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AN ENERGETIC APPROACH FOR INTERFACE DELAMINATION PROBLEM IN SLIDING MODE WITH COHESIVE OR ADHESIVE CONTACT

ENERGETICKÝ PRÍSTUP RIEŠENIA PROBLÉMU DELAMINÁCIE V ŠMYKOVOM MÓDE NA ROZHRANÍ S KOHÉZNYM A ADHÉZNYM KONTAKTOM

Abstract

A new interface delamination model of a layered structure is discussed. The concept of the solution is developed by the mathematical model of interface contact damage. The proposed numerical solution considers the rate independent evolution at small strain and the concept of the energetic solution. Applied energetic formulation governs the debonding process by means of two interface variables. Presented numerical example demonstrates its theoretical applicability of the whole solution approach in the engineering practise.

Keywords

Interface fracture, energetic formulation, sliding crack mode, alternating quadratic minimization, adhesive contact, cohesive contact, mathematical model, delamination process.

Abstrakt

V príspevku je popísaný nový model porušenia rozhrania viacvrstvovej štruktúry. Koncept riešenia predstavuje navrhnutý matematický model porušenia kontaktu na rozhraní. Navrhnuté numerické riešenie využíva rýchlostne nezávislý vývoj napätí a energetický koncept riešenia. Energetická formulácia riadi proces delaminácie prostredníctvom dvoch parametrov rozhrania. Prezentovaný príklad demonštruje teoretický prínos v oblasti aplikácie v inžinierskej praxi.

Kľúčové slová

Porušenie rozhrania, energetická formulácia, šmykový mód porušenia, algoritmus striedavej minimalizácie, adhézny kontakt, kohézny kontakt, matematický model, proces delaminácie.

1 INTRODUCTION

Recently, the analysis of crack propagation process has considerably influenced design and development of engineering constructions and materials. Therefore, considering the numerical approaches and developing of the mathematical models for solving the fracture problems enable its efficient applicability in engineering practice. The general objective of this study is the mutual comparison of the adhesive and cohesive contact model behaviour at the delamination process. Thus, the main motivation was to develop a useful numerical tool based on energetic principle and variational formulations. The interface as a zone of interest of this study is commonly represented by

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a thin layer which can be partially or fully damaged. Such a process is frequently referred to as *delamination* [2]. The aforementioned failure mechanism is given by the crack initiation and propagation which occurs in an interface and depends on the applied load. The process of quasistatic evolution is motivated by *Griffith criterion*, see Section 4.1 [1], [5].

In this study, the numerical approach for investigation of the interface failure mechanism based on microscopic analysis and plasticity is discussed, see [8],[14]. The presented model of interface damage determines two active interface variables which conveniently describe a plasticity of the interface and its failure. First, a *damage parameter* ζ defines the level of rupture of the interface due to delamination. Second, a *plastic slip* π describes a plastic deformation that may appear in the interface tangent direction before the rupture of adjacent bulks as it is discussed in [4], [7], [10]. The presented contribution discusses the effect of crack propagation by means of comparison of analysed models with either type of contact, respectively in the sliding crack mode (Mode II).

The comprehensive mathematical solution was acquired by assuming the *variational methods* which suppose the equilibrium between energy stored in structure and energy which is dissipated from the system in consequence of delamination process [13], [15], [16]. The essential idea of this study is based on the hypothesis that the failure mechanism is governed by a time dependent Gibbs's *stored energy functional* E_S (involving the external loading) and *dissipation potential* R, which reflects the *rate-independence* of the process. Both of mentioned functionals are defined on suitable state space. We consider that R involves only ζ and π components of a state $q = (u, \zeta, \pi)$. Where u represents the *displacement* field, ζ expresses the *internal variable* of the damage process and π describes the influence of plastic slip, see [5], [15]. The frequent effect of numerical algorithms is the *non-convex character* of the total *energy functional* [5], [13], [15], [16]. Additionally, the functional has to be considered with constraints given by the character of the aforementioned variables π and ζ. Therefore, applying the special numerical treatment (*alternative minimization algorithm*), the expected quadratic character of the energy functional was obtained and it refers to *minimum dissipation-potential principle.* The obtained solution exploits the advantageous properties of *conjugate gradient algorithms* for minimizing procedure of non-convex total energy functional H [6]. The suggested approach has been applied and tested in MATLAB [11], see Section 6.

