SCIENTIFIC REPORTS

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A meta-substrate to enhance the bandwidth of metamaterials

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We propose the concept of a meta-substrate to broaden the bandwidth of left-handed metamaterials. The meta-substrate, which behaves like an inhomogeneous magnetic substrate, is composed of another kind of magnetic metamaterials like metallic closed rings. When conventional metamaterial rings are printed on this kind of meta-substrate in a proper way, the interaction of the metamaterials units can be greatly enhanced, yielding an increased bandwidth of negative permeability. An equivalent circuit analytical model is used to quantitatively characterize this phenomenon. Both numerical and experimental demonstrations are carried out, showing good agreement with theoretical predictions.

With tremendous development in the past decade, the performance of the left-handed metamaterial (LHM) still needs improvement before efficient applications can be developed. In particular, bandwidth is one of the items frequently mentioned as needing the most attention. Some applications like invisibility cloaking[1–5] are calling for metamaterial with broad frequency band or lower dispersions. As metamaterial with negative permeability is always involved with magnetic resonant structure and is highly dispersive, negative permeability is limited in a certain frequency band above the magnetic resonance frequency[6–9]. In this paper, we address the specific problem of bandwidth by reporting both theoretical and experimental results on a meta-substrate that can be used to extend the bandwidth of the LHMs. The meta-substrate is composed of another kind of magnetic metamaterial and behaves like an inhomogeneous magnetic substrate. Unlike conventional substrate realized with natural magnetic materials where the magnetic activity tends to tail off at frequencies of even a few gigahertz, the proposed meta-substrate can extend the magnetic activity to much higher frequencies, and therefore is very useful to design broadband left-handed metamaterials. We put the LHM on the meta-substrate in such a way that the substrate with higher effective permeability locates in the interior area of the LHM metallic patterns, while substrate with lower effective permeability locates in the outside area of the LHM pattern. The coupling of the metamaterial units on the meta-substrate can be strongly enhanced, and therefore, the bandwidth of the left-handed metamaterials can be remarkably broadened. Both numerical and experimental results confirm the theoretical predictions. To show that the concept is applicable to a wide range of LHM structures, we discuss both various split rings as well as S-string resonators.

Results

To better illustrate the concept of meta-substrate, we firstly analyze the properties of the metamaterials on an inhomogeneous magnetic substrate. Fig. 1(a) shows the schematic of the split ring resonator (SRR)[7] periodically arrayed on the substrate with a periodicity of a in the x and z directions. Different from the previous SRRs[7], here the array of SRR is printed on an inhomogeneous magnetic substrate where the permeability in the interior area of the SRR patterns is different from that outside of the SRR pattern. The substrates with the SRR are then repeated in the y direction with a periodicity of l to form a bulk metamaterial. The metamaterial formed by the SRR array exhibits a negative permeability in a certain frequency region above the magnetic resonance[7,10]. The equivalent circuit model[11] is used to analyze the impact of the inhomogeneous magnetic substrate on the bandwidth of the SRR. Suppose the substrate material encircled by the SRRs has a permeability of μ_1, while the one outside of the SRRs has a permeability of μ_2. We derived that the effective permeability of the periodic structures can be theoretically written as
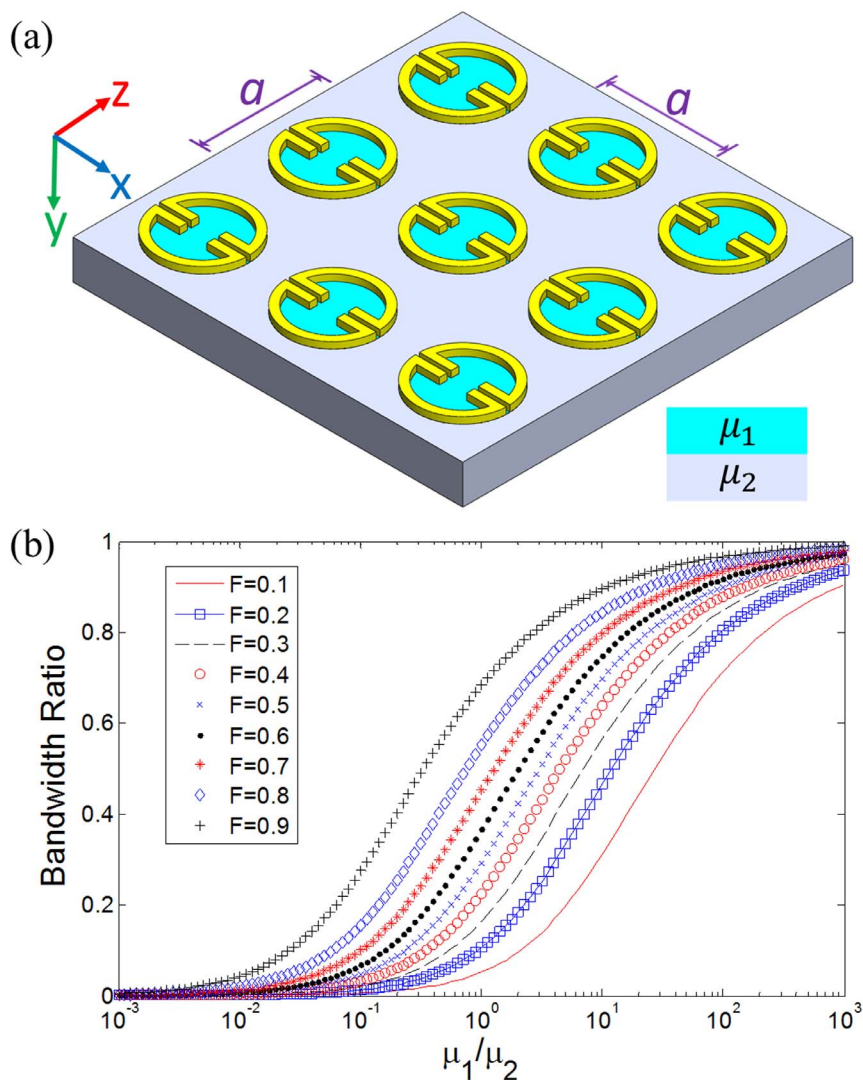
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Figure 1 | (a) Schematic of the split ring resonators printed on an inhomogeneous magnetic substrate. (b) The bandwidth of the SRRs as a function of μ_1/μ_2 for different fraction volume of F.

