



Completing the hadronic Higgs boson decay at order α_s^4

Joshua Davies, Matthias Steinhauser*, David Wellmann

Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

Received 10 March 2017; received in revised form 11 April 2017; accepted 12 April 2017

Available online 19 April 2017

Editor: Tommy Ohlsson

Abstract

We compute four-loop corrections to the hadronic decay of the Standard Model Higgs boson which are induced by effective couplings to bottom quarks and gluons, mediated by the top quark. Our numerical results are comparable in size to the purely massless contributions which have been known for a few years. The results presented in this paper complete the order α_s^4 corrections to the hadronic Higgs boson decay. © 2017 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

In particle physics, one of the most important tasks in the coming years is the precise measurement of the couplings of the Higgs boson to fermions and bosons. An important ingredient in this context is the decay rate of the Higgs boson into bottom quarks, which has the by far largest branching ratio. Together with the decay rate into gluons it constitutes almost 70% of the hadronic decay width and it thus has a major influence on all Higgs boson branching ratios.

One-loop¹ QCD corrections to $\Gamma(H \rightarrow b\bar{b})$ have been known for a long time, including the full bottom quark-mass dependence [1]. The massless approximation² at order α_s^2 has been computed in Ref. [2] and the full bottom quark-mass dependence is known from Ref. [3–5]. Three- and four-loop corrections, of order α_s^3 and α_s^4 , have been computed in the massless

* Corresponding author.

E-mail address: matthias.steinhauser@kit.edu (M. Steinhauser).

¹ In the following we count the number of loops needed for the virtual corrections.

² Here “massless” refers to the bottom quark mass in the propagators; the bottom quark Yukawa coupling remains non-zero.

limit in Refs. [6–8]. A summary of further corrections, including top quark-mass-suppressed terms and electroweak effects can be found in recent review articles [9,10] (see also the program HDECAY [11]).

The main aim of this paper is to complete the corrections of order α_s^4 to the total decay rate of the Higgs boson into hadrons. In Ref. [8] only the contribution involving the bottom quark Yukawa coupling was considered. We compute the contributions induced by effective Higgs–bottom quark and Higgs–gluon couplings. The corresponding three-loop calculation, which was performed in Ref. [7], produces a similarly-sized contribution to the α_s^3 coefficient as that of the purely massless contribution. It is therefore necessary also to evaluate the top quark-induced contributions at order α_s^4 .

For the calculation performed in this paper the relevant part the Standard Model (SM) Lagrange density is given by the Yukawa terms supplemented by the strong interaction terms. For the production and decay of the SM Higgs boson it turns out that the effective theory in which the top quark is integrated out provides a good approximation to the full theory. This leads to the following effective Lagrangian [12–14]³

$$\mathcal{L}_{\text{eff}} = -\frac{H^0}{v^0} (C_1[\mathcal{O}'_1] + C_2[\mathcal{O}'_2]) + \mathcal{L}'_{\text{QCD}}, \tag{1}$$

where the primed quantities are defined in the five-flavor theory. H^0 and v^0 are the bare Higgs boson field and vacuum expectation value which can be identified with their renormalized counterparts if, as in this paper, electroweak effects are neglected. In Eq. (1) all dependence on the top quark is contained in the coefficient functions (or effective couplings) C_1 and C_2 . $[\mathcal{O}'_1]$ and $[\mathcal{O}'_2]$ are renormalized effective operators constructed from the light degrees of freedom. Their bare versions read

$$\begin{aligned} \mathcal{O}'_1 &= \left(G_{a,\mu\nu}^{0r}\right)^2, \\ \mathcal{O}'_2 &= m_b^{0r} \bar{b}^{0r} b^{0r}, \end{aligned} \tag{2}$$

where $G_{a,\mu\nu}^{0r}$ is the bare gluon field strength tensor and \bar{b}^{0r} is the bare bottom quark field.

Further corrections to \mathcal{L}_{eff} are suppressed by the inverse top quark mass, contributing terms of order M_H^2/M_t^2 to the decay rate. These terms are available to order α_s^3 [15–17] and are known to be small. For example, at order α_s^2 the M_H^2/M_t^2 term changes the coefficient by less than 1% and thus induces a correction which is of the same order of magnitude as non-suppressed contributions of order α_s^4 . We also restrict ourselves to the leading m_b^2 term and neglect higher powers in the bottom quark mass which are numerically even smaller than the $1/M_t$ terms.

On the basis of the Lagrange density of Eq. (1) we define correlators formed by the operators \mathcal{O}'_1 and \mathcal{O}'_2 ,

$$\Pi_{ij}(q^2) = i \int dx e^{iqx} \langle 0|T[\mathcal{O}'_i, \mathcal{O}'_j]|0\rangle. \tag{3}$$

Sample Feynman diagrams contributing to Π_{11} , Π_{12} and Π_{22} are shown in Fig. 1.

Using the optical theorem, the total decay rate can be obtained from the imaginary part of Π_{ij} . In this context it is convenient to introduce the quantities

$$\begin{aligned} \Delta_{ii} &= K_{ii} \text{Im} \left[\Pi_{ii}(M_H^2) \right], \\ \Delta_{12} &= K_{12} \text{Im} \left[\Pi_{12}(M_H^2) + \Pi_{21}(M_H^2) \right], \end{aligned} \tag{4}$$

³ We follow the notation of Ref. [7].

