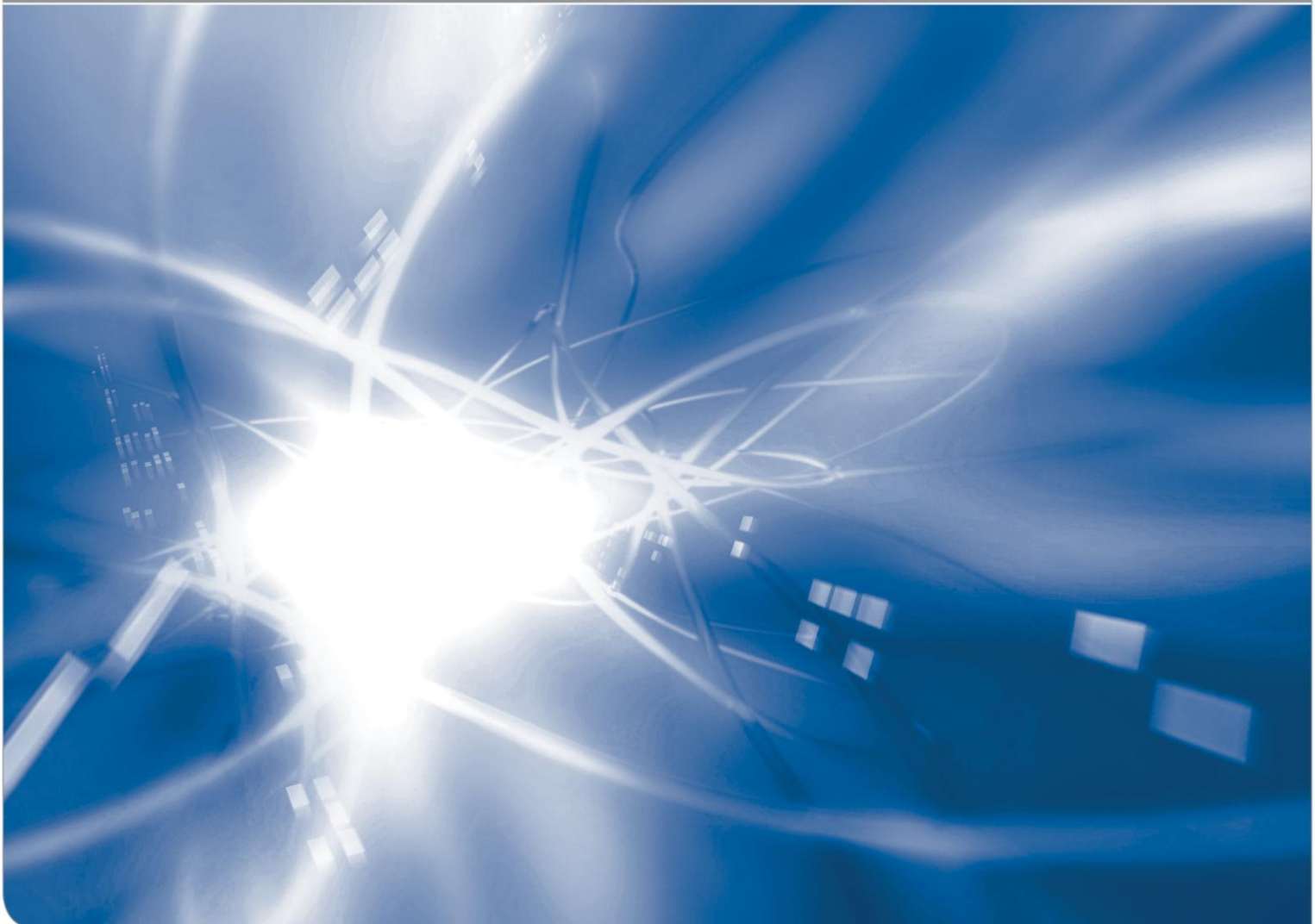# KIT
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The MULTITHIN Option of the Air Shower Simulation Program CORSIKA