$$\mu_{eff} = \mu_{ave}\left(1 - \frac{\kappa}{1 - \frac{1}{\omega^2 LC} + i\frac{R}{\omega L}}\right), \quad (1)$$

where

$$\mu_{ave} = F\mu_1 + (1-F)\mu_2, \quad (2)$$

$$L = \mu_1 \pi r^2 / l \ \text{[H]}, \quad (3)$$

$$\kappa = \frac{F\mu_1}{\mu_{ave}}, \quad (4)$$

$$M = \kappa L = \frac{F\mu_1}{\mu_{ave}}L \ \text{[H]}, \quad (5)$$

In the above equations F is the fractional volume of the periodic unit cell in the xz plane occupied by the interior of the ring, μ_{ave} is the average permeability of the substrate, L is the inductance for an SRR unit, C is the equivalent capacitances in the SRR gaps, R represents the loss in the ring, and κ (with a maximum value equal to 1) is the coupling coefficient between the individual SRRs and was incorporated into the circuit model by using the mutual inductance M[11]. From equation (1), one can find that the magnetic resonant frequency is

$$\omega_{m0} = \sqrt{\frac{1}{LC}}, \quad (6)$$

and the magnetic plasma frequency is

$$\omega_{mp} = \omega_{m0}\sqrt{1 + \frac{\mu_1}{\mu_2}\frac{F}{(1-F)}}. \quad (7)$$

Equation (7) shows that one can either increase the fractional volume F, or increase the value of μ_1/μ_2 to broaden the frequency band of negative permeability. If μ_1 varies, the resonant frequency and the frequency band of negative permeability also vary, as indicated in equations (3) and (6). Therefore, in order to compare the bandwidth for the SRRs working at different frequency bands, we use the parameter of bandwidth ratio (BR), which is defined as

$$BR = \frac{\omega_{mp} - \omega_{m0}}{\omega_{mp}} \quad (8)$$

with an upper limit of 100% for comparison. Fig. 1(b) shows the theoretically calculated bandwidth ratio as a function of μ_1/μ_2 for different F. One can see that the bandwidth ratio increased dramatically when μ_1/μ_2 increased from 1 to 100, indicating the inhomogeneous magnetic substrate is very effective to increase the bandwidth of the metamaterial. Note that if $\mu_1 = \mu_2$, all the equations (1–7) are the same with those obtained from equivalent circuit model[11] for previous SRRs structures[7,8,10,12–14], which are special cases of the configuration proposed here.