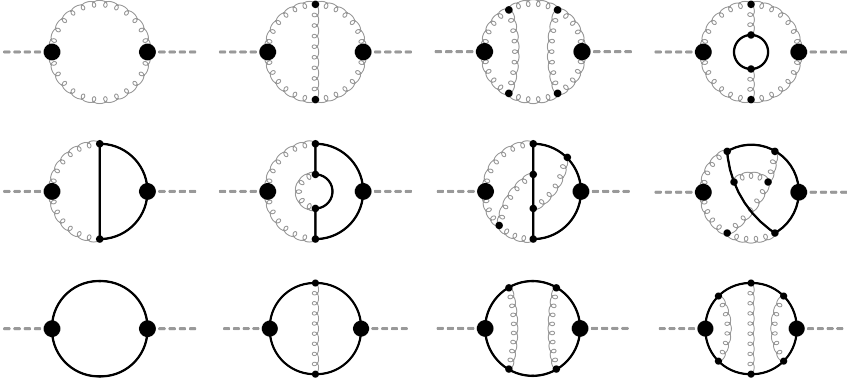


Fig. 1. Sample Feynman diagrams contributing to Π_{11} , Π_{12} and Π_{22} . The curly and straight lines represent gluons and quarks, respectively. The blobs stand for the effective operators \mathcal{O}'_1 and \mathcal{O}'_2 .

with $1/K_{11} = 32\pi M_H^4$ and $1/K_{12} = 1/K_{22} = 6\pi M_H^2 m_b^2$. Note that $\Pi_{12}(M_H^2) = \Pi_{21}(M_H^2)$. The total decay width is then given by

$$\Gamma(H \rightarrow \text{hadrons}) = A_{b\bar{b}} \left[(C_2)^2 (1 + \Delta_{22}) + C_1 C_2 \Delta_{12} \right] + A_{gg} (C_1)^2 \Delta_{11}, \quad (5)$$

where

$$A_{b\bar{b}} = \frac{3G_F M_H m_b^2(\mu)}{4\pi\sqrt{2}},$$

$$A_{gg} = \frac{4G_F M_H^3}{\pi\sqrt{2}}. \quad (6)$$

Note that for clarity, we restrict ourselves in Eq. (5) to the QCD corrections that we compute in this paper; we neglect both electroweak effects and power corrections suppressed by M_H^2/M_t^2 . Furthermore, we concentrate on the decay of the Higgs boson only to bottom quarks and to gluons. The results can easily be extended to include the decay to additional light quark flavors, if necessary. For example, if one wants to consider also the decay to charm quarks, the term proportional to $A_{b\bar{b}}$ in Eq. (5) must be duplicated with its coefficient replaced by $A_{c\bar{c}} = A_{b\bar{b}}|_{m_b \rightarrow m_c}$. A more complete formula can be found in Eq. (10) of Ref. [7]. Note that in Eq. (6), $m_b(\mu)$ refers to the $\overline{\text{MS}}$ bottom quark mass evaluated at the renormalization scale μ .

In Ref. [8] Π_{22} has been computed to five-loop order, yielding order α_s^4 corrections to the Higgs boson decay. For these corrections we have that $C_2 = 1$ and therefore refer to them in the following as “massless contributions”, despite the fact that there is an overall factor of m_b^2 from the bottom quark Yukawa coupling.

The leading-order term of Π_{11} describes the decay of the Higgs boson into gluons. Starting from next-to-leading order (two loops) the gluonic and fermionic decay cannot be separated in the approach based on the optical theorem, since there are diagrams containing both purely gluonic cuts and cuts involving both gluons and quark–antiquark pairs.

The main result of this paper is the extension of [7]. We compute the four-loop correction to Π_{12} which contributes to the hadronic Higgs boson decay at order α_s^4 , along with the five-loop calculation of Ref. [8]. This is because the leading term of C_1 contains a factor α_s .

Note that Π_{22} has an overall prefactor m_b^2 , which comes from the two operators \mathcal{O}'_2 . Π_{12} is also proportional to m_b^2 ; one factor arises from \mathcal{O}'_2 the other from the trace of the bottom quark

loop. In the limit $m_b \rightarrow 0$ the correlator Π_{11} has a non-zero contribution. Terms proportional to m_b^2 appear for the first time at two-loop order, due to the presence of closed bottom quark loops. We compute such terms up to three loops, which give rise to order α_s^4 corrections to the Higgs boson decay. We want to remark that the m_b -independent terms of Π_{11} have been computed to four-loop order in Ref. [18] leading to corrections of order α_s^5 to the hadronic Higgs boson decay.

In the next section we provide several technical details of our calculation. In particular, we discuss the computation of the four-loop integrals and explain the operator mixing and renormalization. We additionally provide explicit expressions for the effective couplings C_1 and C_2 . We present analytic results in Section 3 and discuss the numerical impact of our new corrections. Our conclusions are given in Section 4.

2. Calculation

For the calculation of the Feynman diagrams we use a well-tested automated setup which uses `qgraf` [19] for the generation of the Feynman amplitudes, and `q2e` and `exp` [20–22] for the mapping to one of eleven pre-defined four-loop integral families. The Dirac algebra is performed with `FORM` [23], which also re-writes the amplitude of each diagram as a linear combination of scalar integrals. Next we generate, using `FIRE 5.2` [24,25], tables for the reduction of the integrals of all eleven families to master integrals. We then apply `tsort` [26], in the form of the `FIRE` command `FindRules`, to minimize the number of master integrals among all eleven families and end up with 28 four-loop master integrals, which have been computed in Refs. [27–29].

We have re-computed the one-, two- and three-loop corrections to all correlators using both the setup described above and, as a cross check of our approach, `MINCER` [30]. Both calculations produce identical results, which agree with the literature. As a further check we have performed our calculations using a generic gauge parameter ξ . Our four-loop expressions have been expanded to linear order in ξ which drops out after reducing the master integrals to a minimal set.

We have used this method to compute the four-loop corrections to Π_{12} and the three-loop corrections to Π_{11} which, after taking the imaginary part, lead to the bare quantities Δ_{12}^0 and Δ_{11}^0 . At this point we perform the renormalization of the strong coupling constant and the quark mass in the $\overline{\text{MS}}$ scheme where the renormalization constants are introduced via

$$\begin{aligned} \alpha_s^0 &= Z_{\alpha_s} \alpha_s, \\ m_b^0 &= Z_m m_b. \end{aligned} \tag{7}$$

Z_{α_s} and Z_m are required to third order in α_s and can be found in, e.g., Ref. [31].