2 A CONCEPTUAL MODEL OF INTERFACE DAMAGE AND PLASTICITY

An essential concept of investigated layered structure has been defined by a planar domain $\Omega \subset R^2$ with a bounded *Lipschitz boundary* $\partial\Omega = \Gamma$. Considering its decomposition, we acquire two non-overlapping subdomains Ω^A and Ω^B with respective boundaries Γ^A and Γ^B, for simplification can be expressed in the form Γ^η $\eta = A, B$, see Fig. 1 [2], [3].

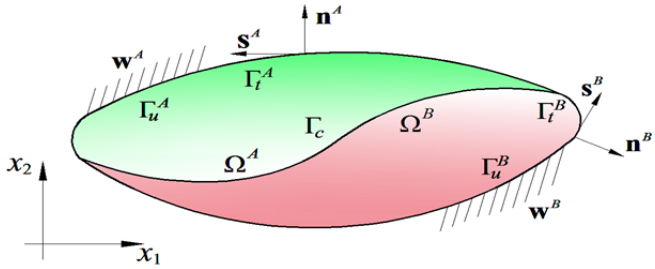

Fig. 1: Layered structure model for interface failure.

Consequently, let us denote the common part of subdomain boundaries Γ^η as an *interface* Γ_C hence $\Gamma_C = \partial\Omega^A \bigcap \partial\Omega^B$ which is usually represented by continuous spring distribution with *tangential* and *normal* elastic stiffnesses k_t and k_n [3]. The interface Γ_C represents a prescribed curve, which is the analysed area. On the smooth part of Γ, we can define the outward normal and

tangential vectors n^{η} and t^{η} [2]. On Γ_u^{η} we imposed time-dependent boundary displacements $w^{\eta}(\tau)$, while the remaining parts Γ_t^{η} are assumed to be traction free $t^{\eta} = 0$. On the Dirichlet part of the boundary Γ_u we apply a time-dependent boundary displacement $u^{\eta}(\tau)$ therefore, any admissible displacements u^{η} are equal to a prescribed *hard-device* $u^{\eta}(\tau)$ on Γ_u^{η} and they represent *the boundary conditions* [2], [5]. It is considered that the crack can be initiated and propagated along the interface Γ_C, this delamination process is assumed as rate-independent evolution, i.e. *no inertial time scale is considered*. Due to the delamination process, the material of the bonding layer is damaged and the interface is ruptured.

This failure mechanism is modelled by two interface parameters:

- *The scalar damage variable* $\zeta : \Gamma_C \to [1,0]$

This parameter has the meaning of the level of rupture interface: $\zeta(x) = 0$ has a meaning of a complete delamination at particular point and $\zeta(x) = 1$ corresponding to undamaged interface means 100 % perfect bonding [5].

- *The tangential plastic slip variable* π :

Considered at the interface and allows for a difference between crack *opening* and *sliding modes* in view of experimental observations of interface crack growth. The investigations confirm experimental observations that the *energy dissipated* from the system in sliding crack mode is significantly greater than in the opening mode. And moreover, corresponding plastic deformations of the structure are larger in sliding mode than in opening mode, see [8], [14].

3 ANALYSIS OF INTERFACE CONTACT MODEL

The study discusses a mathematical modelling of crack propagation along the interface in order to compare the failure process for adhesive and cohesive contacts, respectively. Both contacts were modelled numerically and consequently analysed and tested in MATLAB.