Numerical simulations are carried out to confirm the theoretical results. In the simulation, we take the broad side coupled SRRs[8] shown in the inset of Fig. 2(a) as an example. From the parameters of the structures indicated in the figure caption, the fractional volume of the SRR is calculated to be $F = 0.17$. CST Microwave studio is used to simulate the SRR printed on the substrate with different value of μ_1 while keeping $\mu_2 = \mu_0$ constant. The effective permeability of SRR can be found from the refractive index n and wave impedance Z as $\mu_{eff} = nZ$, where n and Z can be retrieved[15] from the numerical measurement of the reflection coefficients and transmission coefficients of a wave normally incident onto a slab of SRR. For example,

for the SRRs printed on a conventional nonmagnetic substrate, i.e. the substrate encircled by SRRs has the same permeability $\mu_1 = \mu_0$ as those outside the ring, the retrieved results (as shown in the inset of Fig. 2(b)) show the magnetic resonant frequency is at 6.4 GHz, and the frequency band of negative permeability is from 6.4 GHz to 6.8 GHz, yielding a bandwidth ratio of 6.3%. While for the SRRs printed on an inhomogeneous magnetic substrate, where the substrate encircled by SRRs has a permeability of $\mu_1 = 4\mu_0$ and the substrate outside the SRRs has a permeability of $\mu_2 = \mu_0$, the retrieved results (as shown in the inset of Fig. 2(b)) show the magnetic resonant frequency is at 3.3 GHz, and the frequency band of negative permeability is from 3.3 GHz to 4.6 GHz, yielding a bandwidth ratio of 28%. The curves of the resonant frequency ω_{m0} and the plasma frequency ω_{mp} as a function of μ_1 are shown in Fig. 2(a). We set the horizontal axis to be $1/\sqrt{\mu_1}$ so that one can see clearly the fact that ω_{m0} is proportional to $1/\sqrt{\mu_1}$. Because the plasma frequency ω_{mp} decreases much slower than the resonant frequency ω_{m0} as μ_1 increases, the bandwidth increases as μ_1 increases. Fig. 2(b) shows the bandwidth ratio obtained from the simulation results as a function of μ_1. The theoretical results from equation (8) are also presented and
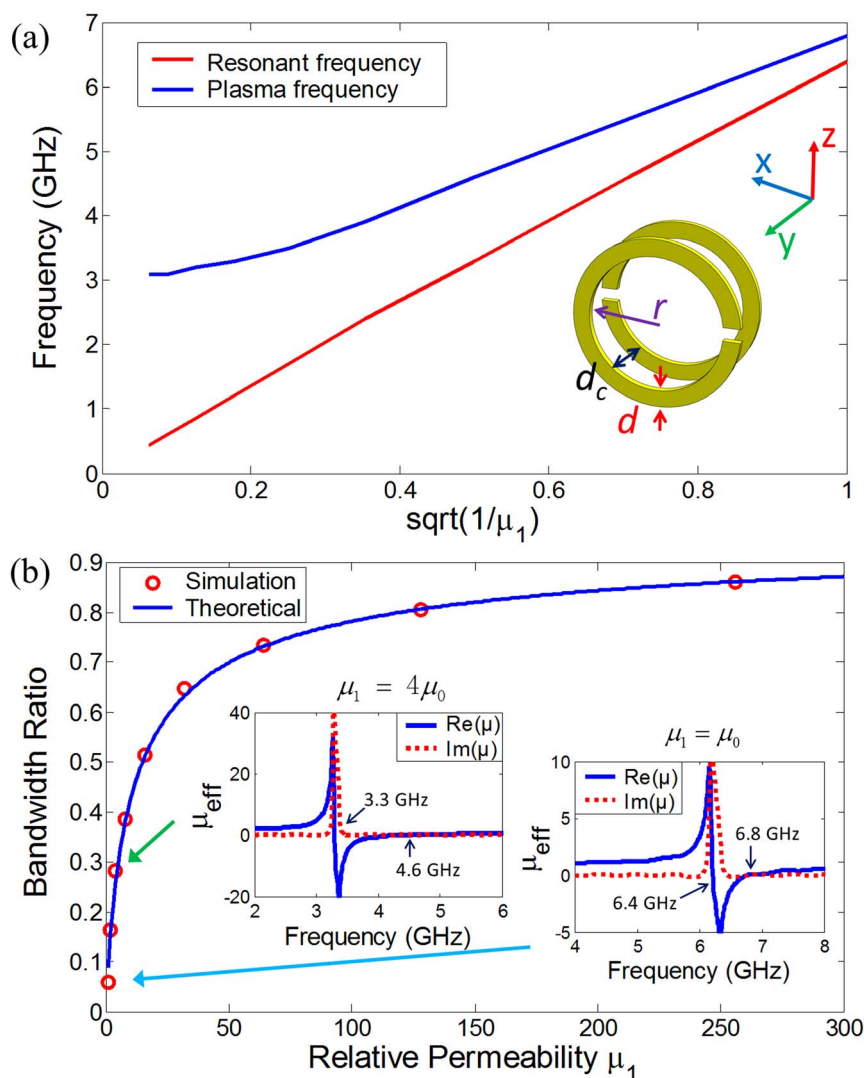