Afterwards, we have to take into account that the operators \mathcal{O}'_1 and \mathcal{O}'_2 mix under renormalization according to [32,7]

$$\begin{aligned} [\mathcal{O}'_1] &= Z_{11} \mathcal{O}'_1 + Z_{12} \mathcal{O}'_2, \\ [\mathcal{O}'_2] &= \mathcal{O}'_2. \end{aligned} \tag{8}$$

The renormalization constants Z_{11} and Z_{12} are obtained from Z_{α_s} and Z_m as follows,

$$\begin{aligned} Z_{11} &= 1 + \frac{\alpha_s \partial}{\partial \alpha_s} \log Z_{\alpha_s}, \\ Z_{12} &= -4 \frac{\alpha_s \partial}{\partial \alpha_s} \log Z_m. \end{aligned} \tag{9}$$

In terms of these renormalization constants, the renormalized correlators Δ_{ij} are given by

$$\begin{aligned}\Delta_{11} &= (Z_{11})^2 \Delta_{11}^0 + 2Z_{11}Z_{12}\Delta_{12}^0 + (Z_{12})^2 \Delta_{22}^0, \\ \Delta_{12} &= Z_{11}\Delta_{12}^0 + Z_{12}\Delta_{22}^0, \\ \Delta_{22} &= \Delta_{22}^0.\end{aligned}\tag{10}$$

Note that the contributions of Δ_{12}^0 and Δ_{22}^0 are proportional to m_b^2 whereas Δ_{11}^0 contains both m_b^2 and m_b -independent terms. Since $Z_{12} \propto \alpha_s$ only the $(n-1)$ -loop terms of Δ_{12}^0 and the $(n-2)$ -loop terms of Δ_{22}^0 enter the n -loop renormalization of Δ_{11} . Similarly, the n -loop renormalization of Δ_{12} requires the $(n-1)$ -loop terms of Δ_{22}^0 .

For completeness we also provide explicit expressions for the effective couplings C_1 and C_2 , which are available in the literature up to fifth order [33–35]. It is convenient to parametrize the perturbative expansion in terms of

$$a_s \equiv \frac{\alpha_s^{(5)}(\mu)}{\pi},\tag{11}$$

where the superscript indicates the number of active quark flavors used for the running, and the on-shell top quark mass. To obtain corrections of order a_s^4 to the decay rate, C_1 is needed to third order and C_2 to fourth order. In the following we present C_1 to order a_s^4 since we include a_s^5 corrections when evaluating the decay rate numerically. The analytic results read

$$\begin{aligned}C_1 &= -a_s \frac{1}{12} - a_s^2 \frac{11}{48} - a_s^3 \left[\frac{2777}{3456} + \frac{19}{192} L_t - n_l \left(\frac{67}{1152} - \frac{1}{36} L_t \right) \right] \\ &\quad + a_s^4 \left[\frac{2761331}{497664} - \frac{897943}{110592} \zeta_3 - \frac{2417}{3456} L_t - \frac{209}{768} L_t^2 \right. \\ &\quad \left. - n_l \left(\frac{58723}{248832} - \frac{110779}{165888} \zeta_3 + \frac{91}{648} L_t + \frac{23}{384} L_t^2 \right) \right. \\ &\quad \left. + n_l^2 \left(\frac{6865}{373248} - \frac{77}{20736} L_t + \frac{1}{216} L_t^2 \right) \right] + \mathcal{O}(a_s^5),\end{aligned}\tag{12}$$

$$\begin{aligned}&\approx -0.08333 a_s - 0.2292 a_s^2 - a_s^3 \left[0.7391 - 0.07624 n_l \right] \\ &\quad - a_s^4 \left[3.8715 - 0.6328 n_l - 0.02277 n_l^2 \right] + \mathcal{O}(a_s^5),\end{aligned}\tag{13}$$

$$\begin{aligned}C_2 &= 1 + a_s^2 \left[\frac{5}{18} - \frac{1}{3} L_t \right] + a_s^3 \left[-\frac{841}{1296} + \frac{5}{3} \zeta_3 - \frac{79}{36} L_t - \frac{11}{12} L_t^2 + n_l \left(\frac{53}{216} + \frac{1}{18} L_t^2 \right) \right] \\ &\quad + a_s^4 \left[\frac{609215}{186624} - \frac{4}{3} \zeta_2 + \frac{374797}{13824} \zeta_3 - \frac{4123}{144} \zeta_4 - \frac{575}{36} \zeta_5 + \frac{62}{9} \text{Li}_4 \left(\frac{1}{2} \right) - \frac{4}{9} \ln 2 \zeta_2 \right. \\ &\quad \left. - \frac{31}{18} (\ln 2)^2 \zeta_2 + \frac{31}{108} (\ln 2)^4 - \left[\frac{4645}{144} - \frac{55}{4} \zeta_3 \right] L_t - \frac{91}{8} L_t^2 - \frac{121}{48} L_t^3 \right. \\ &\quad \left. + n_l \left(-\frac{11557}{15552} + \frac{2}{9} \zeta_2 - \frac{221}{288} \zeta_3 + \frac{163}{72} \zeta_4 - \frac{4}{9} \text{Li}_4 \left(\frac{1}{2} \right) + \frac{1}{9} (\ln 2)^2 \zeta_2 \right) \right. \\ &\quad \left. - \frac{1}{54} (\ln 2)^4 + \frac{9535}{2592} L_t + \frac{109}{144} L_t^2 + \frac{11}{36} L_t^3 \right) \\ &\quad \left. + n_l^2 \left(\frac{3401}{23328} - \frac{7}{54} \zeta_3 - \frac{31}{324} L_t - \frac{1}{108} L_t^3 \right) \right] + \mathcal{O}(a_s^5),\end{aligned}\tag{14}$$

