

# Direct CP violation in $K \rightarrow \pi\pi$ decays and supersymmetry

**Teppei Kitahara**

*Institute for Theoretical Particle Physics (TTP)  
and Institute for Nuclear Physics (IKP)  
Karlsruhe Institute of Technology (KIT)  
76131 Karlsruhe, Germany  
E-mail: [teppei.kitahara@kit.edu](mailto:teppei.kitahara@kit.edu)*

**Ulrich Nierste\***

*Institute for Theoretical Particle Physics (TTP)  
Karlsruhe Institute of Technology (KIT)  
76131 Karlsruhe, Germany  
E-mail: [ulrich.nierste@kit.edu](mailto:ulrich.nierste@kit.edu)*

**Paul Tremper**

*Institute for Theoretical Particle Physics (TTP)  
Karlsruhe Institute of Technology (KIT)  
76131 Karlsruhe, Germany  
E-mail: [paul.tremper@kit.edu](mailto:paul.tremper@kit.edu)*

The quantities  $\epsilon'_K$  and  $\epsilon_K$  measure the amount of direct and indirect CP violation in  $K \rightarrow \pi\pi$  decays, respectively. Using the recent lattice results from the RBC and UKQCD Collaborations and a new compact implementation of the  $\Delta S = 1$  renormalization group evolution we predict

$$\text{Re} \frac{\epsilon'_K}{\epsilon_K} = (1.06 \pm 5.07) \times 10^{-4}$$

in the Standard Model. This value is  $2.8\sigma$  below the experimental value of

$$\text{Re} \frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4}.$$

In generic models of new physics the well-understood  $\epsilon_K$  precludes large contributions to  $\epsilon'_K$ , if the new contributions enter at loop level. However, one can resolve the tension in  $\epsilon'_K/\epsilon_K$  within the Minimal Supersymmetric Standard Model. To this end two features of supersymmetry are crucial: First, one can have large isospin-breaking contributions (involving the strong instead of the weak interaction) which enhance  $\epsilon'_K$ . Second the Majorana nature of gluinos permits a suppression of the MSSM contribution to  $\epsilon_K$ , because two box diagrams interfere destructively.

*38th International Conference on High Energy Physics  
3-10 August 2016  
Chicago, USA*

---

\*Speaker.

## 1. Formalism and Standard-Model prediction

Flavour-changing neutral current (FCNC) transitions of Kaons are extremely sensitive to new physics and probe mass scales far above the reach of current high- $p_T$  experiments.  $K \rightarrow \pi\pi$  decays give access to two CP-violating quantities, which are related to FCNC amplitudes changing strangeness  $S$  by one or two units, respectively. To define these quantities  $\varepsilon'_K$  and  $\varepsilon_K$  one first combines the decay amplitudes  $A(K^0 \rightarrow \pi^+\pi^-)$  and  $A(K^0 \rightarrow \pi^0\pi^0)$  into  $A_0 \equiv A(K^0 \rightarrow (\pi\pi)_{I=0})$  and  $A_2 \equiv A(K^0 \rightarrow (\pi\pi)_{I=2})$  where  $I$  denotes the strong isospin. Indirect CP violation (stemming from the  $\Delta S = 2$  box diagrams) is quantified by

$$\varepsilon_K \equiv \frac{A(K_L \rightarrow (\pi\pi)_{I=0})}{A(K_S \rightarrow (\pi\pi)_{I=0})} = (2.228 \pm 0.011) \cdot 10^{-3} \cdot e^{i(0.97 \pm 0.02)\pi/4} \quad (1.1)$$

and was discovered in 1964 [1]. The measure of direct CP violation, which originates from the  $\Delta S = 1$  Kaon decay amplitude, is<sup>1</sup>

$$\varepsilon'_K \simeq \frac{\varepsilon_K}{\sqrt{2}} \left[ \frac{A(K_L \rightarrow (\pi\pi)_{I=2})}{A(K_L \rightarrow (\pi\pi)_{I=0})} - \frac{A(K_S \rightarrow (\pi\pi)_{I=2})}{A(K_S \rightarrow (\pi\pi)_{I=0})} \right] = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \varepsilon_K. \quad (1.2)$$

This experimental result was established in 1999 and constituted the first measurement of direct CP violation in any decay [2]. Adopting the standard phase convention for the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the real parts of the isospin amplitudes are experimentally determined as

$$\text{Re}A_0 = (3.3201 \pm 0.0018) \times 10^{-7} \text{ GeV}, \quad \text{Re}A_2 = (1.4787 \pm 0.0031) \times 10^{-8} \text{ GeV}. \quad (1.3)$$