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Abstract

The MULTITHIN Option of the Air Shower Simulation Program CORSIKA

This report describes the MULTITHIN option implemented in the CORSIKA versions 7.4002 and later. It performs the simulation of Extensive Air Showers in the unthinned mode. In parallel it gives additionally the weight factors for up to 6 different thinning modes. If the particle under consideration would not have survived in the respected thinning mode, the weight factor is given with a negative sign. This MULTITHIN option can be combined especially with the PARALLEL option for the simulation up to the highest energies $> 10^{18}$ eV, but can also be used together with the CURVED and the SLANT option.

Zusammenfassung

Die MULTITHIN Option des Luftschauer-Simulationsprogramms CORSIKA

Dieser Bericht beschreibt the MULTITHIN Option, die in CORSIKA ab der Version 7.4002 implementiert ist. Sie führt die Simulation von ausgedehnten Luftschauern ohne Dünnen aus. Parallel dazu gibt sie zusätzlich für bis zu 6 verschiedene Dünnmoden die Gewichtsfaktoren an. Würde ein Teilchen im entsprechenden Dünnmodus nicht überlebt haben, wird der Gewichtsfaktor mit negativem Vorzeichen ausgegeben. Diese MULTITHIN Option kann insbesondere mit der PARALLEL Option für die Simulation bis zu den höchsten Energien $> 10^{18}$ eV kombiniert werden, aber auch zusammen mit der CURVED und der SLANT Option benützt werden.

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1 Introduction

The MULTITHIN option implemented in CORSIKA [1] handles the simulation of Extensive Air Showers (EAS) without the 'thinning' procedure [2]. Simultaneously the thinning procedure is applyed to the particles (i.e. generated with the same random numbers) in different thinning modes, i.e. with different thinning parameters. The resulting weight factors are written to the particle output file in an extra entry which follows immediately each particle data entry. Thus the behaviour of weights attributed to an identical particle for the different thinning modes can be studied, especially the benefits from 'optimized' thinning with weight limitation [3]. Also the validation of re-sampling procedures [4, 5] to simulate the detector response of the EAS particles can be tested for different thinning modes, especially to compare such re-sampling procedures with the particles steming from fully simulated data sets without thinning. This will give hints to any biases and artificial fluctuations [6] on particle parameters introduced by the different thinning algorithms [3].

Nowadays the simulation of showers initialized by ultra-high energy cosmic particles with energies above 10^{18} eV without thinning is possible with CORSIKA employing the PARALLEL option [7] on multi-core computers. Without the PARALLEL option for those energies usually the thinning of showers is indispensable because of the extreme long CPU times. The MULTITHIN option is implemented into CORSIKA in a manner that it can be combined with the PARALLEL option. Thus the problems arising from thinning algorithms can be studied also at the highest energies, if a multi-core architecture of CPU's can be used.

A similar approach with up to 10 'multi-weights' implemented in CORSIKA vers. 6.200 has been developed by the Leeds group [8] in 2005 and several tests have been performed [9]. The disadvantage of this approach was the lack of the PARALLEL option, so all simulations with energies at 10^{19} eV had to be decomposed after the first interactions into more than 1200 subshowers with maximum energies of 10^{16} eV to be treated in individual jobs on a computer farm.

In the following sections the sequences of the data entries written to the particle output are described for the normal case and for combining the MULTITHIN option with the 'additional muon information' (see the CORSIKA User's Guide [10]). In the combination with the EHISTORY and/or the MUPROD option [11] a single particle entry is accompanied by several additional entries in a unique sequence as explained in Sects. 2.3 and 2.4. After having written 39 entries on the particle data output file the remaining entries belonging to the actual particle are continued on the next data sub-block.