$$\begin{aligned} &\approx 1 + 0.494759 a_s^2 + a_s^3 \left[2.3946 + 0.2689 n_l \right] \\ &\quad - a_s^4 \left[6.0125 + 1.1543 n_l - 0.05480 n_l^2 \right] + \mathcal{O} \left(a_s^5 \right), \end{aligned} \tag{15}$$

where ζ_n is the Riemann Zeta function, $\text{Li}_n(z)$ is the Polylogarithm function and we have defined $L_t = \log(\mu^2/M_t^2)$. The numerical expressions are given at the renormalization scale $\mu^2 = M_H^2$, for $n_l = 5$ massless flavors running in fermion loops, and for $M_H = 125.09$ GeV and $M_t = 173.21$ GeV [36]. Since μ is of the order of the Higgs boson mass one generates potentially large logarithms which should be resummed [14]. In practice, however, the numerical effect is small and we have decided to consider only the fixed-order result here.

3. Results

We use this section to present our results. The new ingredients of Eq. (5) required to complete the order α_s^4 corrections to $\Gamma(H \rightarrow \text{hadrons})$ are the four-loop corrections to Δ_{12} and the bottom mass-dependent three-loop corrections to Δ_{11} . For convenience we also present the lower-order contributions. The general expressions in terms of the Casimir invariant color factors can be found in Appendix A. For the SU(3) case, for which $C_A = 3$ and $C_F = 4/3$, we obtain

$$\begin{aligned} \Delta_{11} = & 1 + a_s \left[\frac{73}{4} + \frac{11}{2} L_H - n_l \left(\frac{7}{6} + \frac{1}{3} L_H \right) \right] \\ & + a_s^2 \left[\frac{37631}{96} - \frac{363}{8} \zeta_2 - \frac{495}{8} \zeta_3 + \frac{2817}{16} L_H + \frac{363}{16} L_H^2 \right. \\ & \quad \left. + n_l \left(-\frac{7189}{144} + \frac{11}{2} \zeta_2 + \frac{5}{4} \zeta_3 - \frac{263}{12} L_H - \frac{11}{4} L_H^2 \right) \right. \\ & \quad \left. + n_l^2 \left(\frac{127}{108} - \frac{1}{6} \zeta_2 + \frac{7}{12} L_H + \frac{1}{12} L_H^2 \right) \right] \\ & + \left(\frac{m_b^2}{M_H^2} \right) \left\{ 6a_s + a_s^2 \left[\frac{697}{3} - 6\zeta_2 + 6\zeta_3 + \frac{169}{2} L_H + 3L_H^2 - n_l \left(\frac{15}{2} + 3L_H \right) \right] \right\} \\ & + \mathcal{O} \left(a_s^3 \right) \end{aligned} \tag{16}$$

$$\begin{aligned} &\approx 1 + a_s \left[18.2500 - 1.1667 n_l \right] + a_s^2 \left[242.9734 - 39.3739 n_l + 0.9018 n_l^2 \right] \\ & + \left(\frac{m_b^2}{M_H^2} \right) \left\{ 6a_s + a_s^2 \left[229.6761 - 7.5000 n_l \right] \right\} + \mathcal{O} \left(a_s^3 \right) \end{aligned} \tag{17}$$

and

$$\begin{aligned} \Delta_{12} = & + a_s \left[-\frac{92}{3} - 8 L_H \right] \\ & + a_s^2 \left[-\frac{15073}{18} + 76 \zeta_2 + 156 \zeta_3 - \frac{1028}{3} L_H - 38 L_H^2 \right. \\ & \quad \left. + n_l \left(\frac{283}{9} - \frac{8}{3} \zeta_2 - \frac{16}{3} \zeta_3 + \frac{112}{9} L_H + \frac{4}{3} L_H^2 \right) \right] \\ & + a_s^3 \left[-\frac{8957453}{432} + 4150 \zeta_2 + \frac{131389}{18} \zeta_3 - 815 \zeta_5 \right] \end{aligned}$$

$$\begin{aligned}
& - \left[\frac{65267}{6} - 855 \zeta_2 - 1755 \zeta_3 \right] L_H - 2075 L_H^2 - \frac{285}{2} L_H^3 \\
& + n_l \left(\frac{279451}{162} - \frac{1003}{3} \zeta_2 - 446 \zeta_3 + 10 \zeta_4 + \frac{100}{3} \zeta_5 \right. \\
& \quad \left. + \left[\frac{15973}{18} - 68 \zeta_2 - 118 \zeta_3 \right] L_H + \frac{1003}{6} L_H^2 + \frac{34}{3} L_H^3 \right) \\
& + n_l^2 \left(- \frac{25627}{972} + \frac{56}{9} \zeta_2 + \frac{20}{3} \zeta_3 - \left[\frac{407}{27} - \frac{4}{3} \zeta_2 - \frac{8}{3} \zeta_3 \right] L_H \right. \\
& \quad \left. - \frac{28}{9} L_H^2 - \frac{2}{9} L_H^3 \right) + \mathcal{O}(a_s^4) \tag{18}
\end{aligned}$$

$$\begin{aligned}
& \approx -30.6667 a_s + a_s^2 \left[-524.8530 + 20.6470 n_l \right] \\
& + a_s^3 \left[-5979.1838 + 684.320 n_l - 8.1164 n_l^2 \right] + \mathcal{O}(a_s^4), \tag{19}
\end{aligned}$$

where $L_H = \log(\mu^2/M_H^2)$. As above, n_l counts the number of light quarks running in fermion loops. For the numerical evaluation we have set $\mu^2 = M_H^2$ and $n_l = 5$. For both Δ_{11} and Δ_{12} we observe a rapid growth of the coefficients, however, we postpone discussion of the convergence properties to the decay rate, since Δ_{11} and Δ_{12} do not themselves represent physical quantities.

