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A. Kirsch

A. Rieder

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Anschriften der Verfasser:

Prof. Dr. Andreas Kirsch
Institut für Algebra und Geometrie
Karlsruher Institut für Technologie (KIT)
D-76128 Karlsruhe

Prof. Dr. Andreas Rieder
Institut für Angewandte und Numerische Mathematik
Karlsruher Institut für Technologie (KIT)
D-76128 Karlsruhe

SEISMIC TOMOGRAPHY IS LOCALLY ILL-POSED

ANDREAS KIRSCH AND ANDREAS RIEDER

Abstract. In this note we show that the nonlinear problem of seismology in the acoustic regime is locally ill-posed in the strict mathematical sense.

1. Introduction

Seismic tomography entails the reconstruction of subsurface material parameters (e.g. mass density, Lamé constants) from reflected waves measured on a part of the propagation medium (typically an area on the earth's surface or in the ocean). From a mathematical point of view we have to solve a nonlinear parameter identification problem for the elastic wave equation, see, e.g., [9] for an overview on the mathematical aspects of reflection seismology.

If we assume that the medium only supports normal stress we are in the regime of acoustic waves which are solutions u of the acoustic wave equation

$$c\,\partial_t^2 u - \nabla_{\mathbf{x}} \cdot \big(r \nabla_{\mathbf{x}} u\big) = f(\mathbf{x}, t) \tag{1}$$

with

$$c = \frac{1}{\varrho\,\nu^2} \quad \text{and} \quad r = \frac{1}{\varrho}$$

where ν and ϱ are the wave speed and the mass density of the medium, respectively. Both coefficients are real valued and spatially varying: $\nu = \nu(\mathbf{x})$, $\varrho = \varrho(\mathbf{x})$. The forcing term f is the energy source which excites wave propagation. Under appropriate requirements as well as initial and boundary conditions, (1) admits a unique solution (see next section).

In the present work we study the parameter-to-solution map $F\colon (c, r) \mapsto u$ on meaningful function spaces. Our main result (Theorem 3.5 below) is the local ill-posedness of the inverse problem of seismology, i.e., of the operator equation

$$\Psi F(c, r) = \Psi u$$

where the operator Ψ models the measurement procedure taking into account that the wave u can only be observed in a small area of the propagation medium.

This paper is organized as follows. In the next section we recall the definition of local ill-posedness of operator equations on abstract Banach spaces. Further, we slightly modify a well-known criterion to diagnose local ill-posedness by means of properties of the involved spaces and the nonlinear operator. Then, we provide the functional analytic framework in which we validate all properties of the parameter-to-solution map F to convict seismic tomography of ill-posedness. To our knowledge this is the first mathematically rigorous presentation which, moreover, requires only application-oriented assumptions.

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2. The abstract setting

We consider nonlinear inverse problems of the following kind: Given $y \in Y$ find $x \in X$ such that

$$F(x) = y \tag{2}$$

where the mapping $F\colon \mathsf{D}(F) \subset X \to Y$ operates between the infinite dimensional Banach spaces X and Y.

Following Hofmann [2] (see also [3] and [7, Def. 3.15]) we call (2) *locally ill-posed* in $x^+ \in \mathsf{D}(F)$ satisfying $F(x^+) = y$ if in any neighborhood of x^+ a sequence $\{x_k\}_{k\in\mathbb{N}} \subset \mathsf{D}(F)$ can be found such that

$$\lim_{k\to\infty} \|F(x_k) - F(x^+)\|_Y = 0, \text{ however } \|x_k - x^+\|_X \not\to 0 \text{ for } k \to \infty.$$

This definition provides the right mathematical description of the effect typically associated with ill-posedness: the solution x^+ does not depend continuously on the data y; small perturbations in y might be amplified in the reconstruction process.

The following criterion for local ill-posedness is a simple variation of Proposition 3.16 from [7] which we adapted to our needs. To this end we suppose X to be the dual of a further Banach space Z: $X = Z'$, that is, the weak-$\star$ topology is defined on X. The concrete situation we have in mind is $X = L^\infty(\Omega)$ and $Z = L^1(\Omega)$: a sequence $\{w_k\} \subset L^\infty(\Omega)$ converges to w in the weak-$\star$ topology ($w_k \overset{\star}{\rightharpoonup} w$) if

$$\int_\Omega w_k(\mathbf{x})v(\mathbf{x})\mathrm{d}\mathbf{x} \xrightarrow{k\to\infty} \int_\Omega w(\mathbf{x})v(\mathbf{x})\mathrm{d}\mathbf{x} \quad \text{for any } v \in L^1(\Omega). \tag{3}$$

For the duality relation $L^\infty(\Omega) = L^1(\Omega)'$, see, e.g., [5, Chap. VII §2].