2   Particle Output File Structures

In the MULTITHIN option the shower is simulated without thinning, the particle output file structure resembles that of the normal particle output file structure (see CORSIKA User's Guide [10]), but each particle represented by 7 words as given in Table 1 is followed by the extension entry containing the weights for the 6 different thinning modes. The first word of this extension entry contains the number 8888888. which identifies the extension entry in a unique manner, see Table 2. This identifyer is followed by 6 weights given for the different thinning modes which are defined in corresponding input lines with the keyword MTHINH. This keyword has to appear once for each thinning mode to be defined. At maximum up to 6 thinning modes are possible. Their sequence is arbitrary.

The MTHINH keyword is followed by 4 parameters:

```
MTHINH    EFRCTHN    WMAX    THINRAT    WEITRAT
```

Format = (A6, 4F), Defaults = 1.E-4, 1.E30, 1., 1.
EFRCTHN : Factor ε_{th} which defines the energy fraction of the primary energy below which the thinning algorithm becomes active. If the fraction is selected in a manner that this energy is below the lowest energy threshold of ELCUT(i), $i = 1...4$, thinning will not become active.
WMAX : Weight limit for thinning. If the weight of a particle exceeds WMAX, no further thinning is performed.
THINRAT : Defines hadronic thinning limit differing from em-thinning limit EFRCTHN by the ratio of $\varepsilon_{th_{em}}/\varepsilon_{th_{hadr}}$ which gives the ratio between the energy of the em-particles and the energy of the hadronic particles below which the thinning algorithm becomes active for these particle species.
WEITRAT : Defines hadronic weight limit differing from em-weight limit WMAX by the ratio of weight limit of em-particles to weight limit of hadronic particles in case of thinning.
Further explanations on the thinning are given in the CORSIKA User's Guide [10] and Ref. [12].

If in a thinning mode a particle is dropped before reaching the observation level the weight for this mode is set negative. Its absolute value is the weight at the place of the particle's dropping. If less than 6 thinning modes are activated, the unused weight factors are set to zero.

2.1   Standard case including PARALLEL option

The sequence of data sub-block entries for the standard case holds also for simulations with the PARALLEL option distributed on many cores. The particle data entry Table 1 is followed by the extension entry Table 2 with the unique identifyer 8888888. used as first word.

Table 1: Structure of particle data entry.

Particle data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as :
	part. id$\times 1000$ + hadr. generation$\times 10$ + no. of obs. level
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c
$7 \times (n-1) + 5$	x position coordinate in cm
$7 \times (n-1) + 6$	y position coordinate in cm
$7 \times (n-1) + 7$	t time since first interaction (or since entrance into
	atmosphere) in nsec
	for $n = 1 \ldots 39$

Table 2: Structure of particle data entry extension for different weights.

Weight factors entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	multithin identifyer (8888888.E0)
$7 \times (n-1) + 2$	weight for thinning mode 1
$7 \times (n-1) + 3$	weight for thinning mode 2
$7 \times (n-1) + 4$	weight for thinning mode 3
$7 \times (n-1) + 5$	weight for thinning mode 4
$7 \times (n-1) + 6$	weight for thinning mode 5
$7 \times (n-1) + 7$	weight for thinning mode 6
	for $n = 1 \ldots 39$

2.2   Combination with 'Additional Muon Information'

In the combination of the MULTITHIN option with the MUADDI keyword the additional muon information Table 3 precedes the particle data entry Table 4, while the extension entry with the different weights Table 5 follows immediately after the particle data entry.

Table 3: Structure of 'muon at production' data.

Additional muon information data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as : part. id$\times 1000$ + hadr. generation$\times 10$ + no. of obs. level (part. id is 75 for μ^+ rsp. 76 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at production
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at production
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at production
$7 \times (n-1) + 5$	x position coordinate (North) in cm at production
$7 \times (n-1) + 6$	y position coordinate (West) in cm at production
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm at production
	for $n = 1 \ldots 39$

Table 4: Structure of particle data entry.