For the numerical evaluation of the decay rate it is convenient to cast Eq. (5) in the form

$$\Gamma(H \rightarrow \text{hadrons}) = A_{b\bar{b}} \left(1 + \Delta_{\text{light}} + \Delta_{\text{top}} + \Delta_{gg}^{m_b=0} \right), \tag{20}$$

where we have chosen $A_{b\bar{b}}$ as a common prefactor so that we can easily compare the relative sizes of the individual contributions. Δ_{light} contains all corrections obtained for $C_1 = 0$ and $C_2 = 1$. They have already been presented and discussed in Ref. [8]. Δ_{top} contains the top quark-induced corrections obtained from the contributions proportional to C_1 and $(C_2 - 1)$. For completeness we also list the corrections from Δ_{11} which have no factor m_b^2 . They are collected in $\Delta_{gg}^{m_b=0}$. Note that these terms have already been computed in Ref. [37]. For convenience we provide the formulae which relate the quantities in Eq. (20) to the ones in Eq. (5):

$$\begin{aligned}
\Delta_{\text{light}} &= \Delta_{22}, \\
\Delta_{\text{top}} &= \left[(C_2)^2 - 1 \right] (1 + \Delta_{22}) + C_1 C_2 \Delta_{12} + \frac{16M_H^2}{3m_b^2} (C_1)^2 \Delta_{11}^{m_b^2}, \\
\Delta_{gg}^{m_b=0} &= \frac{16M_H^2}{3m_b^2} (C_1)^2 \Delta_{11}^{m_b=0}. \tag{21}
\end{aligned}$$

For the numerical evaluation we use $\alpha_s^{(5)}(M_Z) = 0.1181$ [36] and $m_b(m_b) = 4.163$ GeV [38] which leads to $m_b(M_H) = 2.773$ GeV and $\alpha_s^{(5)}(M_H) = 0.1127$ using RunDec [39,40] with four-loop accuracy. Numerical values for M_H and M_t are already given at the end of Section 2. We expand the expressions of Eq. (21) in a_s and obtain

$$\begin{aligned}
\Delta_{\text{light}} &\approx 5.6667 a_s + 29.1467 a_s^2 + 41.7576 a_s^3 - 825.7466 a_s^4 \\
&\approx 0.2033 + 0.03752 + 0.001929 - 0.001368, \tag{22} \\
\Delta_{\text{top}} &\approx a_s^2 [2.5556_{12} + 0.9895_{22}] \\
&\quad + a_s^3 [0.2222_{11} + 42.1626_{12} + 13.0855_{22}]
\end{aligned}$$

$$\begin{aligned}
 &+ a_s^4 [8.3399_{11} + 338.9021_{12} + 50.6346_{22}] \\
 &\approx 0.003290_{12} + 0.001274_{22} \\
 &+ 0.00001026_{11} + 0.001947_{12} + 0.0006043_{22} \\
 &+ 0.00001382_{11} + 0.0005616_{12} + 0.00008390_{22},
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 \Delta_{gg}^{m_b=0} &\approx \frac{M_H^2}{27m_b^2} \left(a_s^2 + 17.9167a_s^3 + 153.0921a_s^4 + 392.6176a_s^5 \right), \\
 &\approx 0.09699 + 0.06235 + 0.01911 + 0.001759,
 \end{aligned} \tag{24}$$

where the subscripts in the expression for Δ_{top} indicate the origin of each term. For $\Delta_{gg}^{m_b=0}$ we have included the corrections of order a_s^5 from Ref. [18].

From Eqs. (22), (23) and (24) we observe that the a_s^2 term of $\Delta_{gg}^{m_b=0}$ amounts to almost 50% of the a_s term in Δ_{light} . Furthermore, the a_s^5 term of $\Delta_{gg}^{m_b=0}$ has the same order of magnitude as the a_s^4 term of Δ_{light} . Note that the latter is only about twice as large as the a_s^4 contribution to Δ_{top} , obtained from the sum of the three numbers in the last line of Eq. (23); this amounts to 0.0006593.

It is a disturbing feature of Δ_{light} that the a_s^3 and a_s^4 terms deviate by less than 30%. Furthermore they have opposite signs. Therefore, it is interesting to add Δ_{light} and Δ_{top} which leads to

$$1 + \Delta_{\text{light}} + \Delta_{\text{top}} \approx 1 + 0.2033 + 0.04208 + 0.004490 - 0.0007090, \tag{25}$$

where the different loop orders are kept separate. We observe a reduction by a factor of about six between the three- and four-loop contributions; the convergence of the sum is significantly better than that of the individual expressions.

In Fig. 2 we show the dependence of $\Gamma(H \rightarrow \text{hadrons})$ on the renormalization scale μ . We plot $\Gamma(H \rightarrow \text{hadrons})/A_{b\bar{b}}(\mu = M_H)$, which means that for the leading order (short-dashed) curve we have $\Gamma(H \rightarrow \text{hadrons})/A_{b\bar{b}}(\mu = M_H) = 1$ for $\mu = M_H$. The six curves represent (from bottom to top, i.e. from the short-dashed to the solid curve) the predictions of order $\alpha_s^0, \dots, \alpha_s^5$, where α_s^5 terms are only included for $\Delta_{gg}^{m_b=0}$. μ is varied between 10 GeV and 500 GeV which

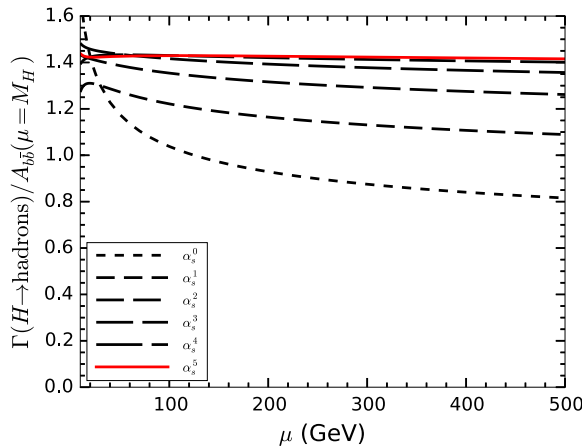


Fig. 2. $\Gamma(H \rightarrow \text{hadrons})/A_{b\bar{b}}(\mu = M_H)$ as a function of the renormalization scale μ .