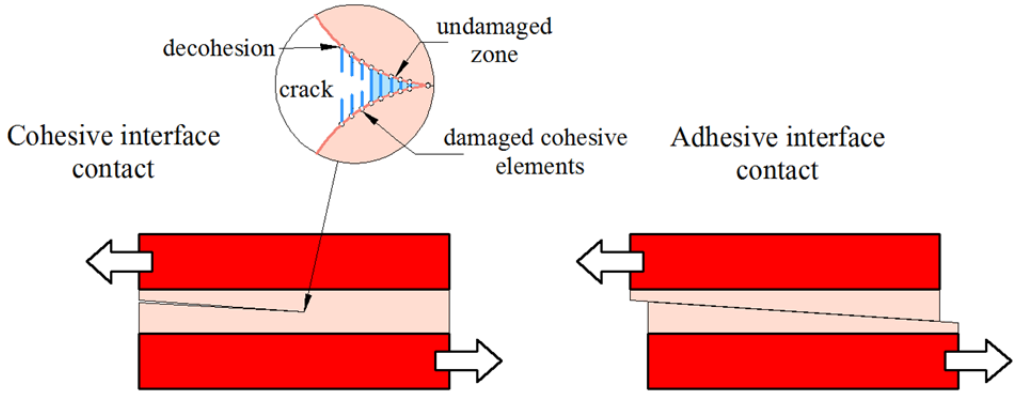

Fig. 2: Depiction of interface damage with cohesive and adhesive contact at the same loadstep λ_{τ}.

3.1 Adhesive-type contact interface model

The *adhesive-type contact*, see Fig. 3, yields a basic scenario which provides a discontinuous response of the *damage parameter* ζ and the *mechanical stress* σ in the form:

$$\sigma = \zeta\, k\, u\,, \tag{1}$$

with the stiffness parameter k and displacement u. The process starts from unstressed state. Applying the load, the stress linearly increases with u until *the crack driving force* $\sigma_d = \frac{1}{2} k u^2$ reaches the activation threshold G_d. Consequently the damage parameter ζ and also the mechanical stress σ change its values *abruptly*. In many practical aspects, the behaviour of adhesive contact can be modelled by a thin adhesive layer as a continuous elastic spring distribution with normal and tangential stiffness parameters. This layer is usually called *weak interface* [3], [5].

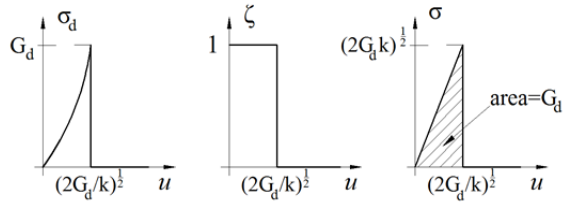

Fig. 3: Adhesive contact response of the crack driving force σ_d, the damage ζ and the stress σ.

σ_d - is a crack driving driving force [J/mm^2], ζ – is a interface damage parameter [-] and

σ - is a mechanical stress [MPa].

The linear response of the adhesive refers to the conception of the so-called *weak solution* given by the *energetic formulation*. These formulations are controlled by *stored energy* functional E_S and also *dissipation potential* R. The mathematical formulation of the energy approach of delamination with adhesive type contact is inducted in Section 4.

3.1 Cohesive-type contact interface model

Another possibility is to consider an engineering approach which supposes the continuous response of the mechanical stress σ. It is referred to as a **cohesive-type model**, see [5], [9], [12]. Assuming the cohesive-type contact requires some modifications in energy approach formulation of the aforementioned adhesive model. One effective option how to achieve the cohesive effect is to modify the *energy stored functional* E_S from the adhesive model, for more details, see Section 4.2 and also [2], [5]. The failure mechanism starts, when the mechanical stress σ, linearly increasing with u until the crack driving force σ_d, reaches activation threshold *fracture energy* G_d. Then ζ starts to evolve from one non-linearly until it arrives at zero [3], [5].

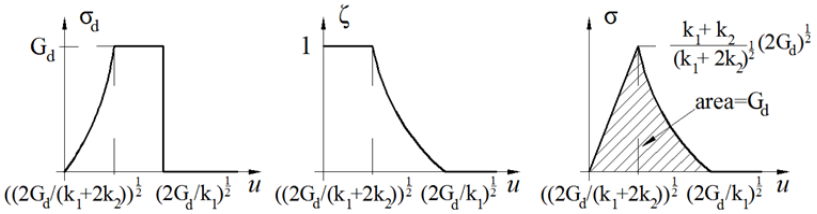

Fig. 4: Adhesive contact response of the crack driving force σ_d, the damage ζ and the stress σ, with denoted adhesive k_1 and cohesive k_2 stiffness parameters, respectively.