Particle data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as : part. id$\times 1000$ + hadr. generation$\times 10$ + no. of obs. level (part. id is 5 for μ^+ rsp. 6 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at observ. level
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at observ. level
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at observ. level
$7 \times (n-1) + 5$	x position coordinate in cm at observ. level
$7 \times (n-1) + 6$	y position coordinate in cm at observ. level
$7 \times (n-1) + 7$	t time since first interaction (or since entrance into atmosphere) in nsec
	for $n = 1 \ldots 39$

Table 5: Structure of particle data entry extension for different weights.

Weight factors entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	multithin identifyer (8888888.)
$7 \times (n-1) + 2$	weight for thinning mode 1
$7 \times (n-1) + 3$	weight for thinning mode 2
$7 \times (n-1) + 4$	weight for thinning mode 3
$7 \times (n-1) + 5$	weight for thinning mode 4
$7 \times (n-1) + 6$	weight for thinning mode 5
$7 \times (n-1) + 7$	weight for thinning mode 6
	for $n = 1 \ldots 39$

2.3 Combination with EHISTORY option

In the combination of the MULTITHIN option with the EHISTORY option [11] (keyword MUADDI activated) the additional muon information Table 6 precedes the mother particle data Table 7, the grandmother particle data Table 8, and the particle data entry Table 9, while the extension entry with the different weights Table 10 follows immediately after the particle data entry.

Table 6: Structure of 'muon at production' data.

Additional muon information data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as : part. id$\times 1000$ + hadr. generation$\times 10$ + no. of obs. level (part. id is 75 for μ^+ rsp. 76 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at production
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at production
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at production
$7 \times (n-1) + 5$	x position coordinate (North) in cm at production
$7 \times (n-1) + 6$	y position coordinate (West) in cm at production
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm at production
	for $n = 1 \ldots 39$

Table 7: Structure of mother particle data.

Mother particle data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as : **-** part. id$\times 1000$
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c
$7 \times (n-1) + 5$	x position coordinate (North) in cm
$7 \times (n-1) + 6$	y position coordinate (West) in cm
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm
	for $n = 1 \ldots 39$

Table 8: Structure of grandmother particle data.

Grandmother particle data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as : **-** part. id$\times 1000$
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c
$7 \times (n-1) + 5$	generation counter of **mother particle**
$7 \times (n-1) + 6$	χ, penetrated matter thickness (g/cm^2) between production and interaction point of grandmother particle
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm
	for $n = 1 \ldots 39$

Table 9: Structure of particle data entry.

Particle data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as :
	part. id$\times$1000 + hadr. generation$\times$10 + no. of obs. level
	(part. id is 5 for μ^+ rsp. 6 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at observ. level
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at observ. level
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at observ. level
$7 \times (n-1) + 5$	x position coordinate in cm at observ. level
$7 \times (n-1) + 6$	y position coordinate in cm at observ. level
$7 \times (n-1) + 7$	t time since first interaction (or since entrance into
	atmosphere) in nsec
	for $n = 1 \ldots 39$

Table 10: Structure of particle data entry extension for different weights.

Weight factors entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	multithin identifyer (8888888.E0)
$7 \times (n-1) + 2$	weight for thinning mode 1
$7 \times (n-1) + 3$	weight for thinning mode 2
$7 \times (n-1) + 4$	weight for thinning mode 3
$7 \times (n-1) + 5$	weight for thinning mode 4
$7 \times (n-1) + 6$	weight for thinning mode 5
$7 \times (n-1) + 7$	weight for thinning mode 6
	for $n = 1 \ldots 39$

2.4   Combination with MUPROD option

In the combination of the MULTITHIN option with the MUPROD option [11] (keyword MUADDI activated) the additional muon information Table 11 gives the muon parameters at the muon production point. This entry is eventually followed by the entries of the mother and grandmother particles if the EHISTORY option had been activated (see preceding Section 2.3). The 'muon at track end' entry Table 12 gives the parameters of the muon at the point of its decay or interaction. The immediately following entry Table 13 giving the weight factors for the different thinning modes finalizes the entry sequence.