The master equation for  $\varepsilon'_K/\varepsilon_K$  (see e.g. Ref. [3]) reads:

$$\frac{\varepsilon'_K}{\varepsilon_K} = \frac{\omega_+}{\sqrt{2}|\varepsilon_K^{\text{exp}}|\text{Re}A_0^{\text{exp}}} \left\{ \frac{\text{Im}A_2}{\omega_+} - (1 - \hat{\Omega}_{\text{eff}}) \text{Im}A_0 \right\}. \quad (1.4)$$

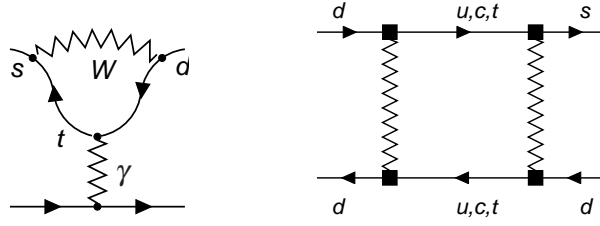
Here  $\omega_+ \simeq \frac{\text{Re}A_2}{\text{Re}A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$  is determined from the charged counterparts of  $\text{Re}A_{0,2}$  and  $\hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \cdot 10^{-2}$  quantifies isospin breaking. The quantities  $|\varepsilon_K^{\text{exp}}|$  and  $\text{Re}A_0^{\text{exp}}$  are also taken from experiment, as indicated.

The important theoretical ingredients encoding potential new-physics effects are  $\text{Im}A_0$  and  $\text{Im}A_2$ , which are calculated from the effective hamiltonian  $H^{|\Delta S|=1}$  describing  $s \rightarrow d\bar{q}q$  decays. This hamiltonian is known for a while at the level of next-to-leading-order (NLO) in QCD [4] and a precise prediction of  $\varepsilon'_K/\varepsilon_K$  is challenged by the difficulty to calculate the hadronic matrix elements of the operators in  $H^{|\Delta S|=1}$ . Within the Standard Model (SM)  $\text{Im}A_0$  is dominated by gluon penguins, with roughly 2/3 stemming from the matrix element  $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$  with the operator

$$Q_6 = \bar{s}_L^j \gamma_\mu d_L^k \sum_q \bar{q}_R^k \gamma^\mu q_R^j. \quad (1.5)$$

About 3/4 of the contribution to  $\text{Im}A_2$  stems from  $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$  with

$$Q_8 = \frac{3}{2} \bar{s}_L^j \gamma_\mu d_L^k \sum_q e_q \bar{q}_R^k \gamma^\mu q_R^j. \quad (1.6)$$



**Figure 1:** Sample diagrams of electroweak penguins and boxes, which contribute to the Wilson coefficient of  $Q_8$ .

The Wilson coefficient of  $Q_8$  stems from electroweak penguins and box diagrams (Fig. 1). Lattice-gauge theory has  $\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle$  (and thereby  $\text{Im}A_2$ ) under good control for some time [5], while lattice calculations of  $\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle$  and the other matrix elements entering  $\text{Im}A_0$  are new [6]. The results are consistent with earlier analytic calculations in the large- $N_c$  “dual QCD” approach [7]. Using these matrix elements from lattice QCD we find [8]

$$\frac{\varepsilon'_K}{\varepsilon_K} = (1.06 \pm 4.66_{\text{Lattice}} \pm 1.91_{\text{NNLO}} \pm 0.59_{\text{IV}} \pm 0.23_{m_t}) \times 10^{-4}. \quad (1.7)$$

The various sources of errors are indicated in the subscripts, with “NNLO” referring to unknown higher-orders of the perturbative expansion and “IV” meaning isospin violation. Adding the errors in quadrature gives the result in the abstract. We use the methodology of [9], which exploits the CP-conserving data of Eq. (1.3) to constrain the matrix elements. To arrive at Eq. (1.7) we have implemented a novel compact solution of the renormalization group equations; the result is in full agreement with the calculation in Ref. [3]. Eq. (1.7) disagrees with the experimental number in Eq. (1.2) by 2.8 standard deviations. The original lattice paper, Ref. [6], quotes a smaller discrepancy. The discussion at this conference has indicated that the combination of Eq. (1.3) with Fierz identities between different matrix elements has led to the sharper prediction in Refs. [3, 8].