Proposition 2.1. *Let F be compact*[1] *and weak-$\star$-to-weak continuous. Further, let $x^+ \in \mathsf{D}(F)$ satisfy $F(x^+) = y$ and assume that there exists a sequence $\{e_k\}_{k\in\mathbb{N}} \subset \mathsf{D}(F)$, which is normalized, $\|e_k\|_X = 1$, and which converges weakly-$\star$ to 0 such that $\{x^+ + re_k\} \subset \mathsf{D}(F)$ for any $r \in]0,1]$.*

Then, (2) is locally ill-posed in x^+.

Proof. Define $x_k := x^+ + re_k$ for $r \in]0,1]$. Given a neighborhood of x^+ we can find an r such that $\{x_k\}$ is contained in this neighborhood. We have $x_k \overset{\star}{\rightharpoonup} x^+$, but $\|x_k - x^+\|_X = r \not\to 0$. Further, $F(x_k) \rightharpoonup F(x^+)$ due to the weak-$\star$-to-weak continuity and this weak convergence is even strong as F is compact. □

3. Seismic tomography

3.1. **The forward model.** Acoustic wave propagation is modeled by the acoustic wave equation (1) which we consider in $\Omega \subset \mathbb{R}^d$, an open, measurable and bounded set with a smooth boundary. Moreover, we require a Dirichlet boundary condition,

$$u|_{\partial\Omega} = 0, \tag{4}$$

as well as initial data

$$u(\cdot,0) = u_0; \quad \partial_t u(\cdot,0) = u_1. \tag{5}$$

[1] F maps bounded sets to relatively compact ones.

Remark 3.1. *The Dirichlet boundary restriction* (4) *is quite meaningful in the framework of seismic wave propagation. According to the finite wave speed and the finite observation time, homogeneous boundary conditions can be assumed if* Ω *is chosen sufficiently large. For more details see* [4, Remark 2.2].

Let $V = H_0^1(\Omega)$ and $H = L^2(\Omega)$. We search for the solution u in the space

$$X := \mathcal{C}^0([0,T],V) \cap \mathcal{C}^1([0,T],H)$$

equipped with the canonical norm

$$\|u\|_X := \left(\max_{0\le t\le T} \|u(t)\|_V^2 + \max_{0\le t\le T} \|\dot u(t)\|_H^2 \right)^{1/2}$$

which makes X a Banach space. The dot on top of a time dependent function indicates the time derivative.

The weak formulation of (1) with (4) and (5) reads:

Given $c, r \in L^\infty(\Omega)$ and $u_0 \in V$ and $u_1 \in H$ and $f \in L^2\big([0,T]\times\Omega\big) = L^2\big((0,T),H\big)$ find $u \in X$ with $u(0) = u_0$ and $\dot u(0) = u_1$ such that

$$\int_0^T \Big(a_r\big(u(t), v(t)\big) - \langle c\dot u(t), \dot v(t)\rangle_H \Big) \mathrm{d}t \;=\; \int_0^T \langle f(t), v(t)\rangle_H \,\mathrm{d}t \tag{6}$$

for all $v \in \mathcal{C}_0^\infty([0,T],V)$,

see e.g. Lions and Magenes [6, Chap. 3.8] or Stolk [8, Sec. 2.4]. Here, $\langle\cdot,\cdot\rangle_H$ denotes the inner product in H and

$$a_r \colon V \times V \to \mathbb{R}, \quad a_r(\psi,\varphi) = \int_\Omega r \nabla_{\mathbf{x}}\psi \cdot \nabla_{\mathbf{x}}\varphi \,\mathrm{d}\mathbf{x}.$$

We assume that the real-valued coefficients $c, r \in L^\infty(\Omega)$ are bounded away from zero; that is,

$$c(\mathbf{x}) \ge \mathrm{c}_- \text{ a.e. and } r(\mathbf{x}) \ge \mathrm{r}_- \text{ a.e.} \tag{7}$$

for positive constants c_- and r_-.