Figure 2 | (a) The simulated resonant frequency and plasma frequency of the SRR shown in the inset as a function of $1/\sqrt{\mu_1}$ while keeping $\mu_2 = \mu_0$ constant. The parameters of the SRR are: $r = 1.6$ mm, $d = 0.4$ mm, $d_c = 0.4$ mm, the period of the structure is 6 mm, 1.2 mm and 8 mm in the x, y and z directions, respectively. The fraction volume is $F = 0.17$. (b) Bandwidth ratio of the SRR as a function of μ_1, here $\mu_2 = \mu_0$. The insets show the effective permeability retrieved from numerical simulations for the two configurations of SRR: one is printed on a conventional nonmagnetic substrate with $\mu_1 = \mu_0$ and the other is printed on an inhomogeneous magnetic substrate with $\mu_1 = 4\mu_0$.

one can see that they are in good agreement with the simulation results. Both the simulation results and the theoretical results show that the bandwidth of the SRR structure is significantly increased by increasing the value of μ_1/μ_2.

From previous analysis, one can see that increasing the contrast value of μ_1/μ_2 is very helpful to increase the bandwidth. At low frequency, some materials like ferrite have a permeability with a value more than hundred[16–18], and therefore, metamaterial with broad bandwidth can be easily realized at lower frequencies. However, the magnetic activity in most materials tends to tail off at high frequencies of a few gigahertz. This makes it difficult to implement broadband metamaterials at higher frequencies. In the following we propose the concept of meta-substrate composed of artificial closed ring metamaterials[19] to broaden the bandwidth of the metamaterial.

Still based on equation (8), but this time we keep $\mu_1 = \mu_0$ constant, while decreasing μ_2 less than unity. An effective permeability less than unity can be achieved in a broad frequency band with periodically arrayed closed rings. One layer of the closed rings array is shown in Fig. 3(a). The effective permeability of such a closed ring array is $1 - F_c$, where F_c is the fractional volume of the periodic unit occupied by the closed ring[19]. The schematic of a meta-substrate is shown in Fig. 3(b). It is an inhomogeneous substrate composed of two parts: the one marked in light blue is a conventional substrate with permeability of $\mu_1 = \mu_0$, and the other part is printed with the array of the closed rings behaving like an effective magnetic material with permeability of $\mu_2 = 1 - F_c$. The arrayed SRR structure shown in Fig. 3(c) is then printed on the meta-substrate shown in Fig. 3(b) and we can finally achieve the composite SRRs on the meta-substrate, as shown in Fig. 3(d). Layers of the SRRs on the meta-substrate shown in Fig. 3(d) are then stacked to form a bulk metamaterial, whose effective permeability can be calculated as follows:

$$\mu_{eff} = [(1-F_c)(1-F)+F] - \frac{F}{1 - \frac{1}{\omega^2 LC} + i\frac{R}{\omega L}} \quad (9)$$

and

$$\frac{\omega_{mp}}{\omega_{m0}} = \sqrt{1 + \frac{1}{1-F_c}\frac{F}{(1-F)}}. \quad (10)$$

From equation (10), one can see that by increasing F_c, the frequency band of such structure with negative permeability can also be increased.