In consequence of modification in energy formulation, i.e. addition of the delamination term ζ^2 and cohesive stiffness parameter k_2 the mechanical stress decays as

$$\sigma = \left(k_1 \zeta + k_2 \zeta^2\right) u. \tag{2}$$

Discussed cohesive contact theory in Fig. 4 does not reflect the influence of the interface *plastic slip* π. This additional effect is included in energy formulation in sense of assuming Mode II [3]. The main feature of cohesive contact theory is that it is separately quadratic both in the u and ζ

variable. So we can conveniently apply *quadratic programming algorithms* for solving *global minimization problem,* see Section 4.2 and [2], [5].

4 MATHEMATICAL CONCEPT OF THE DELAMINATION PROCESS

This section reviews the mathematic formulations of the energetic conception of failure mechanism with both aforementioned defined contacts. The energetic solution is acquired by variational formulation (*Ritz method*), which exploit developed numerical treatment of inelastic process. This treatment has been evolved in accordance with energetic approaches.

4.1 Energetic formulation of interface contact model, Griffith concept

To define the energetic conception of the delamination contact problem, let us consider the *energy stored* (involving external loading) of the system at time τ, obeying the interface damage (Mode II) and a *kinematic-hardening-plasticity model*, with defined the *plastic slope* k_h [2]. The numerical analysis defines two following mathematical forms of stored energy functional E_S:

- ***The adhesive contact formulation***

$$E_S(\tau,u,\zeta,\pi)=\sum_{\eta=A,B}\left[\int_{\Omega^\eta}\frac{1}{2}C^\eta_{ijkl}\,\varepsilon_{ij}\left(u^\eta\right)\varepsilon_{kl}\left(u^\eta\right)\mathrm{d}\Omega + \int_{\Gamma_C}\left(\frac{1}{2}\zeta\,k_n\left[u\right]_n^2+\frac{1}{2}\zeta\,k_t\left(\left[u\right]_t-\pi\right)^2+\frac{1}{2}k_H\pi^2\right)\mathrm{d}\Gamma\right]. \tag{3}$$

- ***The cohesive contact formulation***

In accordance with conception introduced in Section 3.2, we can adduce the modified formulation of the stored energy functional of adhesive model. In consequence of adding the cohesive contact parameters (see Section 3.1), acquiring required *non-linear dependence* of the investigated parameters [3], [5]. The following stored energy functional is obtained:

$$E_S(\tau,u,\zeta,\pi)=\sum_{\eta=A,B}\left[\int_{\Omega^\eta}\frac{1}{2}C^\eta_{ijkl}\,\varepsilon_{ij}\left(u^\eta\right)\varepsilon_{kl}\left(u^\eta\right)\mathrm{d}\Omega + \int_{\Gamma_C}\left(\frac{1}{2}\zeta\,k_n\left[u\right]_n^2+\frac{1}{2}\left(\zeta\,k_{t_1}+\zeta^2k_{t_2}\right)\left(\left[u\right]_t-\pi\right)^2+\frac{1}{2}k_H\pi^2\right)\mathrm{d}\Gamma\right]. \tag{4}$$

k_H – is the hardening stiffness [MPa/mm], u – are the displacements [mm],

π – is the plastic tangential deformation [mm], E_S – is a stored energy of the system [J].

The inducted energetic formulation is assumed in the sense of *adhesive and cohesive contact theory* presented in [5], and it is valid for the state variables u, ζ satisfying:

- *The initial conditions*

$$u(0)=u_0,\quad \zeta(0)=\zeta_0,\quad \pi(0)=\pi_0. \tag{5}$$

- The condition of *Signorini unilateral contact* $\left[u\right]_n\geq 0$, where the *relative normal displacement* $\left[u\right]_n=\left(u^A-u^B\right).n^A$ is introduced.