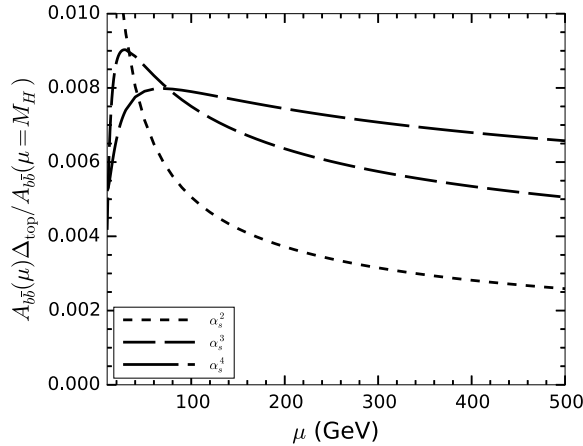


Fig. 3. $A_{bb}(\mu)\Delta_{top}/A_{bb}(\mu=M_H)$ as a function of the renormalization scale μ .

is significantly larger than the usual range spanned between $M_H/2$ and $2M_H$. Nevertheless, one observes a steady flattening of the curves when including higher order corrections; the result represented by solid line is almost μ -independent.

Finally, Fig. 3 shows the contributions to Δ_{top} , which start at order α_s^2 and are separately renormalization-scale independent (up to higher non-calculated orders). Δ_{top} is dominated by Δ_{12} as can be seen from Eq. (23). From bottom to top the lines include α_s^2 , α_s^3 and α_s^4 corrections. One again observes that the dependence on μ becomes weaker after including higher-order corrections.

4. Conclusions

We complete the corrections of order α_s^4 to the hadronic decay rate of the Standard Model Higgs boson by computing the top quark-induced contributions in an effective field-theory framework. This requires the calculation of four-loop propagator-type integrals. The new results of this paper are the order α_s^3 terms of Δ_{12} in Eq. (18) and the m_b^2/M_H^2 terms of Δ_{11} in Eq. (16). Both terms are required to complete the order α_s^4 contribution to the hadronic decay rate of the Standard Model Higgs boson given in Eq. (5). Our new corrections are numerically of the same order of magnitude as the purely massless contribution [8], however they have an opposite sign. We provide all analytic results presented in this paper in a computer-readable format [41], making it straightforward to implement the corrections in existing computer codes which evaluate decay rates of the Higgs boson. Finally, we want to mention that $\Gamma(H \rightarrow \text{hadrons})$ is one of very few physical quantities for which five terms of the perturbative expansion are known and the perturbative expansion can be studied, see Eq. (25).

Acknowledgements

We thank Konstantin Chetyrkin for providing analytic results for Δ_{11} and Δ_{22} containing the full μ dependence and Vladimir Smirnov for analytical results for the four-loop master integrals. This work is supported by the BMBF grant 05H15VKCCA.

Appendix A. Δ_{11} and Δ_{12} in terms of Casimir color factors

In terms of the Casimir invariants of $SU(N)$, Δ_{11} is given by

$$\begin{aligned} \Delta_{11} = & 1 + a_s \left[C_A \left(\frac{73}{12} + \frac{11}{6} L_H \right) - n_l \left(\frac{7}{6} + \frac{1}{3} L_H \right) \right] \\ & + a_s^2 \left[C_A^2 \left(\frac{37631}{864} - \frac{121}{24} \zeta_2 - \frac{55}{8} \zeta_3 + \frac{313}{16} L_H + \frac{121}{48} L_H^2 \right) \right. \\ & \quad + n_l C_F \left(-\frac{131}{48} + \frac{3}{2} \zeta_3 - \frac{1}{2} L_H \right) \\ & \quad + n_l C_A \left(-\frac{6665}{432} + \frac{11}{6} \zeta_2 - \frac{1}{4} \zeta_3 - \frac{85}{12} L_H - \frac{11}{12} L_H^2 \right) \\ & \quad \left. + n_l^2 \left(\frac{127}{108} - \frac{1}{6} \zeta_2 + \frac{7}{12} L_H + \frac{1}{12} L_H^2 \right) \right] \\ & + \left(\frac{m_b^2}{M_H^2} \right) \left\{ 6a_s + a_s^2 \left[C_A \left(55 + 6\zeta_3 + \frac{33}{2} L_H \right) \right. \right. \\ & \quad \left. \left. + C_F \left(\frac{101}{2} - \frac{9}{2} \zeta_2 - 9\zeta_3 + \frac{105}{4} L_H + \frac{9}{4} L_H^2 \right) - n_l \left(\frac{15}{2} + 3L_H \right) \right] \right\} \\ & + \mathcal{O}(a_s^3), \end{aligned} \tag{26}$$