Under the above hypotheses Stolk [8, Lemma 2.4.1] has shown that (6) admits a unique solution $u \in X$ which depends continuously on u_0, u_1, and f. Moreover, an energy estimate for u holds[2]:

$$\|u(t)\|_V^2 + \|\dot u(t)\|_H^2 \lesssim \|u_0\|_V^2 + \|u_1\|_H^2 + \int_0^T \|f(\tau)\|_H^2 \,\mathrm{d}\tau, \tag{8}$$

see also [6, Chap. 3.8]. Note that the constant in the above estimate only depends on c_-, r_-, $\|\mathrm{c}\|_\infty$, and $\|\mathrm{r}\|_\infty$. We recall from [4, Sec. 2] that, for almost all $s \in]0,T[$,

$$a_r(u(s), w) + \langle c\ddot u(s), w\rangle_{V'\times V} = \langle f(s), w\rangle_H \quad \text{for all } w \in V, \tag{9}$$

where $\langle\cdot,\cdot\rangle_{V'\times V}$ is the duality paring in $V' \times V$.

Now we are able to define the forward map

$$F \colon \mathsf{D}(F) \subset L^\infty(\Omega) \times L^\infty(\Omega) \to L^2([0,T]\times\Omega) \quad (c,r) \mapsto u, \tag{10}$$

[2] $A \lesssim B$ indicates the existence of a generic constant m such that $A \le mB$ uniformly in all relevant parameters of the expressions A and B. The respective context will define the meaning of 'relevant parameters'.

where $u \in X$ solves (1) in the weak sense (6) and the domain of definition of F is

$$\mathsf{D}(F) = \{(c, r) \in L^\infty(\Omega) \times L^\infty(\Omega) : c(\mathbf{x}) \geq \mathrm{k}_-,\ r(\mathbf{x}) \geq \mathrm{k}_-,\ \text{a.e.}\}$$

where $\mathrm{k}_- := \max\{\mathrm{c}_-, \mathrm{r}_-\}$.

3.2. **The inverse problem.** To frame the inverse problem of seismic tomography in the acoustic regime we introduce a continuous measurement operator $\Psi\colon L^2([0,T] \times \Omega) \to L^2([0,T] \times M)$, $M \subset \Omega$. For instance, $\Psi u = \int \psi(\cdot, \cdot, t, \mathbf{x}) u(t, \mathbf{x}) \mathrm{d}t \mathrm{d}\mathbf{x}$ where the kernel function ψ models the measurement equipment. One can think of $\psi(\cdot, \cdot, t, \mathbf{x})$ as a family of approximate Dirac distribution. The inverse problem under investigation now reads: Given $\widetilde{u} = \Psi u$ find $(c, r) \in \mathsf{D}(F)$ such that

$$\Psi F(c, r) = \widetilde{u}. \tag{11}$$

In the following we show that the above problem is locally ill-posed anywhere in $\mathsf{D}(F)$ by checking the hypotheses of Proposition 2.1.

Proposition 3.2. *The mapping F as defined in* (10) *is weak-$\star$-to-weak continuous.*

Proof. We follow the line of arguments of the proof of Theorem 2.8.1 in [8].

Let $\{(c_m, r_m)\}_{m \in \mathbb{N}} \subset \mathsf{D}(F)$ converge weakly-$\star$ to $(c, r) \in \mathsf{D}(F)$ in $L^\infty(\Omega)^2$. Recall that weakly-$\star$ and weakly convergent sequences are bounded which is a consequence of the uniform boundedness principle.

Denote by $u_m \in X$ and $u \in X$ the solutions of (6) with parameters (c_m, r_m) and (c, r), respectively. By (8) both sequences $\{u_m\}$ and $\{\dot{u}_m\}$ are bounded in $L^2([0,T], V)$ and $L^2([0,T], H)$, respectively. As both spaces are reflexive, each sequence admits weakly convergent subsequences $\{u_{m_l}\}_{l \in \mathbb{N}}$ and $\{\dot{u}_{m_l}\}_{l \in \mathbb{N}}$ with limits η and ξ, respectively. Since the derivative is defined weakly, we even have that $\dot{\eta} = \xi$.