Table 11: Structure of decaying 'muon at production' data.

Decaying muon start data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as :
	part. id$\times$1000 + hadr. generation$\times$10 + fate index
	(par. id is 85 for μ^+ rsp. 86 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at production
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at production
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at production
$7 \times (n-1) + 5$	x position coordinate (North) in cm at production
$7 \times (n-1) + 6$	y position coordinate (West) in cm at production
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm at production
	for $n = 1 \ldots 39$

. . .

Here the entries for mother and grandmother data (see Tables 7 and 8) appear, if the EHISTORY option is selected.

. . .

Table 12: Structure of decaying 'muon at track end' data.

Decaying muon track end data entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	particle description encoded as :
	part. id$\times$1000 + hadr. generation$\times$10 + fate index
	(part. id is 95 for μ^+ rsp. 96 for μ^-)
$7 \times (n-1) + 2$	px, momentum in x direction in GeV/c at decay point
$7 \times (n-1) + 3$	py, momentum in y direction in GeV/c at decay point
$7 \times (n-1) + 4$	pz, momentum in -z direction in GeV/c at decay point
$7 \times (n-1) + 5$	x position coordinate (North) in cm at decay point
$7 \times (n-1) + 6$	y position coordinate (West) in cm at decay point
$7 \times (n-1) + 7$	z position coordinate (vertical) in cm
	at (estimated) end of muon track
	for $n = 1 \ldots 39$

Table 13: Structure of particle data entry extension for different weights.

Weight factors entry : (7 words each)	
No. of word	Contents of word (as real numbers R*4)
$7 \times (n-1) + 1$	multithin identifyer (8888888.)
$7 \times (n-1) + 2$	weight for thinning mode 1
$7 \times (n-1) + 3$	weight for thinning mode 2
$7 \times (n-1) + 4$	weight for thinning mode 3
$7 \times (n-1) + 5$	weight for thinning mode 4
$7 \times (n-1) + 6$	weight for thinning mode 5
$7 \times (n-1) + 7$	weight for thinning mode 6
	for $n = 1 \ldots 39$

3   Storage and CPU-time Requirements

For the example presented in Table 14 the storage and CPU-time requirements have been determined for three different program versions:

- standard version, no option selected, MUADDI not activated

- MULTITHIN option selected, MUADDI not activated

- MULTITHIN & MUPROD & EHISTORY options selected, MUADDI activated

One single shower has been simulated for each of the different versions. In all cases identical random numbers have been used to produce showers with bit-identical particle parameters. This enables a precise determination of the storage requirements for the output particle files. The file size for the MULTITHIN option is enlarged to the double of the standard version as now each particle data entry is followed by the MULTITHIN extension entry. The enlargement of the MULTITHIN & MUPROD & EHISTORY case relative to the MULTITHIN case has to be attributed to the muon additional information (activated by keyword MUADDI). The increase in file size reaches about 10% coming from 8661 muons reaching the observation level, which produce now 5 entries instead of 2, and from 23366 muons which decay or are eliminated before reaching ground; they also contribute with 5 entries each.

Table 14: Storage and CPU-time requirements.

program options	particle output file size (MB)	CPU-time (min)
(standard, no options)	18.7	≈ 38
MULTITHIN (without MUADDI)	37.3	≈ 38
MULTITHIN & MUPROD & EHISTORY (with MUADDI)	41.3	≈ 38

primary proton, $E_0 = 10^{15}$ eV, vertical incidence, observation at 110 m a.s.l.
$E_{cut,hadr}$=0.1 GeV, $E_{cut,\mu}$=0.1 GeV, $E_{cut,electr}$=250 eV, $E_{cut,\gamma}$=250 eV

For the example of Table 14 the grid engine of the Institut für Kernphysik of the KIT has been used. The CPU-times are dominated by the availability of the data transfer channels to the output files. For all three program versions they vary between ≈ 25 min and ≈ 50 min, depending on the load of the grid engine.