where $C_A = N$ and $C_F = (N^2 - 1)/(2N)$. Δ_{12} reads

$$\begin{aligned} \Delta_{12} = & + a_s \left[C_F \left(-23 - 6L_H \right) \right] \\ & + a_s^2 \left[C_F^2 \left(-\frac{907}{8} + 18\zeta_2 + 18\zeta_3 - \frac{123}{2} L_H - 9L_H^2 \right) \right. \\ & \quad + C_A C_F \left(-\frac{3815}{24} + 11\zeta_2 + 31\zeta_3 - \frac{175}{3} L_H - \frac{11}{2} L_H^2 \right) \\ & \quad \left. + n_l C_F \left(\frac{283}{12} - 2\zeta_2 - 4\zeta_3 + \frac{28}{3} L_H + L_H^2 \right) \right] \\ & + a_s^3 \left[C_F^3 \left(-\frac{29545}{64} + 135\zeta_2 + \frac{663}{4} \zeta_3 - \frac{135}{2} \zeta_5 \right. \right. \\ & \quad \left. \left. - \left[\frac{8631}{32} - \frac{81}{2} \zeta_2 - \frac{81}{2} \zeta_3 \right] L_H - \frac{135}{2} L_H^2 - \frac{27}{4} L_H^3 \right) \right. \\ & \quad + C_A C_F^2 \left(-\frac{108241}{96} + \frac{657}{2} \zeta_2 + \frac{3189}{8} \zeta_3 - \frac{435}{4} \zeta_5 \right. \\ & \quad \left. - \left[\frac{23585}{32} - \frac{297}{4} \zeta_2 - \frac{477}{4} \zeta_3 \right] L_H - \frac{657}{4} L_H^2 - \frac{99}{8} L_H^3 \right) \\ & \quad \left. + C_A^2 C_F \left(-\frac{5886949}{5184} + \frac{1039}{6} \zeta_2 + \frac{3187}{8} \zeta_3 - \frac{25}{4} \zeta_5 \right. \right. \\ & \quad \left. \left. - \left[\frac{18923}{36} - \frac{121}{4} \zeta_2 - \frac{341}{4} \zeta_3 \right] L_H - \frac{1039}{12} L_H^2 - \frac{121}{24} L_H^3 \right) \right] \end{aligned}$$

$$\begin{aligned}
& + n_l C_F^2 \left(\frac{5803}{24} - \frac{225}{4} \zeta_2 - \frac{207}{2} \zeta_3 - \frac{9}{2} \zeta_4 + 30 \zeta_5 \right. \\
& \quad \left. + \left[\frac{1067}{8} - \frac{27}{2} \zeta_2 - 27 \zeta_3 \right] L_H + \frac{225}{8} L_H^2 + \frac{9}{4} L_H^3 \right) \\
& + n_l C_A C_F \left(\frac{209815}{648} - \frac{703}{12} \zeta_2 - \frac{131}{2} \zeta_3 + \frac{9}{2} \zeta_4 - 5 \zeta_5 \right. \\
& \quad \left. + \left[\frac{11705}{72} - 11 \zeta_2 - \frac{35}{2} \zeta_3 \right] L_H + \frac{703}{24} L_H^2 + \frac{11}{6} L_H^3 \right) \\
& + n_l^2 C_F \left(-\frac{25627}{1296} + \frac{14}{3} \zeta_2 + 5 \zeta_3 \right. \\
& \quad \left. - \left[\frac{407}{36} - \zeta_2 - 2 \zeta_3 \right] L_H - \frac{7}{3} L_H^2 - \frac{1}{6} L_H^3 \right) + \mathcal{O}(a_s^4). \quad (27)
\end{aligned}$$

References

- [1] E. Braaten, J.P. Leveille, Phys. Rev. D 22 (1980) 715, <http://dx.doi.org/10.1103/PhysRevD.22.715>.
- [2] S.G. Gorishnii, A.L. Kataev, S.A. Larin, L.R. Surguladze, Mod. Phys. Lett. A 5 (1990) 2703, <http://dx.doi.org/10.1142/S0217732390003152>.
- [3] K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, Nucl. Phys. B 505 (1997) 40, [http://dx.doi.org/10.1016/S0550-3213\(97\)00481-1](http://dx.doi.org/10.1016/S0550-3213(97)00481-1), arXiv:hep-ph/9705254.
- [4] R. Harlander, M. Steinhauser, Phys. Rev. D 56 (1997) 3980, <http://dx.doi.org/10.1103/PhysRevD.56.3980>, arXiv:hep-ph/9704436.
- [5] K.G. Chetyrkin, R. Harlander, M. Steinhauser, Phys. Rev. D 58 (1998) 014012, <http://dx.doi.org/10.1103/PhysRevD.58.014012>, arXiv:hep-ph/9801432.
- [6] K.G. Chetyrkin, Phys. Lett. B 390 (1997) 309, [http://dx.doi.org/10.1016/S0370-2693\(96\)01368-8](http://dx.doi.org/10.1016/S0370-2693(96)01368-8), arXiv:hep-ph/9608318.
- [7] K.G. Chetyrkin, M. Steinhauser, Phys. Lett. B 408 (1997) 320, [http://dx.doi.org/10.1016/S0370-2693\(97\)00779-X](http://dx.doi.org/10.1016/S0370-2693(97)00779-X), arXiv:hep-ph/9706462.
- [8] P.A. Baikov, K.G. Chetyrkin, J.H. Kühn, Phys. Rev. Lett. 96 (2006) 012003, <http://dx.doi.org/10.1103/PhysRevLett.96.012003>, arXiv:hep-ph/0511063.
- [9] D. de Florian, et al., LHC Higgs Cross Section Working Group, arXiv:1610.07922 [hep-ph].
- [10] M. Spira, arXiv:1612.07651 [hep-ph].
- [11] A. Djouadi, J. Kalinowski, M. Spira, Comput. Phys. Commun. 108 (1998) 56, [http://dx.doi.org/10.1016/S0010-4655\(97\)00123-9](http://dx.doi.org/10.1016/S0010-4655(97)00123-9), arXiv:hep-ph/9704448.
- [12] T. Inami, T. Kubota, Y. Okada, Z. Phys. C 18 (1983) 69, <http://dx.doi.org/10.1007/BF01571710>.
- [13] K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, Phys. Rev. Lett. 78 (1997) 594, <http://dx.doi.org/10.1103/PhysRevLett.78.594>, arXiv:hep-ph/9610456.
- [14] K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, Nucl. Phys. B 490 (1997) 19, [http://dx.doi.org/10.1016/S0550-3213\(97\)00051-5](http://dx.doi.org/10.1016/S0550-3213(97)00051-5), arXiv:hep-ph/9701277.
- [15] K.G. Chetyrkin, A. Kwiatkowski, Nucl. Phys. B 461 (1996) 3, [http://dx.doi.org/10.1016/0550-3213\(95\)00616-8](http://dx.doi.org/10.1016/0550-3213(95)00616-8), arXiv:hep-ph/9505358.
- [16] S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, Phys. Lett. B 362 (1995) 134, [http://dx.doi.org/10.1016/0370-2693\(95\)01192-S](http://dx.doi.org/10.1016/0370-2693(95)01192-S), arXiv:hep-ph/9506465.
- [17] M. Schreck, M. Steinhauser, Phys. Lett. B 655 (2007) 148, <http://dx.doi.org/10.1016/j.physletb.2007.08.080>, arXiv:0708.0916 [hep-ph].
- [18] P.A. Baikov, K.G. Chetyrkin, Phys. Rev. Lett. 97 (2006) 061803, <http://dx.doi.org/10.1103/PhysRevLett.97.061803>, arXiv:hep-ph/0604194.
- [19] P. Nogueira, J. Comput. Phys. 105 (1993) 279, <http://cfif.ist.utl.pt/~paulo/qgraf.html>.
- [20] R. Harlander, T. Seidensticker, M. Steinhauser, Phys. Lett. B 426 (1998) 125, arXiv:hep-ph/9712228.
- [21] T. Seidensticker, arXiv:hep-ph/9905298.
- [22] <http://sfb-tr9.ttp.kit.edu/software/html/q2exp.html>.