## 2. A supersymmetric solution

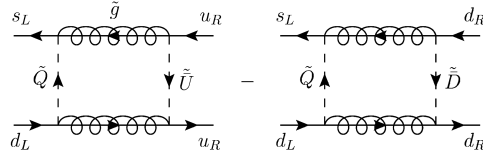
The large factor  $1/\omega_+$  multiplying  $\text{Im}A_2$  in Eq. (1.4) renders  $\varepsilon'_K/\varepsilon_K$  especially sensitive to new physics in the  $\Delta I = 3/2$  decay  $K \rightarrow (\pi\pi)_{I=2}$ . This feature makes  $\varepsilon'_K/\varepsilon_K$  special among all FCNC processes. However, it is difficult to place a large effect into  $\varepsilon'_K$  without overshooting  $\varepsilon_K$ : The SM contributions to both quantities are governed by the CKM combination

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - i0.6) \cdot 10^{-3}. \quad (2.1)$$

Our quantities scale as

$$\varepsilon_K^{\text{SM}} \propto \text{Im} \frac{\tau}{M_W^2} \quad \text{and} \quad \varepsilon'_K^{\text{SM}} \propto \text{Im} \frac{\tau^2}{M_W^2}. \quad (2.2)$$

<sup>1</sup>Accidentally,  $\varepsilon'_K/\varepsilon_K$  is essentially real.



**Figure 2:** “Trojan penguin” diagrams [12]. The difference of the two boxes contributes to the  $\Delta I = 3/2$  amplitude and increases with the mass difference among right-handed up-type ( $\tilde{U}$ ) and down-type ( $\tilde{D}$ ) squark.  $\tilde{Q}$  denotes a left-handed squark, which is a strange-down mixture.

In new-physics scenarios  $\tau$  is replaced by some new  $\Delta S = 1$  parameter  $\delta$  and  $M_W$  is replaced by some particle mass  $M \gg M_W$ . The new-physics contributions scale as

$$\varepsilon_K^{\prime\text{NP}} \propto \text{Im} \frac{\delta}{M^2}, \quad \text{and} \quad \varepsilon_K^{\text{NP}} \propto \text{Im} \frac{\delta^2}{M^2}. \quad (2.3)$$

If new-physics enters through a loop, the only chance to have a detectable effect in  $\varepsilon_K'$  is a scenario with  $|\delta| \gg |\tau|$ . Using Eqs. (2.2) and (2.3) the experimental constraint  $|\varepsilon_K^{\text{NP}}| \leq |\varepsilon_K^{\text{SM}}|$  entails

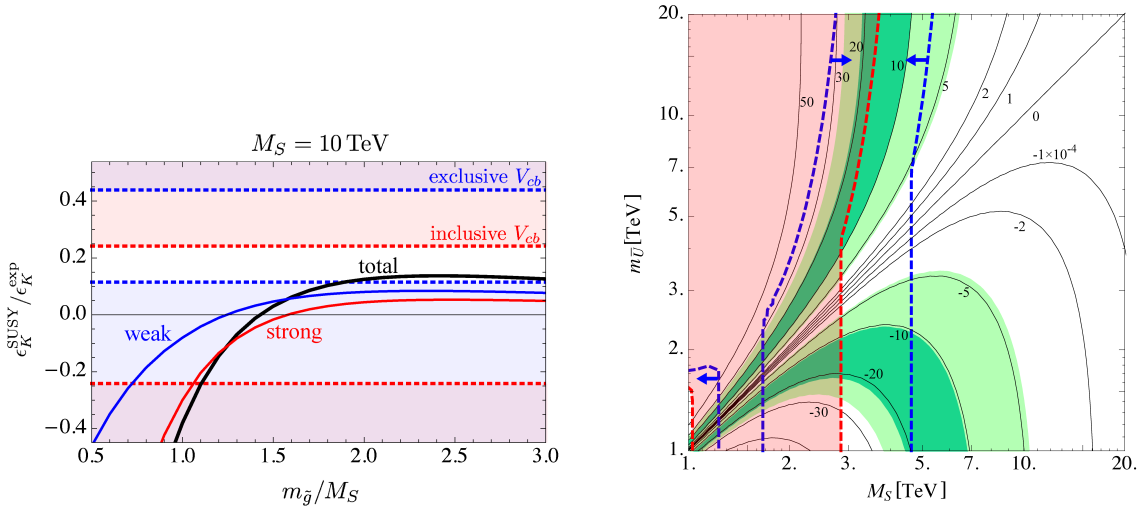
$$\left| \frac{\varepsilon_K^{\prime\text{NP}}}{\varepsilon_K^{\prime\text{SM}}} \right| \leq \left| \frac{\varepsilon_K^{\text{NP}}/\varepsilon_K^{\text{SM}}}{\varepsilon_K^{\text{NP}}/\varepsilon_K^{\text{SM}}} \right| = \mathcal{O} \left( \frac{\text{Re} \tau}{\text{Re} \delta} \right). \quad (2.4)$$

Thus large effects in  $\varepsilon_K'$  from loop-induced new physics are seemingly forbidden. Many studies of  $\varepsilon_K'$  indeed involve new-physics scenarios with tree-level contributions to  $\varepsilon_K'$  [10], in which the requirement  $|\delta| \gg |\tau|$  can be relaxed.