- *The prescribed boundary conditions for displacements and tractions*

$$u^\eta=w^\eta(t)\ on\ \Gamma_u^\eta,\quad t^\eta=0. \tag{6}$$

The interface scalar *damage variable* ζ satisfies the constraints $0\leq\zeta\leq 1$. The C^η_{ijkl} expresses the fourth-order tensor of elastic stiffness and ε_{ij} is a small tensor related to bulk displacements **u** [2]. For activation of delamination the process requires a specific energy G_d [J/mm^2]. The Dissipated energy (well known as a dissipation distance) is then

$$R(A_1, A_2) := \int_{A_2 \setminus A_2} G_d(x)\mathrm{d}S \quad if\ A_1 \subset A_2 \subset \Gamma_C \tag{7}$$

In particular, the dissipated energy does not depend on particular modes. The philosophy of such quasistatic evolution is related with the Griffith criterion [1], [5], implying that the crack grows as soon as the energy release is bigger than the toughness, determined by G_d in (7) [1], [5]. The *dissipation potential R* can be defined by degree 1 positively homogeneous functional and reflects the rate-independence of the pertinent delamination variables [3].

$$R(\dot{\zeta}, \dot{\pi}) = \int_{\Gamma_C} \left(G_d |\dot{\zeta}| + \sigma_{yield} |\dot{\pi}| \right) \mathrm{d}\Gamma \tag{8}$$

$\dot{\zeta}$ – is a partial derivative of the damage parameter according to time τ,

$\dot{\pi}$ – is a partial derivative of the plasticity according to time τ,

G_d – is a minimum interface fracture energy required to initiate a unit interface crack [J/mm^2],

σ_{yield} – is a yield shear stress [MPa], R – is a dissipated energy of the system [J].

We consider the evolution process in the fixed finite time interval [0, T] [2]. The required energetic solution is a type of a *weak solution* and can be obtained by an implicit time discretization of (3), (4) and commonly acquired by solving the following *global-minimization problem:*

$$minimize\ (u, \zeta, \pi) \rightarrow E(\lambda\tau, u, \zeta, \pi) + R(\zeta - \zeta_\tau^{\lambda-1}, \pi - \pi_\tau^{\lambda-1}), \tag{9}$$

with respect to (u, ζ, π) and subjected to aforementioned conditions. The symbol λ defines the appropriate loadstep.

4.2 Mathematical model implementation for the interface failure by sliding slip

Essentially, the function v depends on time step τ such $v(\tau)$ is the function of x_1. Anyhow, it can be prescribed by a gently changing function in order to consider the imperfections and to take it into account (changing of the thickness or of the bulk shapes) [2].

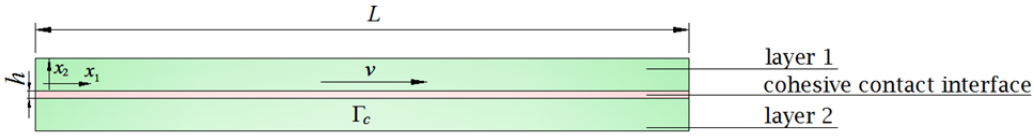

Fig. 5: Contact model for tangential loading by sliding slip.

In agreement with the conception initiated in [5], we can adduce the formulation of the stored energy as (3) or (4). Assuming the loading by sliding mode and considering both bulk layers rigid E_S, removes the dependence of the crack evolution on the bulk material properties [2]. Thus, we can suggest the reduced forms of *the stored energy functional* E_S from (3), (4), respectively as:

The adhesive type contact

$$E_S(\tau, u, \zeta, \pi) = \int_{\Gamma_C} \left(\frac{1}{2} \zeta k_t (v(\tau) - \pi)^2 + \frac{1}{2} k_H \pi^2 \right) d\Gamma. \tag{10}$$

The cohesive type contact

$$E_S(\tau, u, \zeta, \pi) = \int_{\Gamma_C} \left(\frac{1}{2} \left(\zeta k_{t_1} + \zeta^2 k_{t_2} \right) (v(\tau) - \pi)^2 + \frac{1}{2} k_H \pi^2 \right) d\Gamma. \tag{11}$$

5 COMPUTER IMPLEMENTATION OF THE CONTACT MODEL

To achieve the energetic solution, the numerical implementation commonly requires *time* and *spatial discretization*, separately in consecutive form.