Experiments are carried out to verify the theoretical analysis. The SRR exhibits only negative permeability property and thus shows a stop band over the corresponding negative permeability frequency band. As identifying the properties of a stop band is more difficult than that of a transmission pass band, we use the S-ring resonator for experimental demonstration. The S-ring resonator exhibits simultaneously negative permittivity and permeability and thus a transmission pass band over the corresponding frequency band[9,20]. The metamaterial sample printed on the meta-substrate is shown in Fig. 4. The closed ring structure is printed on an FR4 substrate with thickness of 1 mm and relative permittivity of $\varepsilon = 4$. The periodicity of the closed ring is 2.5 mm in the y direction. A periodic unit cell measures 4 mm $\times$ 4 mm in the xz plane. The parameters of the closed rings are: $d_{out} = 3.6$ mm, $d_{in} = 3.2$ mm, For a volume of such closed ring structure, F_c is equal to 0.57, so the effective permeability, is $\mu_2 = 0.43\mu_0$. The parameters of the S-ring resonator are: t = 1 mm, a = 10.8 mm, b = 5.6 mm, h = 0.8 mm, p_x = 10 mm. The periodicity in the y direction is 2.5 mm. The S-ring resonators printed on a conventional FR4 substrate are also fabricated for comparison.

In the transmission experiments, the plane wave is incident along the x direction with a polarization of E_z. Ten unit cells in the wave propagation direction are measured. The transmission results are shown in Fig. 5. The insets show the experimental setup and the numerically retrieved permittivity and permeability of the two samples. From the results, we see that, for the S-ring resonator printed on the conventional substrate (without the closed rings), the pass band with simultaneously negative permittivity and permeability is from 4.29 GHz to 5.61 GHz, corresponding to a bandwidth ratio of 30.8%; while for the S-ring resonator printed on the meta-substrate (with the closed rings), the frequency band with simultaneously negative permittivity and permeability is from 4.08 GHz to 5.91 GHz, corresponding to a bandwidth ratio of 44.9%. The resonant frequency of the latter is a bit lower than the former because the closed rings in the meta-substrate are helpful to decrease the fringe effect of the magnetic field lines that leak from the edge of the column of the S-rings, resulting in a bigger effective inductance[11]. The experimental results are in good agreement with theoretical results predicted by equation (8) and (10), which are BR = 28% for the S-ring resonator printed on the conventional substrate, while BR = 44% for the S-ring resonator printed on the meta-substrate. Our experimental results confirm that the bandwidth of the left-handed pass band can be remarkable increased by using the inhomogeneous meta-substrate.
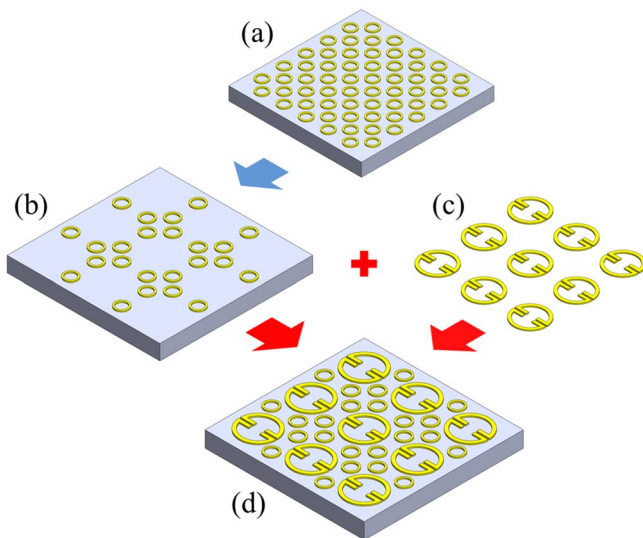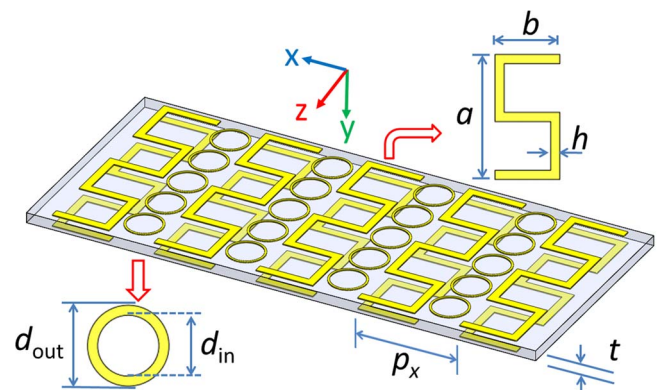

Figure 3 | (a) A magnetic substrate realized with closed rings array. (b) A meta-substrate realized with inhomogeneous closed rings. (c) Schematic of the SRRs to be printed onto the meta-substrate. (d) A metamaterial realized with the SRRs on the meta-substrate.