Here we present an explanation of the measurement in Eq. (1.2) by a supersymmetric loop effect [11]. We circumvent the argument in Eq. (2.4) by exploiting two special features of the Minimal Supersymmetric Standard Model (MSSM): Firstly, the MSSM permits large  $\Delta I = 3/2$  transitions mediated by the strong interaction (“Trojan penguins”) [12]. These enhanced amplitudes occur if the mass splitting between the right-handed up and down squarks is sizable (see Fig. 2). Secondly, the Majorana nature of the gluino permits the suppression of  $\varepsilon_K$ , which receives contributions from two squark-gluino box diagrams (“regular” and “crossed”). These diagrams cancel each other efficiently, once the gluino mass  $m_{\tilde{g}}$  and the squark mass  $m_{\tilde{Q}}$  in the loop satisfy  $m_{\tilde{g}} \geq 1.5m_{\tilde{Q}}$  [13]. In our scenario, the mass scale  $M_S$  of the supersymmetric particles is large, of order 3–7 TeV. Squark flavour mixing appears only among the left-handed doublets. We choose the CP-violating phase of the (2,1) element  $\Delta_{sd}^{LL}$  of the left-handed squark mass matrix equal to  $\arg(\Delta_{sd}^{LL}) = \pi/4$ . The results are shown in Fig. 3

### 3. Summary

Novel lattice results reveal a tension between the measured value of  $\varepsilon_K'$  in Eq. (1.2) and the SM prediction in Eq. (1.7). Within the MSSM one can simultaneously enhance  $\varepsilon_K'$  and suppress unwanted effects in  $\varepsilon_K$ . Our MSSM scenario works with large superpartner masses in the 3–7 TeV range and thereby comply with bounds from collider searches. Crucial elements are  $m_{\tilde{g}} \geq 1.5M_{\tilde{Q}}$ , a sizable mass splitting between right-handed up and down squarks, and flavour mixing among left-handed squarks.



**Figure 3:** Left:  $\varepsilon_K^{\text{SUSY}}/\varepsilon_K^{\text{SM}}$  as a function of  $m_{\tilde{g}}/M_S$  for a common mass  $M_S = 10 \text{ TeV}$  of all superpartners except the gluino. Right: Parameter region explaining  $\varepsilon_K'/\varepsilon_K$  while complying with the measured  $\varepsilon_K$  for the point  $m_{\tilde{g}} = 1.5M_S$  and  $M_S = m_{\tilde{Q}} = m_{\tilde{D}}$ . The lines labeled with negative values of the MSSM contribution  $\varepsilon_K^{\text{SUSY}}/\varepsilon_K$  correspond to correct (positive) solutions if the CP phase is appropriately adjusted. The SM prediction for  $\varepsilon_K$  strongly depends on  $|V_{cb}|$ . The blue (red) lines in both plots delimit the region which complies with  $\varepsilon_K$  if  $|V_{cb}|$  is determined from exclusive (inclusive)  $b \rightarrow c\ell\nu$  decays. If the exclusive determination is correct, some new physics in  $\varepsilon_K$  is welcome. In the inclusive case the forbidden region is marked with the red shading. For more details see Ref. [11], from which the plots are taken.

## Acknowledgements

I thank the organizers for quickly letting me give this talk in a vacant slot of the timetable. The presented work has profited from discussions with Andrzej Buras, Chris Sachrajda, Christoph Bobeth, Martin Gorbahn, and Sebastian Jäger. The work of UN is supported by BMBF under grant no. 05H15VKKB1.