- ***The time discretization*** considers the equidistant partition of $[0,T]$ and provides the solution at time steps defined by an increment δ, $\delta > 0$ thus $\tau_\lambda = \lambda\delta$ for $\lambda = 1,2,\ldots,T/\delta$, starting from the initial conditions (5).
- ***The spatial discretization***, also needed for the computer implementation, exploits the choice of *finite elements* for u and ζ on the boundary Γ_C.

For the explanation of discretization process in both models, see [2],[3]. The minimization problem (8) then provides the solution at the successive time steps defined by the parameter λ as

$$\textit{minimize } H^\lambda(\zeta,\pi) = E_S(\lambda\delta,u,\zeta,\pi) + R\left(\zeta-\zeta^{\lambda-1},\pi-\pi^{\lambda-1}\right). \tag{12}$$

Indeed, the functional H^λ is non-convex, so it requires applying a special numerical treatment in the minimization process. By *Alternative Minimization Algorithm* (AMA), we achieved to split the minimization to alternation between minimization with respect to both interface variables. Each of these being a minimization of a convex quadratic functional, so the *quadratic programming methods* can be applicable properly [3], [6]. For more detail see [2], [16]. The code of the applied algorithm (AMA) was implemented in MATLAB [11]. The convexity of both acquired quadratic functionals effectively enables to exploit bound constrained minimization methods (conjugate gradient method (CGM) with constraints) for reaching the global minimum. The *Polyak´s algorithm* was used as an appropriate CGM for minimization procedure [2].

6 NUMERICAL EXAMPLE OF THE INTERFACE CONTACT MODEL

6.1 Description of assumed contact model properties

The aforementioned simplified case of the interface contact model (see Section 4.2) with defined both types of contact have been subjected to numerical analysis in MATLAB. The investigated unknown interface variables (damage ζ^λ, plastic slip π^λ) remain along the interface for each load step λ [2]. The loading of the layered structure is considered along the interface in the sense of Section 4.2, and it is prescribed by slightly changing tangential displacements $v(\lambda\delta) = v_\lambda$ as

$$v_\lambda(x_1) = \sin\ \lambda\pi/50\ \left(1+5\sin(\pi x_1)\right) v_0, \tag{13}$$

with $v_0 = 0.002\,\text{mm}$ and *Ludolf´s constant* π. The prescribed tangential displacements $v_\lambda = (x_1)$ govern the whole loading process for each of loadsteps $\zeta = 100$ by prescribed *sin*-function v for a period, see [2]. The material of the interface is epoxy resin, with following elastic properties: Young´s modulus $E = 2.4\times10^3\,\text{MPa}$, Poisson´s ratio $\nu = 0.33$. The length of interface layer is $l = 1000\,\text{mm}$ and we consider layer thickness $h = 0.2\,\text{mm}$. The corresponding stiffness parameters were suggested according to the assumed model:

- For the *adhesive contact model* was considered $k_a = 4.5\times10^3\,\text{MPamm}^{-1}$.
- For the *cohesive contact model* we split the stiffness into two parts: $k_c = k_{t_1} + k_{t_2}$, $k_{t_1} = 0.45\times10^3\,\text{MPamm}^{-1}$, $k_{t_2} = 4.05\times10^3\ \text{MPamm}^{-1}$. For more parameter detail see [10].

The essential parameters that govern the crack propagation in the interface are: *the elastic brittle fracture energy* $G_d = 10^{-2}\,\text{mJmm}^{-2}$, *plastic yield stress* $\sigma_{yield} = 5.3\,\text{MPa}$, *hardening slope for plastic slip* is $k_H = 5\times10^2\,\text{MPamm}^{-1}$ [2],[10].

6.2 Results of the analysis of the interface rupture by sliding slip

The energy evolution process

The energy evolution of the delamination process for both analysed models is shown in Fig. 6 and Fig. 7. The presented three curves depict: the *energy stored* in the interface, *dissipated energy* due to plastic deformation and rupture of the interface and the *total energy* which is the sum of

previous components [3]. The non-monotone behaviour of the total energy was acquired in consequence of loading and unloading process by λ-dependent *sin function* of (12), see also [2], [3].