- [23] J. Kuipers, T. Ueda, J.A.M. Vermaseren, J. Vollinga, *Comput. Phys. Commun.* 184 (2013) 1453, <http://dx.doi.org/10.1016/j.cpc.2012.12.028>, arXiv:1203.6543 [cs.SC].
- [24] A.V. Smirnov, *Comput. Phys. Commun.* 189 (2015) 182, <http://dx.doi.org/10.1016/j.cpc.2014.11.024>, arXiv:1408.2372 [hep-ph].
- [25] <https://bitbucket.org/feynmanIntegrals/fire>.
- [26] A.V. Smirnov, V.A. Smirnov, *Comput. Phys. Commun.* 184 (2013) 2820, <http://dx.doi.org/10.1016/j.cpc.2013.06.016>, arXiv:1302.5885 [hep-ph].
- [27] P.A. Baikov, K.G. Chetyrkin, *Nucl. Phys. B* 837 (2010) 186, <http://dx.doi.org/10.1016/j.nuclphysb.2010.05.004>, arXiv:1004.1153 [hep-ph].
- [28] A.V. Smirnov, M. Tentyukov, *Nucl. Phys. B* 837 (2010) 40, <http://dx.doi.org/10.1016/j.nuclphysb.2010.04.020>, arXiv:1004.1149 [hep-ph].
- [29] R.N. Lee, A.V. Smirnov, V.A. Smirnov, *Nucl. Phys. B* 856 (2012) 95, <http://dx.doi.org/10.1016/j.nuclphysb.2011.11.005>, arXiv:1108.0732 [hep-th].
- [30] S.A. Larin, F.V. Tkachov, J.A.M. Vermaseren, NIKHEF-H-91-18.
- [31] K.G. Chetyrkin, *Nucl. Phys. B* 710 (2005) 499, <http://dx.doi.org/10.1016/j.nuclphysb.2005.01.011>, arXiv:hep-ph/0405193.
- [32] V.P. Spiridonov, IYaI-P-0378.
- [33] Y. Schroder, M. Steinhauser, *J. High Energy Phys.* 0601 (2006) 051, <http://dx.doi.org/10.1088/1126-6708/2006/01/051>, arXiv:hep-ph/0512058.
- [34] K.G. Chetyrkin, J.H. Kühn, C. Sturm, *Nucl. Phys. B* 744 (2006) 121, <http://dx.doi.org/10.1016/j.nuclphysb.2006.03.020>, arXiv:hep-ph/0512060.
- [35] T. Liu, M. Steinhauser, *Phys. Lett. B* 746 (2015) 330, <http://dx.doi.org/10.1016/j.physletb.2015.05.023>, arXiv:1502.04719 [hep-ph].
- [36] C. Patrignani, et al., Particle Data Group, *Chin. Phys. C* 40 (10) (2016) 100001, <http://dx.doi.org/10.1088/1674-1137/40/10/100001>.
- [37] K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, *Phys. Rev. Lett.* 79 (1997) 353, <http://dx.doi.org/10.1103/PhysRevLett.79.353>, arXiv:hep-ph/9705240.
- [38] K.G. Chetyrkin, J.H. Kühn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser, C. Sturm, *Phys. Rev. D* 80 (2009) 074010, <http://dx.doi.org/10.1103/PhysRevD.80.074010>, arXiv:0907.2110 [hep-ph].
- [39] K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, *Comput. Phys. Commun.* 133 (2000) 43, arXiv:hep-ph/0004189.
- [40] B. Schmidt, M. Steinhauser, *Comput. Phys. Commun.* 183 (2012) 1845, <http://dx.doi.org/10.1016/j.cpc.2012.03.023>, arXiv:1201.6149 [hep-ph].
- [41] <https://www.ttp.kit.edu/preprints/2017/ttp17-010/>.