## References

- [1] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett. **13** (1964) 138.
- [2] L. K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1203 (1993); G. D. Barr *et al.* [NA31 Collaboration], Phys. Lett. B **317**, 233 (1993); A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **83**, 22 (1999) [hep-ex/9905060]; V. Fanti *et al.* [NA48 Collaboration], Phys. Lett. B **465**, 335 (1999) [hep-ex/9909022]; J. R. Batley *et al.* [NA48 Collaboration], Phys. Lett. B **544**, 97 (2002) [hep-ex/0208009]; E. Abouzaid *et al.* [KTeV Collaboration], Phys. Rev. D **83**, 092001 (2011) [arXiv:1011.0127 [hep-ex]].
- [3] A. J. Buras, M. Gorbahn, S. Jäger and M. Jamin, JHEP **1511**, 202 (2015) [arXiv:1507.06345 [hep-ph]].
- [4] F. J. Gilman and M. B. Wise, Phys. Lett. B **83**, 83 (1979). J. M. Flynn and L. Randall, Phys. Lett. B **224**, 221 (1989); Erratum: [Phys. Lett. B **235**, 412 (1990)]. A. J. Buras, M. Jamin, M. E. Lautenbacher and P. H. Weisz, Nucl. Phys. B **370**, 69 (1992); Addendum: [Nucl. Phys. B **375**, 501 (1992)]. A. J. Buras, M. Jamin, M. E. Lautenbacher and P. H. Weisz, Nucl. Phys. B **400**, 37

- (1993) [hep-ph/9211304]. M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Nucl. Phys. B **415**, 403  
(1994) [hep-ph/9304257]. A. J. Buras, M. Jamin and M. E. Lautenbacher, Nucl. Phys. B **400**, 75  
(1993) [hep-ph/9211321].
- [5] T. Blum *et al.*, Phys. Rev. Lett. **108**, 141601 (2012) [arXiv:1111.1699 [hep-lat]]; Phys. Rev. D **86**, 074513 (2012) [arXiv:1206.5142 [hep-lat]]; Phys. Rev. D **91**, no. 7, 074502 (2015) [arXiv:1502.00263 [hep-lat]].
- [6] Z. Bai *et al.* [RBC and UKQCD Collaborations], Phys. Rev. Lett. **115**, no. 21, 212001 (2015) [arXiv:1505.07863 [hep-lat]].
- [7] A. J. Buras and J. M. Gérard, Nucl. Phys. B **264**, 371 (1986); A. J. Buras and J. M. Gérard, Phys. Lett. B **192**, 156 (1987); W. A. Bardeen, A. J. Buras and J. M. Gérard, Nucl. Phys. B **293**, 787 (1987); W. A. Bardeen, A. J. Buras and J. M. Gérard, Phys. Lett. B **192**, 138 (1987); J. M. Gérard, Acta Phys. Polon. B **21**, 257 (1990); A. J. Buras, J. M. Gérard and W. A. Bardeen, Eur. Phys. J. C **74**, 2871 (2014) [arXiv:1401.1385 [hep-ph]]. A. J. Buras and J. M. Gérard, JHEP **1512**, 008 (2015) [arXiv:1507.06326 [hep-ph]].
- [8] T. Kitahara, U. Nierste and P. Tremper, JHEP **1612** (2016) 78 [arXiv:1607.06727 [hep-ph]].
- [9] A. J. Buras, M. Jamin and M. E. Lautenbacher, Nucl. Phys. B **408**, 209 (1993) [hep-ph/9303284].
- [10] A. J. Buras, F. De Fazio and J. Girschbach, Eur. Phys. J. C **74**, no. 7, 2950 (2014) [arXiv:1404.3824 [hep-ph]]. A. J. Buras, D. Buttazzo and R. Knegjens, JHEP **1511**, 166 (2015) [arXiv:1507.08672 [hep-ph]]. A. J. Buras, JHEP **1604**, 071 (2016) [arXiv:1601.00005 [hep-ph]]. A. J. Buras, F. De Fazio and J. Girschbach-Noe, JHEP **1408**, 039 (2014) [arXiv:1405.3850 [hep-ph]]. A. J. Buras and F. De Fazio, JHEP **1603**, 010 (2016) [arXiv:1512.02869 [hep-ph]]. A. J. Buras and F. De Fazio, JHEP **1608** (2016) 115 [arXiv:1604.02344 [hep-ph]]. M. Blanke, A. J. Buras and S. Recksiegel, Eur. Phys. J. C **76**, no. 4, 182 (2016) [arXiv:1507.06316 [hep-ph]].
- [11] T. Kitahara, U. Nierste and P. Tremper, Phys. Rev. Lett. **117** (2016) no.9, 091802 [arXiv:1604.07400 [hep-ph]].
- [12] A. L. Kagan and M. Neubert, Phys. Rev. Lett. **83**, 4929 (1999) [hep-ph/9908404].
- [13] A. Crivellin and M. Davidkov, Phys. Rev. D **81**, 095004 (2010) [arXiv:1002.2653 [hep-ph]].