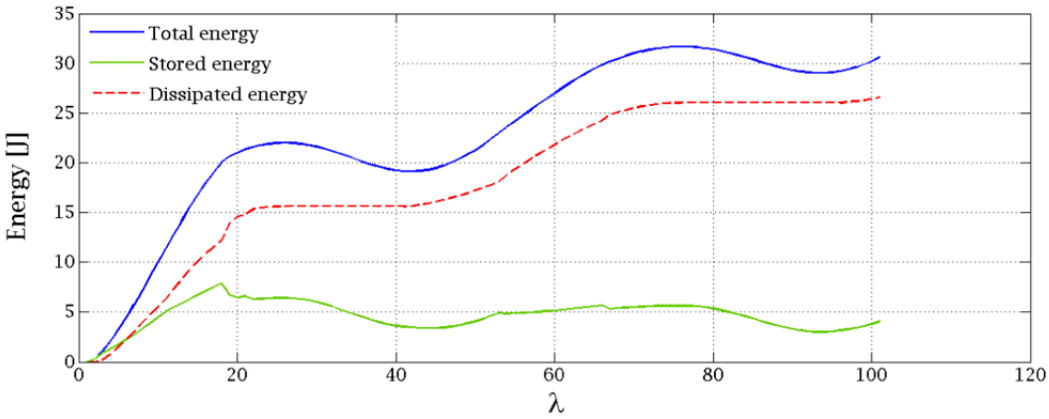

Fig. 6: Energy evolution of crack propagation at the adhesive contact model.

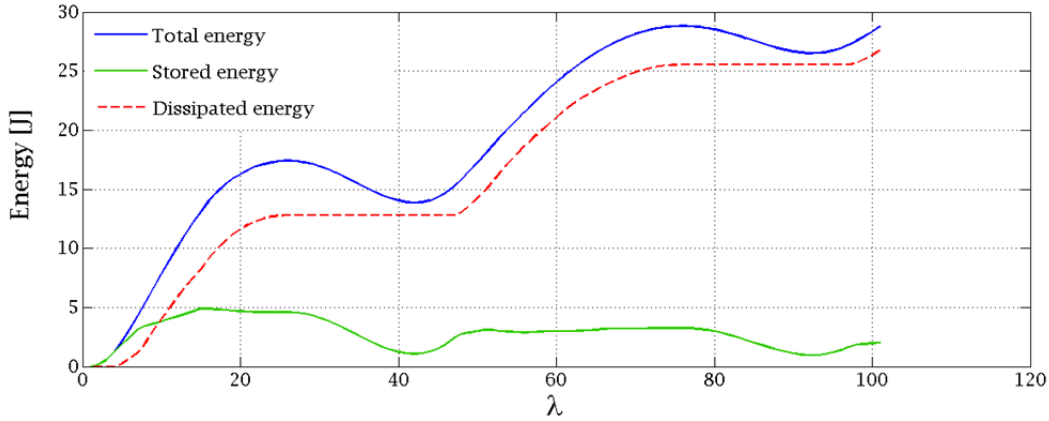

Fig. 7: Energy evolution of crack propagation at the cohesive contact model.

The evolution of the interface variables

The achieved solution of model behaviour is depicted in following figures. The initiation and propagation of the interface crack can be conspicuous in all the graphs. The main feature occurred in the graphs pertinent to the cohesive model is the continuous non-linear. It can be obviously observed the difference between coherency response of the interface variables in adhesive and cohesive model [2], [3]. So in agreement with consequent graphs let us observe that the damage parameter ζ changes from one to zero abruptly for adhesive model, while for the cohesive more or less continuously, see Fig. 8. The plastic deformation remains constant after the initiation of the crack, Fig. 10. The parts of the undamaged interface layer still hold two rigid bulks together, as it can be evidently observed in Fig. 9 and Fig. 10.