We will show now that η solves (6). To this end let $v \in \mathcal{C}_0^\infty([0,T], V)$ and consider

$$\int_0^T \Big(a_{r_{m_l}}\big(u_{m_l}(t), v(t)\big) - a_r\big(\eta(t), v(t)\big) \Big) \mathrm{d}t = \int_0^T a_{r_{m_l} - r}\big(u_{m_l}(t), v(t)\big) \mathrm{d}t + \int_0^T a_r\big(u_{m_l}(t) - \eta(t), v(t)\big) \mathrm{d}t.$$

The second term on the right hand side converges to zero because $u_{m_l} \rightharpoonup \eta$ in $L^2([0,T], V)$. For the first term we get

$$\begin{aligned}\Big| \int_0^T a_{r_{m_l} - r}\big(u_{m_l}(t), v(t)\big) \mathrm{d}t \Big| &\leq \|(r_{m_l} - r) \nabla_{\mathbf{x}} v\|_{L^2([0,T], H^d)} \|u_{m_l}\|_{L^2([0,T], V)} \\ &\lesssim \|(r_{m_l} - r) \nabla_{\mathbf{x}} v\|_{L^2([0,T], H^d)}\end{aligned}$$

where the latter estimate holds true due to the boundedness of $\{u_{m_l}\}$. As $\{r_{m_l}\}$ converges weakly-$\star$ to r, it, in particular, converges to r point-wise almost everywhere which follows immediately from (3). Further, $|(r_{m_l} - r) \nabla_{\mathbf{x}} v|^2 \lesssim |\nabla_{\mathbf{x}} v|^2$ a.e. in $\Omega \times [0,T]$. Applying the dominated convergence theorem yields that

$$\int_0^T \Big(a_{r_{m_l}}\big(u_{m_l}(t), v(t)\big) - a_r\big(\eta(t), v(t)\big) \Big) \mathrm{d}t \xrightarrow{l \to \infty} 0.$$

Analogously,

$$\int_0^T \Big(\big\langle c_{m_l} \dot{u}_{m_l}(t), \dot{v}(t) \big\rangle_H - \big\langle c \dot{\eta}(t), \dot{v}(t) \big\rangle_H \Big) \mathrm{d}t \xrightarrow{l \to \infty} 0.$$

Hence, η satisfies the wave equation in the weak form (6). Moreover, η attains the initial values

$$\eta(0) = u_0 \text{ and } \dot\eta(0) = u_1 \tag{12}$$

which we validate by a standard argument, see, e.g., [1, Chap. 7.2]: We emphasize that (9) holds for η. It also holds for u_{m_l} when we replace c and r by c_{m_l} and r_{m_l}, respectively. Both resulting equations we test with $w = w(s) = \phi(s)v$, $v \in V$, $\phi \in \mathcal{C}^2(0,T)$, $\phi(T) = \dot\phi(T) = 0$, integrate then with respect to s over $[0,T]$, and, finally, integrate in parts twice to obtain

$$\int_0^T \Big(a_r(\eta(s), w(s)) + \langle c\eta(s), \ddot w(s)\rangle_H\Big)\mathrm{d}s = \int_0^T \langle f(s), w(s)\rangle_H \mathrm{d}s \\ + \phi(0)\,\langle c\dot\eta(0), v\rangle_H - \dot\phi(0)\,\langle c\eta(0), v\rangle_H$$

as well as

$$\int_0^T \Big(a_{r_{m_l}}(u_{m_l}(s), w(s)) + \langle c_{m_l} u_{m_l}(s), \ddot w(s)\rangle_H\Big)\mathrm{d}s = \int_0^T \langle f(s), w(s)\rangle_H \mathrm{d}s \\ + \phi(0)\,\langle c_{m_l} u_1, v\rangle_H - \dot\phi(0)\,\langle c_{m_l} u_0, v\rangle_H.$$

In the limit we conclude (12) since v and ϕ are arbitrary.