Figure 4 | Sample of the S-ring resonator printed on the meta-substrate composed of the closed rings.
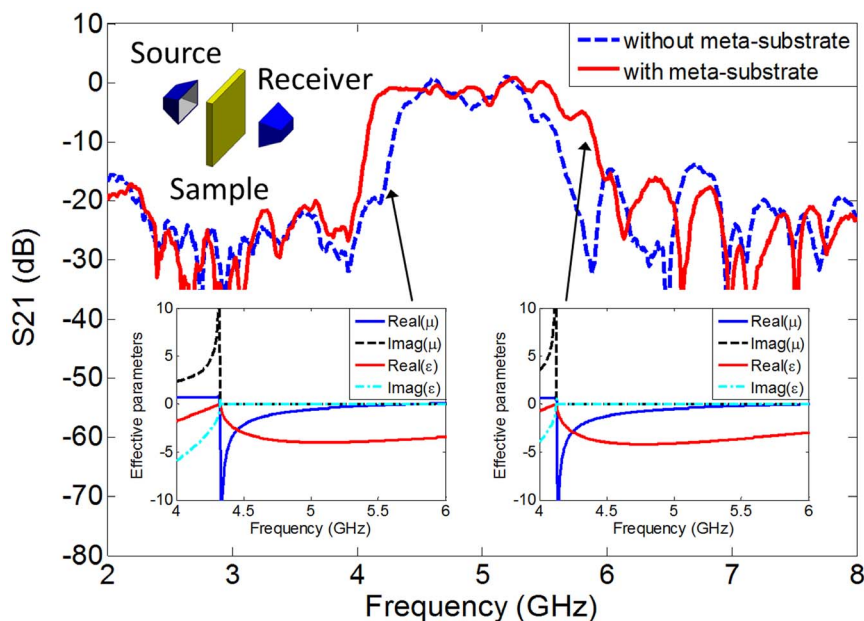
Figure 5 | Transmission experimental results measured for the S-ring on the conventional substrate and that on the meta-substrate. The insets show the experimental setup and the numerically retrieved constitutive parameters for the two samples.

Discussion

The reason why the bandwidth can be broadened by the meta-substrate can be explained as follows: when a time-varying external field is applied on the magnetic resonant metamaterial, e.g. SRRs, currents flow in the resonant units and generate depolarization magnetic field. The depolarization magnetic fields will fall both into the area encircled by other resonant units and into the area encircled by the closed rings. The bandwidth of the metamaterial is determined by the coupling between the metamaterial resonant units, i.e. by how many fraction of the depolarization magnetic fields falling into other metamaterial resonant units, not those falling into the closed rings. When the time varying depolarization fields are falling to the closed rings, the closed rings can generate magnetic fields opposite to the depolarization field to repel the depolarization fields by inducing current loops. As a result, these repelled depolarization fields will be added into the area encircled by other metamaterial resonant units and therefore, the bandwidth is broadened. The closed rings are not necessarily in the same layer of the SRRs. As long as the magnetic fields generated by the current flowing around the closed rings are in the external area of the magnetic resonant rings, the bandwidth of the metamaterial can be broadened. Reversely, if the closed rings locate inside of the SRRs, the bandwidth will decrease. The concept of the meta-substrate can be readily extended to THz and optical frequencies as S-strings and metallic closed and split rings have been realized using micro- and nanofabrication techniques[14,20,21]. It should also be noted that the closed rings also exhibit electrical responses. When applying this methodology, one should keep the closed rings working in their long operating wavelength region so that their electric response is not in the resonance region and may not deteriorate the magnetic response of the SRRs.

In conclusion, a meta-substrate with closed rings, behaving like an inhomogeneous magnetic substrate, is proposed to broaden the bandwidth of the metamaterial. The relation between the bandwidth and the meta-substrate parameters are shown theoretically and verified by both numerical and experimental results. As the magnetic activity in most materials substrates tends to tail off at high frequencies of even a few gigahertz, the meta-substrate with closed rings shows superior advantage that it can be easily extended to higher frequencies, and therefore is very useful to broaden the frequency band of metamaterials.

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Acknowledgments

Author contributions

H.C. conceived the idea of the study. H.C. and Z.W. performed the analysis. Z.W. and R.Z. carried out the experiment. H.C., Z.W., H.W., S.L., F.Y. and H.M. contributed in the calculation and interpretation. H.C. wrote the manuscript with input from others.

Additional information

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Chen, H.S. *et al.* A meta-substrate to enhance the bandwidth of metamaterials. *Sci. Rep.* **4**, 5264; DOI:10.1038/srep05264 (2014).