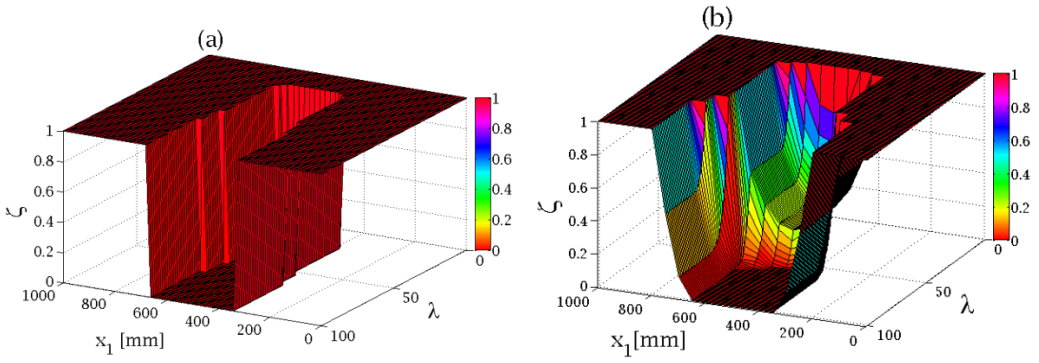

Fig. 8: The damage parameter ζ (a) adhesive model, (b) cohesive model.

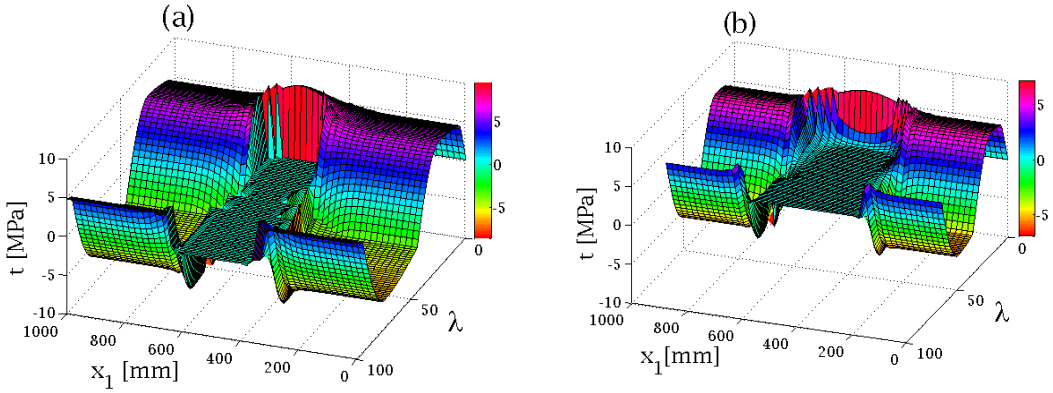

Fig. 9: The mechanical stress (a) adhesive model, (b) cohesive model.

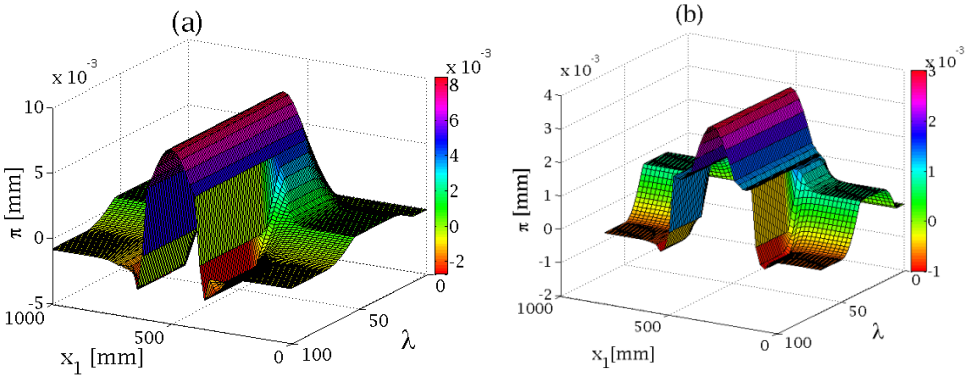

Fig. 10: The plastic slip π (a) adhesive model, (b) cohesive model.

7 CONCLUSIONS

The published study presents the comparison of the model response with adhesive and cohesive type contact. The investigated models yield a sensitive approach to the crack mode which has been reached by considering the interface contact by two interface variables. The proposed cohesive-type contact provides in many situations a more realistic behaviour than the adhesive and was acquired by mere adding of a new delamination term. The proposed numerical models confirm the expected response in accordance with the applied theory and enable its applicability in many aspects of engineering practise.

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