Thus, $u = \eta$ and the whole sequence $\{u_m\}$ converges weakly to u because all convergent subsequences of $\{u_m\}$ have the limit u. □

Remark 3.3. *Note that we have actually validated the weak-⋆-to-weak continuity of F as a mapping from $L^\infty(\Omega)^2$ to $L^2([0,T],V)$. However, a sequence, which converges weakly in $L^2([0,T],V)$, converges, in particular, weakly in $L^2([0,T],H)$. This can be seen from the fact that $\langle v, w\rangle_{V'\times V} = \langle v, w\rangle_H$ in case $v \in H$.*[3]

Proposition 3.4. *The map F as defined in* (10) *is compact.*

Proof. Let $Q \subset \mathsf{D}(F)$ be bounded, that is, there exist $\mathrm{k}_+ > 0$ with $\mathrm{k}_- \le c(\mathbf{x}) \le \mathrm{k}_+$ and $\mathrm{k}_- \le r(\mathbf{x}) \le \mathrm{k}_+$ for almost all $\mathbf{x} \in \Omega$. Let $F(c,r) = u \in X$ be the corresponding solution. We show that $F(Q)$ is relatively compact in $\mathcal{C}([0,T],H)$ by the (general) theorem of Arcela-Ascoli, see, e.g., [5, Chap. III §3]. Indeed, for any $t \in [0,T]$ we have, in view of (8), that $\{u(t) : u \in F(Q)\} \subset \{v \in V : \|\nabla_{\mathbf{x}} v\|_{L^2(\Omega)^d} \le \hat c\}$ for some $\hat c$, and the latter set is relatively compact in $H = L^2(\Omega)$. Furthermore, $F(Q)$ is equi-continuous because, for $u \in F(Q)$,

$$\|u(t_2) - u(t_1)\|_H = \sup_{\|\psi\|_H = 1} \big\langle u(t_2) - u(t_1), \psi\big\rangle_H = \sup_{\|\psi\|_H=1} \int_{t_1}^{t_2} \frac{\mathrm{d}}{\mathrm{d}s}\big\langle u(s), \psi\big\rangle_H \,\mathrm{d}s \\ = \sup_{\|\psi\|_H=1} \int_{t_1}^{t_2} \big\langle \dot u(s), \psi\big\rangle_H \,\mathrm{d}s \le |t_2 - t_1|\,\|\dot u\|_{\mathcal{C}([0,T],H)}$$

and $\|\dot u\|_{\mathcal{C}([0,T],H)}$ is uniformly bounded for $(c,r) \in Q$ due to (8).

The continuous embedding $\mathcal{C}([0,T],H) \hookrightarrow L^2([0,T],H)$ finishes the proof. □

Now we are prepared for the main result.

[3]The three Hilbert spaces V, H, and V' constitute a Gelfand triple: the spaces are continuously embedded, $V \hookrightarrow H \hookrightarrow V'$, and the duality pairing $\langle\cdot,\cdot\rangle_{V'\times V}$ is an extension of the inner product in H.

Theorem 3.5. *Let the mapping F be defined in* (10). *Then, the inverse problem of seismic imaging* (11) *is locally ill-posed in any point* $(r_0, c_0) \in \mathsf{D}(F)$ *satisfying* $\Psi F(r_0, c_0) = \widetilde{u}$.

Proof. The assertion follows readily from Proposition 2.1 as soon as we have found a sequence $\{e_n\} \subset L^\infty(\Omega)$ with $e_n \geq 0$ a.e., $\|e_n\|_\infty = 1$, and $e_n \overset{\star}{\rightharpoonup} 0$.

Take a point $\xi \in \Omega$. Then there is a ball $B_\rho(\xi)$ with radius $\rho > 0$ completely contained in Ω. Let $\{\rho_n\}$ be a monotone zero sequence with $\rho_1 \leq \rho$. Define e_n to be the indicator function of $B_{\rho_n}(\xi)$. Obviously, $e_n \geq 0$ and $\|e_n\|_\infty = 1$. It remains to show that $\{e_n\}$ converges weakly-$\star$ to 0. Let $v \in L^1(\Omega)$. The convergence of $\int_\omega e_n v \,\mathrm{d}\mathbf{x}$ to 0 as $n \to \infty$ is a straightforward consequence of the dominated convergence theorem. □

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Department of Mathematics, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany

E-mail address: andreas.kirsch@kit.edu

E-mail address: andreas.rieder@kit.edu

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Karlsruher Institut für Technologie (KIT)
Institut für Wissenschaftliches Rechnen
und Mathematische Modellbildung

Prof. Dr. Christian Wieners
Geschäftsführender Direktor

Campus Süd
Engesserstr. 6
76131 Karlsruhe

E-Mail:Bettina.Haindl@kit.edu

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Herausgeber

Karlsruher Institut für Technologie (KIT)
Kaiserstraße 12 | 76131 Karlsruhe

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