A   Particle Stacks and Parameters in the Stack

The Table 15 gives the different registers which are used in CORSIKA. The registers denoted by xxxPAR contain just one particle and the meanings of the array positions are explained in Table 16. Several of the array extensions depend on the used options. They are marked by different colours.

Table 15: Particle registers/stacks used in CORSIKA

register / stack name	number of particles	content	located in common / routine
PRMPAR	1	primary particle	common PARPAR
CURPAR	1	current particle	common PARPAR
PROPAR	1	propagated particle	AAMAIN, BOX3, MUTRAC
SECPAR	1	secondary particle	common PARPAR
OUTPAR	1	output / propagated particle	common PARPAR
AMUPAR	1	additional muon info	common MUPART
CUTPAR	1	cutted particle (PARALLEL)	common PARPAR
STACKINT	200 000	intermediate stack for thinning	common STACKINT
STACKI	680 624 (THIN) 512 (CURVED) 50 000 (PARALLEL)	intermediate stack for ext. file	common STACKF
STACKJ	50 000 (PARALLEL)	intermediate stack	common STACKF
DATAB	39	output buffer	common BUFFS
(MEXST)	∞	particle stack	(scratch file)
(STACKE)	60	em-particle stack	common STACKE

Table 16: Particle stack with extensions for THIN, CURVED, INTTEST, PARALLEL, and MULTITHIN options.

current particle stack position	equivalence	mother particle stack position	grandma particle stack position	meaning
CURPAR(0)	(ITYPE)	SECPAR(17)	SECPAR(28)	particle identifier
CURPAR(1)	GAMMA	SECRAR(18)	SECPAR(29)	γ Lorentz factor (or energy)
CURPAR(2)	COSTHE	SECPAR(19)	SECPAR(30)	$\cos(\theta)$ vertical direction cosine [1]
CURPAR(3)	PHIX	SECPAR(20)	SECPAR(31)	$\sin(\theta)\cdot\cos(\phi)$ horizontal direction cosine [1]
CURPAR(4)	PHIY	SECPAR(21)	SECPAR(32)	$\sin(\theta)\cdot\sin(\phi)$ horizontal direction cosine [1]
CURPAR(5)	H	SECPAR(22)	SECPAR(33)	height [cm]
CURPAR(6)	T	SECPAR(23)	SECPAR(34)	time [sec] (since first interation)
CURPAR(7)	X	SECPAR(24)	SECPAR(35)	horizontal position [cm]
CURPAR(8)	Y	SECPAR(25)	SECPAR(36)	horizontal position [cm]
CURPAR(9)	(GEN)			generation counter of mother particle
CURPAR(10)	CHI			χ depth to next interaction [g·cm^{-2}]
CURPAR(11)	BETA			β =v/c fraction of speed of light
CURPAR(12)	GCM			γ Lorentz factor in cms-system
CURPAR(13)	ECM			energy in cms-system [GeV]
CURPAR(13)	WEIGHT	SECPAR(26)	SECPAR(37)	weight for thinning (THIN option)
CURPAR(16)	COSTEA			$\cos(\theta_E)$ angle at Earth center cosine (CURVED option)
CURPAR(15)	COSTAP			$\cos(\theta*)$ apparent zenith angle cosine (CURVED option)
CURPAR(14)	HAPP			apparent height [cm] (CURVED option)
CURPAR(17)				transverse momentum [GeV/c] (INTTEST option)
CUTPAR(18)		SECPAR(27)	SECPAR(38)	seed (PARALLEL option)
				(not used)
CURPAR(39)				ECT-flag (PARALLEL option)
CURPAR(40)				(not used)
CURPAR(41 - 46)				weights (MULTITHIN option)

[1] θ = zenith angle, ϕ = azimuth angle

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